HW4 PHY950

1. $\log L(\nu, n) = n \log \nu - \nu$

$\Rightarrow \dfrac{\partial \log L(\nu, n)}{\partial \nu} = \dfrac{n}{\nu} - 1 = 0 \Rightarrow \hat{\nu} = n$

$E[\hat{\nu}] = \int \hat{\nu} f(n) dn = \int n f(n) dn = n = \hat{\nu}$

therefore, the estimator $\hat{\nu}$ is unbiased. $b = E[\hat{\nu}] - \hat{\nu} = 0$

For Poisson distribution, $E = \sigma^2 = \hat{\nu}$

$\Rightarrow V[\hat{\nu}] = \dfrac{\hat{\nu}}{n}$

The Fisher information matrix : $I_{ij} = E[-\dfrac{\partial^2 \ln L}{\partial \nu^2}]$

From the derivation on notebook, when $b=0$,

the minimum variance bound : $E^{-1}[-\dfrac{\partial^2 \ln L}{\partial \nu^2}] = E^{-1}[\dfrac{n}{\nu^2}] = \int n f(n) dn = \hat{\nu}$

$\left(\dfrac{\partial^2 \ln L(\nu)}{\partial \nu^2} = \dfrac{\partial}{\partial \nu}\left(\dfrac{n}{\nu} - 1\right) = -\dfrac{n}{\nu^2} \right)$

Therefore, $\hat{\nu}$ = the minimum variance bound !

2. (a) $\alpha = P(x \in W \mid \text{pions}) = \frac{1}{2}(1-68\%) = 16\%$

(b) power of the test: prob. to accept electrons, $\varepsilon_e = 1-16\% = 84\%$

prob. e^- be accepted as π, $\beta = 16\%$

(c) $P(\text{electrons} \mid T<1) = \dfrac{\varepsilon_e \pi_e}{\varepsilon_e \pi_e + \varepsilon_\pi \pi_\pi} = \dfrac{84\% \times 1\%}{84\% \times 1\% + 16\% \times 99\%} = 5\%$

(d) By coding, I get the purity $\geq 95\%$ when $T < -2.52$

3.(a) ① Hypothesis 1: $\chi_1^2 =$ ~~28.9955 for bin = 25~~ ~~[struck]~~ 82.3507 $\rightarrow$ (for bin number 100)

The fitted curve is a straight line. It's clear to see that there are lots of places unmatched with the data points. Therefore, it makes sense that χ^2 is kind of big.

② Hypothesis 2: $\chi_2^2 =$ 16.1991 $\rightarrow$ bin = 25 ~~[struck]~~ 68.9578 $\rightarrow$ bin = 100

Obviously, hypothesis 2 fits the data points better than 1. Hence, χ^2 goes smaller. While, there are still many ~~places~~ data points not being covered. Hence, χ^2 is not small enough.

(c) For a giving number of bins: 25

Number of fitted parameters of hypothesis 1: 2

$\Rightarrow$ $n_d = 25 - 2 = 23$

(d) $p_1 = 0.18046$ with $\chi_1^2 = 28.9955$

$p_2 = 0.805881$ with $\chi_2^2 = 16.1991$

They make sense! The lower p corresponds to a worse fit and larger χ^2