Modified Nodal Analysis

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1 Circuit Notation

For a circuit with n nodes, m voltage sources, and k independent current sources:

Circuit Component	Symbol
Ground	0
Node	$1 \rightarrow$ n
Nodal Voltage	$v_1 \rightarrow v_n$
Independent Voltage Source	$V_1 \rightarrow V_m$
Current through Independent Voltage Source	$i_1 \rightarrow i_m$
Independent Current Source	$I_1 \rightarrow I_k$

2 Modified Nodal Analysis (MNA)

MNA will reduce circuits that only have passive components and independent voltage or current sources into the form:

$$\mathbf{Ax} = \mathbf{z}$$

For a circuit with n nodes and m voltage sources:

2.1 The A Matrix

- **Size**: $(n + m) \times (n + m)$
- Contains **4 sub-matrices**: the conductance matrix (**G**), the voltage source matrices (**B** and **C**), and the dependent source matrix (**D**).

$$\begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

- **The G Matrix**:
  - **Size**: $(n \times n)$

  - Each diagonal term is equal to the sum of the conductance of elements connected to the corresponding node. *Example*: The first diagonal term is the sum of conductances connected to node 1.
  - Each off-diagonal term is the negative conductance of the element connected to the pair of corresponding nodes. *Example*: A resistor connected to nodes 2 and 3 will be placed in the **G** matrix at positions (2, 3) and (3, 2).

- **The B Matrix**:
  - **Size**: $(n \times m)$
  - Contains only the values -1, 0, and 1.
  - If the nth node is connected to the mth voltage source's positive terminal, then the element at (n, m) is 1.
  - If the nth node is connected to the mth voltage source's negative terminal, then the element at (n, m) is -1.
  - Otherwise, the entry is 0.

- **The C Matrix**:
  - **Size**: $(m \times n)$
  - Transpose of the **B** matrix.

- **The D Matrix**:
  - **Size**: $(m \times m)$
  - Contains all 0s.

2.2 The x Vector

- **Size**: $(n + m) \times 1$
- Contains **2 vectors**: **v** and **j**

$$\mathbf{x} = \begin{bmatrix} \mathbf{v} \\ \mathbf{j} \end{bmatrix}$$

- **The v Vector**:
  - Each entry of the vector is the node voltage of the nth node (no entry for ground, node 0).
  - *Example*:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

- **The j Vector**:
  - Each entry of the vector is the current flowing into the mth voltage source.
  - *Example*:

$$\mathbf{j} = \begin{bmatrix} i_1 \\ \vdots \\ i_m \end{bmatrix}$$

2.3 The z Vector

- **Size**: $(n + m) \times 1$
- Contains **2 vectors**: **i** and **e**

$$\mathbf{z} = \begin{bmatrix} \mathbf{i} \\ \mathbf{e} \end{bmatrix}$$

- **The i Vector**:
  - **Size**: $(n \times 1)$
  - The nth element is the sum of the current sources into the nth node (Node 0 isn't included). If no current source is connected to the nth node, then $(n, 1) = 0$.
- **The e Vector**:
  - **Size**: $(m \times 1)$
  - The mth entry contains the value of the mth voltage source.