# Estimation of the CubeSat's available energy for free-orientation scenario

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**Abstract**

The power budget of CubeSat is difficult to estimate, especially when the law of its orientation while moving in orbit is unknown. If the orientation system is absent, failed, or an orientation failure has occurred, then the satellite usually begins to rotate; in this case, the axis of its own rotation can be any. The amount of available energy in free-orientation mode determines how quickly it can restore its functionality and whether it can at all. Thus, the available energy can be obtained using a statistical estimate.

This article provides estimates for popular CubeSat designs. To estimate the energy, one need to substitute the parameters of the solar cells used (at maximum power point), as well as the fill factor of the solar array panels.

*Keywords:* CubeSat, electrical power system, power budget, attitude Control System, free orientation, attitude failure, solar battery illumination

## 1. Introduction

When designing an electrical power system for CubeSats, the available energy is usually estimated first, and then the energy budget is calculated, taking into account the load consumption. This is a simple operation, but in some cases it is difficult to estimate the amount of available energy.

For example, how to estimate the energy budget for a CubeSats without an orientation system? Or for the mode when the orientation system failed? When the spacecraft is released from the container, it inevitably begins to rotate, and this rotation occurs around a random axis with a random value

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of the angular velocity; so how much energy can we get from the solar array in this case?
Undoubtedly, these are quite important cases that require an estimation of the available energy. A search on this issue yielded little information, regardless of the fact that the CubeSat format is very popular [1].

In many articles, reports and dissertations describing the results of designing the satellite power system, usually two modes of attitude control system (ACS) operation are selected as the design case – the orientation of the solar array (SA) normal to the Sun, and the orientation of the satellite build axis along the local vertical for the payload operation. These modes have fairly strictly specified orientation parameters – the angular velocity and the axis of rotation of the satellite. But when considering the free-orientation mode, the calculation is made for some randomly selected parameters, by which it is difficult to estimate whether the chosen case is the worst; in fact, such a combination of orientation parameters may develop that the solar array energy will be insufficient, and the energy budget will become negative.

The closest situation is discussed in [2]. By determining the attitude of a 3U CubeSat over the orbit, authors estimated the incident solar energy. These estimations where performed for three orientation scenarios: nadir-pointing, Sun-pointing and free-orientation. The estimated incident average solar energy for the three scenarios indicated that the Sun-pointing and free-orientation scenarios harvest more energy than the nadir-pointing one.

For each orientation scenario, authors estimated the incident solar radiation as a function of time, the orbit average power and the orbit average energy. According to these results, more radiation can be harvested by the solar cells in the free-orientation and Sun-pointing scenarios. The nadir-pointing scenario was the worst case because the incident solar radiation reaches only one 1U side during some periods. However, the free-orientation scenario depends on the initial torque; the authors do not describe which initial value they chose.

In [3] authors propose a method for calculating the power budget and optimizing the satellite design, including the case with deployable solar array panels to increase power production.
The model uses a spacecraft-body-fixed spherical coordinate system to analyze the complex geometry of a satellite's self-induced shadowing (with Open Graphics Library). As an example design problem, a 3U CubeSat configured as a space-dart with four deployable panels is optimized. Since the satellite has the shuttlecock shape, its orientation is aerodynamically stabilized.

Simulation results are presented for a variety of orbit scenarios (differs with orbital elements), and then the total power generation minimum and maximum are sought among them.

Using a similar approach, let's try to calculate what will be the most probable value of the CubeSat solar array power, if its orientation parameters are unknown in advance.

## 2. Mathematical model

As you know [4], to estimate the energy budget we need to know the solar array generated power. In the case under consideration, the satellite in the form of a parallelepiped rotates around a certain unknown axis; as a result the illumination of each face, and the solar battery panel located on it changes, which in turn affects the output power of the entire solar array.

Thus, we have to present two models:

- Model of the solar cell output power, depending on its position in relation to the luminous flux (illumination);
- Model of satellite rotation from a certain initial position around an arbitrary axis, which allows calculating the illumination of each face of a CubeSat.

The result of the calculation should be an estimate of the generated power of the solar array.

### *2.1. Calculation of the solar cell power, operating at MPP (first approximation)*

On each solar panel usually placed two solar cells (made from 4-in wafer), or panels are almost completely filled with triangular or rectangular solar cells. From the point of view of energy budget, these cases differ in the filling factor of the panel with solar cells, and of course in the characteristics of the solar cells used.

Since almost all small satellite, starting with micro-size and below, use maximum power point trackers (MPPT, see the review of electrical power systems topology in [5]), it makes sense to consider the solar array external characteristics at the maximum power point. And as shown in [6, 7, 8], the maximum power depends on the illumination almost linearly (it is also true for modern types such as dye-sensitized and perovskite solar cells), that is

$$P_{MPP} = P^{*}_{MPP} \cdot L. \tag{1}$$

In this formula, $P^{*}_{MPP}$ is the maximum power of a solar cell under AM0 conditions, and the illumination $L$ takes values in the interval [0..1] (dimensionless), where 0 is completely shaded, and 1 is fully (AM0) illuminated solar cell.

Physically, the illumination $L$ reflects the power contained in the luminous flux, incident on the surface of the element. It depends both on the angle of incidence of the luminous flux and on its intensity:

$$L = L_{REL} \cdot \cos(\beta), \tag{2}$$

where $L_{REL} = G/G_{AM0}$ is relative illumination and also takes values in the range [0..1], $G$ and $G_{AM0}$ are luminous fluxes – active and for AM0 ($1370\,W/m^2$), and $\beta$ is the angle of flow deflection from the normal to the solar cell surface.

The value of $L_{REL}$ in the formula 2 usually differs from one in the following cases:

- In case of characteristics degradation (turbidity) of the protective radiation coating, due to natural aging under the influence of outer space harmful factors.
- When the sunlight flux changes naturally, for example, due to the difference in distances to the Sun at the aphelion and perihelion points of the Earth's orbit.
- When the solar battery is shaded by the elements of the satellite structure, including mesh ones, which create penumbra.

In the case of a CubeSat with a 6-12 month lifespan, that does not have structural elements capable of creating significant shading, $L$ can be assumed to be equal to one with sufficient accuracy.

The factor $\cos(\beta)$ physically means that the "visible to the luminous flux" area of the solar cell has decreased. Or, in other words, the projection of the solar cell onto the plane, normal to the luminous flux, is proportional to the cosine of the deflection angle.

To be precise, the illumination is multiplied not by the usual cosine, but by the so-called Kelly cosine (see [9] and fig. 11 in Supplementary Data for [7]). It differs in that at large deflection angles (more than 80°) it vanishes. In our case, we can use the usual cosine.

Thus, in the first approximation, the maximum power of the solar cell when operating at MPP will be directly proportional to the relative illumination $L_{REL}$ (taken equal to one) and the cosine of the normals deviation angle $\beta$ from the luminous flux.

### *2.2. Calculation of the rotating CubeSat faces illumination*

Developers solve a similar problem when designing a electrical power system for full-size satellites; approaches to its solution are described in sufficient detail in [10, 9, 4, 2]. Usually, several variants of the spatial position (nominal and worst positions) and sometimes the spacecraft rotation axis are selected. For each variant, the illumination of each solar array panels (and then the energy budget) is calculated.

But in the case under consideration, there is one important difference – neither the initial position of the CubeSat nor the axis of rotation are known. We can say that any position and axis are equally probable.
Therefore, for *any* initial spatial position model must calculate the illumination of all faces, per revolution around *any* axis of rotation.

To do this, in fact, one need to calculate how the angle $\beta$ changes for each face of the CubeSat. This calculation is very convenient to do using the mathematical apparatus of quaternions [11].

Let at the *origin* position the axes of the CubeSat body coordinate system coincide with the inertial axises; from the *initial* position it starts rotation around a given axis. Let the rotation from the origin position to the initial one be specified by the quaternion $\boldsymbol{p}$, and the rotation around a certain axis by the quaternion $\boldsymbol{q}$. The rotation scheme is shown in the figure 1.

Let's set the origin position of each normal by vectors $\vec{n_i}$, where $i = 1 \dots 6$. The quaternionic representation of each normal vector is given by the formula

$$\boldsymbol{n} = [0, n_x, n_y, n_z] \, .$$

The initial position of the CubeSat is set by the vector $\vec{b}$, with which the body axis $Ox_B$ (it is the first normal $\vec{n_1}$) should be aligned after the shortest rotation from the origin position. The quaternion of this rotation is

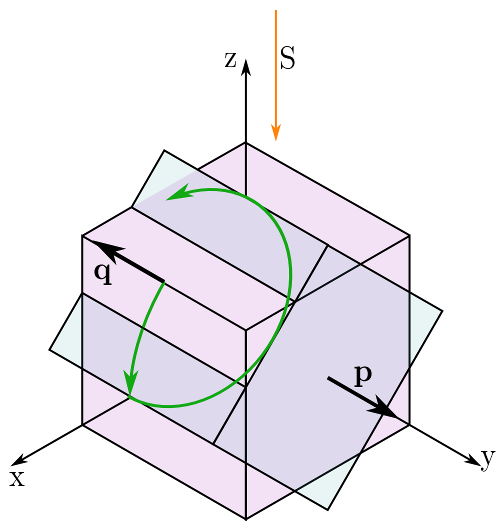


Figure 1: Rotation scheme

calculated by the formula $\boldsymbol{p} = \sqrt{\boldsymbol{u}}$, where

$$\boldsymbol{u} = \frac{\boldsymbol{b}}{|\boldsymbol{b}|} \circ \frac{\boldsymbol{n}_1}{|\boldsymbol{n}_1|}.$$

The quaternionic representation $\boldsymbol{b}$ of the corresponding vector is obtained in the same way as for the normals.

Let the rotation axis be specified by the vector $\vec{q} = [q_x, q_y, q_z]$, and $\alpha$ is the angle of rotation around this axis. Then one can write the corresponding quaternion

$$\boldsymbol{q} = \left[\cos\left(\frac{\alpha}{2}\right), q_x \sin\left(\frac{\alpha}{2}\right), q_y \sin\left(\frac{\alpha}{2}\right), q_z \sin\left(\frac{\alpha}{2}\right)\right].$$

According to the rule of addition of successive rotations, the resulting rotation $\boldsymbol{r}$ will be obtained as a result of quaternionic multiplication $\boldsymbol{r} = \boldsymbol{p} \circ \boldsymbol{q}$. The coordinates of the normal vectors $\vec{m}$ after the rotations are found by the formula

$$\vec{m} = \mathrm{vect}\left(\boldsymbol{r} \circ \boldsymbol{n} \circ \boldsymbol{r}^{-1}\right).$$

Finally, the angles between the sunlight direction and normals to each face are calculated using the dot product

$$(\vec{m}, \vec{S}) = |\vec{m}|\,|\vec{S}|\,\cos(\beta),$$

from which it is easy to express the $\cos(\beta)$ of interest to us, required in the formula 1. If $\cos(\beta)$ is negative, then this means that the angle $\beta$ is greater than 90°, that is, the face front side is turned away from the light flux (vector $\vec{S}$), and hence face is completely shaded ($L = 0$, same formula).

### *2.3. The solar array generated power estimation*

Let the CubeSat rotates around some axis, and for each face normal $\vec{n_i}$ we know $\cos(\beta_i(\varphi))$ for any angular position $\varphi = 0 \ldots 2\pi$ within this rotation. Then the average generated power for this turn will be

$$P^\circ = P^*_{MPP} \sum_{i=1}^{6} w_i \cdot \frac{1}{2\pi} \int_0^{2\pi} \cos(\beta_i(\varphi)) d\varphi. \tag{3}$$

Here, the power is summed up on all sides of the . For the case of unequal panels area on the CubeSats sides, or if there is no solar panel on one of the sides, one can use weights $w_i$ for them. It is clear that for a 1U cubesat with solar panels on all sides, the vector of weights $w$ contains only 1's, and for the 3U format with a solar panel missing on one of the end sides one should take $w = [3, 3, 1, 3, 3, 0]$ (the normal vectors $\vec{n_i}$ must be in the order $[x, y, z, \text{-}x, \text{-}y, \text{-}z]$).

The above formula is valid for a known axis; but if we do not know the axis in advance, we have to be content with only a statistical estimate of the power. It can be obtained as follows:

1. Let's create a set of vectors $\vec{v}$, uniformly distributed in all possible directions ($N$ directions in total). Such a distribution can be obtained, for example, using the Fibonacci algorithm (see fig. 2). Let's number these vectors.
2. From this set we will choose vectors $\vec{b}$ and $\vec{q}$, which after the above transformations will give the quaternions $\boldsymbol{p}_j$ and $\boldsymbol{q}_k$. We will choose vectors in such a way as to iterate over all possible combinations of pairs of vectors $\vec{v_j}$ and $\vec{v_k}$ (these are just two nested loops for $j$ and $k$). Each quaternion will correspond to one of the $\vec{v}$ vectors. Thus, we will take into account all possible variants for both the initial position of the CubeSat and the axis of its rotation in three-dimensional space.
3. For each pair of quaternions $\boldsymbol{p}_j$ and $\boldsymbol{q}_k$ and for each normal $\vec{n_i}$, we calculate $\cos(\beta_i(\varphi))$, and then the average generated power $P^\circ_{jk}$ per revolution, along each of the normals. Let's enter them into the $N \times N$ matrix containing all the calculated powers $P^\circ_{jk}$.

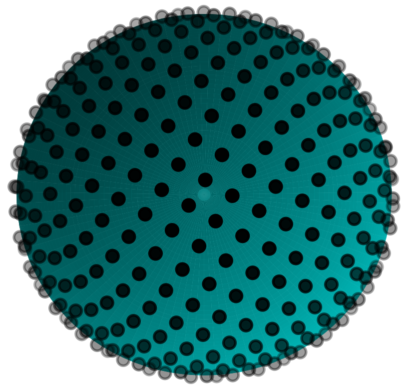

Figure 2: A set of vectors, uniformly distributed in all possible directions (points on the sphere are the ends of these vectors)

4. Finally, let's find the statistical parameters – the probability distribution for the entire $P^{\circ}$ sample, the expected (mean) value and the standard deviation.

The summation of weighted illumination with weights in formula 3, corresponding to the CubeSat configuration, is convenient to take into a separate operation. Then the integration of illumination can be performed in advance (for the normals of the unit cube), saving the data for each normal to an intermediate file.

## 3. Results and discussion

In order to apply the resulting estimation for any type of CubeSat solar cells, let us take $P^{*}_{MPP}$ in the formula 3 equal to one. Then, for the used type and shape of the solar cells, one need to calculate the power of one panel, and then multiply by the relative illumination for your case of the CubeSat solar array panels configuration.

The results for different configurations of solar array panels are given below – for the 1U format in figure 3, for the 2U and 3U formats in tables 1 and 2. The sides with the solar array panels are marked in purple, the empty sides are in gray. The Shuttlecock (3U) structure is similar to Legs, except that there are not two, but four folding panels.

The example of the 1U format shows that the shape of the probability distribution is quite different and, in general, corresponds to both the expected values and the physics of the process. The mean illumination value

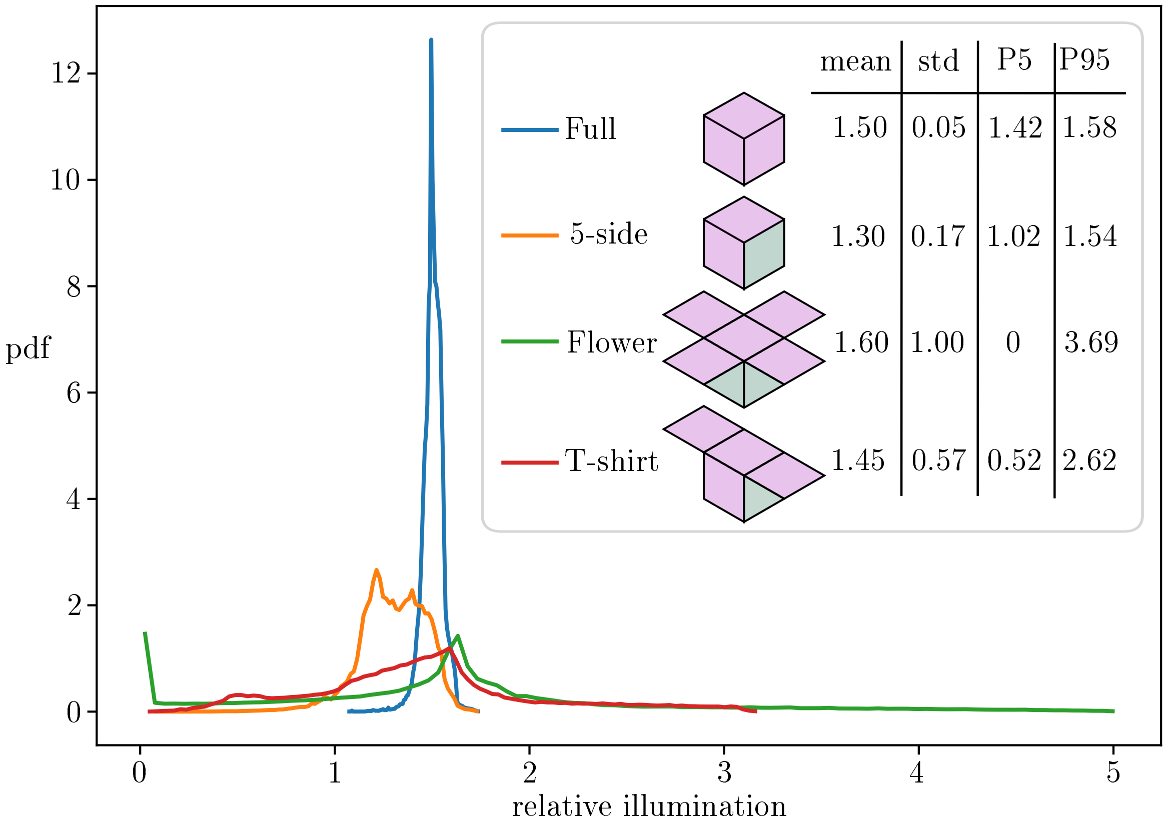


Figure 3: Probability distribution and statistics of relative illumination for 1U CubeSats

corresponds to its expected value; for example, in the case of an undirected 1U CubeSat (or in a loss of orientation case) with panels on all its sides (Full structure), one should expect 1.5 times more energy than from one panel. The P5 and P95 values are the 5th and 95th percentiles; the range of values between them will include 90% of all possible illumination.

If one panel fails (5-Side structure), the most probable value decreases to 1.3, but the actual value with a probability of 0.9 can be in the range from 1.02 to 1.54. The full range of the possible actual value is from 0 (if the CubeSat is directed by the failed side to the Sun and rotates around this direction) to 1.74 (the maximum value is the same as that of the Full structure).

The developed model does not take into account the magnitude of the angular velocity, therefore, it is impossible to calculate the number of revolutions of the CubeSat for a certain time, for example, in the lighted part of the orbit. As a consequence, the most probable value can be used on only in orbits without shadows, or when the period of the CubeSat rotation is much (more than an order of magnitude) less than the orbital period. Otherwise, one need to focus on the statistically worst case for the selected CubeSat configuration.

Python3 was used in the calculations, the code and intermediate results are given in the repository GitHub[1].

## 4. Conclusion

To calculate the energy budget in the case of free-oriented CubeSat, one can use the above estimates of the mean available energy, adjusted for the used solar cells and the solar panels filling degree.

Analyzing the obtained values of the mean illumination, we can say that, in fact, we obtained the average integral value of the CubeSat shadow area at various axes of rotation. The physics of solar cells is such that we calculated the average area of a shadow spot, created by a CubeSat of a certain structure and format.

One can take into account the impact of panel (or part of panel) failure to energy budget; to do this, it is enough to properly set the corresponding weights $w_i$ during the summation.

The proposed method can be generalized for the case of a more complex satellite in shape, only it must be convex so that the sides do not shade on each other; you just need to set other directions of the normals.

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Table 1: Statistics of relative illumination for 2U CubeSats

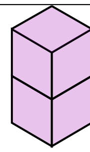
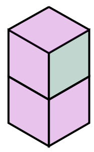
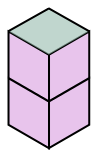
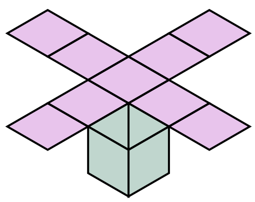
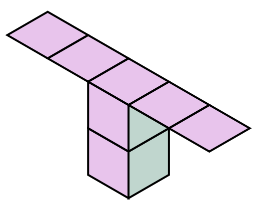
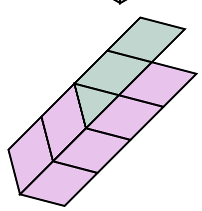
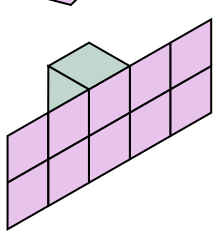
| structure | scheme | mean | std | P5 | P95 |
|---|---|---|---|---|---|
| Full |  | 2.48 | 0.17 | 2.21 | 2.71 |
| Fail1 |  | 2.24 | 0.22 | 1.88 | 2.58 |
| 5 side |  | 2.27 | 0.24 | 1.88 | 2.61 |
| Flower |  | 2.88 | 1.80 | 0.00 | 6.65 |
| T-shirt |  | 2.58 | 0.95 | 1.02 | 4.51 |
| Legs |  | 3.25 | 0.42 | 2.51 | 3.89 |
| Foldscreen |  | 2.44 | 1.73 | 0.00 | 5.53 |

Table 2: Statistics of relative illumination for 3U CubeSats

| structure | mean | std | P5 | P95 |
|---|---|---|---|---|
| Full | 3.46 | 0.31 | 2.95 | 3.90 |
| Fail1 | 3.21 | 0.32 | 2.71 | 3.68 |
| 5 side | 3.25 | 0.36 | 2.67 | 3.78 |
| Flower | 4.16 | 2.60 | 0.00 | 9.60 |
| T-shirt | 3.70 | 1.33 | 1.53 | 6.39 |
| Legs | 4.72 | 0.64 | 3.57 | 5.73 |
| Shuttlecock | 6.18 | 0.76 | 4.94 | 7.32 |
| Foldscreen | 3.67 | 2.60 | 0.00 | 8.29 |