List of formulas

Universal constants

- electrons charge: $e = 1.6 \cdot 10^{-19}\,\mathrm{C}$
- electrons mass: $m_\mathrm{e} = 9.11 \cdot 10^{-31}\,\mathrm{kg}$
- protons mass: $m_\mathrm{p} = 1.66 \cdot 10^{27}\,\mathrm{kg} \Rightarrow m_\mathrm{p} \approx 1836 \times m_\mathrm{e}$
- Coulombs constant: $k = 9 \cdot 10^9\,(\mathrm{N \cdot m^2/C^2})$
- Avogadros number: $N_\mathrm{A} = 6.022 \cdot 10^{23}\,(1/\mathrm{mol})$
- atomic mass unit: $u = m_\mathrm{p} = 1.66 \cdot 10^{-27}\,\mathrm{kg}$
- electric permittivity of vacuum: $\varepsilon_0 = 8.854 \cdot 10^{-12}\,\mathrm{F/m} \Rightarrow k = \dfrac{1}{4\pi\varepsilon_0}$

Mathematical instruments

1) Polar coordinates: $\{\rho, \varphi\}$

- elementary surface: $\mathrm{d}S = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi \Rightarrow$
  elementary arc along a circle of radius R: $\mathrm{d}L = R\,\mathrm{d}\varphi$;
- elementary surface of a ring: $\mathrm{d}S = 2\pi\rho\,\mathrm{d}\rho$.

2) Cylindrical coordinates: $\{\rho, \varphi, z\}$

- elementary volume: $\mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z \Rightarrow$
  elementary surface on a cylindrical surface of radius R: $\mathrm{d}S = R\,\mathrm{d}\varphi\,\mathrm{d}z$;
- elementary surface of a band on a cylindrical surface of radius R: $\mathrm{d}S = 2\pi R\,\mathrm{d}z$.

3) Spherical coordinates: $\{\rho, \vartheta, \varphi\}$

- elementary volume: $\mathrm{d}V = \rho^2 \sin(\vartheta)\,\mathrm{d}\rho\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \Rightarrow$
  elementary surface on a spherical surface of radius R: $\mathrm{d}S = R^2 \sin(\vartheta)\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$;
- elementary volume of a thin shell: $\mathrm{d}V = 4\pi\rho^2\,\mathrm{d}\rho$.

Electrostatics

- Colomb's force exerted by a point charge q_a at **a** over a point charge q_b at **b**:

$$\vec{F} = k\,\frac{q_\mathrm{a}\,q_\mathrm{b}}{|\vec{r}_\mathrm{b} - \vec{r}_\mathrm{a}|^3}\,(\vec{r}_\mathrm{b} - \vec{r}_\mathrm{a}) = k\,\frac{q_\mathrm{a}\,q_\mathrm{b}}{r_\mathrm{ab}^2}\hat{r}_\mathrm{ab}$$

  with $\vec{r}_\mathrm{ab} = \vec{r}_\mathrm{b} - \vec{r}_\mathrm{a}$, $r_\mathrm{ab} = |\vec{r}_\mathrm{ab}|$ and $\hat{r}_\mathrm{ab} = \vec{r}_\mathrm{ab}/r_\mathrm{ab}$

- Electric field (strength) at a point:

  $$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q}$$

  with $\vec{F}$ being the force exerted on a test point charge q.

- Electric field at **b** due to a point charge q_a at **a**:

  $$\vec{E} = k\,\frac{q_\mathrm{a}}{|\vec{r}_\mathrm{b} - \vec{r}_\mathrm{a}|^3}\,(\vec{r}_\mathrm{b} - \vec{r}_\mathrm{a}) = k\,\frac{q_\mathrm{a}}{r_\mathrm{ab}^2}\hat{r}_\mathrm{ab}$$

- Electric field at **a** due to a collection of point charges q_i at $\vec{r}_i$, $i = 1, \ldots, N$:

  $$\vec{E}(\vec{r}_\mathrm{a}) = k\sum_{i=1}^{N}\frac{q_i}{|\vec{r}_\mathrm{a} - \vec{r}_i|^3}\,(\vec{r}_\mathrm{a} - \vec{r}_i) = k\sum_{i=1}^{N}\frac{q_i}{r_{\mathrm{a},i}^2}\hat{r}_{\mathrm{a},i}$$

- Electric field at **a** due to a volume charge distribution:

  $$\vec{E}(\vec{r}_\mathrm{a}) = k\int_{\mathcal{V}}\frac{\rho(\vec{r}\,')}{|\vec{r}_\mathrm{a} - \vec{r}\,'|^3}\,(\vec{r}_\mathrm{a} - \vec{r}\,')\,\mathrm{d}V(\vec{r}\,') = k\int_{\mathcal{V}}\frac{\rho(\vec{r}\,')}{|\vec{r}_\mathrm{a} - \vec{r}\,'|^2}\,\frac{\vec{r}_\mathrm{a} - \vec{r}\,'}{|\vec{r}_\mathrm{a} - \vec{r}\,'|}\mathrm{d}V(\vec{r}\,')$$

- Electric field at **a** due to a surface charge distribution:

  $$\vec{E}(\vec{r}_\mathrm{a}) = k\int_{\mathcal{S}}\frac{\sigma(\vec{r}\,')}{|\vec{r}_\mathrm{a} - \vec{r}\,'|^2}\,\frac{\vec{r}_\mathrm{a} - \vec{r}\,'}{|\vec{r}_\mathrm{a} - \vec{r}\,'|}\mathrm{d}S(\vec{r}\,')$$

- Electric field at **a** due to a line charge distribution:

  $$\vec{E}(\vec{r}_\mathrm{a}) = k\int_{\mathcal{L}}\frac{\lambda(\vec{r}\,')}{|\vec{r}_\mathrm{a} - \vec{r}\,'|^2}\,\frac{\vec{r}_\mathrm{a} - \vec{r}\,'}{|\vec{r}_\mathrm{a} - \vec{r}\,'|}\mathrm{d}L(\vec{r}\,')$$

- Electric dipole moment – two point charges $+q$ and $-q$ at a small distance d:

  $$\vec{p} = qd\,\hat{l} = p\,\hat{l}$$

  with $\hat{l}$ oriented from the $-q$ point charge towards the $+q$ one.

- Electric field at $\vec{r}$ due to an electric dipole $\vec{p}$ centred at $\vec{r}_\mathrm{o}$ (the centre of the $d\,\hat{l}$ line segment); the observation point is located such that $r_p \gg d$, with $\vec{r}_p = \vec{r} - \vec{r}_\mathrm{o}$:

  – on the perpendicular bisector ($\vec{r}_p \cdot \hat{l} = 0$):

  $$\vec{E}(\vec{r}) = -k\,\frac{p}{r_p^3}\,\hat{l}$$

  – along $\hat{l}$ ($\vec{r}_p \cdot \hat{l} = r_p$):

  $$\vec{E}(\vec{r}) = 2k\,\frac{p}{r_p^3}\,\hat{l}$$

- Torque experienced by an electric dipole $\vec{p}$ in a (uniform) electric field $\vec{E}$:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

- Electric flux through an arbitrary surface $\mathcal{S}$:

$$\Phi = \int_{\mathcal{S}} \vec{E} \cdot \mathrm{d}\vec{A} = \iint_{\mathcal{S}} \vec{E} \cdot \mathrm{d}\vec{A} = \iint_{\mathcal{S}} \vec{E} \cdot \hat{n}\,\mathrm{d}A$$

  with $\hat{n}$ being the normal to the surface $\mathcal{S}$

- Gauss's law for a *closed* surface $\mathcal{S}$:

$$\int_{\mathcal{S}} \vec{E} \cdot \mathrm{d}\vec{A} = \iint_{\mathcal{S}} \vec{E} \cdot \hat{n}\,\mathrm{d}A = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

  with $\hat{n}$ being the *outward oriented* normal to the surface $\mathcal{S}$

- Gauss's law in the case of charge distributions inside of a volume $\mathcal{V}_{\mathcal{S}}$ enclosed by a *closed* surface $\mathcal{S}$:

$$\iint_{\mathcal{S}} \vec{E} \cdot \mathrm{d}\vec{A} = \iint_{\mathcal{S}} \vec{E} \cdot \hat{n}\,\mathrm{d}A = \frac{1}{\varepsilon_0} \iiint_{\mathcal{V}_{\mathcal{S}}} \rho(\vec{r}')\mathrm{d}V$$

- Electric field at the surface of a charged, perfectly conducting surface:

$$E_n = \vec{E} \cdot \hat{n} = \frac{\sigma}{\varepsilon_0}$$

- Electric potential between the points **a** and **b**:

$$\Delta V_{\text{ab}} = V_{\text{b}} - V_{\text{a}} = \frac{\Delta U_{\text{ab}}}{q} = -\int_a^b \vec{E} \cdot \mathrm{d}\vec{l}$$

  with $\mathrm{d}\vec{l}$ along an arbitrary curve between **a** and **b**

- Electric potential between the points **a** and **b** in a uniform field:

$$\Delta V_{\text{ab}} = V_{\text{b}} - V_{\text{a}} = -\vec{E} \cdot \Delta\vec{l}$$

- Electric potential at **b** due to a point charge q_{a} at **a**:

$$V = k\,\frac{q_{\text{a}}}{|\vec{r}_{\text{b}} - \vec{r}_{\text{a}}|} = k\,\frac{q_{\text{a}}}{r_{\text{ab}}}$$

- Electric potential at **a** due to a collection of point charges q_i at $\vec{r}_i$, $i = 1, \ldots, N$:

$$V(\vec{r}_{\text{a}}) = k \sum_{i=1}^{N} \frac{q_i}{|\vec{r}_{\text{a}} - \vec{r}_i|} = k \sum_{i=1}^{N} \frac{q_i}{r_{\text{a},i}}$$

- Electric potential at **a** due to a volume charge distribution:

$$V(\vec{r}_{\rm a}) = k \int_{\mathcal{V}} \frac{\rho(\vec{r}')}{|\vec{r}_{\rm a} - \vec{r}'|} \mathrm{d}V(\vec{r}')$$

- Electric potential at **a** due to a surface charge distribution:

$$V(\vec{r}_{\rm a}) = k \int_{\mathcal{S}} \frac{\sigma(\vec{r}')}{|\vec{r}_{\rm a} - \vec{r}'|} \mathrm{d}S(\vec{r}')$$

- Electric potential at **a** due to a line charge distribution:

$$V(\vec{r}_{\rm a}) = k \int_{\mathcal{L}} \frac{\lambda(\vec{r}')}{|\vec{r}_{\rm a} - \vec{r}'|} \mathrm{d}L(\vec{r}')$$

- Electric potential at $\vec{r}$ due to an electric dipole $\vec{p}$ centred at $\vec{r}_{\rm o}$ (the centre of the $d\,\hat{l}$ line segment); the observation point is located such that $r_p \gg d$, with $\vec{r}_p = \vec{r} - \vec{r}_{\rm o}$:
  - in general:

  $$V(\vec{r}) = k\,\frac{\vec{p} \cdot \hat{r}_p}{r_p^2}$$

  - on the perpendicular bisector ($\vec{r}_p \cdot \hat{l} = 0$):

  $$V(\vec{r}) = 0$$

  - along $\hat{l}$ ($\vec{r}_p \cdot \hat{l} = r_p$):

  $$V(\vec{r}) = k\,\frac{p}{r_p^2}$$

- Calculating the electric field from the electric potential:

$$\vec{E} = -\nabla V = \Big(\partial_x \hat{x} + \partial_x \hat{y} + \partial_x \hat{z}\Big) V$$

- Capacitance:

$$C = \frac{Q}{V}$$

- Capacitance of a parallel-plate capacitor (fringing effects neglected)

$$C = \frac{\varepsilon_{\rm r} \varepsilon_0\, A}{d}$$

  with A the aria and d the distance between the plates

- Energy stored in a capacitor:

$$U = \frac{C\,V^2}{2} = \frac{Q^2}{2\,C}$$

- Energy in the electric field electric energy density:

$$u_{\mathrm{e}} = \frac{1}{2}\varepsilon_{\mathrm{r}}\varepsilon_0\, E^2$$

- Integral electric energy:

$$u_{\mathrm{e}} = \frac{1}{2}\int_{\mathcal{V}} \varepsilon_{\mathrm{r}}\varepsilon_0\, E^2 \mathrm{d}V$$

- Instantaneous electric current:

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

- Electric current density:

$$\vec{J} = nq\vec{v}_{\mathrm{d}}$$

  with n being the number of charges per unit volume, q the charge value and $\vec{v}_{\mathrm{d}}$ the drift velocity

- Ohm's law (local form):

$$\vec{J} = \sigma\vec{E} \quad \vec{E} = \frac{\vec{J}}{\rho}$$

  with σ being the conductivity and ρ the resistivity

- Ohm's law (integral form):

$$I = \frac{V}{R}$$

- Resistance of a cylindrical wire of length L and cross section A:

$$R = \frac{\rho\, L}{A}$$

- Electric power:

$$P = I\,V = I^2 R = \frac{V^2}{R}$$