

HW6 PHY 950

1. (a) $a = 20$, $\delta = 7.23$

all codes are posted

(b) $a = 15$, $\delta = 1.63$

(c) $a = 30$, $\delta = 20.8$

2. (a) Binomial prob. $f(n) = C_{18}^n p^n (1-p)^{18-n}$

ML estimate: $L(4, \hat{p}) = f(4) = A \hat{p}^4 (1-\hat{p})^{18-4}$

$$\frac{\partial L}{\partial \hat{p}} = 0 \Rightarrow \hat{p} = \frac{4}{9} = 0.222$$

68.3% central confidence level $\Rightarrow \alpha = \beta = \frac{1}{2} \alpha = \frac{1}{2} (1 - 68.3\%) = 15.85\%$

$$\begin{cases} \alpha = \sum_{n=0}^{18} C_{18}^n a^n (1-a)^{18-n} \\ \beta = \sum_{n=0}^4 C_{18}^n b^n (1-b)^{18-n} \end{cases} \Rightarrow \begin{cases} a = 0.119 \\ b = 0.363 \end{cases} \Rightarrow \begin{cases} a = \hat{p} - 0.023 \\ c = b - \hat{p} = 0.141 \end{cases}$$
$$\Rightarrow \hat{p} = 0.222 \pm 0.023$$

(b) By code, $\sigma^2 = 0.075$
 $\hat{p} \pm \hat{\sigma}_p = 0.222 \pm 0.075$

The interval $[0.119, 0.363]$ from (a) means in 68.3% of experiments the interval will include the true value.

While, the interval $[\hat{p} - \hat{\sigma}_p, \hat{p} + \hat{\sigma}_p]$ means the observed data located in this interval has a probability of 68.3%

(c) Because each events ~~are~~ don't happen with equal probability.