EEC133 HW4

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December 8, 2024

1. The reactance of small loop antennas

(a) $\boxed{Z_{in} = jZ_0 \tan(kl)}$

Given the input impedance of a transmission line,

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan(kl)}{Z_0 + jZ_L \tan(kl)} \right)$$

Then,

$$Z_{in}(Z_L = 0) = Z_0 \left(\frac{jZ_0 \tan(kl)}{Z_0} \right) = jZ_0 \tan(kl)$$

(b) $\boxed{\text{Inductor}}$. $\boxed{Z_{in} \approx jZ_0}$

(c) The loop antenna physically look like a shorted transmission line. Therefore, its input impedance is similar to the shorted transmission line from (1), which means it's purely reactive and similar to an inductor.

(d)

$$\widetilde{A} = j \left(\frac{\mu_0 IAk}{4\pi r} \right) e^{-jkr} \sin(\theta)\hat{\phi}$$

$$\begin{aligned}
\widetilde{H} &= \frac{1}{\mu_0} \nabla \times \widetilde{A} = j \left(\frac{IAk}{4\pi} \right) \nabla \times \left(\frac{1}{r} e^{-jkr} \sin(\theta)\hat{\phi} \right) \\
&= j \left(\frac{IAk}{4\pi} \right) \left(\frac{e^{-jkr}}{r^2} 2\cos(\theta)\hat{r} - \frac{\sin(\theta)}{r} (-jk) e^{-jkr} \hat{\theta} \right)
\end{aligned}$$

$$\begin{aligned}
\widetilde{E} &= \frac{1}{j\omega\epsilon_0} \nabla \times \widetilde{H} \\
&= j \left(\frac{IAk}{4\pi r} \right) \left((jk)^2 e^{-jkr} \sin(\theta)\hat{\theta} + \frac{e^{-jkr}}{r^2} 2\sin(\theta) \right) \hat{\phi}
\end{aligned}$$

Assume that the $\frac{1}{r^2}$ terms dominate in near field, we have

$$\boxed{\widetilde{E}_{ff} = \left(\frac{IA}{4\pi r^3}\sin(\theta)\right) 2\eta_0 e^{-jkr}\hat{\phi}}$$

$$\boxed{\widetilde{H}_{ff} = j\left(\frac{IAk}{4\pi r^2}\cos(\theta)\right) 2e^{-jkr}\hat{r}}$$

(e)

$$\vec{S} = \widetilde{E} \times \widetilde{H}^*$$
$$= \boxed{j\left(\frac{IA}{4\pi r^2}\right)^2 \left(\frac{k}{r}\right) 4\eta_0 \sin(\theta)\cos(\theta)\hat{\theta}}$$

(f)

$$\widetilde{V} = jX_L\widetilde{I}$$
$$S = \widetilde{V}\widetilde{I}^*$$
$$= \boxed{jX_L|\widetilde{I}|^2}$$

(g) Both results are in the form j(reactance)(current)^2. It suggests that the input impedance of a loop antenna is purely reactive.

(h) $\boxed{\vec{S}_{av} = 0}$ because the power is imaginary. The result makes sense because the loop antenna in the near field behaves as a inductor, which stores energy and has imaginary power.

(i) Add a $\boxed{\text{capacitor in parallel}}$ to the inductor because the purely negative reactance will cancel the purely positive reactance.

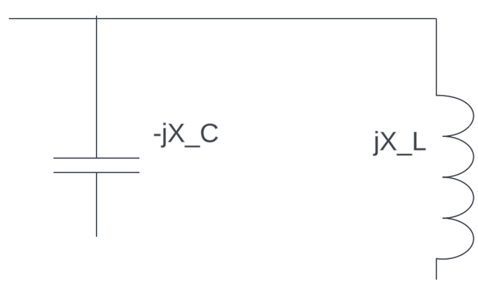

Figure 1: A Capacitor in Parallel with a Loop Antenna

2. The full-wave loop

(a)

$$\gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{150} = \frac{1}{3}$$

$$|\gamma|^2 = \left(\frac{1}{3}\right)^2 = \boxed{\frac{1}{9}}$$

(b) $\boxed{S = \frac{1 + 1/3}{1 - 1/3} = \frac{4/3}{2/3} = 2}$. The SWR quantifies the amount of standing wave. The higher the ratio, the higher the amplitude of reflected wave that will form standing waves.

(c) Current Distribution around the loop

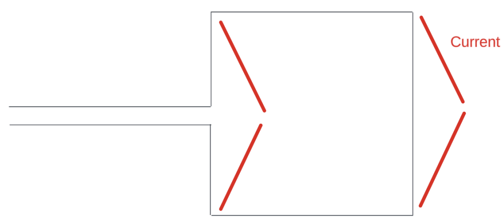

Figure 2: Current Distribution

(d) This is not a good approximation because the length of a Hertzian Dipole is $\frac{\lambda}{50}$, but the antenna here is $\frac{\lambda}{4}$ in length.

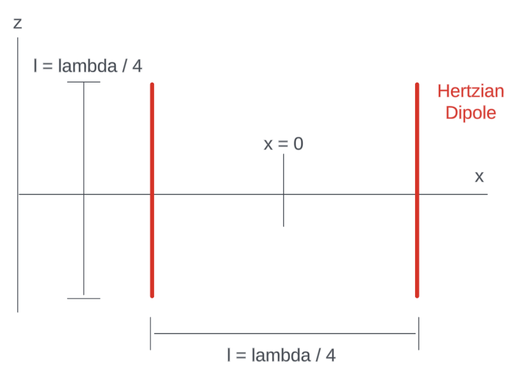

Figure 3: Geometry of the Dipole Antennas

(e)

$$F(\phi) = \frac{1 + 1 + 2\cos(kl\cos(\phi))}{(1+1)^2} = \frac{1}{2}\left(1 + \cos\left(\frac{\pi}{2}\cos(\phi)\right)\right)$$

$$\boxed{F(x,y) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi}{2}\cos\left(\tan^{-1}\left(\frac{y}{x}\right)\right)\right)\right)}$$

(f) The full-wave loop has a lower directivity because its radiation pattern looks closer to that of the isotropic radiator.

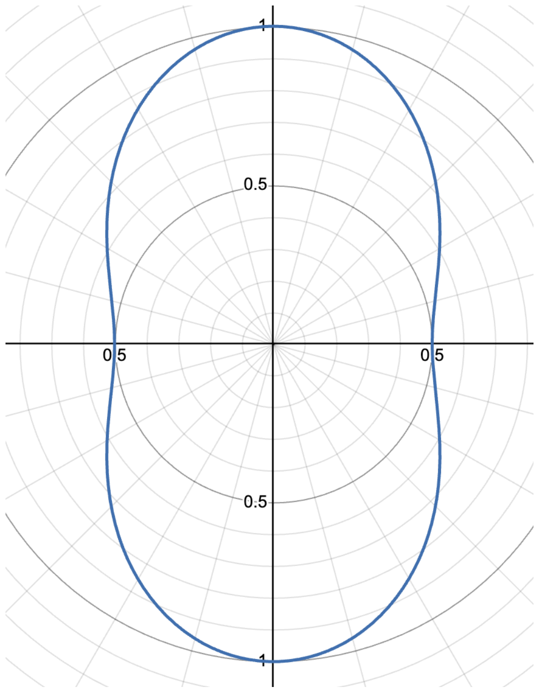

Figure 4: Normalized Radiation Intensity with Respect to ϕ