EE1P21 Electricity and Magnetism

Why Electricity and Magnetism?

Most of EE courses build on E.M. knowledge
Defines the depth of your understanding of other topics
Typically a support discipline (`*you must have done this at E.M!*`´)

Learning Objectives of the course

1. Learn how to use the mathematics you already know to read physics
2. Understanding Fundamentals of electric fields and currents
3. Understanding Fundamentals of magnetic fields

Provides basic tools to understand virtually all aspects of EE

EE1P21 Requirements / intensity

Requires mathematical background knowledge on:

Matrices
Differentiation, partial derivatives
Linear, Surface and Volume integrations

Typically a difficult course in EE

Einstein said
´ Physics should be as easy as possible, not more than that´

You must try to understand lectures

Problems are rarely with the mathematics, but with abstract concepts

Actually first time you actually use the math you have been given

EE1P21 Electromagnetics (5 credits)

Electricity

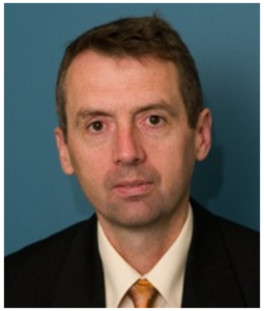
Andrea Neto

Ioan Lager **Giorgio Carluccio**

Magnetism

Henk Polinder

Ioan Lager **Giorgio Carluccio**



Most lectures given in English

Every topic includes lectures and exercises

Special Notes

Black board should already be working from today

Syllabus with the detailed program of all lectures is uploaded already

Notes will be given before the lectures

Lectures based on slides

Slides provided after each lecture

Book Essential University Physics; Vol. 2 Electromagnetism and modern Physics; R. Wolfson; Pearson

We assume that you follow the exercises!!

EE1P21 Exam

50 % from written test on electricity

50 % from written test on Magnetism

With the practice from the exercises it will be easy

Bonus from assignments: they key to success

You will be tested on the learning objectives.

Announced at beginning of each lecture

Truly important items about each topic will be highlighted

Ideal Outcome

You will develop a curiosity for Applied Electromagnetism:
Circuits, Components, Antennas, Microwaves, Radar, Electronics.

Why?

My (biased) vision

Life is much harder than you now imagine….

Soft (ware) backgrounds, at 35, outdated you become managers
…. by 40 you are replaced by younger generations and go to personal Dept.
…... by 45 you are studying again for your second career

If you study E.M. seriously ...
…..you have a chance to be a successful hardware engineer,

Truly permanent jobs,
…..the company can fail but your profession will remain

People

THz Sensing Group

Full professor

Prof. A. Neto

Associate professors

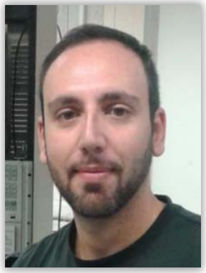
Dr. I.E. Lager

Dr. N. Llombart

Dr. J. Baselmans

Assistant Professors

Dr. A. Endo



Dr. D. Cavallo

Visiting professor

Prof. A. Freni

Postdocs

At LB floor 1 someone always available for questions

Dr. E. Gandini

Dr. G. Carluccio

Dr. W. Syed

Dr. K. Karatsu

PhDs

A. Garufo

O. Yurduseven

S. Dabironezare

D. Toen

S. van Berkel

C. Yepes

N. Marrewijk

X. Tober

Electric Charge, Force and Field

Topic 1

Reminders of vector algebra
Electric Charges
Force: Coulomb's law
Dipoles Moment

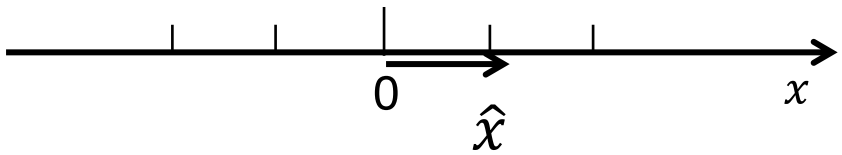
Learning Objectives

Know important electromagnetic quantities
Know Maxwell Equations in Time Domain
Constitutive Relations for simple matter

Vectors Operations

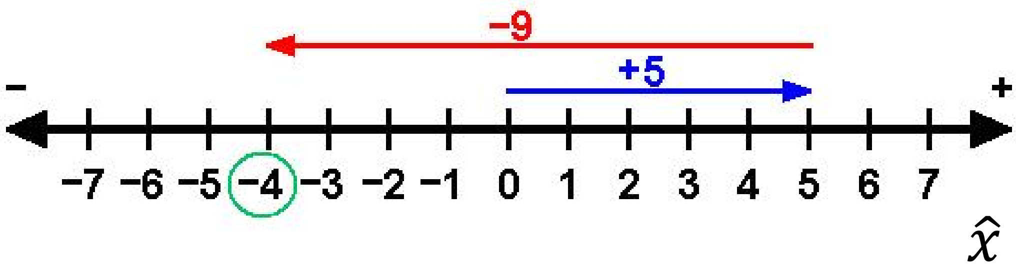
One dimension

$$\vec{r} = x\hat{x}$$



x coordinate $\hat{x}$ unit vector

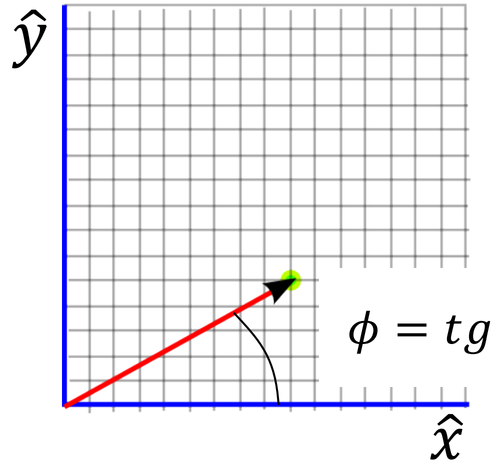
$$\vec{r} = \vec{r}_1 + \vec{r}_2$$
$$= x_1\hat{x} + x_2\hat{x}$$
$$= (x_1 + x_2)\hat{x}$$



Vector Operations

Two dimensions

$$\vec{r} = x\hat{x} + y\hat{y}$$



$$\phi = tg^{-1}\left(\frac{y}{x}\right)$$

Alternative parametrization (1)

$$\vec{r} = r\hat{r}$$

$$r = |\vec{r}| = \sqrt{x^2 + y^2};\ \hat{r} = \frac{\vec{r}}{r}$$

r modulus or amplitude of vector

Alternative parametrization (2)

$$\hat{r} = cos\phi\hat{x} + sin\phi\hat{y}$$

$$\vec{r} = r\ cos\phi\,\hat{x} + rsin\phi\,\hat{y}$$

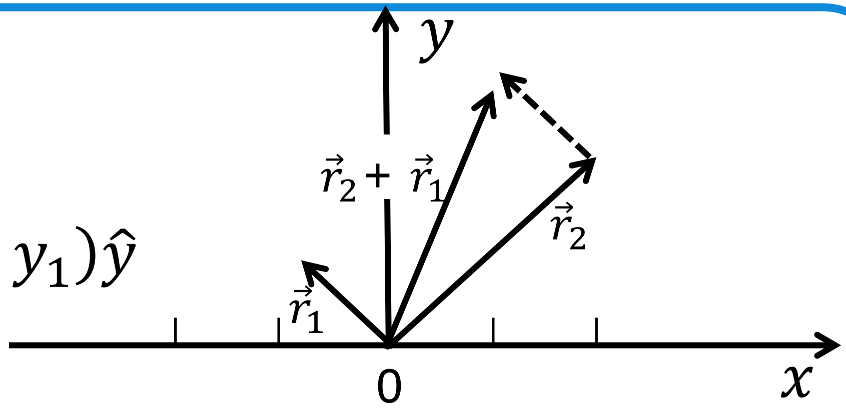
Vector Operations

Two dimensions

$$\vec{r}_1 = x_1\hat{x} + y_1\hat{y} \qquad \vec{r}_2 = x_2\hat{x} + y_2\hat{y}$$

Sum

$$\vec{r}_2 + \vec{r}_1 = (x_2 + x_1)\,\hat{x} + (y_2 + y_1)\hat{y}$$

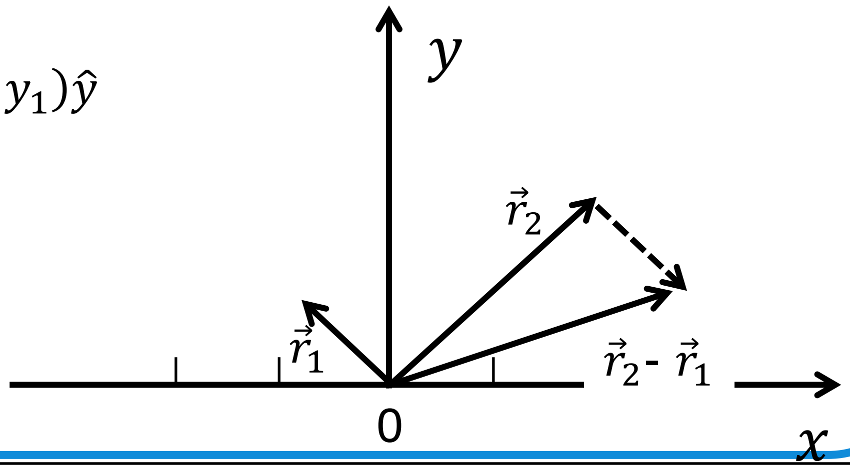


Difference

$$\vec{r}_{12} \equiv \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\,\hat{x} + (y_2 - y_1)\hat{y}$$

$$\vec{r}_{12} = r_{12}\hat{r}_{12}$$

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Vector Operations

In three dimensions

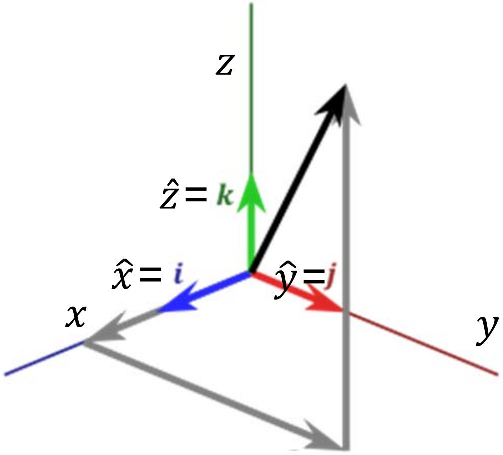
Vectors

3) Three dimensions

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r} = r\hat{r}$$

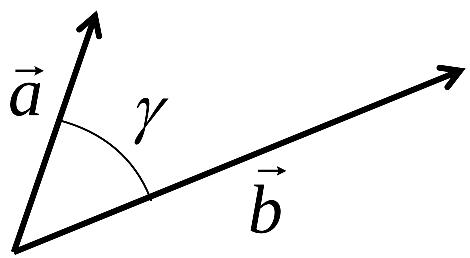
$$r = \sqrt{x^2 + y^2 + z^2};\ \hat{r} = \frac{\vec{r}}{r}$$



Scalar Product
is a scalar

Vectors

$$\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$
$$\vec{b} = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$$



Scalar Product

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\gamma$$

In Cartesian coordinates

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

Vector Product

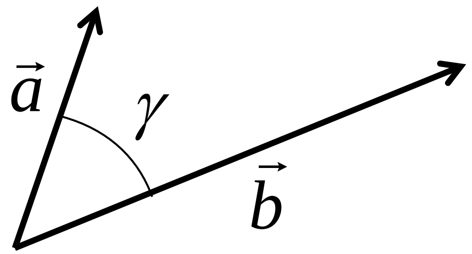
is a vector

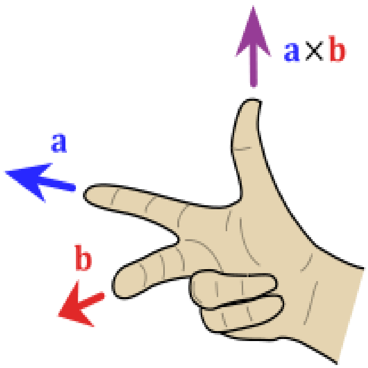
Vectors

$$\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$
$$\vec{b} = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$$

Vector Product

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\gamma \, \hat{n}$$





In Cartesian coordinates

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Determinant of a matrix

3x3

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

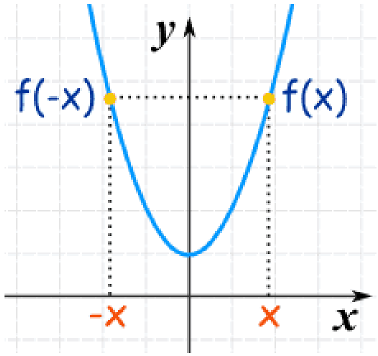
2x2

$$\det\begin{bmatrix} 8 & 3 \\ 4 & 2 \end{bmatrix} = 8 \bullet 2 - 4 \bullet 3 = 16 - 12 = 4$$

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a \bullet d - c \bullet b$$

Fields

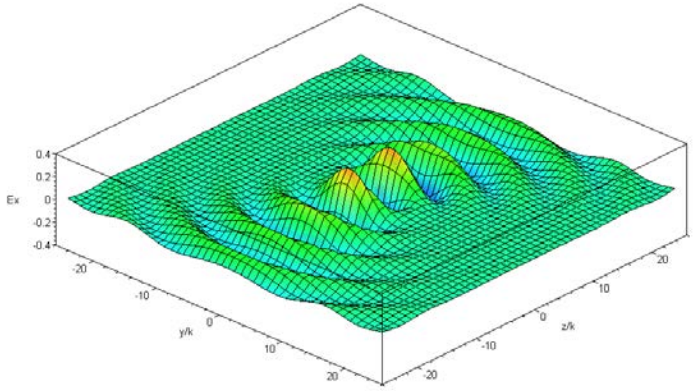
A one-dimensional **function**,



Field: function of more dimensions

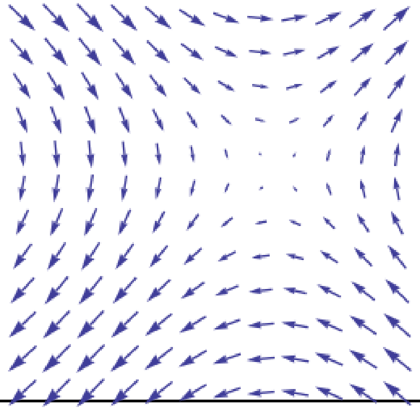
Scalar field

$$f(\vec{r}) = f(x, y)$$



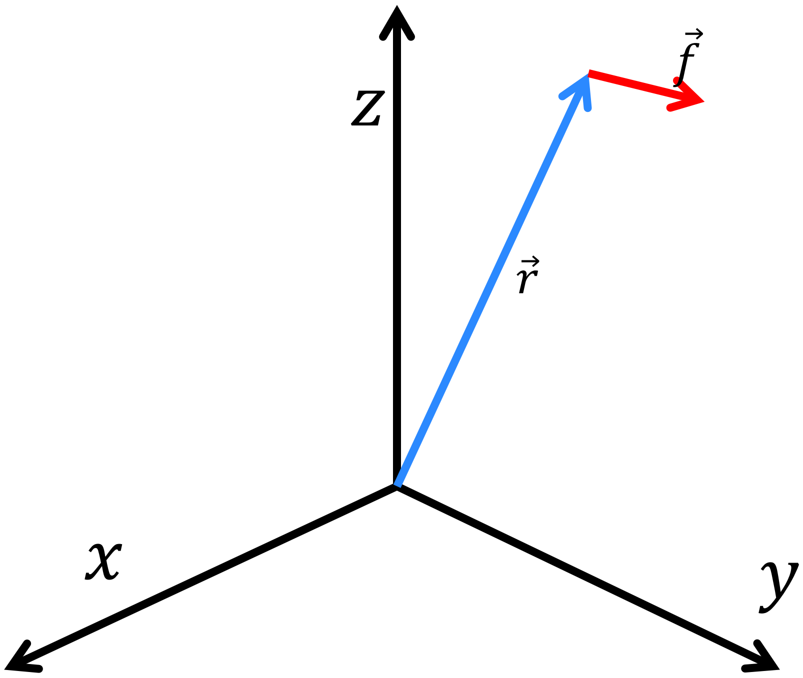
Vector field

$$\vec{f}(\vec{r}) = f_x(x, y)\hat{x} + f_y(x, y)\hat{y}$$



Most difficulties in Electricity and Magnetism

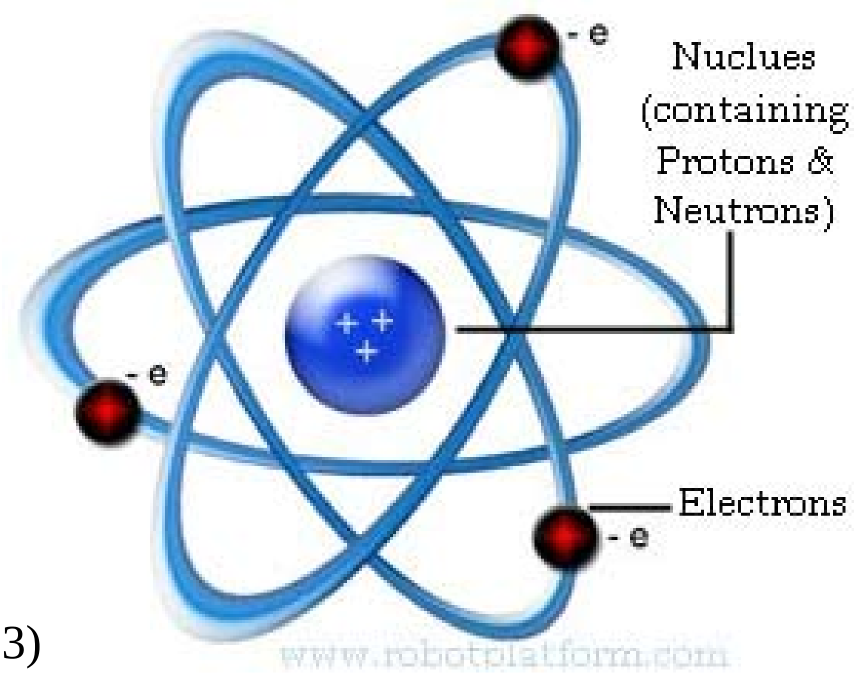
Most difficulties in E.M. arise
from vectorial field nature of quantities



$$\vec{f}(\vec{r}) = f_x(x,y,z)\hat{x} + f_y(x,y,z)\hat{y} + f_z(x,y,z)\hat{z}$$

Electric Charges

Electric charges exist: they are a fundamental property of matter.



Charge comes in two varieties, **positive** and **negative.**

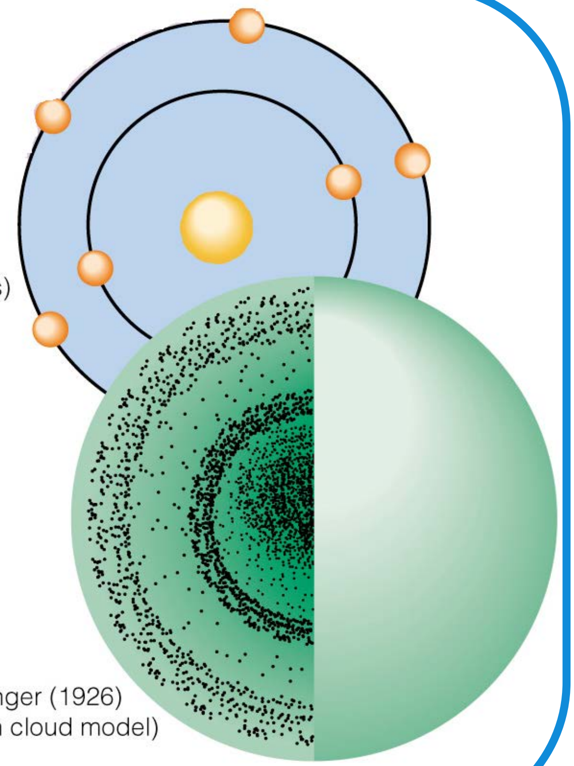
Most charged particles carry exactly one **elementary** charge, *e*, either positive or negative.

The proton carries exactly +*e*, the electron exactly –*e*. *This is the smallest that has ever been measured* R. Millikan (Nobel 1923)

The International System unit of charge is the **coulomb** (C), equal to approximately 6.25×10^{18} elementary charges: *e* is approximately 1.6×10^{-19} C.

Actually …

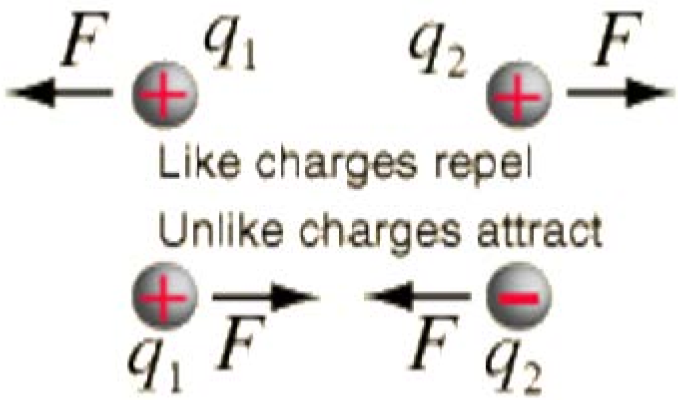
The notion of indivisible charges has been known to be inaccurate for a long time…



Property:
The charge in a closed system is conserved, in that the algebraic sum of charges remains unchanged; they cannot disappear.

Force between Charges

Charges act with forces one on the other.



Coulomb's Law

$$\vec{F}_{12}(\vec{r}, q_1, q_2) = \frac{k_e q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{r_{12}}$$

$$r_{12} = |\vec{r}_2 - \vec{r}_1|$$

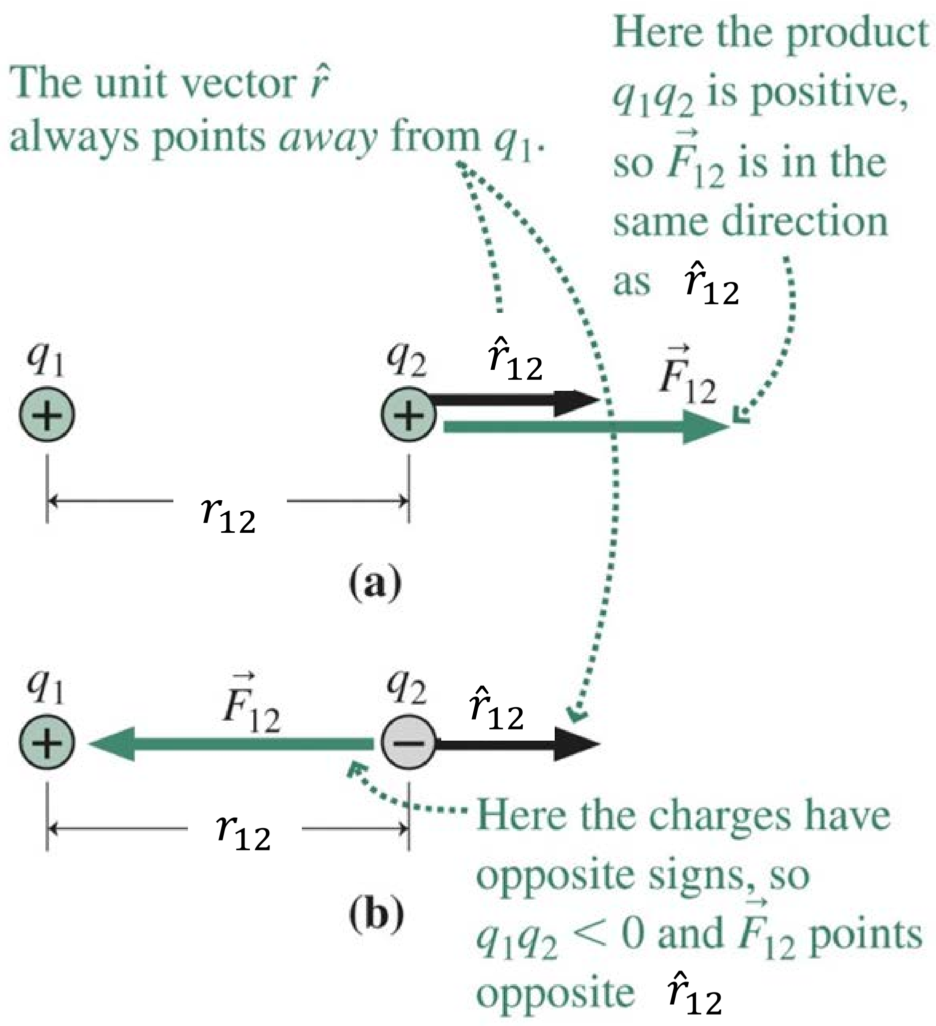
Coulomb's Constant

$$k_e = \frac{F_{12} r_{12}^2}{q_1 q_2}$$

$$k_e \underset{dim.}{=} Nm^2/C^2$$

$$k_e = 8.98\,\frac{10^9 Nm^2}{C^2} = \frac{1}{4\pi\epsilon_0}$$

The sign of the Coulomb's Force



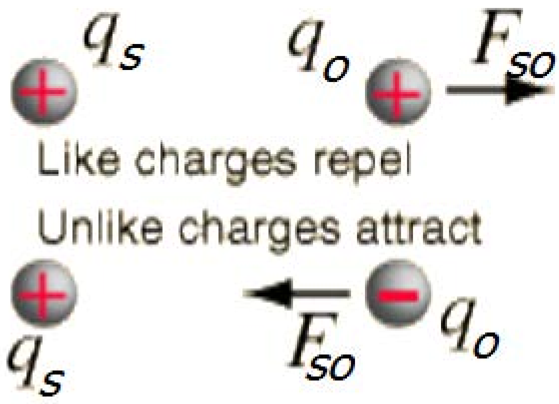
Source and observer Charges

Coulomb's Law

$$\vec{F}_{so}(\vec{r}_o - \vec{r}_s, q_s, q_o) = \frac{k_e q_s q_o}{r_{so}^2}\hat{r}_{so}$$

$$\hat{r}_{so} = \frac{\vec{r}_o - \vec{r}_s}{r_{so}}$$

$$r_{so} = |\vec{r}_o - \vec{r}_s|$$



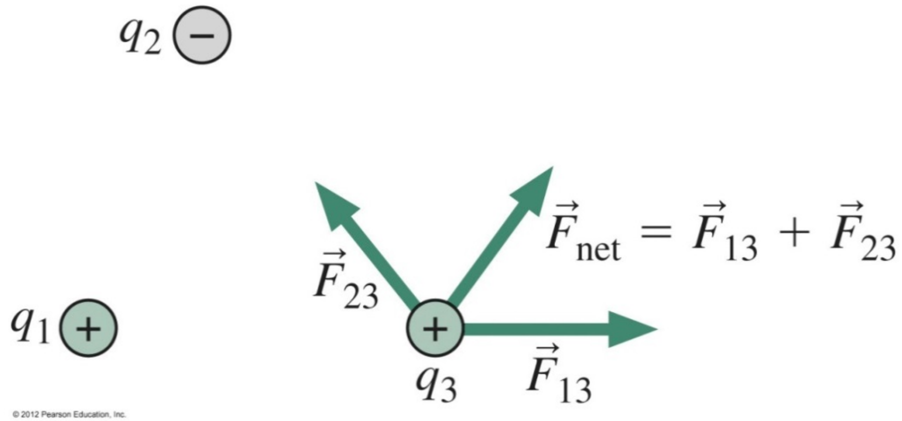
Same thing, it's just useful to imagine
there is always a source and an observer

The Superposition Principle

The **force** acting on a observer particle is the **sum of all the forces** due to all other source charges.

The electric force obeys the **superposition principle.**

- That means the force two charges exert on a third force is just the vector sum of the forces from the two charges, each treated without regard to the other charge.



In source and observer terms, the sources are charges 1 and 2, the observer is charge 3

Mathematics of Superposition Principle

Mathematically the **superposition principle** applied to forces , is expressed by

1) identifying *N* ***active** charges***,** q_i and their **source** location $\vec{r}_i$

2) identifying **an "test" charge,** q_t and its **observation** location $\vec{r}_t$

3) evaluating the force that acts on the test due to all other charges

$$\vec{F}(\vec{r}_t) = \sum_{i=1}^{N} \vec{F}_{it}(\vec{r}_t, \vec{r}_i)$$

The force is obtained as the superposition of the forces associated to each couple

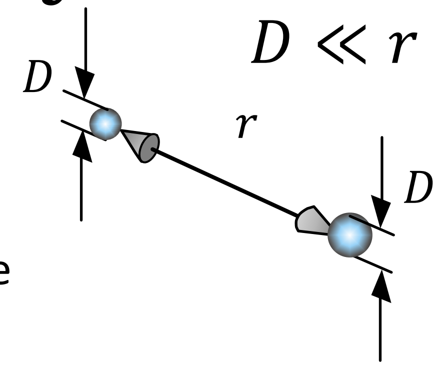
$$\vec{F}_{it}(\vec{r}_t, \vec{r}_i) = q_t \frac{k_e q_i}{|\vec{r}_t - \vec{r}_i|^2} \frac{(\vec{r}_t - \vec{r}_i)}{|\vec{r}_t - \vec{r}_i|}$$

Note that now the reference system can be taken everywhere since also the location of the sources must be explicitly specified

$$\vec{F}(\vec{r}_t) = q_t \sum_{i=1}^{N} \frac{k_e q_i}{|\vec{r}_t - \vec{r}_i|^2} \frac{(\vec{r}_t - \vec{r}_i)}{|\vec{r}_t - \vec{r}_i|}$$

Point charge and charge density

Point charge: Makes sense to think in terms of it when volume is small compared to the distances between charges

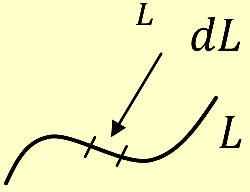


Otherwise Charge densities: distributed on a line/surface/volume large compared to other dimensions

Q = total charge

Linear charge density (C/m)

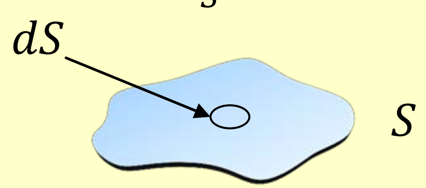
$$Q = \int_L \rho_l dl$$



$$\rho_l = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl}$$

Surface charge density (C/m^2)

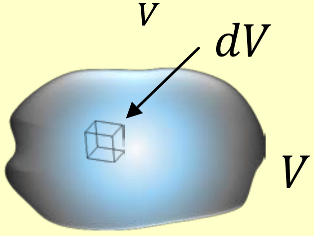
$$Q = \iint_S \rho_S dS$$



$$\rho_S = \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} = \frac{dq}{dS}$$

Volume charge density (C/m^3)

$$Q = \iiint_V \rho_V \, dV$$



$$\rho_V = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} = \frac{dq}{dV}$$

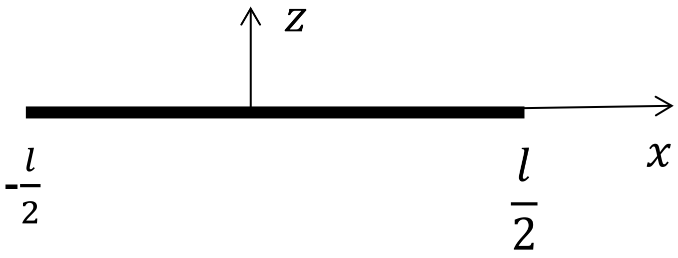
Δq charge contained in Δl

Δq charge contained in ΔS

Δq charge contained in ΔV

Exercises

- Let the line shown in the figure support a constant line charge distribution: $\rho_l(x)$=2C/m.
- What is the total charge if l=2m?



$$Q = \int_{-\frac{l}{2}}^{\frac{l}{2}} \rho_l(x)dx$$

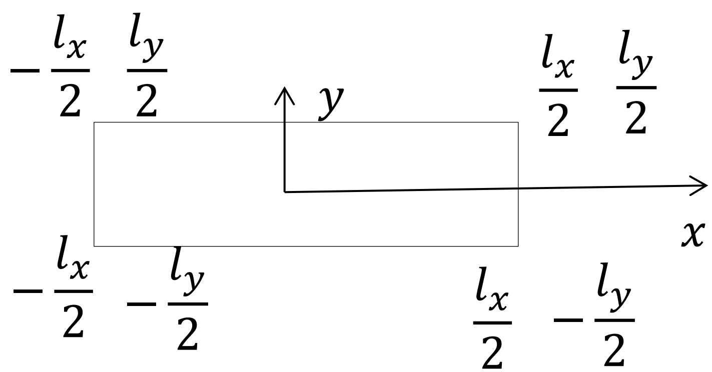
$$Q = \int_{-1}^{1} 2dx = 4C$$

Exercises

- Let the rectangle shown in the figure support a surface charge distribution $\rho_s(x,y) = 5C/m^2$
- What is the total charge if lx=2m, ly=0.5m?

$$Q = \int_{-\frac{l_x}{2}}^{\frac{l_x}{2}} \int_{-\frac{l_y}{2}}^{\frac{l_y}{2}} \rho_s(x,y)dxdy$$

$$Q = \int_{-1}^{1} \int_{-0.25}^{0.25} 5dxdy = 5C$$

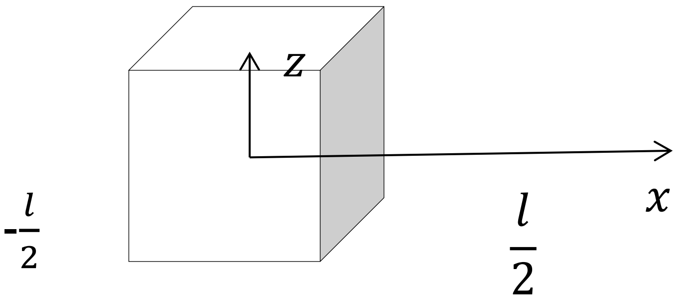


Exercises

- Let the cube shown in the figure support a volumetric charge distribution $\rho_v(x,y,z) = 1C/m^3$
- What is the total charge if lx=2m, ly=2m, lz=2m ?

$$Q = \int_{-\frac{l_x}{2}}^{\frac{l_x}{2}} \int_{-\frac{l_y}{2}}^{\frac{l_y}{2}} \int_{-\frac{l_z}{2}}^{\frac{l_z}{2}} \rho_v(x,y,z)dxdydz$$

$$Q = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} dxdydz = 8C$$



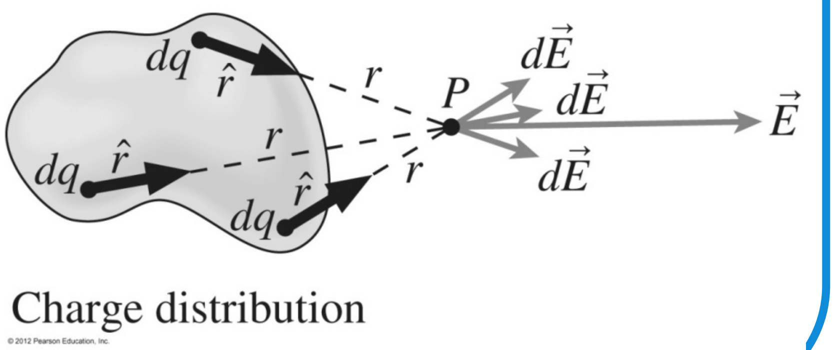
Force due to Charge Distributions

The force on a test point due to a source charge distribution follows by summing the fields of individual charges

$$\vec{F}(\vec{r}_t) = \sum_{i=1}^{N} \vec{F}_{it}(\vec{r}_t, \vec{r}_i)$$

In book you find

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$



When charges are continuously distributed, the force is obtained integrating the fields of individual charge elements *dq*

$$\vec{F}(\vec{r}_t) = \iiint_{Vol} \vec{F}_{it}(\vec{r}_t, \vec{r}\,')d\vec{r}\,'$$

$$\vec{F}(\vec{r}_t) = \iint_{Surf} \vec{F}_{it}(\vec{r}_t, \vec{r}\,')d\vec{r}\,'$$

$$\vec{F}(\vec{r}_t) = \int_{line} \vec{F}_{it}(\vec{r}_t, \vec{r}\,')d\vec{r}\,'$$

$$\vec{F}_{it}(\vec{r}_t, \vec{r}\,') = q_t \frac{k_e \rho(\vec{r}\,')}{|\vec{r}_t - \vec{r}\,'|^2} \frac{(\vec{r}_t - \vec{r}\,')}{|\vec{r}_t - \vec{r}\,'|}$$

Integrals of Vectors

$$\vec{F}(\vec{r}) = F_x(\vec{r})\hat{x} + F_y(\vec{r})\hat{y} + F_z(\vec{r})\hat{z}$$

Also in the case of Force, three components

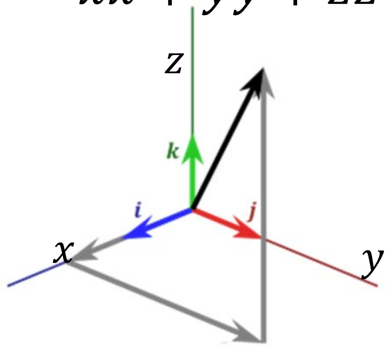
$$F_x(\vec{r}) = q_t \iiint\limits_{Vol} \frac{k_e \rho_v(\vec{r}')}{|\vec{r}-\vec{r}'|^2} \frac{(x-x')}{|\vec{r}-\vec{r}'|} dx'dy'dy'$$

$$F_y(\vec{r}) = q_t \iiint\limits_{Vol} \frac{k_e \rho_v(\vec{r}')}{|\vec{r}-\vec{r}'|^2} \frac{(y-y')}{|\vec{r}-\vec{r}'|} dx'dy'dy'$$

$$F_z(\vec{r}) = q_t \iiint\limits_{Vol} \frac{k_e \rho_v(\vec{r}')}{|\vec{r}-\vec{r}'|^2} \frac{(z-z')}{|\vec{r}-\vec{r}'|} dx'dy'dy'$$

Vectors

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$



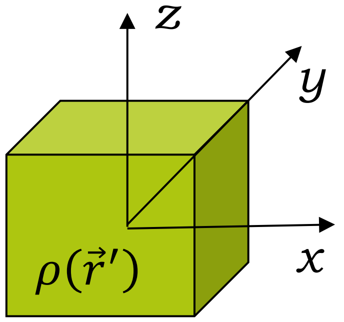
Compact notation

$$\vec{F}(\vec{r}) = q_t \iiint\limits_{Vol} \frac{k_e \rho_v(\vec{r}')}{|\vec{r}-\vec{r}'|^2} \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r}'$$

Force due to Charge Distributions

$\vec{r}_t$

Evaluate the force acting on q_t, due to source charge distribution in Volume, assuming $\vec{r}_t, \vec{r}'$ $such\ that\ r_t \gg r'$



$$\vec{F}(\vec{r}_t) = q_t \iiint_{Vol} \frac{k_e \rho(\vec{r}')}{|\vec{r}_t - \vec{r}'|^2} \frac{(\vec{r}_t - \vec{r}')}{|\vec{r}_t - \vec{r}'|} d\vec{r}'$$

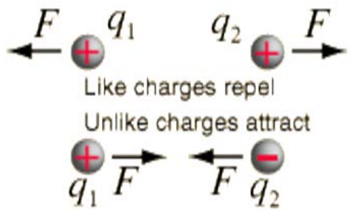
$$\frac{1}{|\vec{r}_t - \vec{r}'|^2} \approx \frac{1}{r_t^2}$$

$$\frac{(\vec{r}_t - \vec{r}')}{|\vec{r}_t - \vec{r}'|} \approx \hat{r}_t$$

$$\vec{F}(\vec{r}_t) = q_t k_e \frac{\hat{r}_t}{r_t^2} \iiint_{Vol} \rho(\vec{r}') d\vec{r}' = q_t k_e \frac{\hat{r}_t}{r_t^2} Q$$

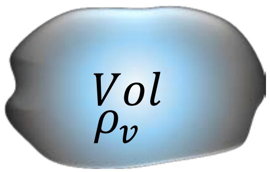
Truly Important

1)



$$\vec{F}_{12}(\vec{r}, q_1, q_2) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

2)



$$\vec{F}(\vec{r}) = q_t \iiint\limits_{Vol} \frac{k_e \rho_v(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^2} \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\vec{r}\,'$$