CAPACITORS

1. OUTLINE

- ✓ Capacitance and capacitors
- ✓ Parallel plate capacitor

2. CAPACITANCE AND CAPACITORS

From previous lectures we know that a conductor in a static electric filed is an equipotential body and that charges deposited on this conductor will distribute themselves on its surface in such a way that the electric field inside vanishes. Suppose that the potential due to a charge Q is V. Obviously, increasing the total charge by some factor a would merely increase the surface charge density ρ_s everywhere by the same factor without affecting the charge distribution because the conductor remains and equipotential body in a static situation. From previous lectures we also know that the electrical potential due to a continuous distribution of charge confined in a given region is obtained by integrating the contribution of an element of charge over the charged region. We have, for a volume charge distribution,

$$V(\vec{r}) = \iiint_{volume} k\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}d\vec{r}' \text{ or } V = \iiint_{V'} k\frac{\rho}{R}dv' \qquad (V) \tag{1}$$

Therefore, for a surface charge distribution the electric potential will be

$$V(\vec{r}) = \iint_{surface} k\frac{\rho_s(\vec{r}')}{|\vec{r}-\vec{r}'|}d\vec{r}' \text{ or } V = \iint_{S'} k\frac{\rho_s}{R}ds' \qquad (V) \tag{2}$$

We may conclude from Eq. (2) that the potential of an isolated conductor is directly proportional to the total charge on it. This may also be seen from the fact that, increasing V by a factor of a, the electric field $\vec{E} = -\nabla V$ increases by a factor of a. But from previous lectures we know

$$E_{normal}(field\ at\ the\ conductor\ surface) = \frac{\rho_s}{\epsilon_0}$$

It follows that ρ_s, and consequently the total charge Q will also increase by a factor of a. The ratio Q/V therefore remains unchanged. We can write

$$Q = CV \tag{3}$$

where the constant of proportionality C is called the **capacitance** of the isolated conducting body. Its SI unit is Coulomb per Volt, or Farad (F).

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Of considerable importance in practice is the capacitor, which consist of two conductors separated by free space or a dielectric medium. The conductors may be of arbitrary shapes as in Fig.1(a), but in this section the simplest realization, the parallel plate capacitor, will be studied (see Fig.1(b)).

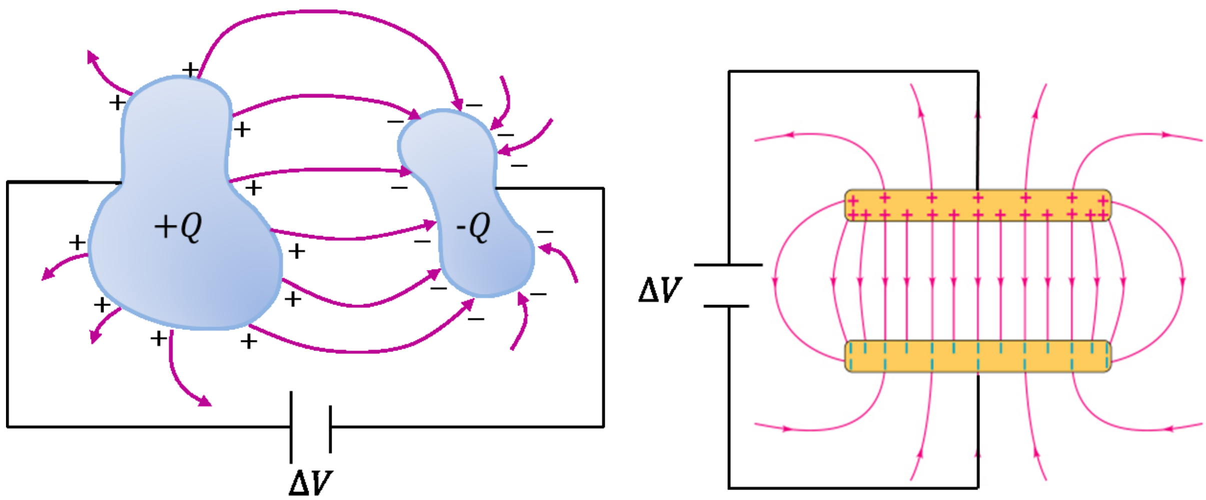

Figure 1. (a) Two-conductor capacitor with an arbitrary shape. (b) Parallel plate capacitor.

When a DC voltage source is connected between the conductors, a charge transfer occurs, resulting in a charge $+Q$ on one conductor and $-Q$ on the other. Electric field lines originating from positive charges and terminating on negative charges are shown in Figure 4. Note that he field lines are perpendicular to the conductor surfaces, which are equipotential surface. If V is the potential difference between the two conductors and Q the charge in each conductor, from Eq.(3) we can write:

$$C = \frac{Q}{\Delta V} \tag{4}$$

The capacitance of a capacitor is a physical property of the two-conductor system. It depends on the geometry of the conductors and on the permittivity of the medium between them; it does *not* depend on either the charge Q or the potential difference ΔV. A capacitor has a capacitance even when no voltage is applied to it and no free charges exist on its conductor.

The energy stored in any system is

$$U_{tot} = \frac{1}{2}\iiint_{V'} \rho V dv = \frac{1}{2}\iiint\limits_{Volume} \rho(\vec{r}')V(\vec{r}')d\vec{r}' \qquad \text{(J)}$$

For a surface charge distribution

$$U_{tot} = \frac{1}{2}\iint_{Surface} \rho_s V ds = \frac{1}{2}\iint_{Surface} \sigma(\vec{r}')V(\vec{r}')d\vec{r}' \qquad \text{(J)} \tag{5}$$

Since there are two conductors in a capacitor

$$U_{tot} = \frac{1}{2}\iint_{Area\ conductor\ 1} \sigma_1(\vec{r}')V(\vec{r}')d\vec{r}' + \frac{1}{2}\iint_{Area\ conductor\ 2} \sigma_2(\vec{r}')V(\vec{r}')d\vec{r}'$$

Assuming that the charge is uniformly distributed on panels we can approximate the surface charge distribution as

$$\sigma_1(\vec{r}') \sim \frac{Q}{A_1} \qquad\qquad \sigma_2(\vec{r}') \sim \frac{Q}{A_2}$$

Let V_1 be the potential on one plate of the capacitor and V_2 the potential on the other plate. The energy of the total system can be written as

$$U_{tot} = \frac{1}{2}\iint_{A_1} \frac{Q}{A_1} V_1 dA - \frac{1}{2}\iint_{A_2} \frac{Q}{A_2} V_2 dA \tag{6}$$

which leads to

$$U_{tot} = \frac{1}{2} Q \Delta V \tag{7}$$

where ΔV is the potential difference between the two conductors.

3. PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two parallel conducting plates of area A separated by a uniform distance d. The space between the plates is filled with a dielectric of a constant relative permittivity ϵ_r (see Fig. 2). We add charges $+Q$ and $-Q$ on the upper and lower conducting plates, respectively. The charges are assumed to be uniformly distributed over the conducting plates with surface densities $+\rho_s$ and $-\rho_s$ where

$$\rho_s = \frac{Q}{A}$$

From previous lectures we can write the field at the conductor surface in a dielectric medium as

$$E_{normal} = \frac{\rho_s}{\epsilon_0 \epsilon_r}$$

Using a Cartesian coordinate system the field inside the capacitor is

$$\vec{E} = -\frac{\rho_s}{\epsilon_0 \epsilon_r}\hat{y} = -\frac{Q}{A\epsilon_0\epsilon_r}\hat{y}$$

which is constant within the dielectric if the fringing of the electric field at the edges of the plates is neglected. From previous lectures we also know that the potential difference can be calculated as

$$\Delta V = -\int_A^B \vec{E}(l)\cdot d\vec{l}$$

Then

$$\Delta V = -\int_{y=0}^{y=d} \vec{E}(l)\cdot d\vec{l} = -\int_0^d \left(-\frac{Q}{A\epsilon_0\epsilon_r}\hat{y}\right)\cdot(\hat{y}\,dy) = \frac{Q}{A\epsilon_0\epsilon_r}d$$

Therefore, for a parallel plate capacitor

$$C = \frac{Q}{\Delta V} = \epsilon_0\epsilon_r\frac{A}{d} \tag{8}$$

which is independent both from Q and ΔV.

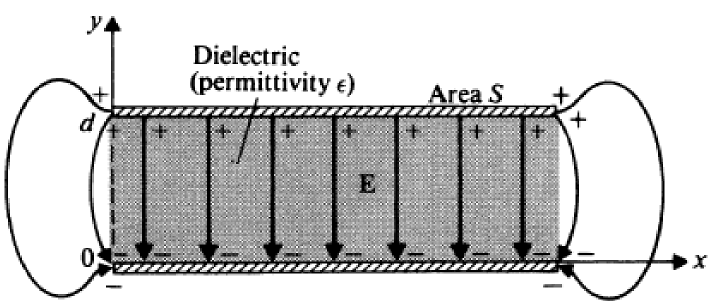

Figure 2. Cross section of a parallel plate capacitor.