Electricity and Magnetism

Overview Magnetism

- 28-5: Introduction, magnetism: field and force
- 1-6: Magnetism: Biot-Savart, Ampere
- 4-6: Electromagnetic induction
- 8-6: Electromagnetic induction
- 11-6: Maxwell’s equations and electromagnetic waves
- 15-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 18-6: available for answering questions, exercises

Equations of electromagnetism

- The four complete laws of electromagnetism are collectively called **Maxwell's equations.** They describe all electromagnetic fields in the universe, outside the realm of quantum physics.
- Gauss for the electric field E
  - Charge produces electric field
  - Field lines begin and end on charges or close

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- Gauss for the magnetic field B
  - No magnetic charges
  - Magnetic field lines close

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Faraday
  - Changing magnetic flux produces electric field

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- Ampere
  - Electric currents and changing electric flux produce magnetic field

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Magnetostatic field: magnetic materials

- Learning objectives
- Introducing the magnetic field strength, the magnetic susceptibility, and the magnetization
- Magnetic materials
  - Diamagnetic/paramagnetic materials
  - Soft magnetic materials
  - Hard magnetic materials (permanent magnets)
- Application to toroid and solenoid
- Rewriting Maxwell's equations including dielectric and magnetic material properties
- Local form of Maxwell's equations
- Boundary conditions

Learning objectives

- Know
  - The magnetic field strength H,
  - The magnetic susceptibility χ_m,
  - The relative magnetic permeability μ_{rm}
  - The magnetization M
- Know and separate between different magnetic materials
- Know the local form of Maxwell's equations
- Know the boundary conditions

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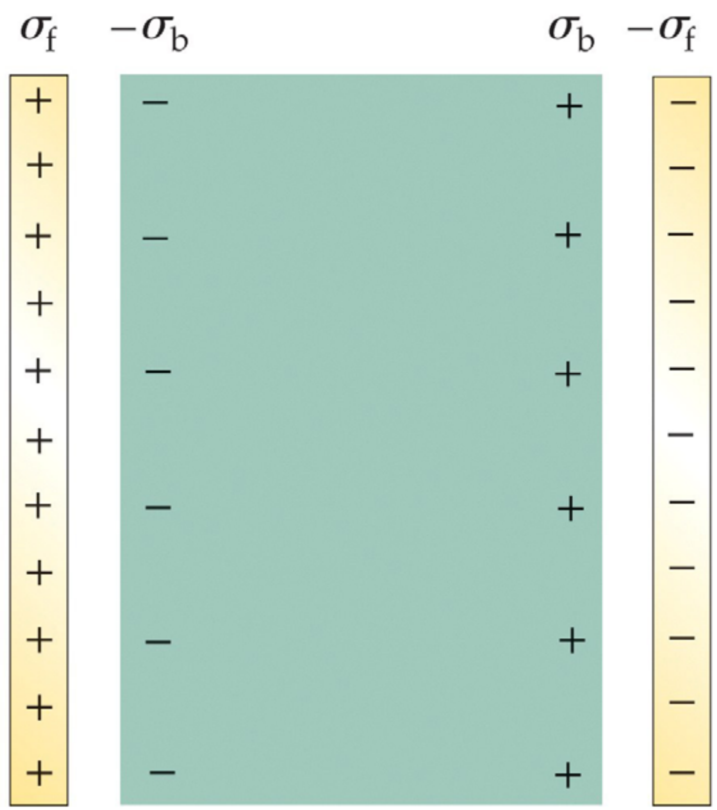
Dielectric material properties

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} = \frac{q_f + q_b}{\varepsilon_0}$$

$$\oint \vec{D} \cdot d\vec{A} = q_f$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 \kappa \vec{E}$$

$$\vec{P} = \varepsilon_0 (\varepsilon_r - 1)\vec{E} = \varepsilon_0 (\kappa - 1)\vec{E} = \varepsilon_0 \chi_e \vec{E}$$



- D is electric flux density
- P is the polarisation
- χ_e is the electric susceptibility
- ε_0 is the permittivity in vacuum
- $\varepsilon_r = \kappa$ is the relative permittivity
- Electric flux density increases with polarisation
- Electric field strength E reduces due to polarisation
- Assuming linearity!!!

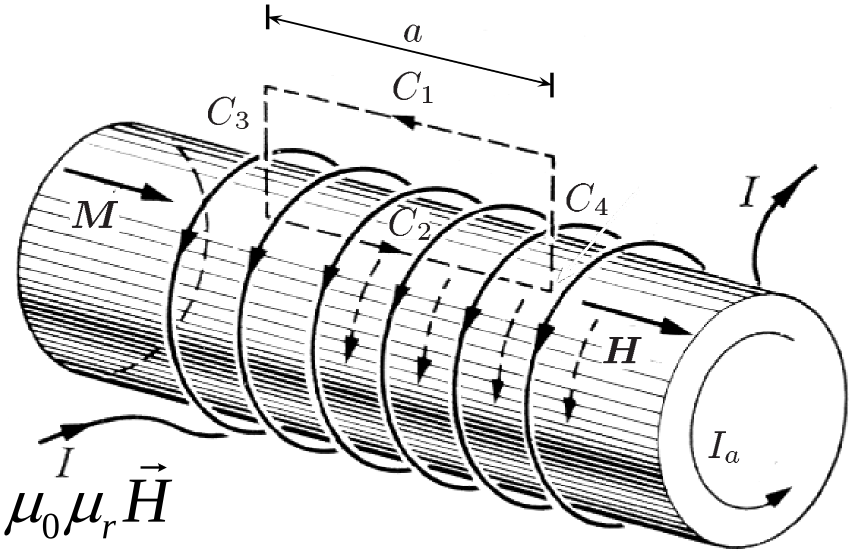
Magnetic material properties (static)

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 I = \mu_0 (I_f + I_a)$$

$$\oint \vec{H}\cdot d\vec{r} = I_f$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$\vec{B} = \mu_0(\vec{H}+\vec{M}) = \mu_0(1+\chi_m)\vec{H} = \mu_0\mu_r\vec{H}$$



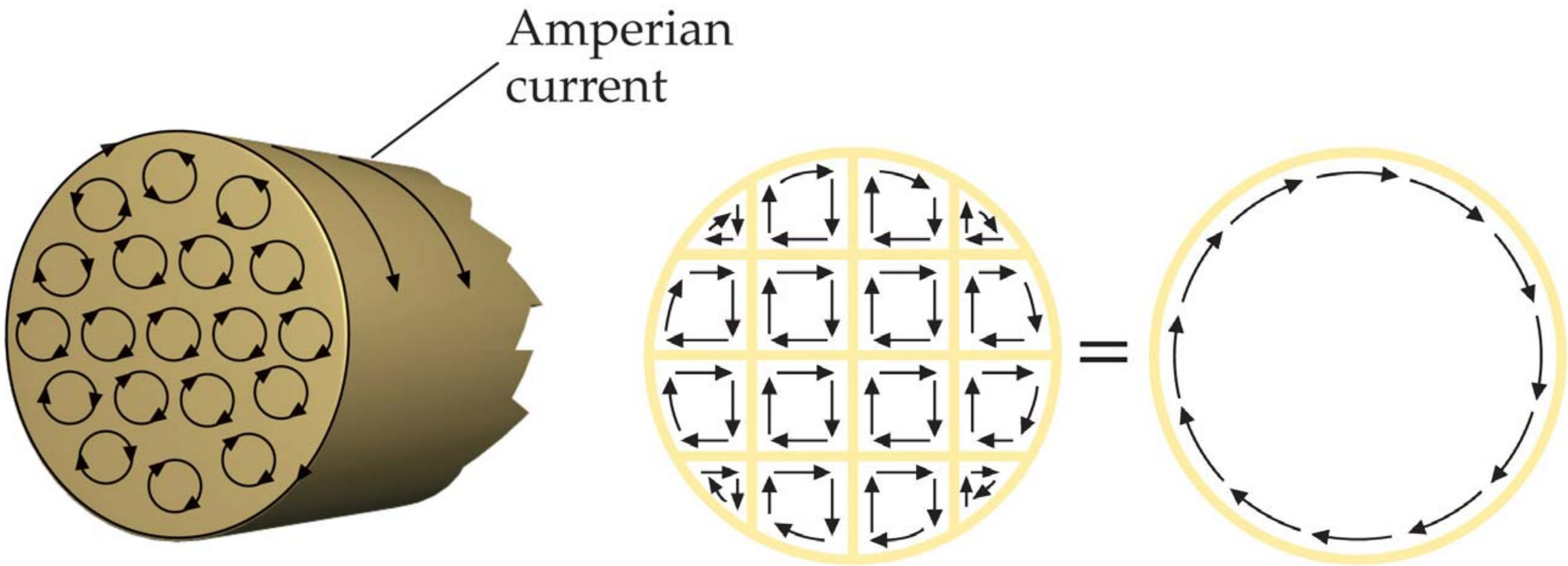
- H is the magnetic field strength
- M is magnetisation
- χ_m is the magnetic susceptibility
- μ_0 is the magnetic permeability in vacuum
- μ_r is the relative permeability
- Magnetic flux density B increases due to magnetisation
- Magnetic field strength H decreases due to magnetisation
- Assuming linearity!!!

Question

- What is the unit of the magnetization M?

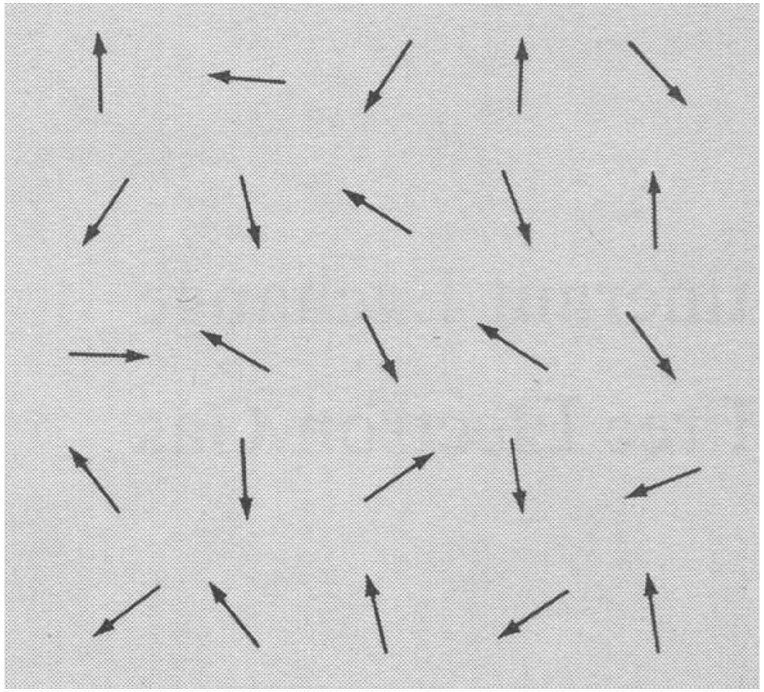
A. T
B. A/m
C. T/m^2
D. Am^2

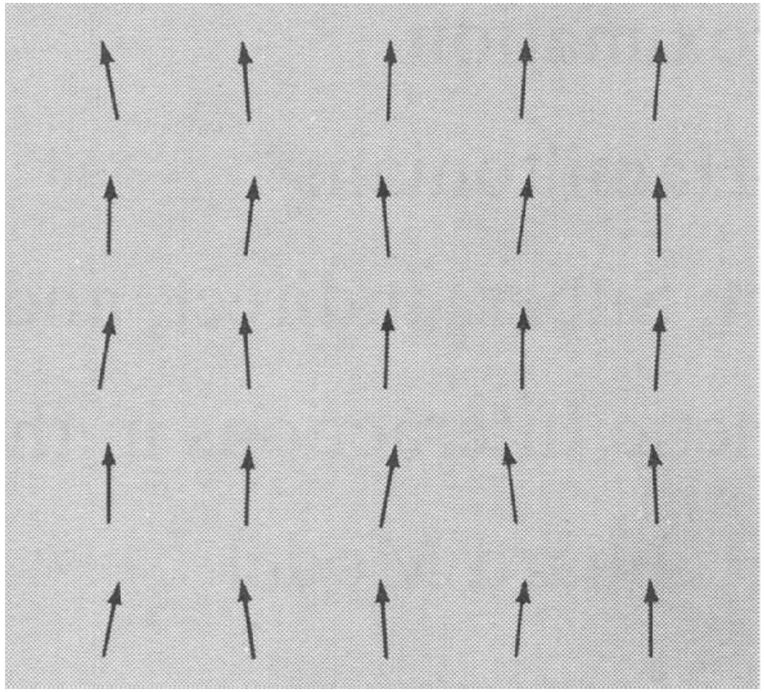
Amperian current



- Electrons circling around atoms
- Electron's intrinsic magnetic dipole moment

Magnetism in materials





Magnetostatic field: magnetic materials

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Magnetic susceptibility diamagnetic and paramagnetic materials

- Magnetisation often negligible

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

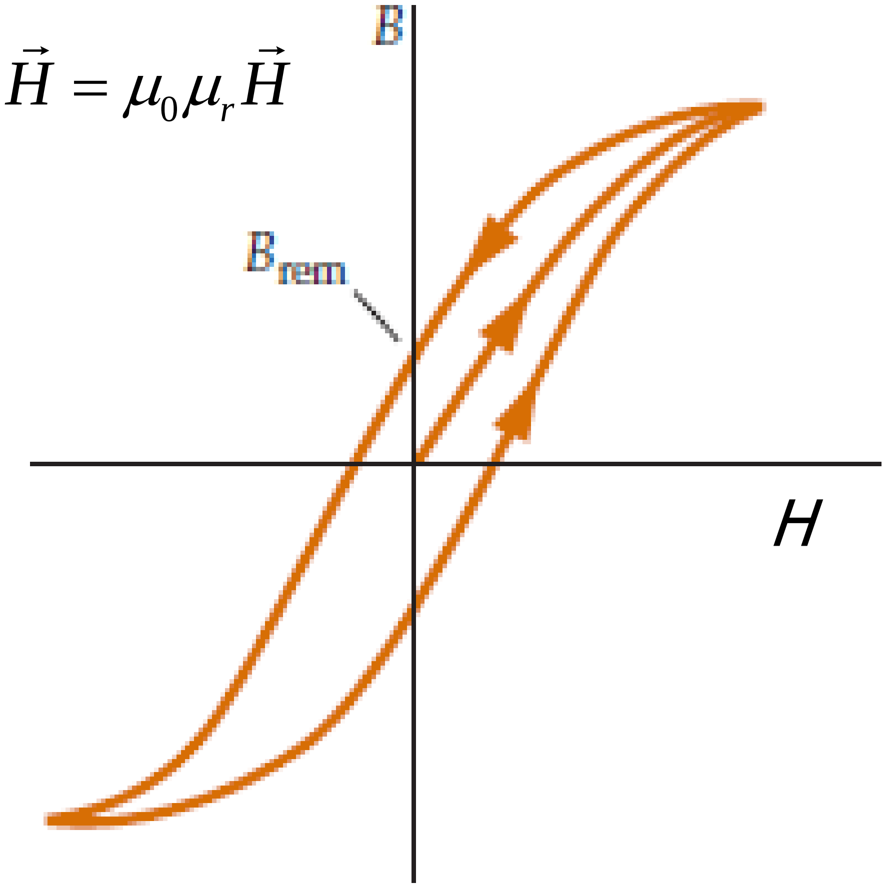
$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu_0\mu_r\vec{H}$$

Material	χ_m
Aluminium	$2.3 \cdot 10^{-5}$
Copper	$-0.98 \cdot 10^{-5}$
Gold	$-3.6 \cdot 10^{-5}$
Silver	$-2.6 \cdot 10^{-5}$
Titanium	$7.06 \cdot 10^{-5}$
Hydrogen (1atm)	$-9.9 \cdot 10^{-9}$
Carbon dioxide (1 atm)	$-2.3 \cdot 10^{-9}$
Nitrogen (1 atm)	$-5 \cdot 10^{-9}$
Oxygen (1 atm)	$2090 \cdot 10^{-9}$

Magnetic susceptibility soft ferromagnetic materials

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu_0\mu_r\vec{H}$$

- Hysteresis characteristic:
  - Saturation
  - Losses



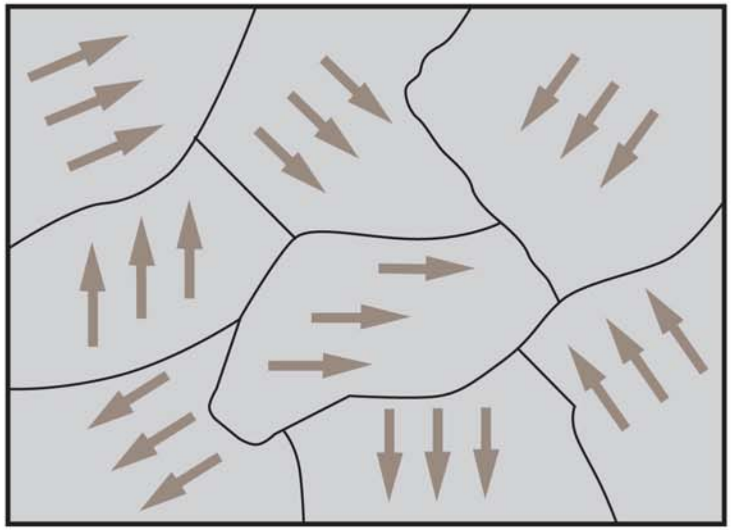
Magnetic susceptibility soft ferromagnetic materials

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu_0\mu_r\vec{H}$$

Material	$\mu_0 M_s$ (T)	χ_m ($\approx\mu_r$)
Iron (annealed)	2.16	5500
Iron-silicon (96% - 4%)	1.95	7000
Permalloy (FeNi)	1.6	25000
Mu-metal (NiFeCuCr)	0.65	100000

- Susceptibility can be very large
- Applied in transformers, motors, generators, coils

Magnetic domains in soft ferromagnetic materials



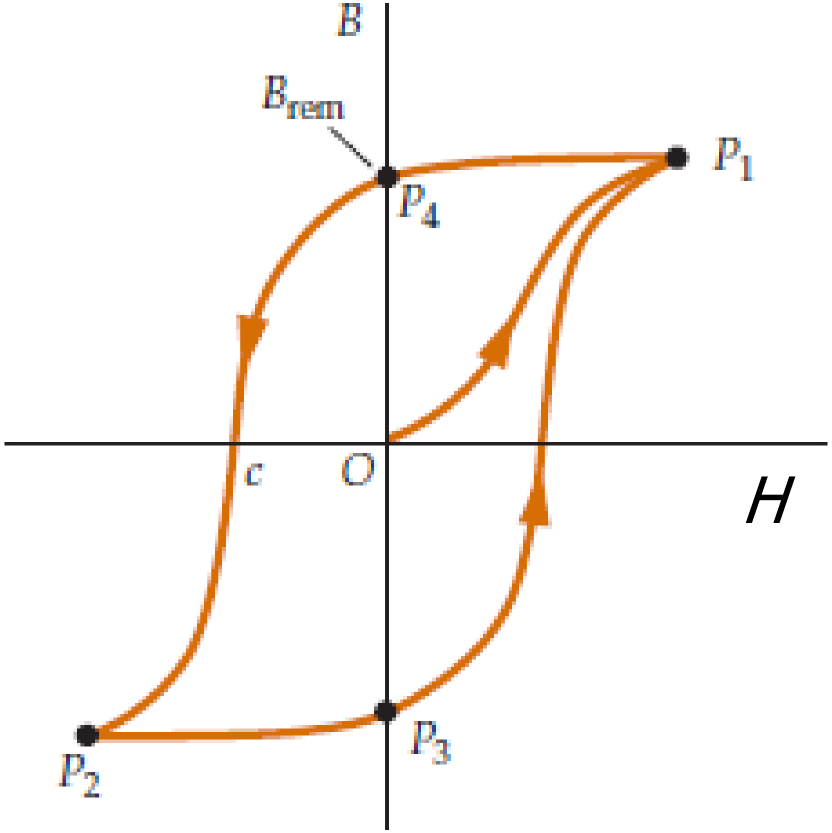
(a) schematic example for domains netto resulting in M=0

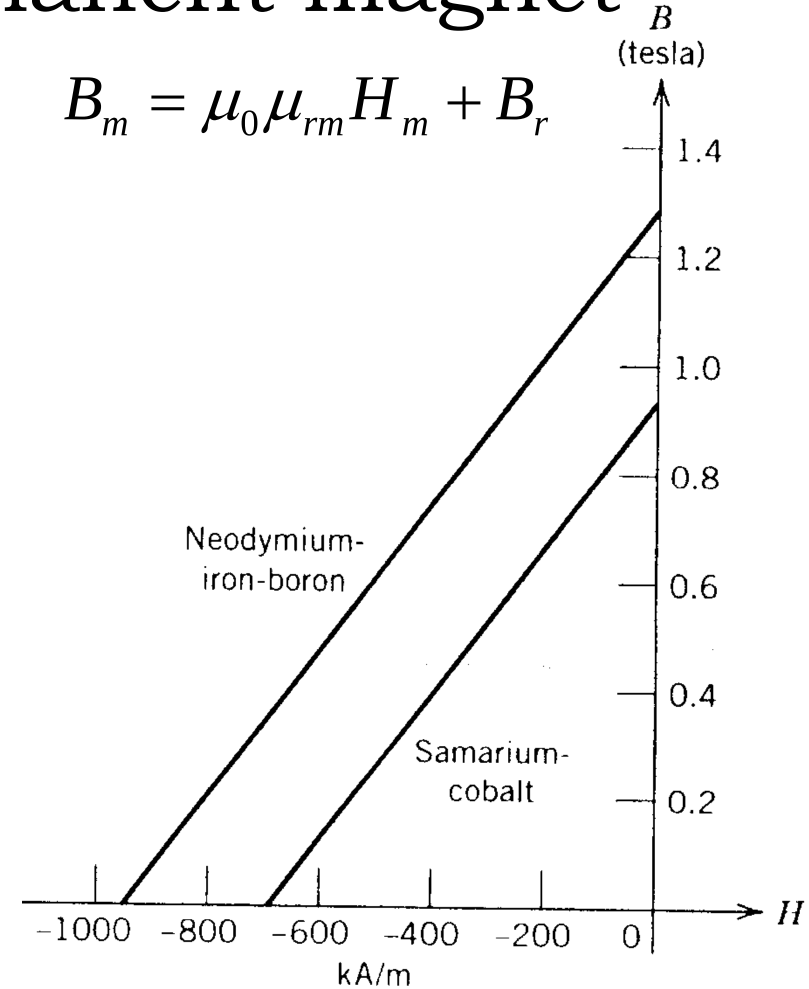
(b) SEM picture of 4-colors of domain orientations in $Fe_{0,97}Si_{0,03}$

Magnetisation hard ferromagnetic materials: permanent magnet

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$B_m = \mu_0 \mu_{rm} H_m + B_r$$





Magnetisation hard ferromagnetic materials: permanent magnet

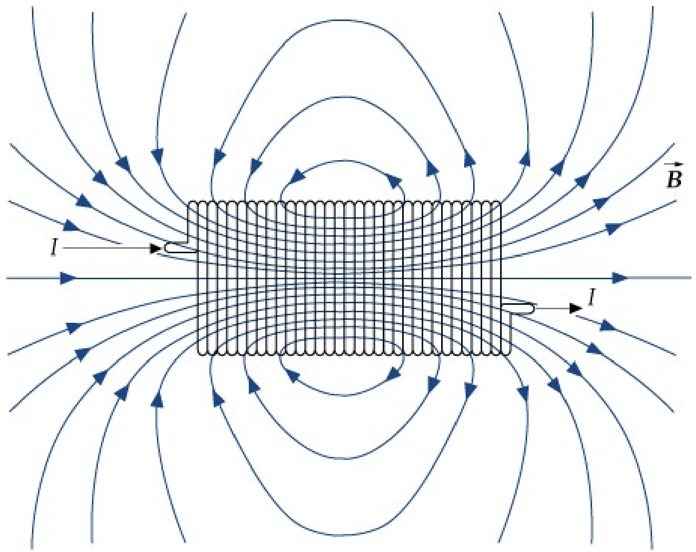
$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

	B_r (T)	H_c (A/m)
Superpermalloy	0,8	Ca. 0,3
Chrome steel	1,0	4000
Ferrite	0,4	250000
Alnico 5	1,2	44000
Platinum cobalt	0,6	290000
SmCo	1,0	750000
NdFeB	1,4	1000000

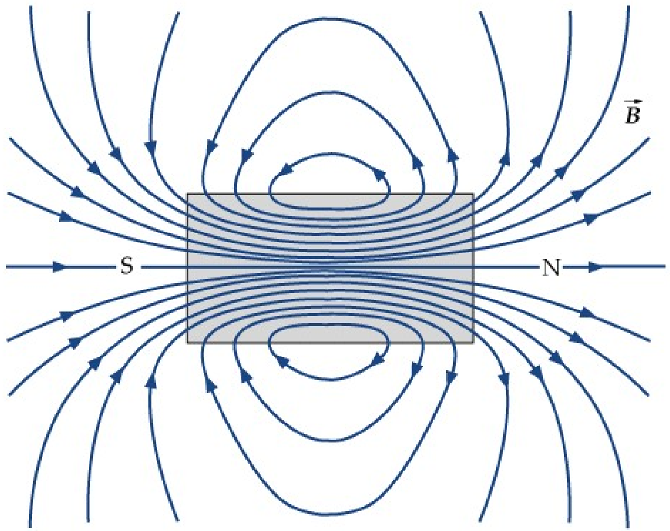
Magnetic field of a solenoid and a magnet

- If a magnet has a constant magnetisation, the external field is the same as for a solenoid.

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$



(a)



(b)

Question

- A strong permanent magnet has

A. A high saturation flux density and a narrow BH curve
B. A high remanent flux density and a wide BH curve
C. A low remanent flux density and a high coercive force

Question

- The magnetic circuit of a transformer has

A. A high saturation flux density and a narrow BH curve
B. A high remanent flux density and a wide BH curve
C. A low remanent flux density and a high coercive force

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Toroid

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 (I_f + I_a)$$

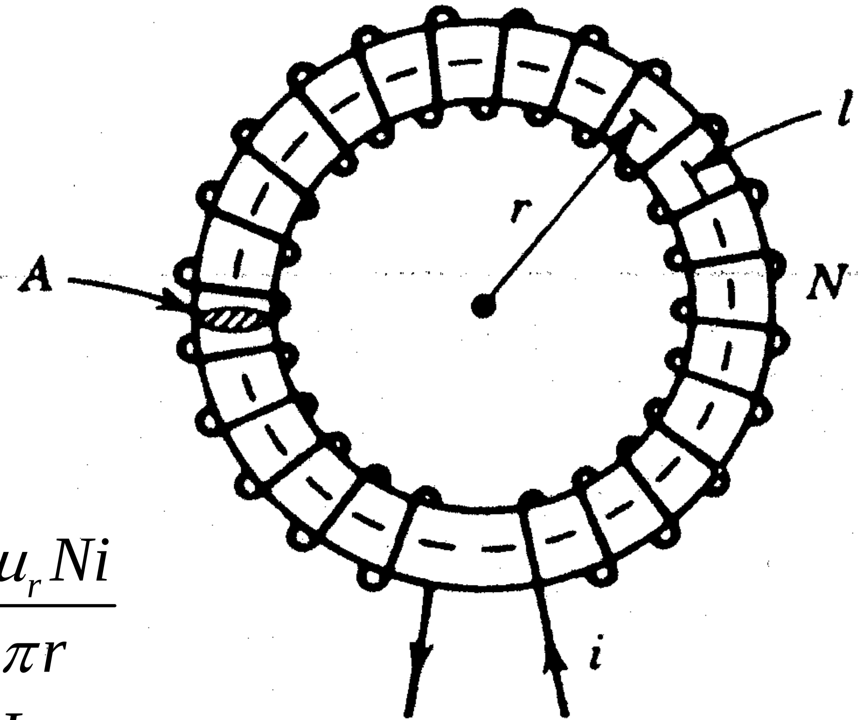
$$\oint \vec{H}\cdot d\vec{r} = I_{encircled}$$

$$2\pi r H = Ni$$

$$B = \mu_0 (H + M) = \mu_0 \mu_r H = \frac{\mu_0 \mu_r Ni}{2\pi r}$$

$$\Phi = \int \vec{B}\cdot d\vec{A} \approx NAB = \frac{\mu_0 \mu_r A N^2 I}{2\pi r}$$

$$L = \frac{\Phi}{I} = \frac{\mu_0 \mu_r A N^2}{2\pi r}$$



Assumptions:
- Symmetry
- B constant over A

Solenoid

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 (I_f + I_a)$$

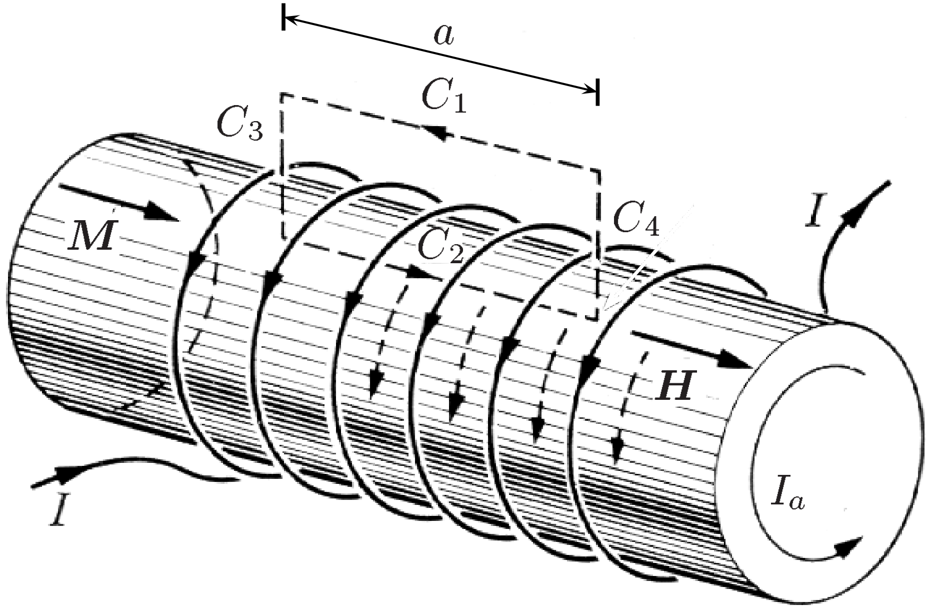
$$\oint \vec{H}\cdot d\vec{r} = I_{encircled}$$

$$lH = Ni$$

$$B = \mu_0 (H + M) = \mu_0 \mu_r H = \frac{\mu_0 \mu_r Ni}{l}$$

$$\Phi = \int \vec{B}\cdot d\vec{A} \approx NAB = \frac{\mu_0 \mu_r AN^2 I}{l}$$

$$L = \frac{\Phi}{I} = \frac{\mu_0 \mu_r AN^2}{l}$$



Assumption:
- H outside negligible
- Problematic. Why?

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Equations of electromagnetism

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  - Charge produces electric field
  - Field lines begin and end on charges

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- Gauss for the magnetic field B
  - No magnetic charges
  - Magnetic field lines close

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Faraday
  - Changing magnetic flux produces electric field

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- Ampere
  - Electric currents and changing electric flux produce magnetic field

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Equations of electromagnetism rewritten

- Gauss for the electric field E
  - Charge produces electric field
  - Field lines begin and end on charges

$$\oint \vec{D}\cdot d\vec{A} = \int \rho_f \, dV$$

- Gauss for the magnetic field B
  - No magnetic charges
  - Magnetic field lines close

$$\oint \vec{B}\cdot d\vec{A} = 0$$

- Faraday
  - Changing magnetic flux produces electric field

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

- Ampere
  - Electric currents and changing electric flux produce magnetic field

$$\oint \vec{H}\cdot d\vec{r} = \int \vec{J}\cdot d\vec{A} + \frac{d}{dt}\int \vec{D}\cdot d\vec{A}$$

- Constitutive relations

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Maxwell's equations in local form

- From integral form to local form using Gauss and Stokes

$$\oint \vec{D}\cdot d\vec{A} = \int \vec{\nabla}\cdot\vec{D}dV = \int \rho_f dV \qquad \vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\oint \vec{B}\cdot d\vec{A} = \int \vec{\nabla}\cdot\vec{B}dV = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\oint \vec{E}\cdot d\vec{r} = \int (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\oint \vec{H}\cdot d\vec{r} = \int (\vec{\nabla}\times\vec{H})\cdot d\vec{A} = \int \vec{J}\cdot d\vec{A} + \frac{d}{dt}\int \vec{D}\cdot d\vec{A} \qquad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

- The constitutive relations

$$\vec{B} = \mu_0\vec{H} + \mu_0\vec{M} \qquad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

Maxwell's equations in local form

- From integral form to local form using Gauss and Stokes

$$\oiint \vec{D}\cdot\hat{n}\,\mathrm{d}A = \iiint \vec{\nabla}\cdot\vec{D}\,\mathrm{d}V = \iiint \rho_f\,\mathrm{d}V \qquad \vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\oiint \vec{B}\cdot\hat{n}\,\mathrm{d}A = \iiint \vec{\nabla}\cdot\vec{B}\,\mathrm{d}V = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\oint \vec{E}\cdot\hat{\tau}\,\mathrm{d}s = \iint (\vec{\nabla}\times\vec{E})\cdot\hat{n}\,\mathrm{d}A = -\frac{\mathrm{d}}{\mathrm{d}t}\iint \vec{B}\cdot\hat{n}\,\mathrm{d}A \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

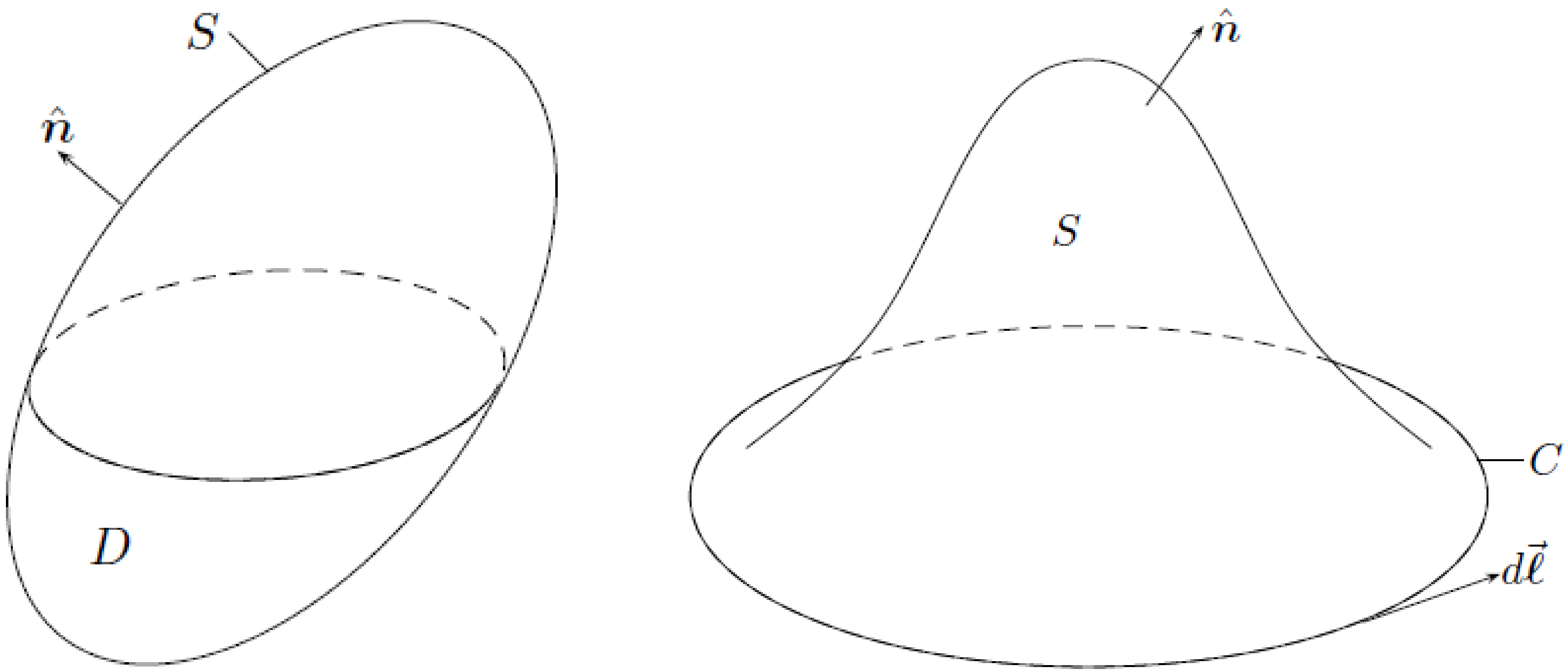
$$\oint \vec{H}\cdot\hat{\tau}\,\mathrm{d}s = \iint (\vec{\nabla}\times\vec{H})\cdot\hat{n}\,\mathrm{d}A = \iint \vec{J}\cdot\hat{n}\,\mathrm{d}A + \frac{d}{dt}\iint \vec{D}\cdot\hat{n}\,\mathrm{d}A$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

- The constitutive relations

$$\vec{B} = \mu_0\vec{H} + \mu_0\vec{M} \qquad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$$

Closed and open surfaces



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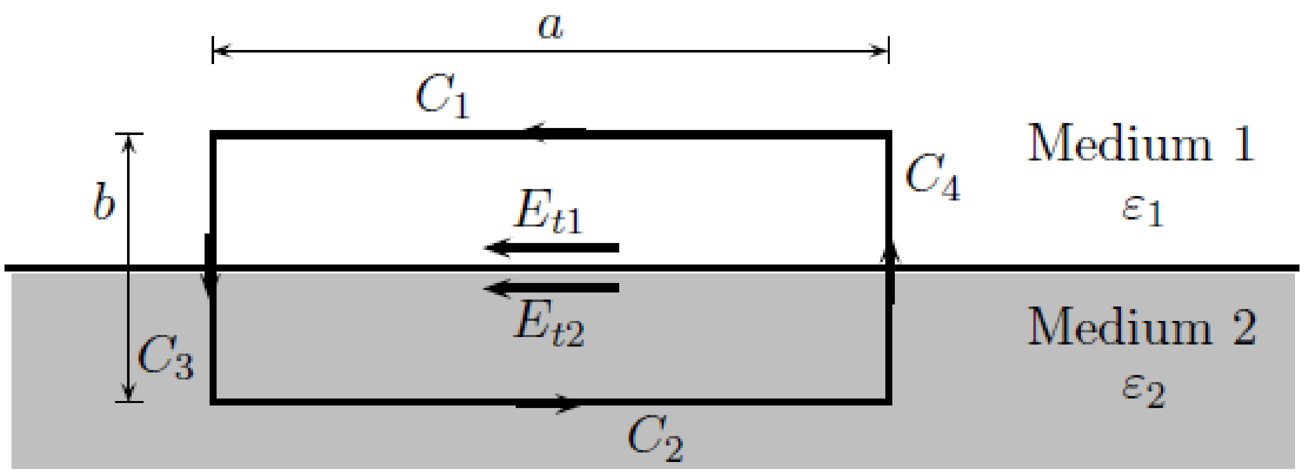
Boundary conditions for the electric field: tangential

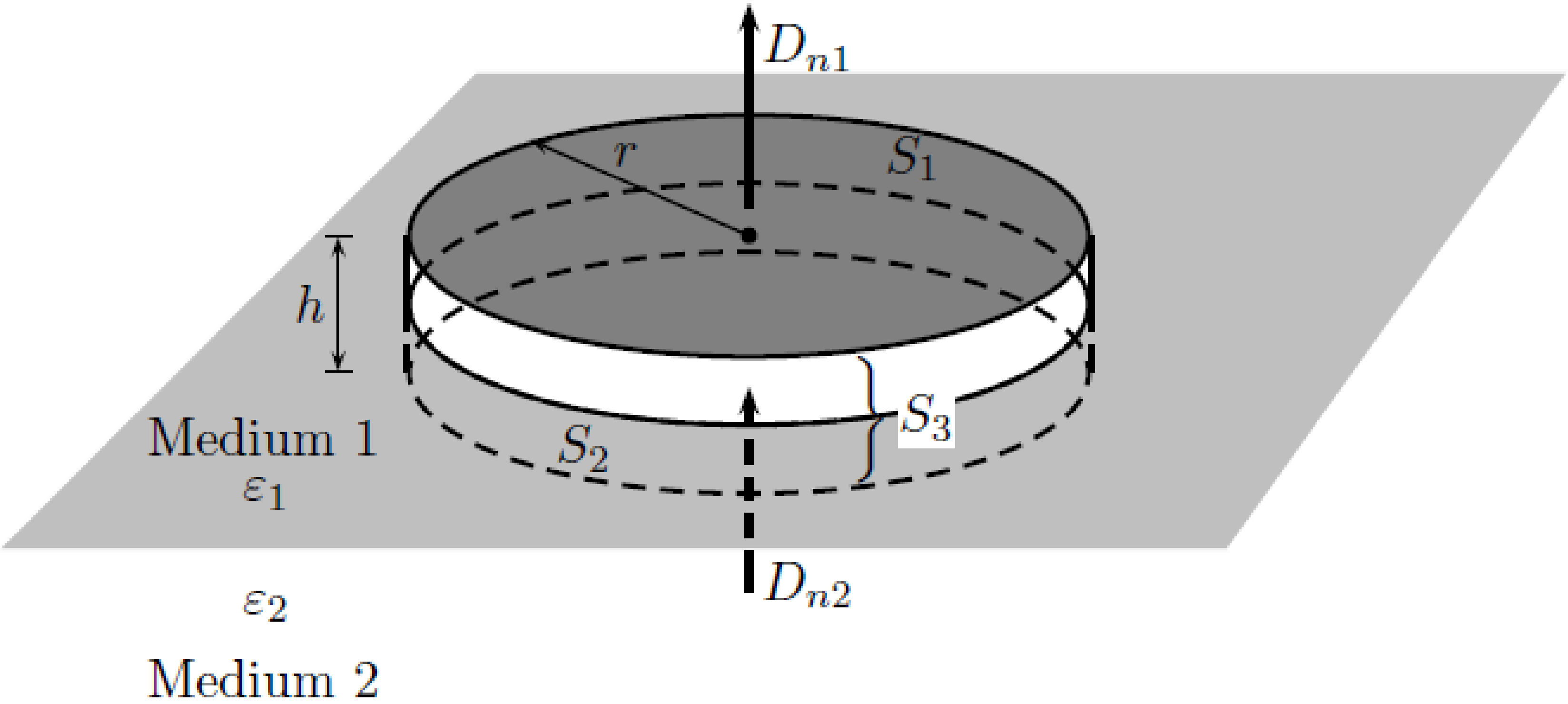


$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

- If *b* goes to zero, Faraday says

$$\oint \vec{E} \cdot d\vec{r} = 0 \quad \Rightarrow \quad E_{t1} = E_{t2}$$

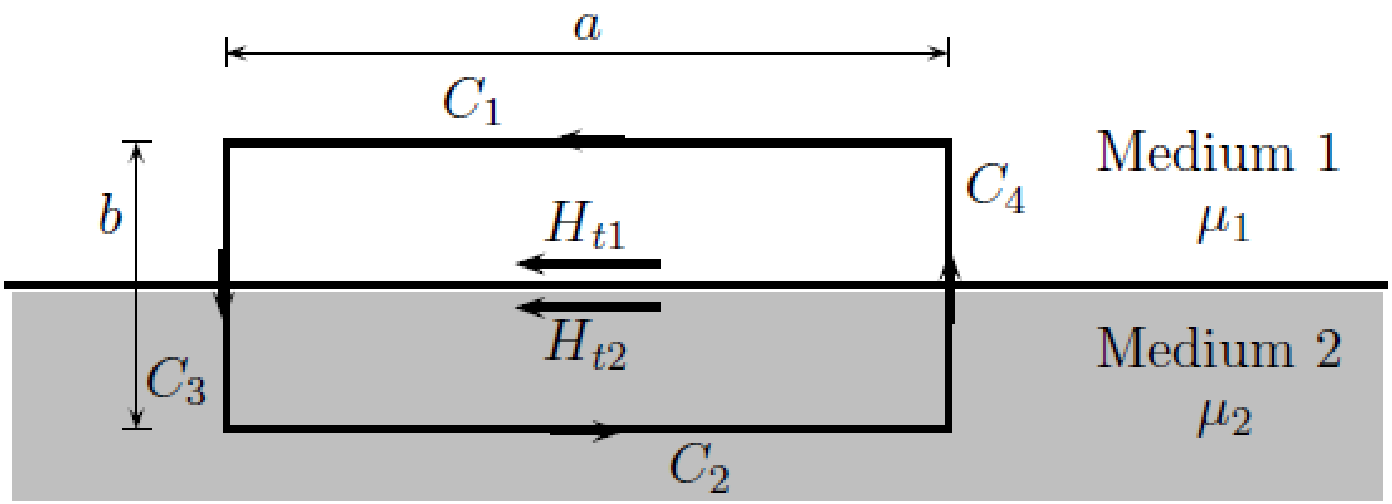
Boundary conditions for the electric field: normal



- If *h* goes to zero, Gauss for the electric displacement says

$$\oint \vec{D} \cdot d\vec{A} = Q_f \quad \Rightarrow \quad D_{n1} - D_{n2} = \sigma_f$$

Boundary conditions for the magnetic field: tangential

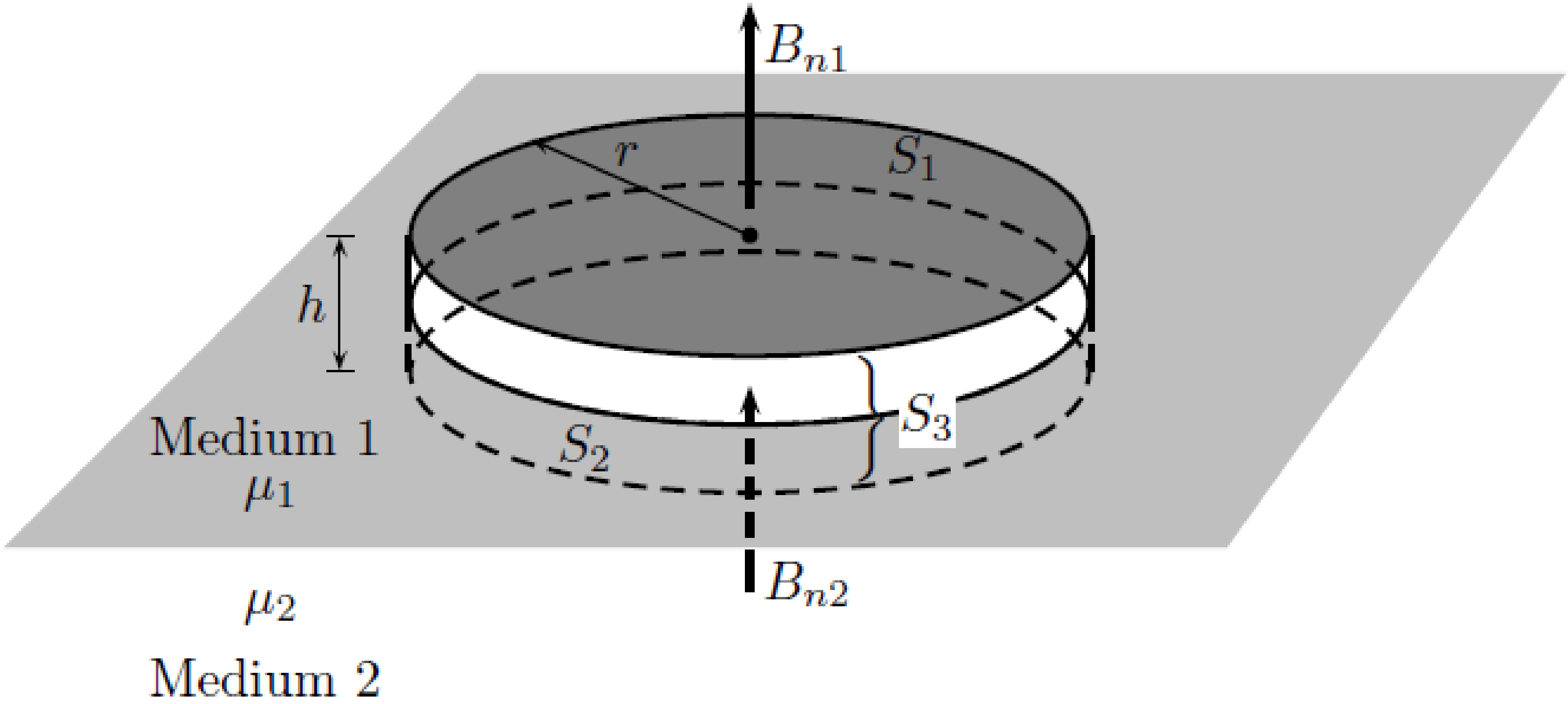


$$\oint \vec{H} \cdot d\vec{r} = \int \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int \vec{D} \cdot d\vec{A}$$

- If *b* goes to zero, Ampere says

$$\oint \vec{H} \cdot d\vec{r} = 0 \quad \Rightarrow \quad H_{t1} = H_{t2}$$

Boundary conditions for the electric field: normal

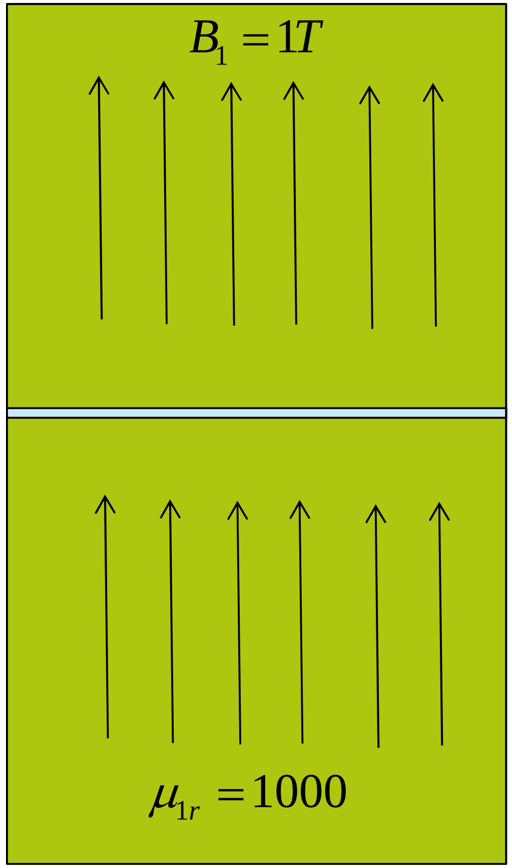


- If h goes to zero, Gauss for the magnetic flux density says

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \Rightarrow \quad B_{n1} = B_{n2}$$

Question

- Calculate the magnetic flux density B and the magnetic field strength H in the gap.



Question

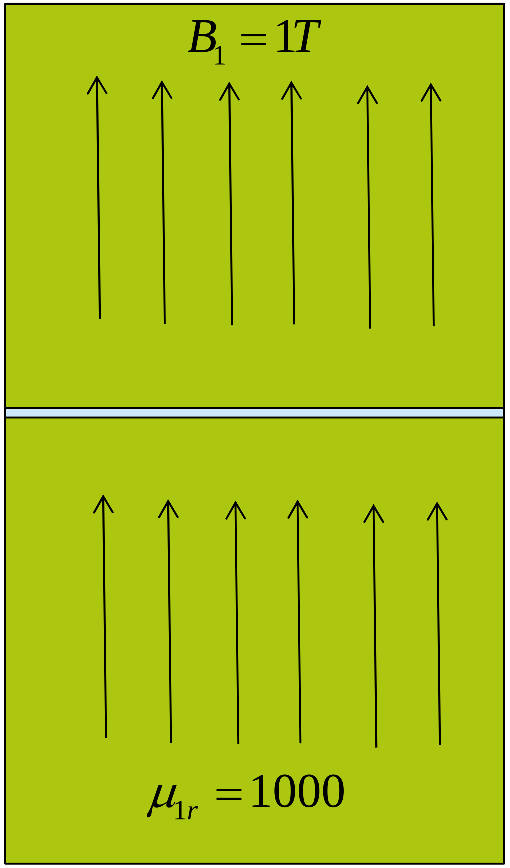
- Calculate the magnetic flux density B and the magnetic field strength H in the gap.

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \Rightarrow \quad B_{n1} = B_{n2} = 1\mathrm{T}$$

$$H_n = \frac{B_n}{\mu_0 \mu_r}$$

$$H_{ng} = \frac{B_{ng}}{\mu_0} = 796\,\mathrm{kA/m}$$

$$H_{nFe} = \frac{B_{nFe}}{\mu_0 \mu_r} = 796\,\mathrm{A/m}$$



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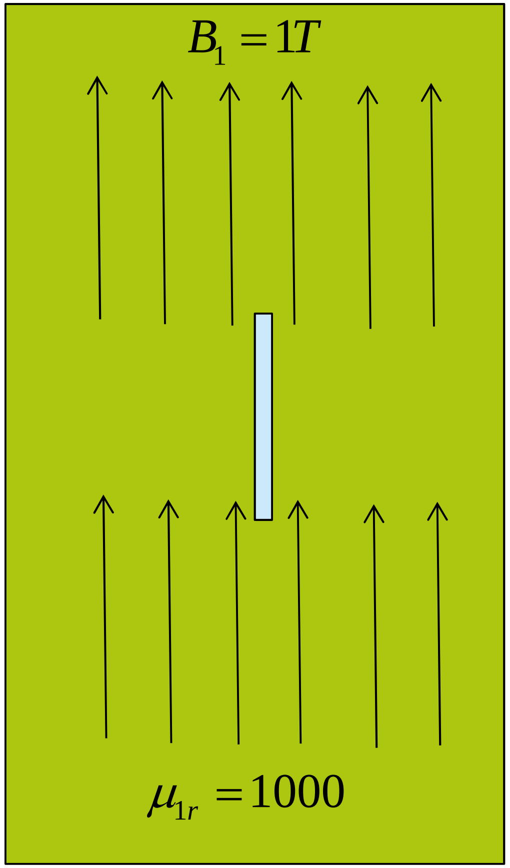


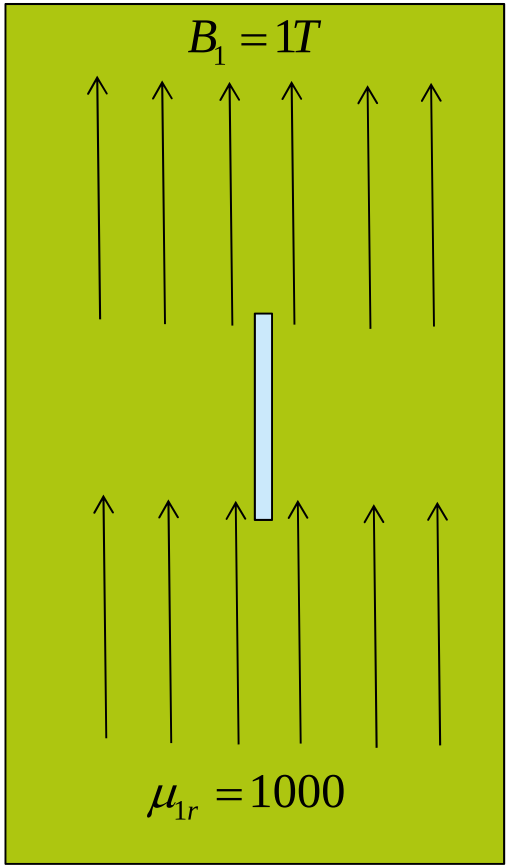
Question

- Calculate the magnetic flux density B and the magnetic field strength H in the gap.

$$\oint \vec{H} \cdot d\vec{r} = 0 \quad \Rightarrow \quad H_{t1} = H_{t2}$$

$$H_{tFe} = \frac{B_{tFe}}{\mu_0 \mu_r} = 796\,\mathrm{A/m}$$

$$B_{tg} = \mu_0 H_{tg} = \mu_0 H_{tFe} = 1\,\mathrm{mT}$$



Maxwell's equations and materials

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$$\oint \vec{D}\cdot d\vec{A} = \int \vec{\nabla}\cdot\vec{D}dV = \int \rho_f dV \qquad \vec{\nabla}\cdot\vec{D} = \rho_f$$

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- The constitutive relations $\quad \vec{B} = \mu_0\vec{H} + \mu_0\vec{M} \qquad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$
- The boundary conditions

$$\oint \vec{H}\cdot d\vec{r} = 0 \quad \Rightarrow \quad H_{t1} = H_{t2} \qquad \oint \vec{E}\cdot d\vec{r} = 0 \quad \Rightarrow \quad E_{t1} = E_{t2}$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \Rightarrow \quad B_{n1} = B_{n2} \qquad \oint \vec{D}\cdot d\vec{A} = Q_f \quad \Rightarrow \quad D_{n1} - D_{n2} = \sigma_f$$

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Summary magnetism force and field 26

- **Magnetism** involves *moving electric charge.*
- Magnetic fields exert forces on moving electric charges:
  - For a moving charge: $\vec{F} = q\vec{v} \times \vec{B}$
  - For a current: $\vec{F} = I\vec{L} \times \vec{B}$
- Magnetic fields arise from moving electric charge, as described by
  - Biot-Savart law: $$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$$
  - Ampère's law: $$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$$
- Magnetic fields encircle the currents and moving charges that are their sources.
  - Magnetic field lines don't begin or end.
  - This is expressed in Gauss's law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$

Summary electromagnetic induction 27

- **Faraday's law** describes electromagnetic induction, most fundamentally the phenomenon whereby **a changing magnetic field produces an electric field:**

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- This induced electric field is nonconservative and its field lines have no beginnings or endings.
- In the presence of a circuit, the induced electric field gives rise to an induced emf and an induced current.
  - Lenz's law states that the direction of the induced current is such that the magnetic field it produces acts to *oppose* the change that gives rise to it.
  - Self-inductance is a circuit property whereby changing current in a circuit results in an induced emf that opposes the change.
- Consideration of current buildup in an inductor shows that all magnetic fields store energy, with energy density $B^2/2\mu_0$.

Equations of electromagnetism 29

- The four complete laws of electromagnetism are collectively called **Maxwell's equations.** They describe all electromagnetic fields in the universe, outside the realm of quantum physics.
- Gauss for the electric field E
  - Charge produces electric field
  - Field lines begin and end on charges or close

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- Gauss for the magnetic field B
  - No magnetic charges
  - Magnetic field lines close

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Faraday
  - Changing magnetic flux produces electric field

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- Ampere
  - Electric currents and changing electric flux produce magnetic field

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's equations and materials

- From integral form to local form using Gauss and Stokes

$$\oint \vec{D}\cdot d\vec{A} = \int \vec{\nabla}\cdot\vec{D}dV = \int \rho_f dV \qquad \vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\oint \vec{B}\cdot d\vec{A} = \int \vec{\nabla}\cdot\vec{B}dV = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\oint \vec{E}\cdot d\vec{r} = \int (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\oint \vec{H}\cdot d\vec{r} = \int (\vec{\nabla}\times\vec{H})\cdot d\vec{A} = \int \vec{J}\cdot d\vec{A} + \frac{d}{dt}\int \vec{D}\cdot d\vec{A} \qquad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

- The constitutive relations $\vec{B} = \mu_0\vec{H} + \mu_0\vec{M}$ $\qquad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$
- The boundary conditions

$$\oint \vec{H}\cdot d\vec{r} = 0 \quad \Rightarrow \quad H_{t1} = H_{t2} \qquad \oint \vec{E}\cdot d\vec{r} = 0 \quad \Rightarrow \quad E_{t1} = E_{t2}$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \Rightarrow \quad B_{n1} = B_{n2} \qquad \oint \vec{D}\cdot d\vec{A} = Q_f \quad \Rightarrow \quad D_{n1} - D_{n2} = \sigma_f$$