HW4 PHY950

1. $\log L(\nu, n) = n \log \nu - \nu \quad \Rightarrow \quad f(n) = \nu^n e^{-\nu}$

$\frac{\partial \log L}{\partial \nu} = \frac{n}{\nu} - 1 = 0 \quad \Rightarrow \quad \hat{\nu} = n$

$E(\hat{\nu}) = \int \hat{\nu} f(n)\, dn = \int n \nu^n e^{-\nu} dn$

$= e^{-\nu} \frac{\nu^n (n \log \nu - 1)}{\log^2 \nu}$ too complicated, why?

2. (a) $\alpha = P(x \in W \mid \text{pions}) = \frac{1}{2}(1-68\%) = 16\%$

(b) power of the test: prob. to accept electrons, $\varepsilon_e = 1-16\% = 84\%$

prob. e^- be accepted as π, $\beta = 16\%$

(c) $$P(\text{electrons} \mid T<1) = \frac{\varepsilon_e \pi_e}{\varepsilon_e \pi_e + \varepsilon_\pi \pi_\pi} = \frac{84\% \times 1\%}{84\% \times 1\% + 16\% \times 99\%} = 5\%$$

(d) By coding, I get the purity $> 95\%$ when $T < -2.52$

28.9955 for bin = 25

3.(a) ① Hypothesis 1: χ_1^2 = ~~[illegible]~~ 82.3507 → (for bin number 100)

The fitted curve is a straight line. It's clear to see that there are lots of places unmatched with the data points. Therefore, it makes sense that χ^2 is kind of big.

16.1991 → bin = 25

② Hypothesis 2: χ_2^2 = ~~[illegible]~~ 68.9578 → bin = 100

Obviously, hypothesis 2 fits the data points better than 1. Hence, χ^2 goes smaller. While, there are still many ~~places~~ data points not being covered. Hence, χ^2 is not small enough.

(c) For a giving number of bins: 25

Number of fitted parameters of hypothesis 1 : 2

$\Rightarrow$ $n_d = 25 - 2 = 23$

(d) $p1 = 0.18046$ with $\chi_1^2 = 28.9955$

$p2 = 0.805881$ with $\chi_2^2 = 16.1991$

They make sense! The lower p corresponds to a worse fit and larger χ^2