Electricity and Magnetism

Overview Magnetism

- 26-5: Introduction, magnetism: field and force
- 30-5: Magnetism: Biot-Savart, Ampere
- 2-6: Electromagnetic induction
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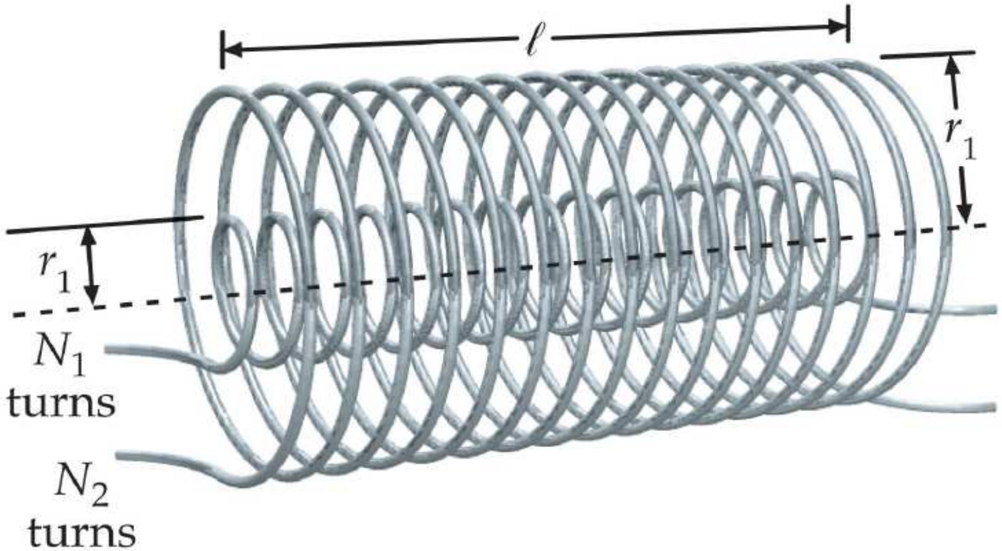
Concentric solenoids



- Two concentric solenoids:
- N1 turns and radius r1
- N2 turns and radius r2
- The current in coil 2 is i2.

- Calculate
- The flux density
- The flux linkage of coil 2
- The voltage induced in coil 2
- The self inductance of coil 2
- The flux linkage of coil 1
- The voltage induced in coil 1
- The mutual inductance between coil 1 and 2

Flux density, flux linkage, voltage and self inductance

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

$$Bl = \mu_0 N_2 i_2 \quad \Rightarrow \quad B = \frac{\mu_0 N_2 i_2}{l}$$

$$\Phi_2 = \int \vec{B} \cdot d\vec{A} = \pi r_2^2 B N_2 = \frac{\mu_0 \pi r_2^2 N_2^2}{l} i_2$$

$$u_2 = R_2 i_2 + \frac{d\Phi_2}{dt} = R_2 i_2 + L_2 \frac{di_2}{dt}$$

$$L_2 = \frac{\Phi_2}{i_2} = \frac{\mu_0 \pi r_2^2 N_2^2}{l}$$

Mutual inductance, voltage equations

$$\Phi_1 = \int \vec{B} \cdot d\vec{A} = \pi r_1^2 B N_1 = \frac{\mu_0 \pi r_1^2 N_2 N_1}{l} i_2$$

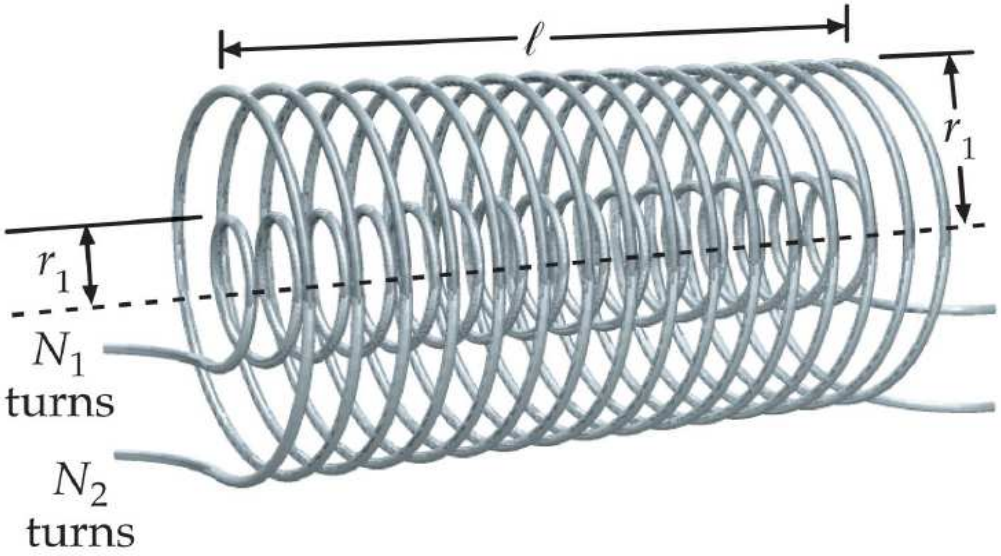
$$u_1 = \frac{d\Phi_1}{dt} = M \frac{di_2}{dt}$$

$$M = \frac{\Phi_1}{i_2} = \frac{\mu_0 \pi r_1^2 N_1 N_2}{l}$$

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Concentric solenoids



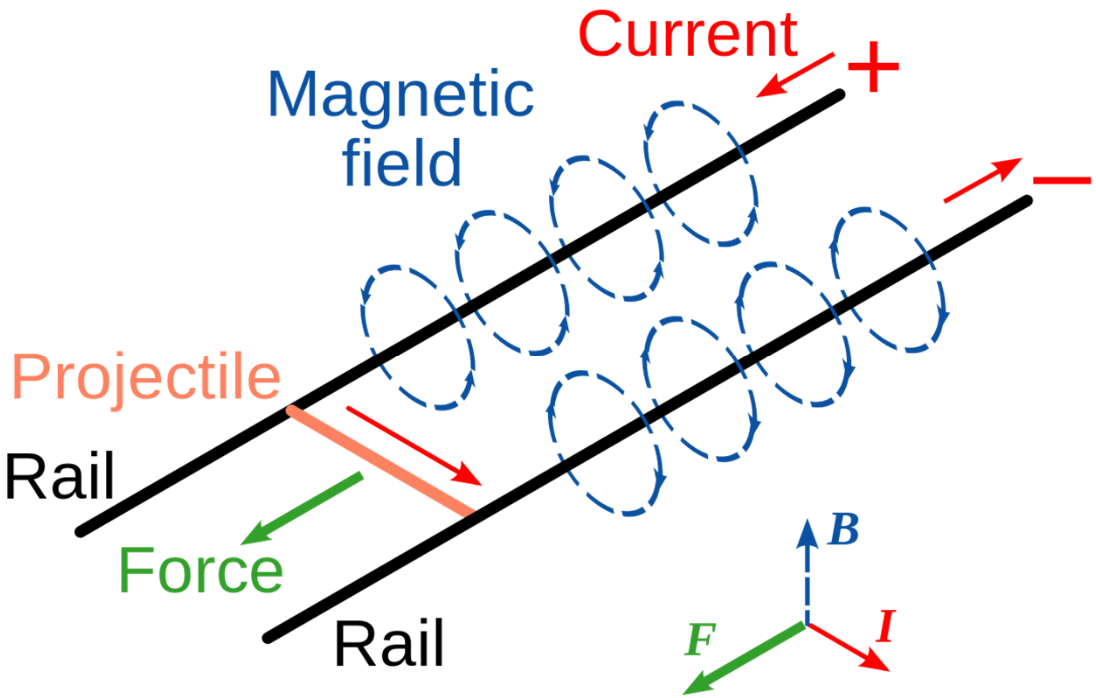
- Two concentric solenoids:
- N1 turns and radius r1
- N2 turns and radius r2
- The current in coil 2 is i2.
- r1=r2; N1=N2
- Coil 1 is short-circuited and has resistance 0
- Calculate the voltage over coil 2
- Calculate the flux density inside the coil

Voltage and flux density

- N1=N2 and r1=r2, therefore, L1=L2=M
- If coil 1 is short circuited and the resistance is zero, it follows from the voltage equations that i1=-i2.
- This can also be concluded from Faradays law: the induced current in coil 1 is such that opposes the change of the flux. Because the resistance is zero, it can do that in a perfect way, so the flux remains zero.
- Therefore, the voltage on coil 2 is zero (assuming this coil also has a zero resistance)
- Therefore, B=0

Railgun

- 2 rails r = 2 mm
- Distance d = 14 mm
- Current 1000 A
- The rails are very long



- Calculate
  - the magnetic flux density between the conducting rails
  - the magnetic force per meter of the rails on each other
  - the magnetic flux density in the middle of the projectile (Biot-Savart)
  - the magnetic force on the projectile
  - the acceleration of a 10 g projectile

Calculate flux density

- Flux density $$2\pi r B = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$
- Force on rails $$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi r} = 14.29\ \mathrm{N/m}$$

Flux density and force

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$$

$$r = \sqrt{x^2 + (d/2)^2}$$

$$B = 2\int_{\infty}^{0} \frac{\mu_0 I}{4\pi} \frac{d/2}{r^3} dx = \frac{\mu_0 I}{2\pi} \int_{\infty}^{0} \frac{d/2}{(x^2 + (d/2)^2)^{3/2}} dx$$

$$= \frac{\mu_0 I}{4\pi} \int_{\infty}^{0} \frac{4x}{d\sqrt{x^2 + (d/2)^2}} dx = \frac{\mu_0 I}{\pi d} = 28.57\,\mathrm{mT}$$

Flux density and force

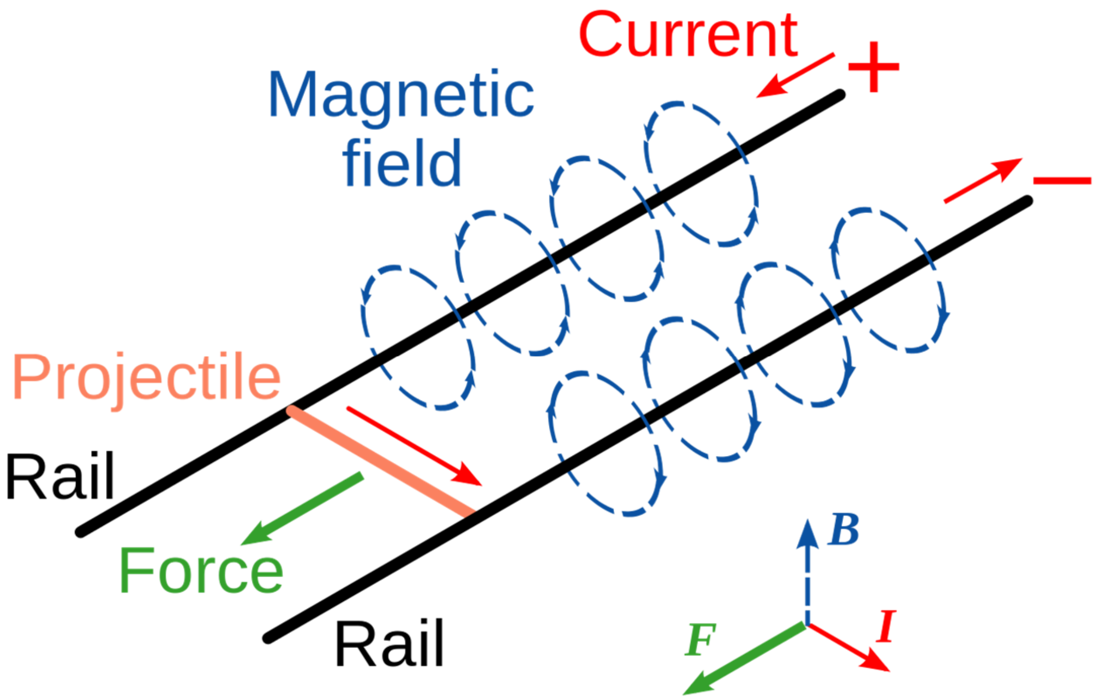
$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2} \qquad r = \sqrt{x^2 + (d/2)^2}$$

$$B = 2\int_{\infty}^{0} \frac{\mu_0 I}{4\pi} \frac{d/2}{r^3} dx = \frac{\mu_0 I}{2\pi} \int_{\infty}^{0} \frac{d/2}{(x^2 + (d/2)^2)^{3/2}} dx$$

$$= \frac{\mu_0 I}{4\pi} \int_{\infty}^{0} \frac{4x}{d\sqrt{x^2 + (d/2)^2}} dx = \frac{\mu_0 I}{\pi d} = 28.57\,\mathrm{mT}$$

$$F = ILB = 0.2857\,\mathrm{N} \qquad a = F/m = 28.57\,\mathrm{m/s^2}$$

Railgun



- 2 rails r = 2 mm
- Distance d = 14 mm
- Current 1000 A
- The rails are very long

- Calculate
  - The inductance per meter
  - Energy stored in the magnetic field per meter
  - Voltage induced in the loop
  - Energy supplied by the source
  - Energy taken by the projectile

Flux and inductance

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int_0^l \int_0^d B dr dl = l \int_0^d B dr$$

$$\frac{\Phi_B}{l} = 2\int_0^d B dr = 2\int_0^{r_{Cu}} B dr + 2\int_{r_{Cu}}^d B dr = 2\int_0^{r_{Cu}} \frac{\mu_0 I r}{2\pi r_{Cu}^2} dr + 2\int_{r_{Uu}}^d \frac{\mu_0 I}{2\pi r} dr$$

$$= \frac{\mu_0 I}{2\pi} + \frac{\mu_0 I}{\pi} \ln\left(\frac{d}{r_{Cu}}\right) = 978\mu\,\mathrm{Wb/\,m}$$

$$\frac{L}{l} = \frac{\Phi_B}{lI} = \frac{\mu_0}{\pi}\left(\frac{1}{2} + \ln\left(\frac{d}{r_{Cu}}\right)\right) = 0.978\mu\,\mathrm{H/\,m}$$

Energy, force from power balance

$$\frac{W_m}{l} = \frac{1}{2}\frac{L}{l}I^2 = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{2} + \ln\left(\frac{d}{r_{Cu}}\right)\right) = 0.489\,\mathrm{J/m}$$

$$E = \frac{d}{dt}\int \vec{B}\cdot d\vec{A} = \frac{dLI}{dt} = I\,\frac{dx}{dt}\frac{dL}{dx}$$

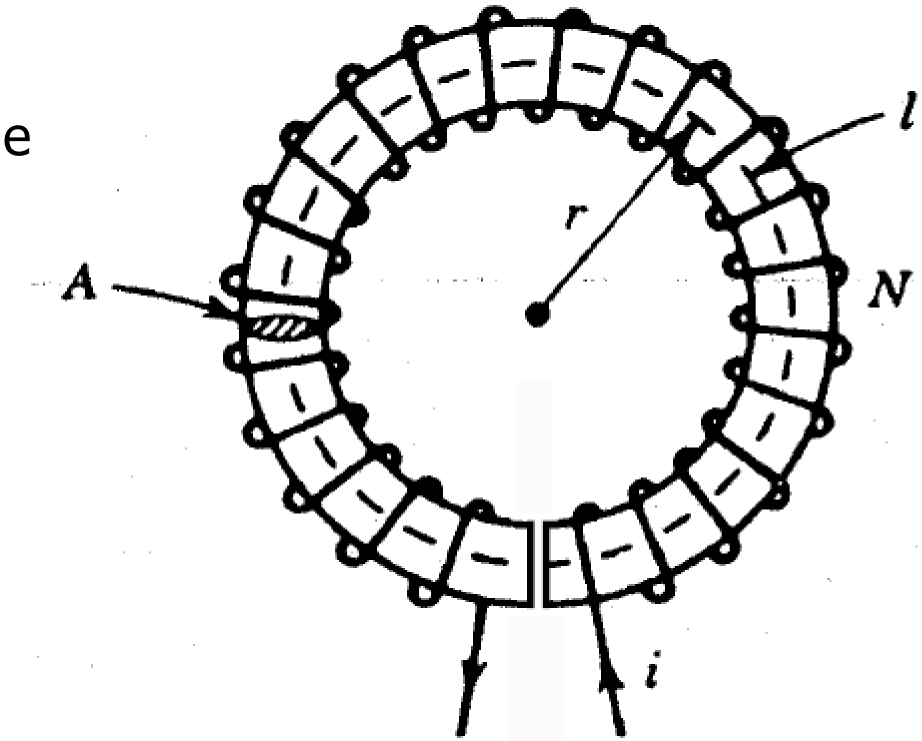
$$= Iv\frac{\mu_0}{\pi}\left(\frac{1}{2} + \ln\left(\frac{d}{r_{Cu}}\right)\right) = v978\mu\,\mathrm{Vs/\,m}$$

$$P_{el} = IE = \frac{\mu_0 I^2 v}{\pi}\left(\frac{1}{2} + \ln\left(\frac{d}{r_{Cu}}\right)\right) = v978\,\mathrm{mWs/\,m}$$

$$F = \frac{P_{mech}}{v} = \frac{1}{v}\left(P_{el} - \frac{dW_m}{dt}\right) = 0.489\,\mathrm{N}$$

Toroid with air gap

- Calculate the magnetic flux density in the core and in the gap
- Calculate the flux linked by the coil
- Calculate the inductance



Magnetic field around a conductor

- A conductor carries a current I. This conductor lies in a flat surface that forms the separation between a half space with air and a half space with a linear magnetic material with permeability μ_r. We assume the magnetic field lines to form circles around the conductor.
- Sketch the situation
- Calculate the magnetic flux density
- Does this satisfy the boundary conditions for the magnetic field?