Vorläufige Folien. Diese Folien werden nach der Vorlesung kommentarlos durch die aus der Vorlesung ersetzt.

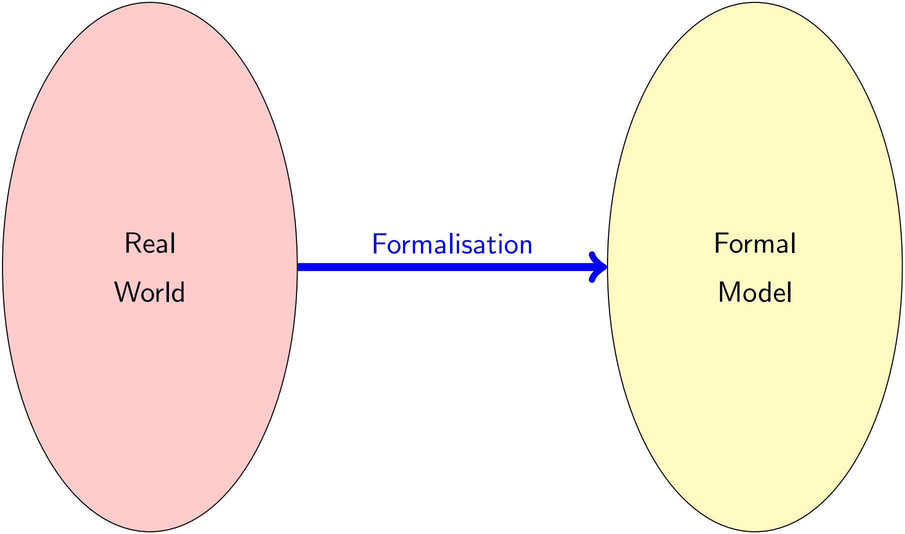
Formale Methoden im Softwareentwurf

Spezifikation mit Linearer Temporaler Logik / Specifying with Linear Temporal Logic

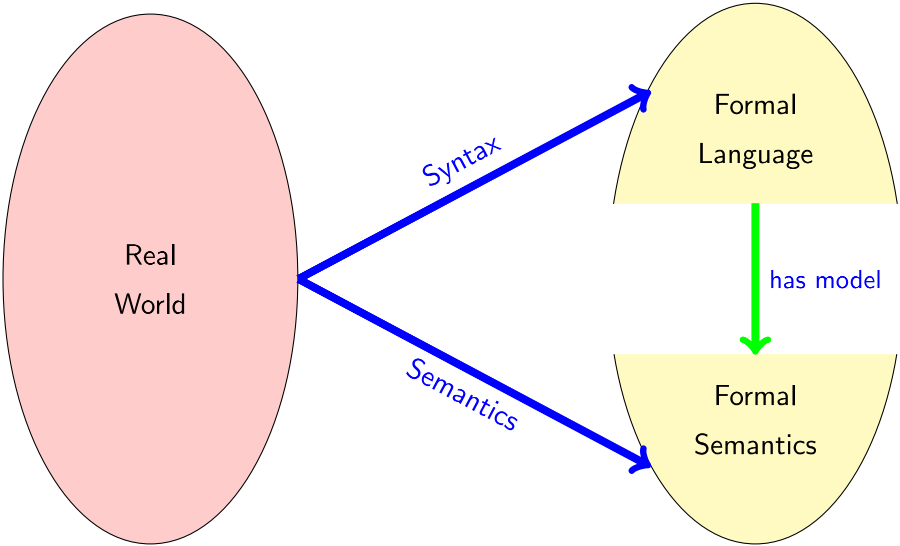
Reiner Hähnle

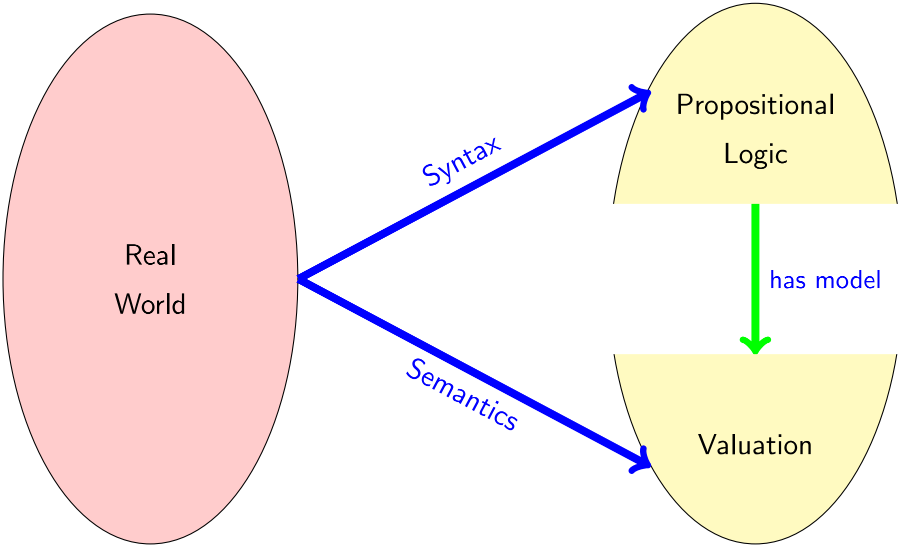
19 November 2018

Formalisation: Syntax, Semantics, Proving



Formal Verification: Model Checking

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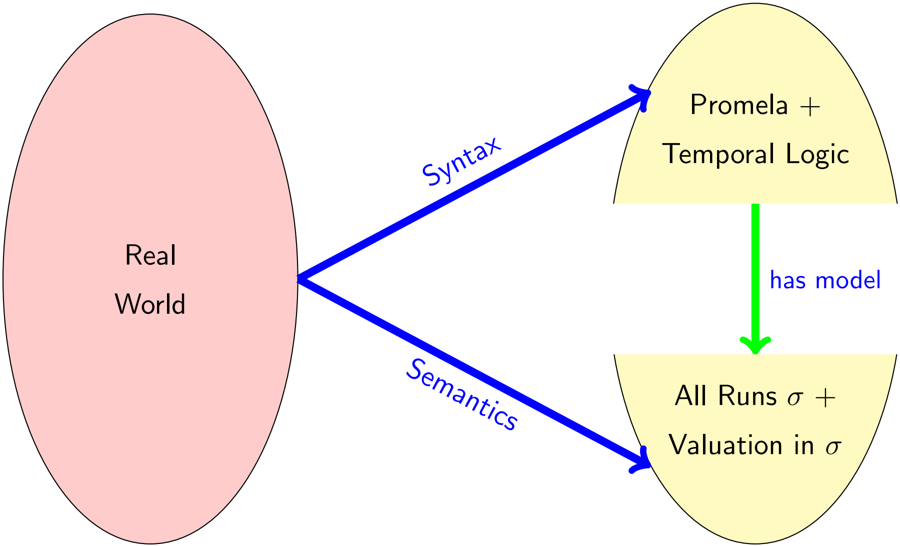
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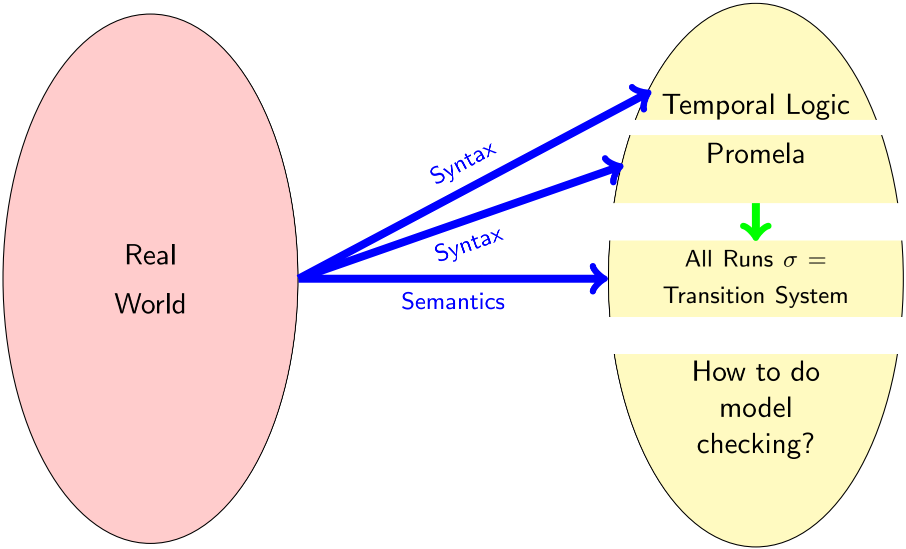
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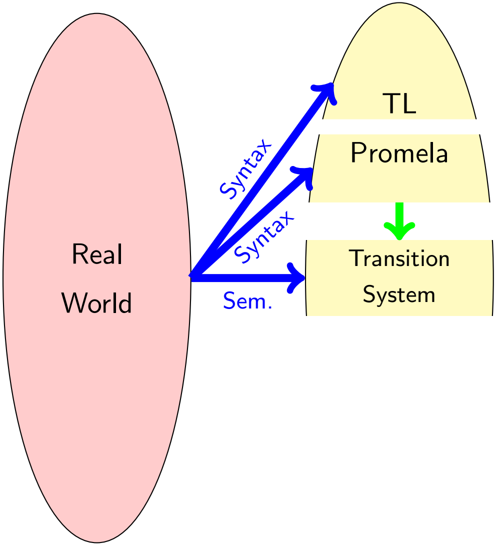
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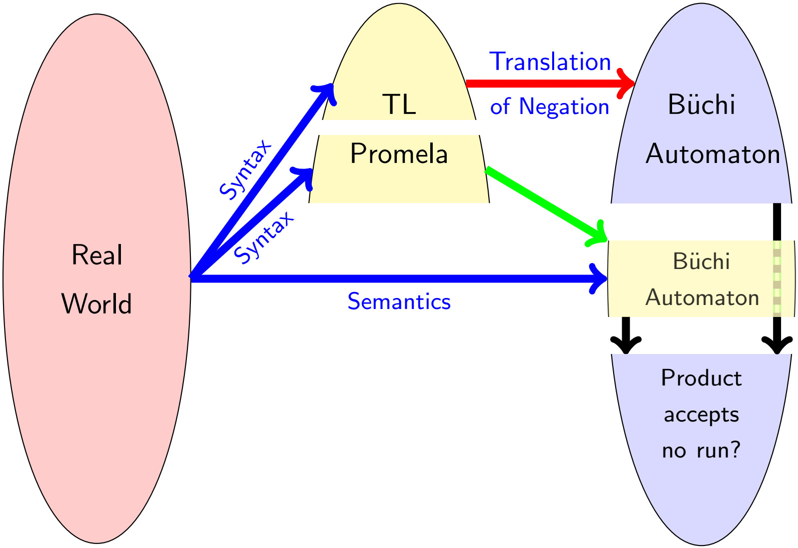
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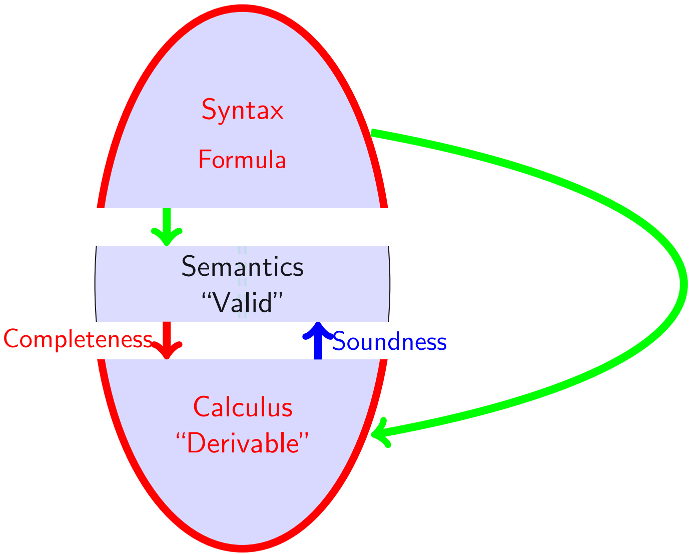
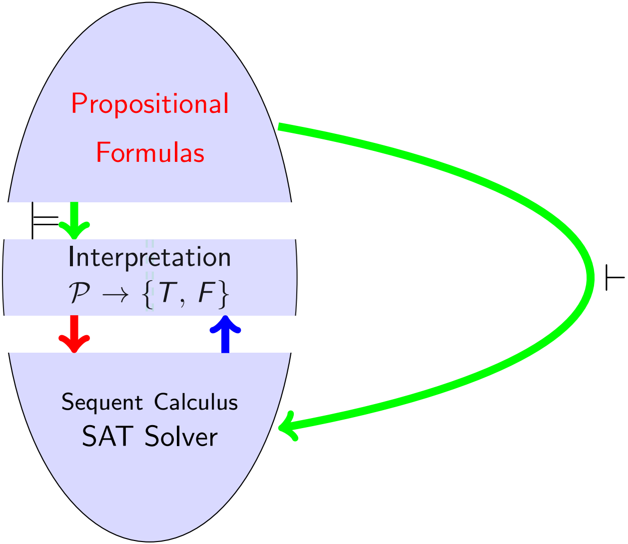
The Big Picture: Syntax, Semantics, Calculus



Simplest Case: Propositional Logic—Syntax



Syntax of Propositional Logic

Signature

A set of Propositional Variables $\mathcal{P}$ (with typical elements $p, q, r, \ldots$)

Propositional Connectives

$\mathrm{true}, \ \mathrm{false}, \ \wedge, \ \vee, \ \neg, \ \rightarrow, \ \leftrightarrow$

Set of Propositional Formulas For_0

1. Truth constants true, false and variables $\mathcal{P}$ are formulas
2. If ϕ and ψ are formulas then
$$\neg\phi, \quad (\phi \wedge \psi), \quad (\phi \vee \psi), \quad (\phi \rightarrow \psi), \quad (\phi \leftrightarrow \psi)$$
are also formulas
3. There are no other formulas (inductive definition)

Remark on Concrete Syntax

	Text book	SPIN
Negation	$\neg$	!
Conjunction	$\wedge$	&&
Disjunction	$\vee$	||
Implication	$\rightarrow$, $\supset$	->
Equivalence	$\leftrightarrow$	<->

We use mostly the textbook notation
Except for tool-specific slides, input files

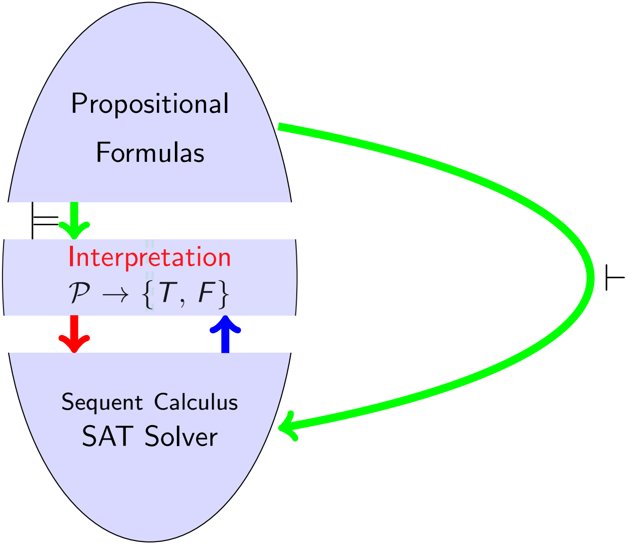
Propositional Logic Syntax: Examples

Let $\mathcal{P} = \{p, q, r\}$ be the set of propositional variables

Are the following character sequences also propositional formulas?

- $(\mathrm{true} \rightarrow p)$ ✔
- $((p(q \wedge r)) \vee p)$ ✘
- $(p \rightarrow (q \wedge))$ ✘
- $(\mathrm{false} \wedge (p \rightarrow (q \wedge r)))$ ✔

Simplest Case: Propositional Logic



Semantics of Propositional Logic

Interpretation $\mathcal{I}$

Assigns a truth value to each propositional variable

$$\mathcal{I} : \mathcal{P} \to \{T, F\}$$

Example

Let $\mathcal{P} = \{p, q\}$

$$(p \ \to \ (q \ \to \ p))$$

	p	q
$\mathcal{I}_1$	F	F
$\mathcal{I}_2$	T	F
$\vdots$	$\vdots$	$\vdots$

Semantics of Propositional Logic

Interpretation $\mathcal{I}$

Assigns a truth value to each propositional variable

$$\mathcal{I} : \mathcal{P} \to \{T, F\}$$

Valuation Function

$val_{\mathcal{I}}$: Continuation of $\mathcal{I}$ on For_0

$$val_{\mathcal{I}} : For_0 \; \to \; \{T, F\}$$

$val_{\mathcal{I}}(p_i) = \mathcal{I}(p_i)$
$val_{\mathcal{I}}(\mathrm{true}) = T$
$val_{\mathcal{I}}(\mathrm{false}) = F$

(cont'd next page)

Semantics of Propositional Logic (Cont'd)

Valuation function (Cont'd)

$$val_{\mathcal{I}}(\neg\phi) = \begin{cases} T & \text{if } val_{\mathcal{I}}(\phi) = F \\ F & otherwise \end{cases}$$

$$val_{\mathcal{I}}(\phi \wedge \psi) = \begin{cases} T & \text{if } val_{\mathcal{I}}(\phi) = T \textbf{ and } val_{\mathcal{I}}(\psi) = T \\ F & otherwise \end{cases}$$

$$val_{\mathcal{I}}(\phi \vee \psi) = \begin{cases} T & \text{if } val_{\mathcal{I}}(\phi) = T \textbf{ or } val_{\mathcal{I}}(\psi) = T \\ F & otherwise \end{cases}$$

$$val_{\mathcal{I}}(\phi \rightarrow \psi) = \begin{cases} T & \text{if } val_{\mathcal{I}}(\phi) = F \textbf{ or } val_{\mathcal{I}}(\psi) = T \\ F & otherwise \end{cases}$$

$$val_{\mathcal{I}}(\phi \leftrightarrow \psi) = \begin{cases} T & \text{if } val_{\mathcal{I}}(\phi) = val_{\mathcal{I}}(\psi) \\ F & otherwise \end{cases}$$

Valuation Examples

Example

Let $\mathcal{P} = \{p, q\}$

$$(p \rightarrow (q \rightarrow p))$$

	p	q
$\mathcal{I}_1$	F	F
$\mathcal{I}_2$	T	F
...		

How to evaluate $(p \rightarrow (q \rightarrow p))$ in $\mathcal{I}_2$?

Valuation in $\mathcal{I}_2$

$val_{\mathcal{I}_2}(\ q \rightarrow p\ ) = T$ because $val_{\mathcal{I}_2}(q) = F$ or $val_{\mathcal{I}_2}(p) = T$

$val_{\mathcal{I}_2}(\ p \rightarrow (q \rightarrow p)\ ) = T$

because $val_{\mathcal{I}_2}(p) = F$ or $val_{\mathcal{I}_2}(\ q \rightarrow p\ ) = T$

Semantic Notions of Propositional Logic

Let $\phi \in For_0$, $\Gamma \subseteq For_0$

Definition (Satisfying Interpretation, Consequence Relation)

$\mathcal{I}$ satisfies ϕ (write: $\mathcal{I} \models \phi$) iff $val_{\mathcal{I}}(\phi) = T$

ϕ follows from Γ (write: $\Gamma \models \phi$) iff for all interpretations $\mathcal{I}$:

$$\text{If } \mathcal{I} \models \psi \text{ for all } \psi \in \Gamma \text{ then also } \mathcal{I} \models \phi$$
$$\text{iff}$$
$$\{\mathcal{I} \mid \mathcal{I} \models \psi \text{ for all } \psi \in \Gamma\} \subseteq \{\mathcal{I} \mid \mathcal{I} \models \phi\}$$

Definition (Satisfiability, Validity)

A formula is satisfiable if it is satisfied by some interpretation.
If every interpretation satisfies ϕ (write: $\models \phi$) then ϕ is called valid.

Semantics of Propositional Logic: Examples

Formula (same as before)

$$p \rightarrow (q \rightarrow p)$$

Is this formula valid?

$$\models p \rightarrow (q \rightarrow p)\ ?$$

Yes. How to prove?

Semantics of Propositional Logic: Examples

$$p \wedge ((\neg p) \vee q)$$

Satisfiable?	✔
Satisfying Interpretation?	$\mathcal{I}(p) = T, \mathcal{I}(q) = T$
Other Satisfying Interpretations?	✘
Therefore, also not valid!	

$$p \wedge ((\neg p) \vee q) \models q \vee r$$

Does it hold? Yes. Why?

An Easy Exercise in Formalisation

Knights & Knaves

In old times there existed an island whose inhabitants were either knights or knaves. A knight always tells the truth, while a knave always lies. Hedwig and Katrin lived on that Island. A historian found in the archives the following statements:

Hedwig: I am a knave if and only if Katrin is a knave.

Katrin: We are of different kind.

(after Raymond Smullyan, Knight and Knaves)

Who is who?

Try to formalise the puzzle in propositional logic

A Harder Exercise in Formalisation

```
// Program P
byte n;
active proctype [2] p() {
  n = 0;
  n = n + 1
}
```

How can we formalize the PROMELA program P in propositional logic?

P is represented by a propositional formula ϕ_{P} provided that an interpretation $\mathcal{I}$ satisfies ϕ_{P} iff $\mathcal{I}$ describes a possible state of P

A Harder Exercise in Formalisation

```
1 // Program P
2 byte n;
3 active proctype [2] p() {
4   n = 0;
5   n = n + 1
6 }
```

$\mathcal{P}$: $N_0, N_1, N_2, \ldots, N_7$ 8-bit representation of **byte** `n`
PCi_j counter when next instruction of process i at line $j \in \{3, 4, 5\}$

Which interpretations do we need to "exclude"?

- The variable `n` is represented by eight bits, all values possible
- A process cannot be at two positions at the same time
- If neither process 0 nor process 1 are at position 5, then `n` is zero
- ...

$$\phi_{\mathrm{P}} := \begin{pmatrix} ((PC0_3 \wedge \neg PC0_4 \wedge \neg PC0_5) \vee \cdots) \wedge \\ ((\neg PC0_5 \wedge \neg PC1_5) \implies (\neg N_0 \wedge \cdots \wedge \neg N_7)) \wedge \cdots \end{pmatrix}$$

Is Propositional Logic Enough?

Can design for a program P a formula Φ_P describing all reachable states

For a given property Ψ the consequence relation

$$\Phi_p \models \Psi$$

holds when Ψ is true in every state reachable in every run of P

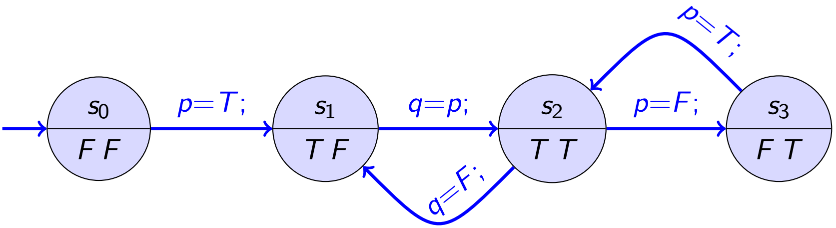
How to Express Properties Involving State Changes?

In every run of a program P

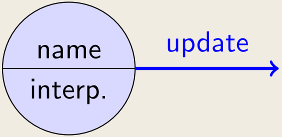
- n will become greater than 0 eventually?
- n changes its value infinitely often
- ...

$\Rightarrow$ Need a more expressive logic: Linear Temporal Logic

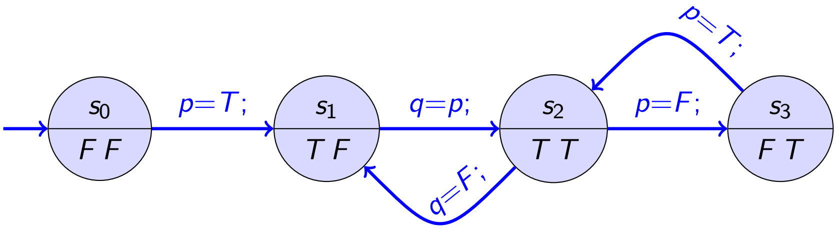
Semantics: Transition systems (Kripke Structures)



Notation



Semantics: Transition systems (Kripke Structures)

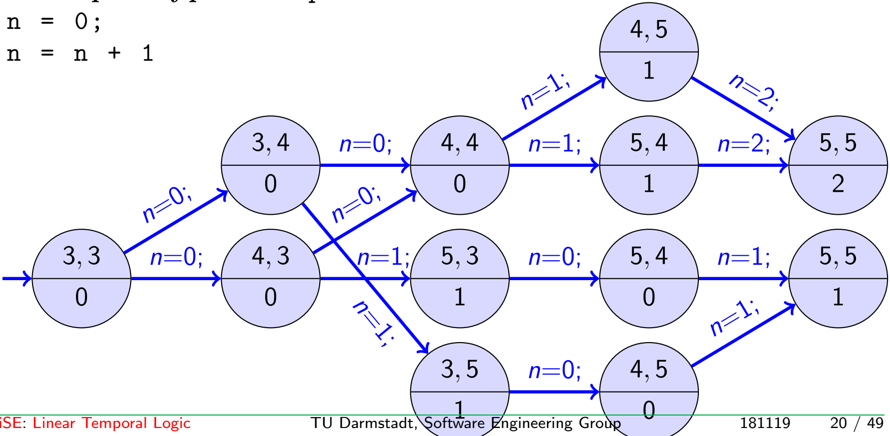


- Each (program) state s_j has its own propositional interpretation $\mathcal{I}_j$
  - Convention: list values of variables in ascending lexicographic order
- Computations, or runs, are infinite paths through states
  - "infinite" not a restriction: let finite run be "stuck" at final state
- In general, infinitely many different runs possible
- How to express (for example) that p changes its value infinitely often in each run?

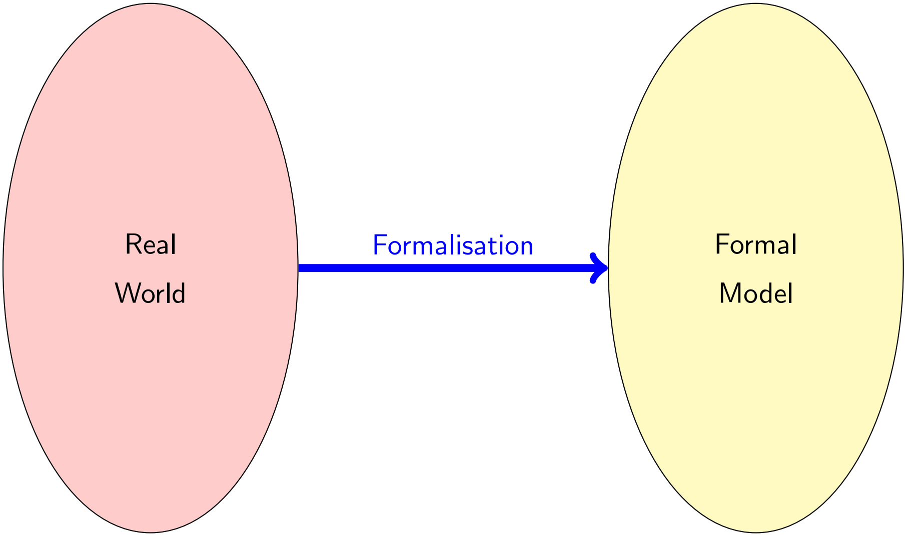
PROMELA Programs and Transition Systems

Runs of PROMELA program ≈ runs of suitable transition system

```
// Program P
byte n;
active proctype [2] p() {
  n = 0;
  n = n + 1
}
```



Formalisation: Syntax, Semantics, Proving



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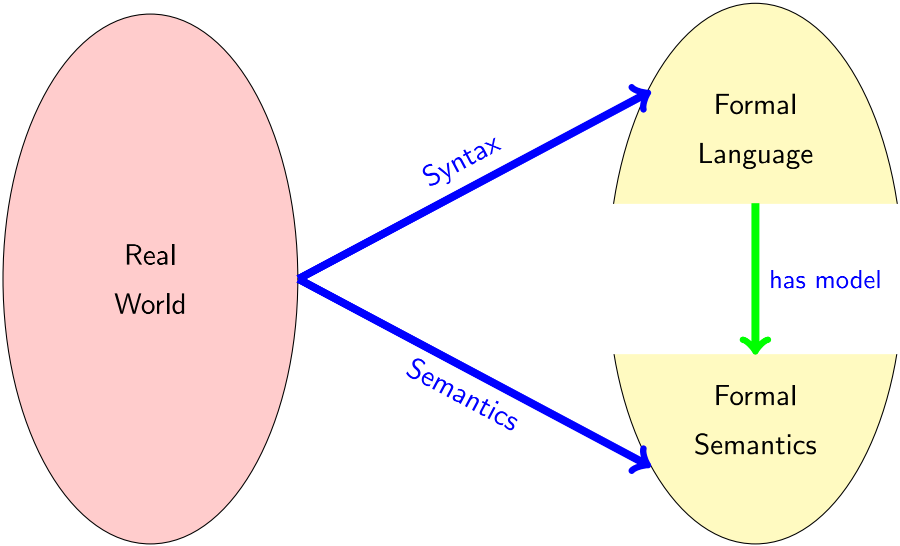

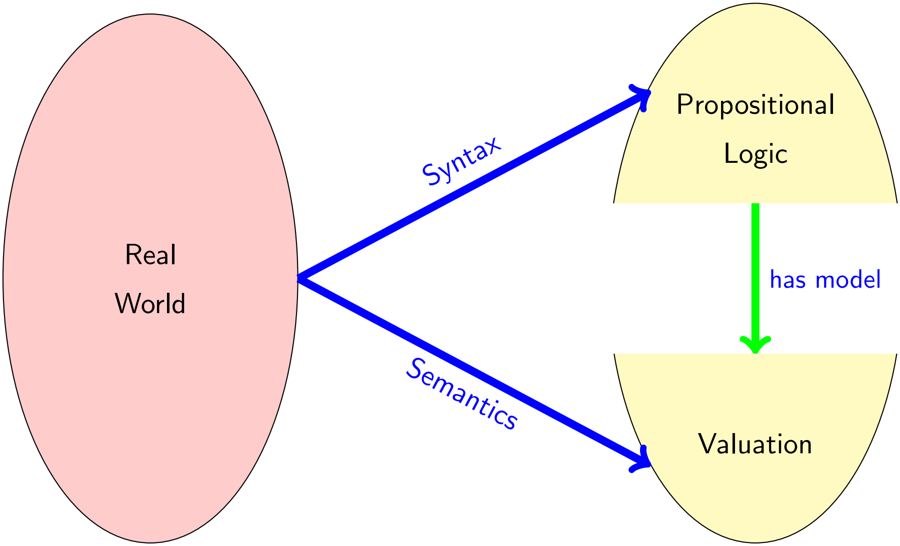
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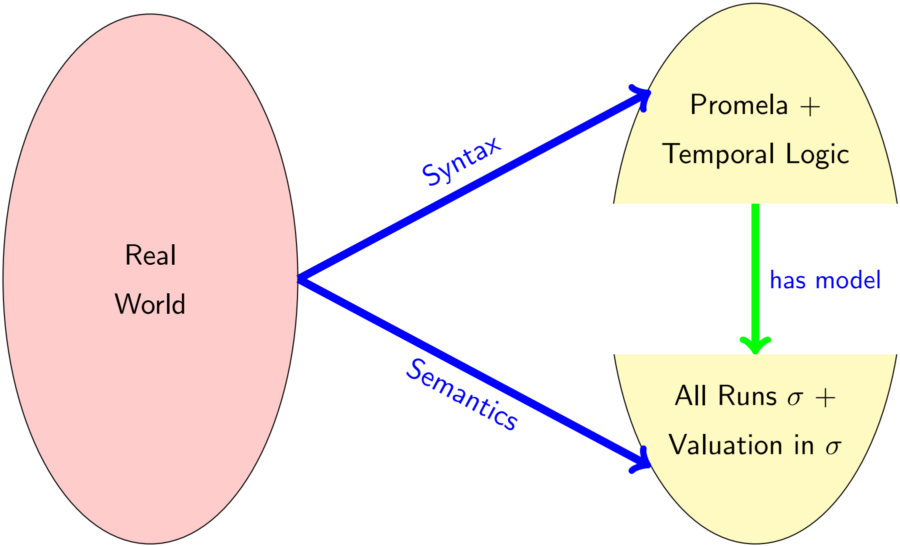
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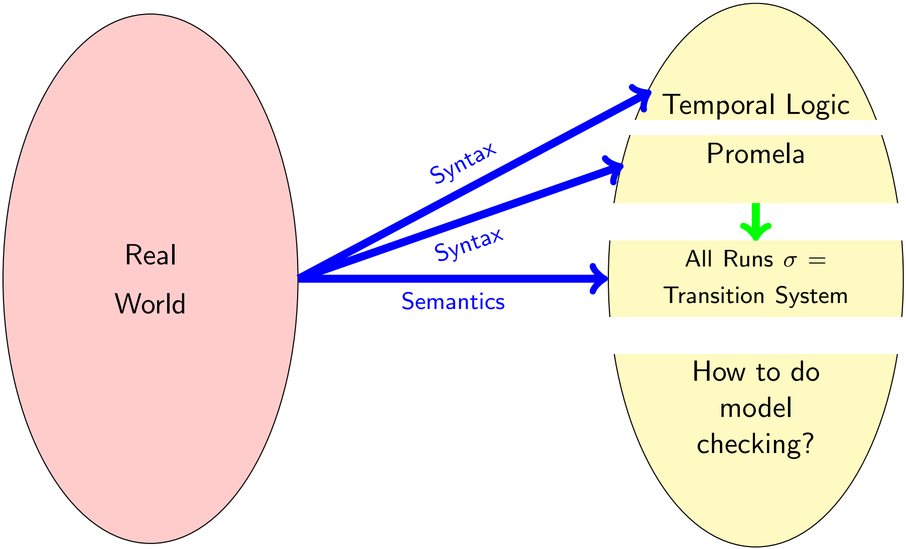
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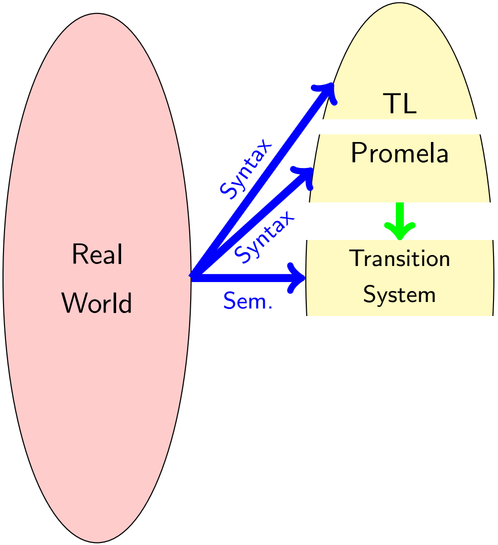
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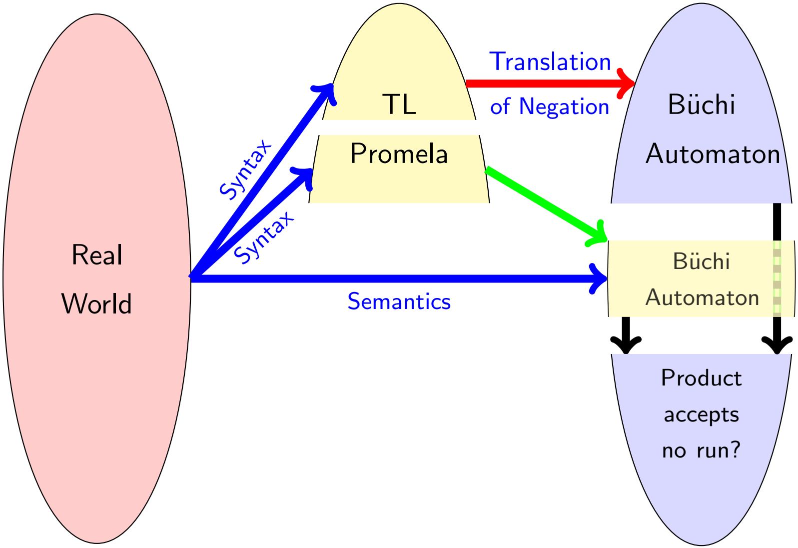
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Temporal Logic—Syntax

An extension of propositional logic that
allows to specify properties of sets of runs

Syntax

Based on propositional signature and syntax

Extension with three connectives:

Always If ϕ is a formula then so is $\Box\phi$

Sometimes If ϕ is a formula then so is $\Diamond\phi$

Until If ϕ and ψ are formulas then so is $\phi\mathcal{U}\psi$

Concrete Syntax

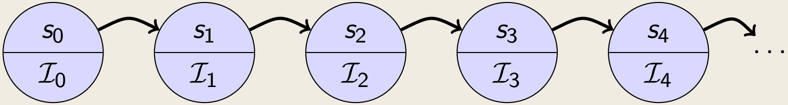
	text book	SPIN
Always	$\Box$	[]
Sometimes	$\Diamond$	<>
Until	$\mathcal{U}$	U

Temporal Logic—Semantics

Need to generalize semantics of propositional logic

- Propositional formula evaluated relative to one interpretation
- Temporal formula evaluated relative to sequence of interpretations

A run σ of a transition system is an infinite chain of states



$\mathcal{I}_j$ propositional interpretation of variables in j-th state
Write run more compactly $s_0\, s_1\, s_2\, s_3 \ldots$

If $\sigma = s_0\, s_1 \cdots$, then $\sigma|_i$ denotes the suffix $s_i\, s_{i+1} \cdots$ of σ

Temporal Logic—Semantics (Cont'd)

Valuation of temporal formula relative to run: infinite sequence of states

Definition (Validity Relation)

Validity of temporal formula depends on runs $\sigma = s_0\, s_1 \ldots$

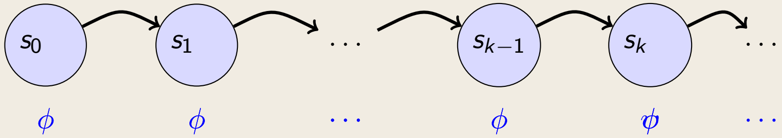
$\sigma \models p$	iff	$\mathcal{I}_0(p) = T$, for $p \in \mathcal{P}$
$\sigma \models \neg\phi$	iff	not $\sigma \models \phi$ (write $\sigma \not\models \phi$)
$\sigma \models \phi \land \psi$	iff	$\sigma \models \phi$ and $\sigma \models \psi$
$\sigma \models \phi \lor \psi$	iff	$\sigma \models \phi$ or $\sigma \models \psi$
$\sigma \models \phi \to \psi$	iff	$\sigma \not\models \phi$ or $\sigma \models \psi$

Propositional formulas evaluated in interpretation of initial state of σ

Temporal connectives?

Temporal Logic—Semantics (Cont'd)

Run σ



Definition (Validity Relation for Temporal Connectives)

Given a run $\sigma = s_0\, s_1 \cdots s_{k-1}\, s_k \cdots$

$\sigma \models \Box\phi$ iff $\sigma|_k \models \phi$ for all $k \geq 0$

$\sigma \models \Diamond\phi$ iff $\sigma|_k \models \phi$ for some $k \geq 0$

$\sigma \models \phi\mathcal{U}\psi$ iff $\sigma|_k \models \psi$ for some $k \geq 0$, and $\sigma|_j \models \phi$ for all $0 \leq j < k$
(if $k = 0$ then ϕ needs never hold)

*

Safety and Liveness Properties

Safety Properties

- Always-formulas called safety property:
  "something bad never happens"
- Let `mutex` ("mutual exclusion") be a variable that is true when two processes do not access a critical resource at the same time
- $\Box\,\texttt{mutex}$ expresses that simultaneous access never happens

Liveness Properties

- Sometimes-formulas called liveness property:
  "something good happens eventually"
- Let `s` be a variable that is true when a process delivers a service
- $\Diamond\,\texttt{s}$ expresses that this service is eventually provided

A Complex Property

What does this mean?Infinitely Often

$$\sigma \models \Box\Diamond\phi$$

"During run σ the formula ϕ becomes true infinitely often"

Validity of Temporal Logic

Definition (Validity)

ϕ is valid, write $\models \phi$, iff ϕ is valid in all runs $\sigma = s_0\, s_1 \cdots$.

Recall that each run $s_0\, s_1 \cdots$ essentially is an infinite sequence of interpretations $\mathcal{I}_0\, \mathcal{I}_1 \cdots$

Representation of Runs

Can represent a set of runs as a sequence of propositional formulas:

- $\phi_0\, \phi_1 \cdots$ represents all runs $s_0\, s_1 \cdots$ such that $\mathcal{I}_j \models \phi_j$ for $j \geq 0$

Semantics of Temporal Logic: Examples

$$\Diamond\Box\phi$$

Valid?

No, there is a run where it is not valid:
$(\neg\phi\,\neg\phi\,\neg\phi\ \cdots)$

Valid in some run?

Yes, for example: $(\neg\phi\,\phi\,\phi\ \cdots)$

$$\Box\phi \rightarrow \phi \qquad (\neg\Box\phi) \leftrightarrow (\Diamond\neg\phi) \qquad \Diamond\phi \leftrightarrow (\mathrm{true}\,\mathcal{U}\phi)$$

All are valid! (proof is exercise)

- $\Box$ is reflexive
- $\Box$ and $\Diamond$ are dual connectives
- $\Box$ and $\Diamond$ can be expressed with $\mathcal{U}$ only

Semantics of Temporal Logic: More Examples

$$(\phi \; \mathcal{U} \; \psi) \rightarrow (\phi \; \mathcal{U} \; \Box\psi)$$

Valid?

No, there is a run where it is not valid:
$(\psi \, \neg\psi \, \neg\psi \; \cdots)$

Valid in some run?

Yes, for example: $(\psi \, \psi \; \cdots)$ or $(\mathrm{false}, \mathrm{false} \; \cdots)$

$$\Box\psi \rightarrow (\phi \; \mathcal{U} \; \psi)$$

Valid! (proof is exercise)

Transition Systems: Formal Definition

Definition (Transition System)

A transition system $\mathcal{T} = (S, Ini, \delta, \mathcal{I})$ is composed of:

- set of states S
- set $\emptyset \neq Ini \subseteq S$ of initial states
- transition relation $\delta \subseteq S \times S$
- labeling $\mathcal{I}$ of each state $s \in S$ with a propositional interpretation $\mathcal{I}_s$

Definition (Run of Transition System)

A run of $\mathcal{T}$ is a sequence of states $\sigma = s_0\, s_1 \cdots$ such that $s_0 \in Ini$ and for all i is $(s_i, s_{i+1}) \in \delta$.

Temporal Logic—Semantics (Cont'd)

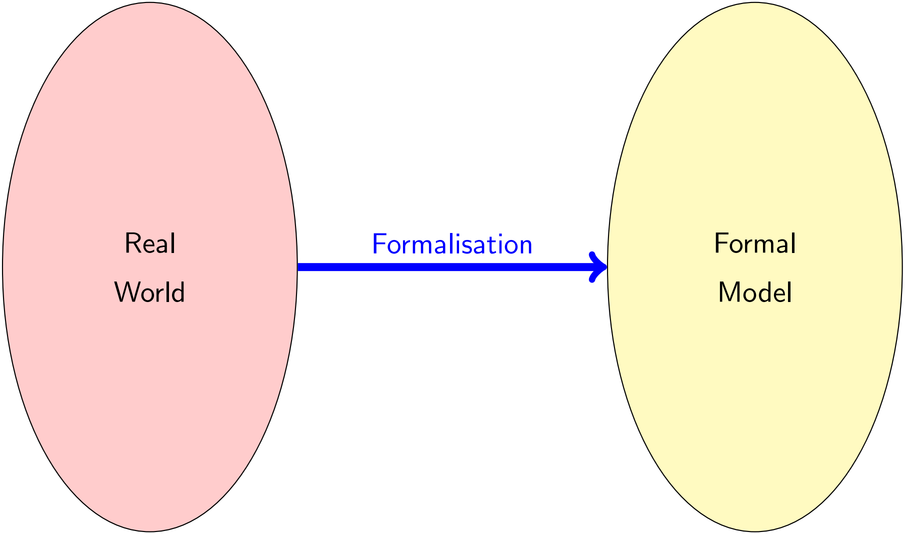
Extension of validity of temporal formulas to transition systems:

Definition (Validity Relation)

Given a transition system $\mathcal{T} = (S, \mathit{Ini}, \delta, \mathcal{I})$, a temporal formula ϕ is valid in $\mathcal{T}$ (write $\mathcal{T} \models \phi$) iff $\sigma \models \phi$ for all runs σ of $\mathcal{T}$.

We could stop here, but transition systems hard to automate efficiently

Formalisation: Syntax, Semantics, Proving



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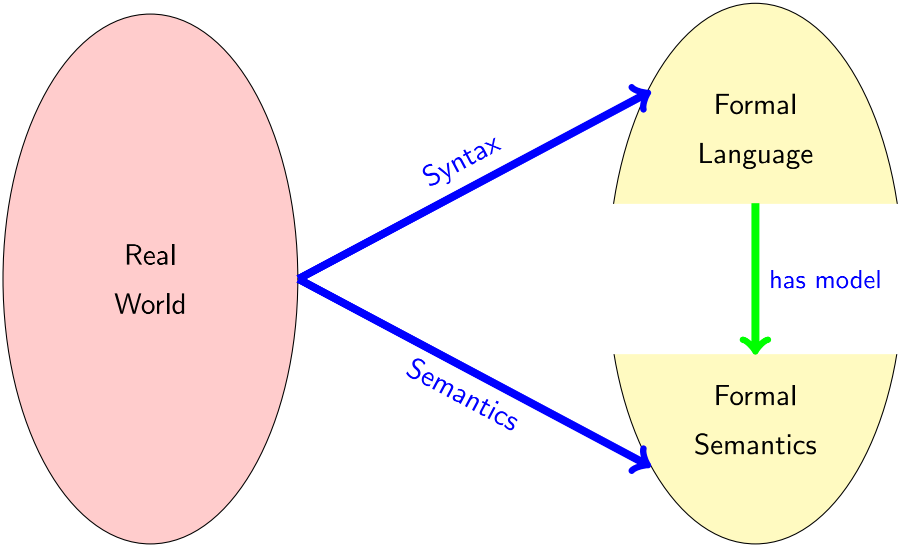

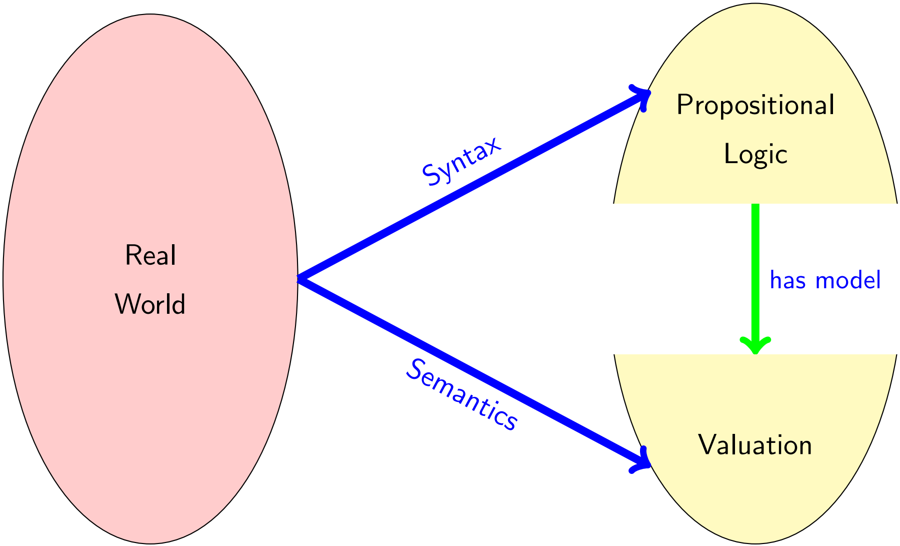
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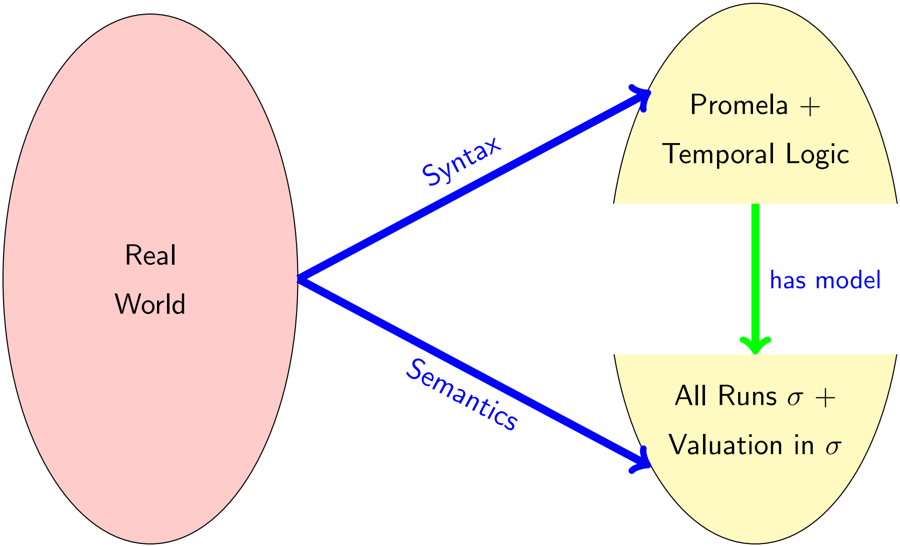
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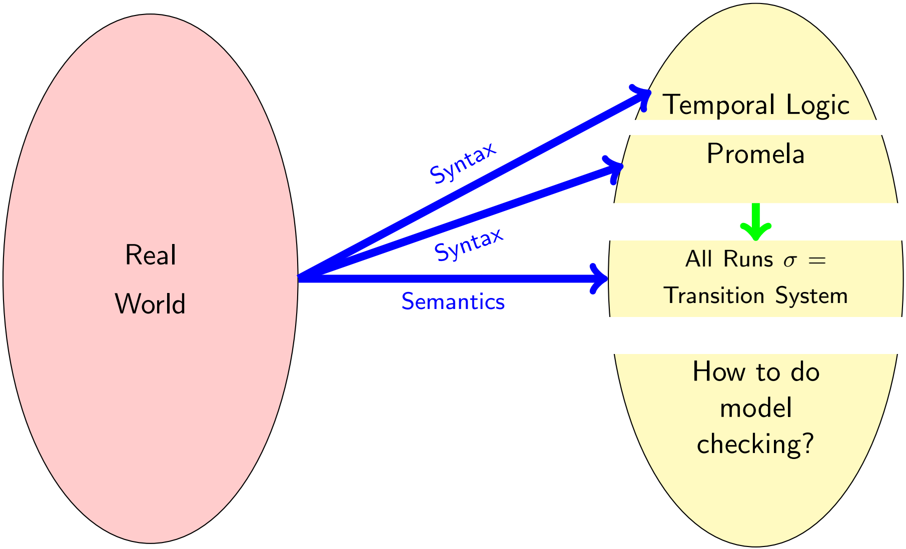
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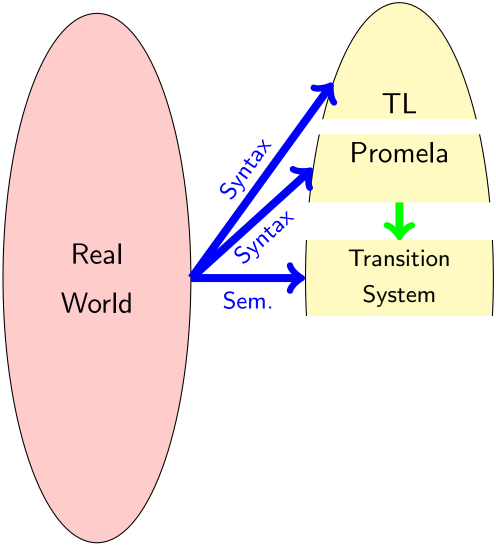
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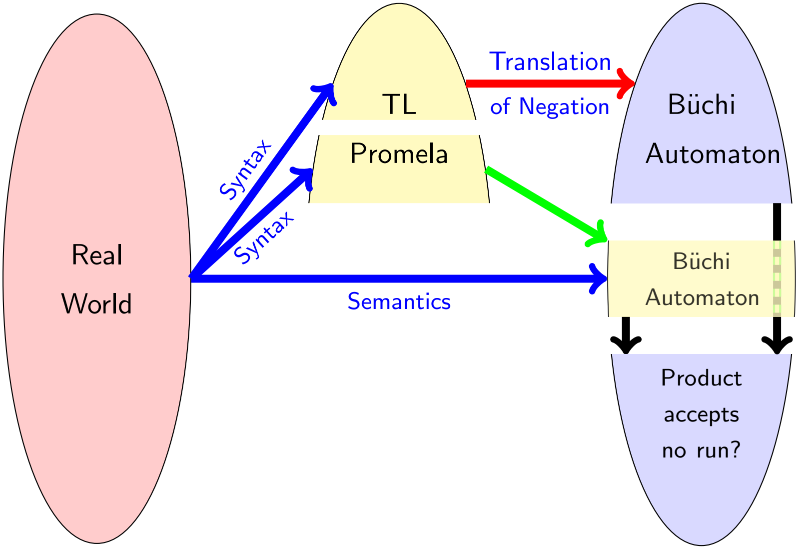
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accepts
no run?

Formal ω-Languages

Given a finite alphabet (vocabulary) Σ

An ω-word $w \in \Sigma^{*\omega}$ is a n infinite sequence

$$w = a_o \cdots a_{nk} \cdots$$

with $a_i \in \Sigma, i \in \{0, \ldots, n\}\mathbb{N}$

$\mathcal{L}^\omega \subseteq \Sigma^{*\omega}$ is called a n ω-language over Σ

Büchi Automaton

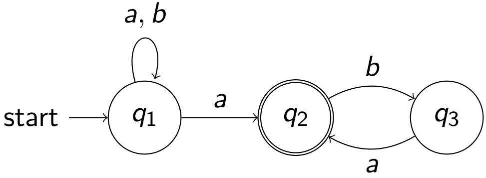
Definition (Büchi Automaton)

A (non-deterministic) Büchi automaton over an alphabet Σ consists of a

- finite, non-empty set of locations (or states) Q
- a non-empty set of initial/start locations $I \subseteq Q$
- a set of accepting/final locations $F = \{F_1, \ldots, F_n\} \subseteq Q$
- a transition relation $\delta \subseteq Q \times \Sigma \times Q$

Example

$\Sigma = \{a, b\}, Q = \{q_1, q_2, q_3\}, I = \{q_1\}, F = \{q_2\}$



Büchi Automaton—Acceptance

Definition (Run and Accepted Run)

An infinite word $w = a_o \cdots a_k \cdots \in \Sigma^\omega$ is a run of a Büchi automaton if

$$q_{i+1} \in \delta(q_i, a_i)$$

for all $i \geq 0$ and some initial location $q_0 \in I$.

A Büchi automaton accepts a run $w \in \Sigma^\omega$, if some accepting location $f \in F$ is infinitely often visited during w.

Let $\mathcal{B} = (Q, I, F, \delta)$ be a Büchi automaton, then

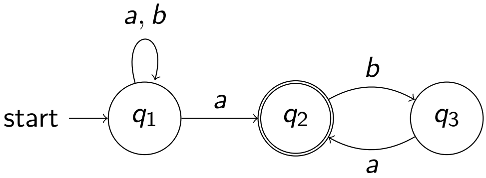
$$\mathcal{L}^\omega(\mathcal{B}) = \{w \in \Sigma^\omega | w \in \Sigma^\omega \text{ is an accepted run of } \mathcal{B}\}$$

denotes the ω-language recognized by $\mathcal{B}$

An ω-language for which an accepting Büchi automaton exists is called ω-regular language

Example, ω-Regular Expression

Which language is accepted by the following Büchi automaton?



Solution: $(a+b)^*(ab)^\omega$ [NB: $(ab)^\omega = a(ba)^\omega$]

ω-regular expressions like standard regular expression

ab a **then** b

$a+b$ a **or** b

a^* arbitrarily, but finitely often a

new: a^ω infinitely often a

Decidability, Closure Properties

Many properties for regular finite automata hold also for Büchi automata

Theorem (Decidability)

It is decidable whether the accepted language $\mathcal{L}^\omega(\mathcal{B})$ of a Büchi automaton $\mathcal{B}$ is empty.

Theorem (Closure properties)

The set of ω-regular languages is closed with respect to intersection, union and complement:

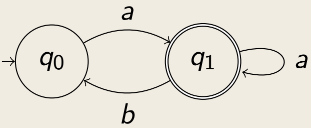
- *if $\mathcal{L}_1, \mathcal{L}_2$ are ω-regular then $\mathcal{L}_1 \cap \mathcal{L}_2$ and $\mathcal{L}_1 \cup \mathcal{L}_2$ are ω-regular*
- *$\mathcal{L}$ is ω-regular then $\Sigma^\omega \backslash \mathcal{L}$ is ω-regular*

But in contrast to regular finite automata

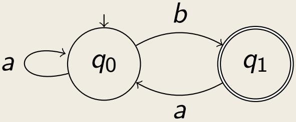
Non-deterministic Büchi automata are strictly more expressive than deterministic ones (latter cannot accept all ω-regular expressions)

Büchi Automata—More Examples

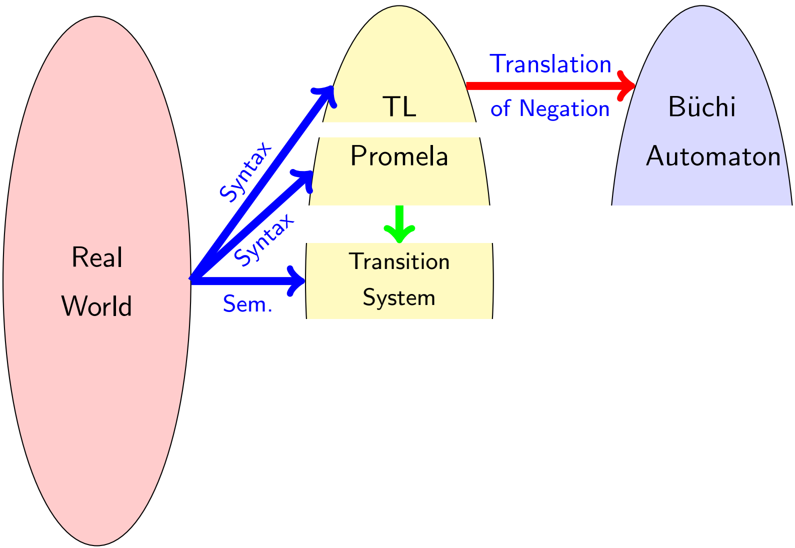
Language? $a(a + ba)^\omega$



Language? $(a^* ba)^\omega$



Formal Verification: Model Checking



Linear Temporal Logic and Büchi Automata

LTL and Büchi Automata are connected

Recall

Definition (Validity Relation)

Given a transition system $\mathcal{T} = (S, Ini, \delta, \mathcal{I})$, a temporal formula ϕ is valid in $\mathcal{T}$ (write $\mathcal{T} \models \phi$) iff $\sigma \models \phi$ for all runs σ of $\mathcal{T}$.

A run of the transition system is an infinite sequence of interpretations I

Intended Connection

Given an LTL formula ϕ:

Construct a Büchi automaton accepting exactly those runs (infinite sequences of interpretations) that satisfy ϕ

Encoding an LTL Formula as a Büchi Automaton

$\mathcal{P}$ set of propositional variables, e.g., $\mathcal{P} = \{r, s\}$

Alphabet Σ of Büchi automaton

A state transition of Büchi automaton must represent an interpretation

Let Σ be set of all interpretations over $\mathcal{P}$, i.e., $\Sigma = 2^{\mathcal{P}}$

Example

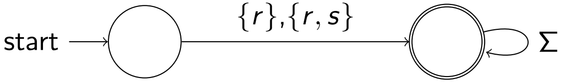
$\Sigma = \big\{\emptyset, \{r\}, \{s\}, \{r, s\}\big\}$

$$I_\emptyset(r) = F, I_\emptyset(s) = F, I_{\{r\}}(r) = T, I_{\{r\}}(s) = F, \ldots$$

Büchi Automaton for LTL Formula By Example

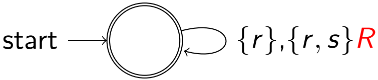
Example (Büchi automaton for formula r over $\mathcal{P} = \{r, s\}$)

A Büchi automaton $\mathcal{B}$ accepting exactly those runs σ satisfying r



In $s_0 \in \sigma$ at least r must hold, the rest is arbitrary

Example (Büchi automaton for formula $\Box r$ over $\mathcal{P} = \{r, s\}$)

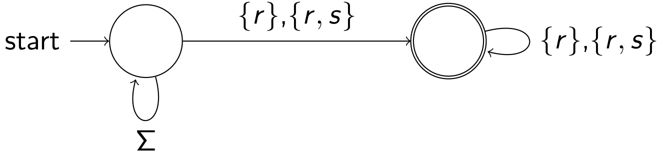


$(R := \{I | I \in \Sigma, r \in I\})$

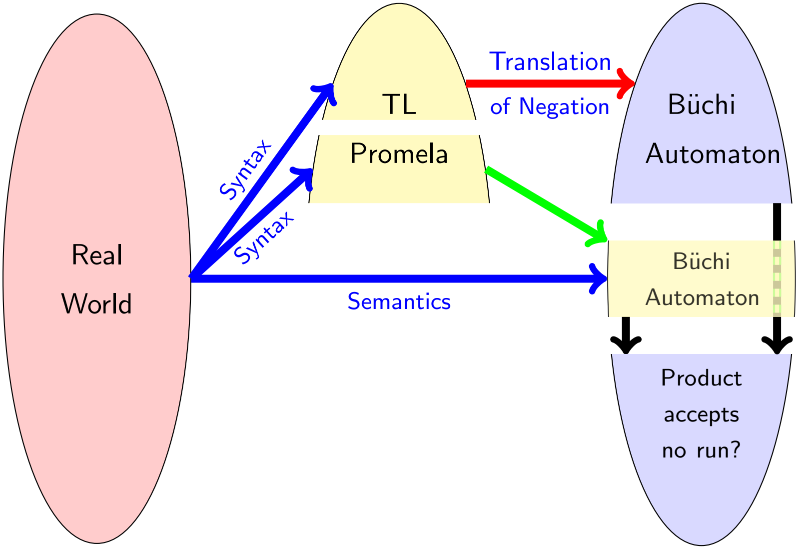
In any $s \in \sigma$ at least r must hold

Büchi Automaton for LTL Formula By Example

Example (Büchi automaton for formula $\Diamond\Box r$ over $\mathcal{P} = \{r, s\}$)



Formal Verification: Model Checking



Model Checking

Check whether an LTL formula is valid in all runs of a transition system

Given a transition system $\mathcal{T}$ (e.g., derived from a PROMELA program)

Verification task: is the LTL formula ϕ satisfied in all runs of $\mathcal{T}$, i.e.,

$$\mathcal{T} \models \phi \quad ?$$

Temporal model checking with SPIN: Topic of next lecture

Today: Basic principle behind SPIN model checking

Next Time: LTS to Büchi

LTS are Büchi Automata:

- Transitions labeled with $p := 1$ ($p := 0$) get label p ($\neg p$)
- Start state is initial state
- States corresponding to final process statements and locations with end labels become final states (with added self transition)
- Possible runs of LTS correspond to accepted words over variables

SPIN Model Checking—Overview

$$\mathcal{T} \models \phi \quad ?$$

1. Represent transition system $\mathcal{T}$ as Büchi automaton $\mathcal{B}_\mathcal{T}$ such that $\mathcal{B}_\mathcal{T}$ accepts exactly those words corresponding to runs through $\mathcal{T}$
2. Construct Büchi automaton $\mathcal{B}_{\neg\phi}$ for negation of formula ϕ
3. If

   $$\mathcal{L}^\omega(\mathcal{B}_\mathcal{T}) \cap \mathcal{L}^\omega(\mathcal{B}_{\neg\phi}) = \emptyset$$

   then ϕ holds, otherwise we have a counterexample

   To check $\mathcal{L}^\omega(\mathcal{B}_\mathcal{T}) \cap \mathcal{L}^\omega(\mathcal{B}_{\neg\phi})$ construct intersection automaton and search for cycle through accepting state

Literature for this Lecture

Ben-Ari Section 5.2.1
(only syntax of LTL)

Baier and Katoen Principles of Model Checking, May 2008,
The MIT Press, ISBN: 0-262-02649-X
Vorhanden in ULB