System and Parallel Programming

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# PARALLEL PERFORMANCE (PART 1)

# Outline

- Performance metrics
- Amdahl's law
- Law of Gustavson
- Asymptotic complexity
- Little's formula

# Primary performance metrics

- Response time, execution time
  - Time between start and completion of an event or program
- Throughput
  - Total amount of work done in a given time
- Energy (to solution)


- Performance = $\dfrac{1}{\text{Resources to solution}}$

# Peak performance

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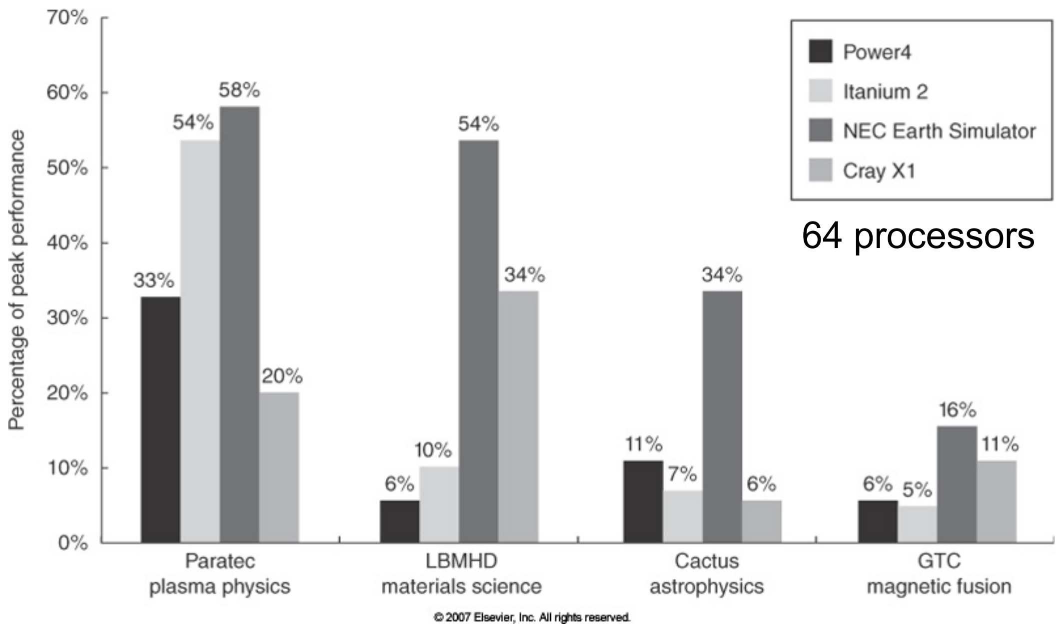
- Peak performance is the performance a computer is guaranteed not to exceed

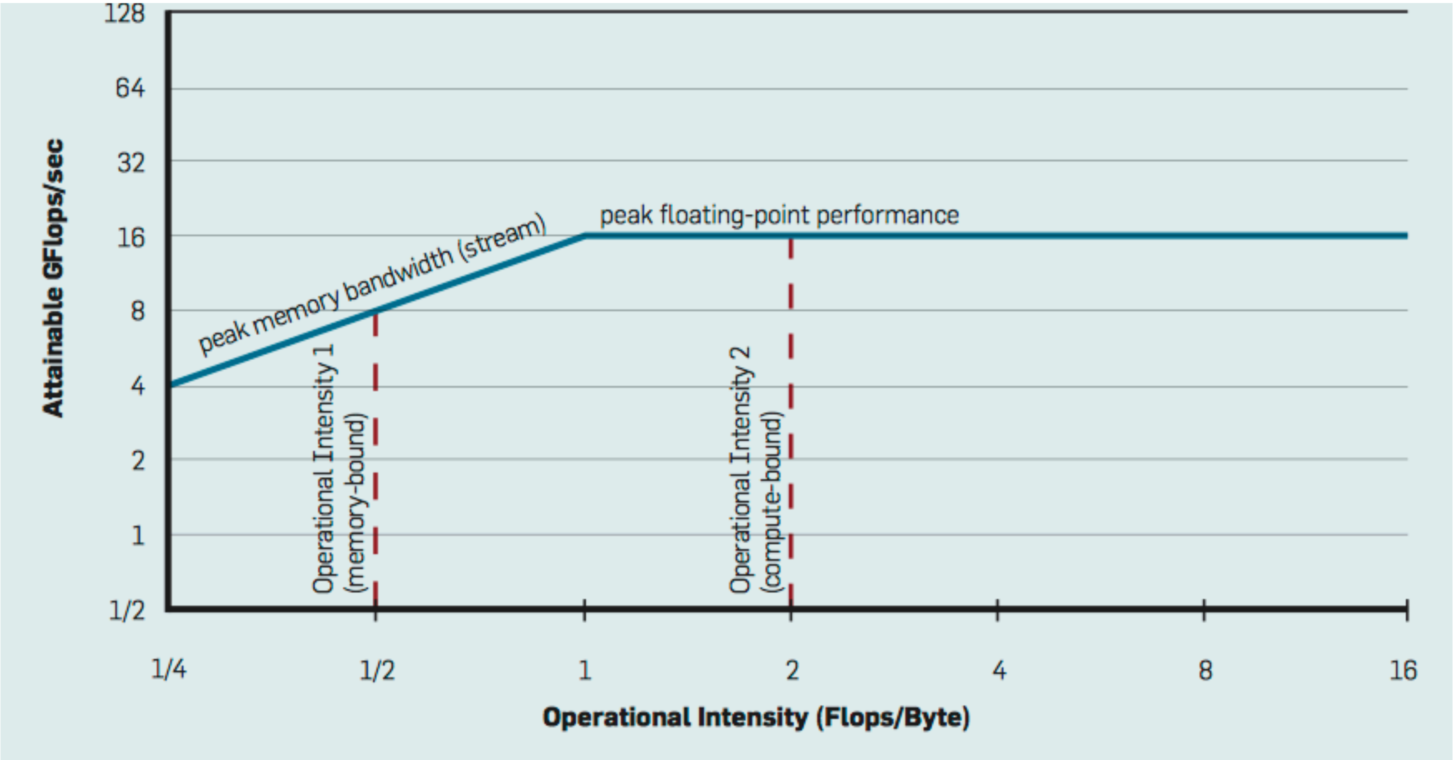



64 processors

Source: Hennessy, Patterson: Computer Architecture, 4th edition, Morgan Kaufmann

# Actual performance on a computer limited by critical resource

Opteron X2




Source: S. Williams, A. Waterman, D. Patterson: Roofline: An Insightful Visual Performance Model for Multicore Architectures, Communications of the ACM, 52(4), 2009.

# Speedup

- Improvement in execution time to be gained from using some faster mode of execution (e.g., parallel execution)

$$\text{Speedup:} \quad S = \frac{\text{Execution time for the entire task without using the enhancement}}{\text{Execution time for the entire task using the enhancement when possible}}$$

- Depends on two factors
  - Fraction f of the original execution that can benefit from the enhancement
  - Improvement gained through enhancement during that fraction

# Amdahl’s law

$$S = \frac{1}{(1 - f_{enhanced}) + \frac{f_{enhanced}}{S_{enhanced}}}$$

$$S < \frac{1}{(1 - f_{enhanced})}$$

# Example



- Function foo() of a program takes 20 % of the overall time
- How is the speedup if the time needed for foo() can be halved?

$$S = \frac{1}{(1-0.2)+\frac{0.2}{2}} = \frac{10}{9}$$

- How is the speedup if the time needed for foo() can almost be eliminated?

$$S = \frac{1}{(1-0.2)+\frac{0.2}{\infty}} = \frac{10}{8}$$

# Amdahl's law for parallelism

- Assumption – program can be parallelized on $p$ processors except for a sequential fraction $f_{seq}$ with $0 \le f_{seq} \le 1$

$$S = \frac{1}{(1 - f_{enhanced}) + \dfrac{f_{enhanced}}{S_{enhanced}}}$$

$$S = \frac{1}{(1 - (1 - f_{seq}) + \dfrac{1 - f_{seq}}{p}}$$

$$S < \frac{1}{f_{seq}}$$

Speedup limited by sequential fraction

# Available parallelism

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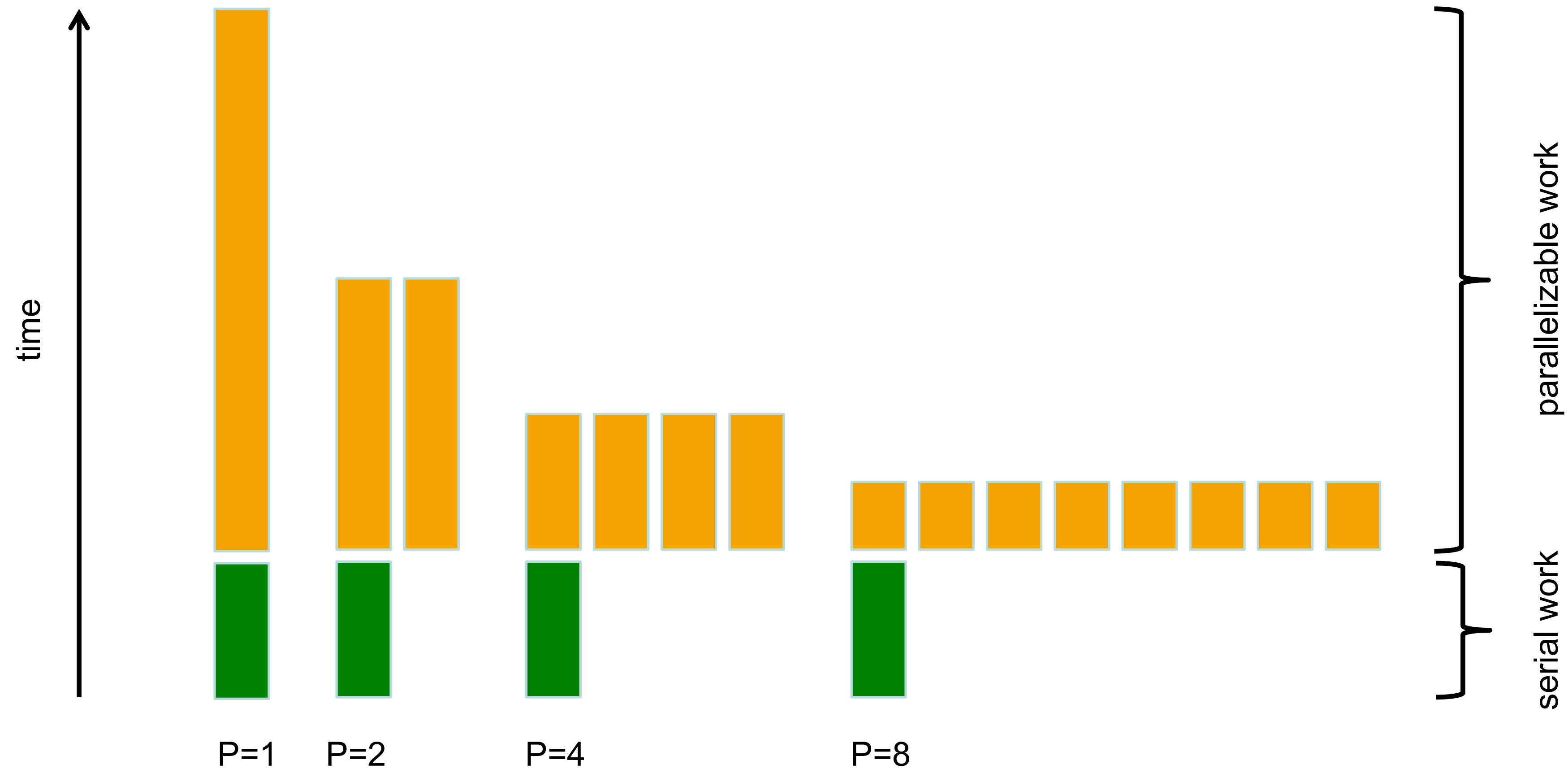
Amdahl's law

$$S = \frac{1}{f_{seq} + \frac{1 - f_{seq}}{p}}$$

Overall speedup of 80 on 100 processors

$$80 = \frac{1}{f_{seq} + \frac{1 - f_{seq}}{100}} \qquad \Rightarrow f_{seq} = 0.0025$$

# Amdahl's law visualized



# Parallel efficiency

$$E(p) = \frac{S(p)}{p}$$

- Metric for cost of parallelization (e.g., communication)
- Without super-linear speedup

$$E(p) \leq 1$$

Super-linear speedup possible

- Critical data structures may fit into the aggregate cache

# Law of Gustafson

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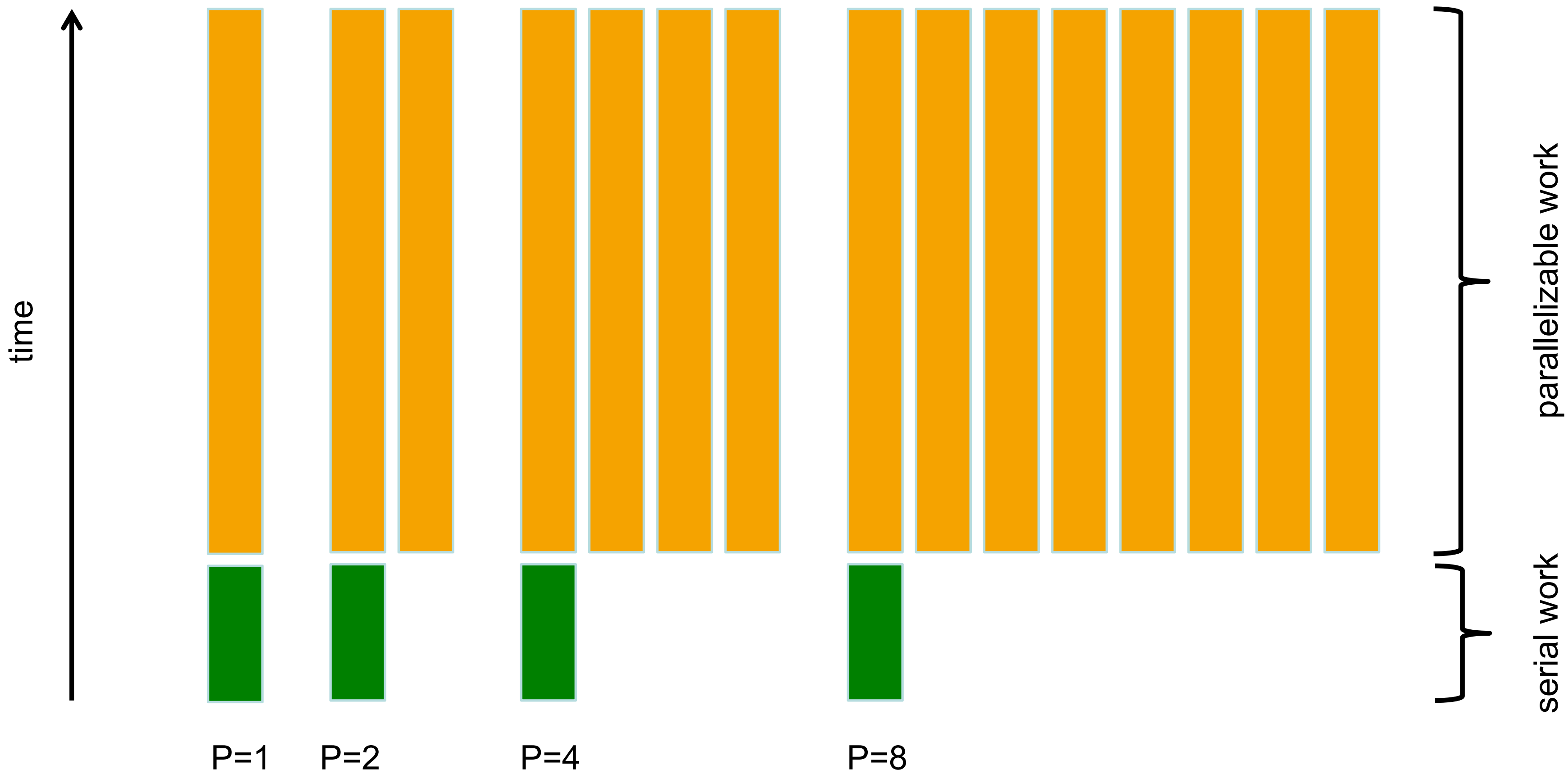
- Amdahl's Law ignores increasing problem size
  - Parallelism often applied to calculate bigger problems instead of calculating a given problem faster
- Fraction of sequential part may be function of problem size
- Assumption
  - Sequential part has constant runtime $\tau_f$
  - Parallel part has runtime $\tau_v(n,p)$
- Speedup

If parallel part can be perfectly parallelized

$$S(n,p) = \frac{\tau_f + \tau_v(n,1)}{\tau_f + \tau_v(n,p)} \xrightarrow[n \to \infty]{} p$$

# Law of Gustafson (2)



# Scalability

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## Weak scaling

- Ability to solve a larger input problem by using more resources (here: processors)
- Problem size per processor remains constant
- Example: larger domain, more particles, higher resolution

## Strong scaling

- Ability to solve the same input problem faster as more resources are used
- Usually more challenging
- Limited by Amdahl's law and communication demand

# Asymptotic complexity

- Example: dot product of two vectors of length *N* ≥ *P*
  - Split vector into pieces of length *N/P*
  - Calculate subproducts in parallel
  - Calculate global sum in tree-like fashion

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} * \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

- Asymptotic complexity

$$T(N,P) = \Theta(N / P + \lg P)$$

# Asymptotic complexity notation

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- Big **O** notation – denotes a set of functions with an upper bound. $O(f(N))$ = set of all functions $g(N)$ such that there exists positive constants $c, N_0$ with $|g(N)| \leq c^* |f(N)|$ for $N \geq N_0$.

- Big **Omega** notation – denotes a set of functions with a lower bound. $\Omega(f(N))$ = set of all functions $g(N)$ such that there exists positive constants $c, N_0$ with $|g(N)| \geq c^* |f(N)|$ for $N \geq N_0$.

- Big **Teta** notation – denotes a set of functions with both lower and upper bounds. $\theta(f(N))$ = set of all functions $g(N)$ such that there exists positive constants $c_1, c_2, N_0$ with $c_1{}^* |f(N)| \leq |g(N)| \leq c_2{}^* |f(N)|$ for $N \geq N_0$.

# Asymptotic speedup

- Asymptotic speedup

$$\frac{T_1}{T_P} = \frac{\Theta(N)}{\Theta(N/P + \lg P)} = \Theta\left(\frac{N}{N/P + \lg P}\right)$$

- Asymptotic efficiency

$$\frac{T_1}{P \cdot T_P} = \Theta\left(\frac{N}{N + P\lg P}\right)$$

# Little's formula

Relates throughput and latency of a system to concurrency

$$C = R \cdot L$$

Applies to system in steady state

- Desired throughput rate is R items per unit time
- Latency to process each item is L
- Number of items concurrently in the system is C

Determines concurrency required to hide latency

# Summary

- Amdahl's law applies to strong scaling
- Law of Gustavson takes weak scaling into account
- Asymptotic complexity is an effective instrument to reason about an algorithm's scalability
- Little's formula tells how to hide latency