1. The program would use numpy to find the rank

rank = np.linalg.matrix_rank(matrix)

if rank is less than 3 then it is not linear independent

in this case the matrix is linearly independent

2. $A = \begin{bmatrix} -1 & 1 \\ 2.5 & 1 \\ 6 & 1 \\ -3.5 & 1 \\ 3 & 1 \\ 8.5 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} -1 & 2.5 & 6 & -3.5 & 3 & 8.5 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

$A^T A = \begin{bmatrix} (-1)(-1) + 2.5^2 + 6^2 + (-3.5)^2 + 9 + 8.5^2 & -1 + 2.5 + 6 & -3.5 + 3 + 8.5 \\ -1 + 2.5 + 6 & -3.5 + 3 + 8.5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 136.75 & 15.5 \\ 15.5 & 6 \end{bmatrix}$

$b = \begin{bmatrix} 8.5 \\ 3 \\ -5.5 \\ 13 \\ 0 \\ -10 \end{bmatrix}$

$(A^T A)^{-1} = \frac{1}{6(136.75) - (15.6)^2} \begin{bmatrix}$

$= \frac{1}{580.25} \begin{bmatrix} 6 & -15.5 \\ -15.5 & 136.75 \end{bmatrix}$

$= \begin{bmatrix} 0.6103 & -0.0267 \\ -0.0267 & 0.2357 \end{bmatrix}$

$A^T b = \begin{bmatrix} -8.5 + 2.5(3) + 6(-5.5) + (-3.5)13 + 3(0) + 8.5(-10) \\ 8.5 + 3 - 5.5 + 13 - 10 \end{bmatrix}$

$= \begin{bmatrix} -164.5 \\ 9 \end{bmatrix}$

$q = \begin{bmatrix} 0.6103 & -0.0267 \\ -0.0267 & 0.2357 \end{bmatrix} \begin{bmatrix} -164.5 \\ 9 \end{bmatrix} = \begin{bmatrix} -1.9347 \\ 6.5135 \end{bmatrix}$

$y = -1.9347x + 6.5135$

3.

$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\det = -1$ Thus is invertible and the vectors v_1, v_2, v_3 span $\mathbb{R}^3$

4.

$M = \begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $R_2 = R_2 + R_1$ $\rightarrow \begin{bmatrix} 0.707 & 0.707 & 0 \\ 0 & 1.414 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\det(M) = 0.99$

yes because the matrix is linearly independent and spans $\mathbb{R}^3$

5.

$M = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

Rank = 3
so the vectors span $\mathbb{R}^3$
but since its 4 vectors
It doesn't form a basis
for $\mathbb{R}^3$