# EEC133 HW3

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## Contents



# 1 A few simple dipole antennas

(a)

$$f = 100 \times 10^6 \text{ Hz}, \lambda = \frac{c}{f} = 3 \text{ m}, l = 5 \times 10^{-2} \text{ m}, I_0 = 0.5 \text{ A}$$

(i) $\boxed{\text{Yes}}$, the antenna could be model as a Hertzian Dipole because $50 < \frac{\lambda}{l} = 60$

(ii) For a Hertzian Dipole,

$$S_{max} = \frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} \approx \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

Here, R = 1 mile = $1.6 \times 10^3$ m

Plug in all values:

$$\boxed{S_{max} = 1.28 \frac{\text{nW}}{\text{m}^2}}$$

(iii) For a Hertzian Dipole

$$|\widetilde{E}| = \frac{Ikl\eta_0}{4\pi R}$$

Therefore,

$$|\widetilde{E}|_{rms} = \frac{I_0 kl\eta_0}{4\pi R\sqrt{2}}$$

$$= \boxed{0.69 \frac{\text{mV}}{\text{m}}}$$

(iv) Since the antenna is a Hertzian Dipole, $F(\theta,\phi) = \sin^2(\theta)$.
Next,we have the total radiated power in all direction as

$$P_{rad} = R^2 S_{max} \int_{\text{sphere}} F(\theta,\phi) d\Omega$$

For Hertzian Dipole, $P_{rad} = \frac{8\pi}{3}$.
To get the total power radiated between $\theta = 85°$ and $95°$, change the integration bounds.

$$P_{85-95} = R^2 S_{max} \int_0^{2\pi} \int_{85°}^{95°} F(\theta,\phi) d\Omega$$

Therefore,

$$\frac{P_{85-95}}{P_{rad}} = \frac{\int_0^{2\pi} \int_{85°}^{95°} F(\theta,\phi) d\Omega}{\int_{\text{sphere}} F(\theta,\phi) d\Omega} = 0.13 = \boxed{13\%}$$

(b)

$$f = 100 \times 10^6 \text{ Hz}, \ \lambda = \frac{c}{f} = 3 \text{ m}, \ l = 10 \times 10^{-2} \text{ m}, \ I_0 = 0.5 \text{ A}$$

(i) $\boxed{\text{No}}$, the antenna could not be model as a Hertzian Dipole because $50 < \frac{\lambda}{l} = 30$.
However, it could be modeled as a *Small Dipole Antenna* because $10 < 30$

(ii) For a Small Dipole,

$$S_{max} = \frac{\eta_0 k^2 I_0^2 l^2}{(4)32\pi^2 R^2} \approx \frac{15\pi I_0^2}{4R^2}\left(\frac{l}{\lambda}\right)^2$$

Here, R = 1 mile = $1.6 \times 10^3$ m
Plug in all values:

$$\boxed{S_{max} = 1.28 \frac{\text{nW}}{\text{m}^2}}$$

(iii) For a Small Dipole

$$|\widetilde{E}| = \frac{Ikl\eta_0}{(2)4\pi R}$$

Therefore,

$$|\widetilde{E}|_{rms} = \frac{I_0 kl\eta_0}{(2)4\pi R\sqrt{2}}$$

$$= \boxed{0.69 \frac{\text{mV}}{\text{m}}}$$

(iv) Since the antenna is a Small Dipole, $F(\theta, \phi) = \sin^2(\theta)$.
Next,we have the total radiated power in all direction as

$$P_{rad} = R^2 S_{max} \int_{\text{sphere}} F(\theta, \phi) d\Omega$$

For Small Dipole, $P_{rad} = \frac{2\pi}{3}$.
To get the total power radiated between $\theta = 85°$ and $95°$, change the integration bounds.

$$P_{85-95} = R^2 S_{max} \int_0^{2\pi} \int_{85°}^{95°} F(\theta, \phi) d\Omega$$

Therefore,

$$\frac{P_{85-95}}{P_{rad}} = \frac{\int_0^{2\pi} \int_{85°}^{95°} F(\theta, \phi) d\Omega}{\int_{\text{sphere}} F(\theta, \phi) d\Omega} = 0.13 = \boxed{13\%}$$

(v) Sketch the current distribution in the 10 cm long Small Dipole Antenna.

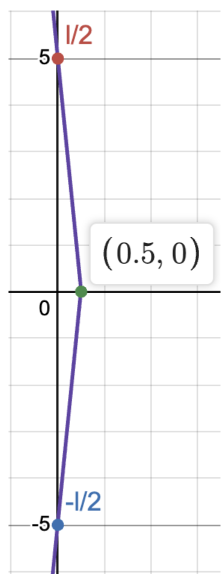


Figure 1: Current Distribution of Small Dipole Antenna

(c)

$$l = \frac{\lambda}{2},\ f = 200 \times 10^6\text{Hz},\ \lambda = 1.5\text{m},\ R_{in} = R_{rad} = 73\Omega,\ D_0 = 1.64 = 2.15\text{dB}$$

(i) $\boxed{l = 0.75}$.

(ii) $\boxed{G_0 = 2.51\text{dB}}$.

(iii) We want $P_{rad} = 100\text{W}$.

For a Half-wave Dipole Antenna,

$$\frac{1}{2} R_{in} |I_{in}|^2 = P_{rad}$$

Therefore,

$$|I_{in}| = \sqrt{\frac{2P_{rad}}{R_{in}}} = \boxed{1.66\text{A}}$$