HW5 PHY950

1.(a) ① $s = 1\,\text{GeV}$, $p_1(\sigma|\alpha,\beta) = \frac{1}{\sqrt{2\pi V_1}} e^{-(\sigma_1-6)^2/2V_1}$

expected mean $\bar{\sigma}_1 = 4\sqrt{1}+2 = 6$ $\qquad (\sigma_1 = \alpha+\beta)$

variance $V_1 = (0.2\,\bar{\sigma}_1)^2 = (1.2)^2 = 1.44$

② energy $s = 2\,\text{GeV}$, $p_2(\sigma|\alpha,\beta) = \frac{1}{\sqrt{2\pi V_2}} e^{-(\sigma_2-7.66)^2/2V_2}$

expected mean $\bar{\sigma}_2 = 4\sqrt{2}+2 = 7.66$ $\qquad (\sigma_2 = \sqrt{2}\,\alpha+\beta)$

variance $V_2 = (0.15\,\bar{\sigma}_2)^2 = 1.15^2 = 1.32$

③ energy $s = 5\,\text{GeV}$, $p_3(\sigma|\alpha,\beta) = \frac{1}{\sqrt{2\pi V_3}} e^{-(\sigma_3-\bar{\sigma}_3)^2/2V_3}$

expected mean $\bar{\sigma}_3 = 4\sqrt{5}+2 = 10.94$ $\qquad (\sigma_3 = \sqrt{5}\,\alpha+\beta)$

variance $V_3 = (0.1\,\bar{\sigma}_3)^2 = 1.09^2 = 1.20$

$$\ln P = N_1 \ln p_1 + N_2 \ln p_2 + N_3 \ln p_3 = -N_1\frac{(\sigma_1-\bar{\sigma}_1)^2}{2V_1} - N_2\frac{(\sigma_2-\bar{\sigma}_2)^2}{2V_2} - N_3\frac{(\sigma_3-\bar{\sigma}_3)^2}{2V_3} + C$$

where $\sigma_1 = \alpha+\beta$, $\sigma_2 = \sqrt{2}\,\alpha+\beta$, $\sigma_3 = \sqrt{5}\,\alpha+\beta$

$$\Rightarrow I_{ij} = E\left[\sum_{m=1}^{3} \frac{N_m}{V_m}\frac{\partial \bar{\sigma}_m}{\partial \theta_i}\frac{\partial \bar{\sigma}_m}{\partial \theta_j}\right] \qquad \theta_i, \theta_j \in \{\alpha, \beta\}$$

$$= \begin{bmatrix} \frac{N_1}{V_1} + 2\frac{N_2}{V_2} + 5\frac{N_3}{V_3} & \frac{N_1}{V_1} + \sqrt{2}\frac{N_2}{V_2} + \sqrt{5}\frac{N_3}{V_3} \\ \frac{N_1}{V_1} + \sqrt{2}\frac{N_2}{V_2} + \sqrt{5}\frac{N_3}{V_3} & \frac{N_1}{V_1} + \frac{N_2}{V_2} + \frac{N_3}{V_3} \end{bmatrix}$$

$V[\hat{\theta}_{ij}] \geq (I^{-1})_{ij}$