EEC133 Midterm Formula Sheet

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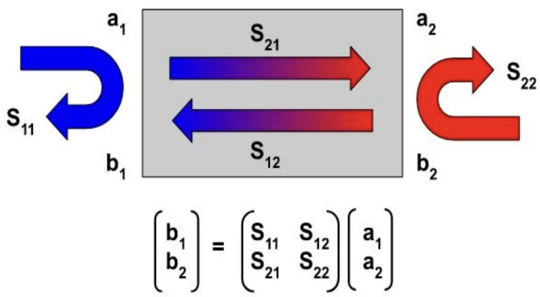

Figure 1: S Parameter

db to dbm

$$P(\text{dBm}) = 10 \times \log(P(w)) + 30$$

Steradian:

$$A = r^2\Omega$$

where A is the surface area patch, r is the radius, and Ω is the solid angle.

Total Radiation Power (W)

$$P_{rad} = \int U(\theta, \phi) d\Omega$$

Half Power Beamwidth:

$$\theta_2 - \theta_1$$

where θ_2, θ_1 are the angles where the normalized radiation intensity is $\frac{1}{2}$.

$$\Omega_A \approx \frac{4\pi}{D_0}$$

Directivity (1/steradian):

$$D_0 = \frac{4\pi}{\Omega_p} = \frac{4\pi}{\int_{sphere} F(\theta, \phi) d\Omega}$$

where Ω_p is the pattern solid angle.

$$\frac{A_e}{D_0} = \frac{\lambda^2}{4\pi}$$

where A_e is the effective area of the antenna, D_0 is the directivity of the antenna, and λ is the wavelength of the wave transmitted by the antenna.

Gain

$$G_0 = e \times D_0$$

where e is the efficiency of the antenna.

Angular Resolution

$$\frac{\lambda}{\text{Aperture Size}} = \frac{\lambda}{d}, \text{ where d is the diameter of the dish}$$

Poynting Vector

$$\vec{S}_{av} = \hat{z}\frac{|\widetilde{E}|^2}{2\eta}$$

Power Radiated from the Antenna

$$|\vec{S}_{avg}| = \frac{e_t P_t}{4\pi r^2} D_t$$

Friss Transmission Equation:

$$\begin{aligned}\frac{P_{rec}}{P_t} &= e_t e_r \left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r \\ &= \left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r \\ &= \frac{e_r e_t A_t A_r}{\lambda^2 R^2}\end{aligned}$$

where subscript t denotes the transmitter's parameter, r denotes the receiver's parameter, and R is the distance between two antennas.

Vector Potential

$$\vec{H} = \frac{1}{\mu_0}\nabla \times \vec{A}$$

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

Gauge

$$\phi_{new} = \phi - \frac{\partial \psi}{\partial t}$$

$$\vec{A}_{new} = \vec{A} + \nabla\psi$$

$$\text{Coulomb Gauge: } \nabla \cdot \vec{A} = 0$$

$$\text{Lorenz Gauge: } \nabla \cdot \vec{A} = \frac{-1}{c^2}\frac{\partial \phi}{\partial t}$$

Retarded Potential

$$\widetilde{A}(\vec{r}) = \int \frac{\mu_0 \widetilde{J}(\vec{r})e^{-jkr}}{4\pi R} dV'$$

Hertzian Dipole Antenna: $l < \frac{\lambda}{50}$
Far Field ($r >> \lambda$)

$$\widetilde{E}_{ff,\theta} = j\eta_0 \frac{Ikl}{4\pi r} e^{-jkr}\sin(\theta)$$

$$\widetilde{H}_{ff,\phi} = \frac{\widetilde{E}_{ff,\theta}}{\eta_0}$$

$$D(\theta,\phi) = 1.5\sin^2(\theta)$$

$$F(\theta,\phi) = \sin^2(\theta)$$

$$P_{rad} = 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

Small dipole Antenna: $l < \frac{\lambda}{10}$
Fields are 1/2 those form the Hertzian Dipole.

$$P_{rad} = 10\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$$

$$\text{Resonance Dipole Length: } L = \frac{\lambda}{2} + n\lambda, n \geq 0$$

Small Loop Antenna: $a << \frac{\lambda}{6\pi}$
Far Fields ($r >> \lambda$)

$$\widetilde{H}_{ff,\theta} = \frac{Ik^2(\pi a^2)}{4\pi r} e^{-jkr} \sin(\theta)$$

$$\widetilde{E}_{ff,\phi} = -\widetilde{H}_\theta \eta_0$$

$$D(\theta, \phi) = 1.5 \sin^2(\theta)$$

$$R_{rad} = 320\pi^6 \left(\frac{a}{\lambda}\right)^4 N^2$$

Choosing Capacitor to be in parallel with loop antenna to cancel antenna's input reactance

$$j\omega C - j\frac{X_A}{R_A^2 + X_A^2} = 0$$

where X_A is the input reactance and R_A is the input resistance of the antenna

Two Hertzian Dipole Antenna

$$\widetilde{E}_{ff}(\vec{r})\Big|_{\theta=\pi/2} = \hat{\theta}\frac{j\eta_0 kld}{4\pi r} e^{-jkr} \left(e^{jk\frac{l}{2}\cos(\phi)} + Ae^{j\alpha} e^{-jk\frac{l}{2}\cos(\phi)}\right), \text{where l is the distance between two Hertzian Dipoles and A is t}$$

$$\widetilde{H}_{ff}(\vec{r})\Big|_{\theta=\pi/2} = \hat{\phi}\frac{jkld}{4\pi r} e^{-jkr} \left(e^{jk\frac{l}{2}\cos(\phi)} + Ae^{j\alpha} e^{-jk\frac{l}{2}\cos(\phi)}\right)$$

$$\vec{S}(\vec{r})\Big|_{\theta=\pi/2} = \hat{r}\eta_0 \left|\frac{kld}{4\pi r}\right|^2 \left|e^{jk\frac{l}{2}\cos(\phi)} + Ae^{j\alpha} e^{-jk\frac{l}{2}\cos(\phi)}\right|^2$$

$$D(\theta, \phi) = \frac{3}{2}\left(\frac{1 + A^2 + 2A\cos[kl\cos(\phi) - \alpha]}{f(A, \phi)}\right)$$

$$F(\theta, \phi) = \left(\frac{1 + A^2 + 2A\cos[kl\cos(\phi) - \alpha]}{(1 + A)^2}\right)$$

Patch Antenna

$$f = \frac{c}{2\sqrt{\epsilon_r}L}$$

$$W = 1.5L$$

Aperture Antenna

$$\beta_{xz} = 2\theta = 0.38\frac{\lambda}{l_x}$$

$$\beta_{yz} = 2\theta = 0.88\frac{\lambda}{l_y}$$

$$D_0 = \frac{4\pi}{\text{Beam Solid Angle}}$$

Other

$$\text{Reflection Coefficient} : \gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\text{Transmission Line Input Impedance} : Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(kl)}{Z_0 + jZ_L \tan(kl)} \right)$$

$$\text{Power Flowing Through a Transmission Line} : P_{avg} = \frac{|V_0^+|^2}{Z_0}(1 - |\gamma|^2)$$

$$\text{Standing Wave Ratio} : S = \frac{1 + |\gamma|}{1 - |\gamma|}$$