Partial Exam Magnetism (Electricity and Magnetism, EE1P21)
28th of June 2016 from 9.00 to 11.00

This exam consists of 3 questions on 4 pages. The number of points per question indicates how many points you can earn by answering that question. A partly correct answer may give a part of the points. This examination has to be made without using a book, old exams, notes, dictionaries or programmable calculators; a simple pocket calculator may be used.

Question 1 (4+4+12+4+6+6+4+4=44 points)

First, we consider an infinitely long conductor with radius *R* and current *I*.

a) Derive an expression for the magnitude of the magnetic field *B* within the conductor generated by the current in that conductor as a function of the distance to the centre of the line r.

b) Derive an expression for the magnitude of the magnetic field *B* outside the conductor generated by the current in that conductor as a function of the distance to the centre of the line r.

When new power transmission lines are built, people are often protesting because they are afraid of being exposed to the electromagnetic fields of the power line. They are often satisfied when the power transmission line is buried in the ground. In this question, we will compare the magnetic fields of a simplified transmission line above the terrain surface (see fig. 1) and simplified transmission cable below the terrain surface (see fig. 2).

The simplified electric power line consists of two parallel wires of radius $R = 10$ mm and length $l = 5$km.

A current of 500 A flows in the wires in opposite direction. Let us assume it is a DC current.

In Configuration (a) the wires are placed in air at a height $h_a = 40$m from the terrain surface and the centres of the wires are distance $d_a = 4$m.

In Configuration (b) the wires are placed in soil at a height $h_b = -1$m, with respect to the terrain surface, and the centers of the wires are distant of a length $d_b = 2$m.

In the next questions, we compare the magnetic fields of the two configurations.

c) Assuming the lines to be infinitely long, what is the magnetic field $\vec{B}$ in the middle of the lines at a height $h_m = 1$m from the terrain for Configuration (a) and Configuration (b), respectively? (Vectorial quantities with the relevant $\hat{x}, \hat{y}, \hat{z},$ components are expected for both Configuration (a) and Configuration (b).)

d) What are the magnetic field energy densities at a height $h_m = 1$m from the terrain for Configuration (a) and for Configuration (b), respectively?

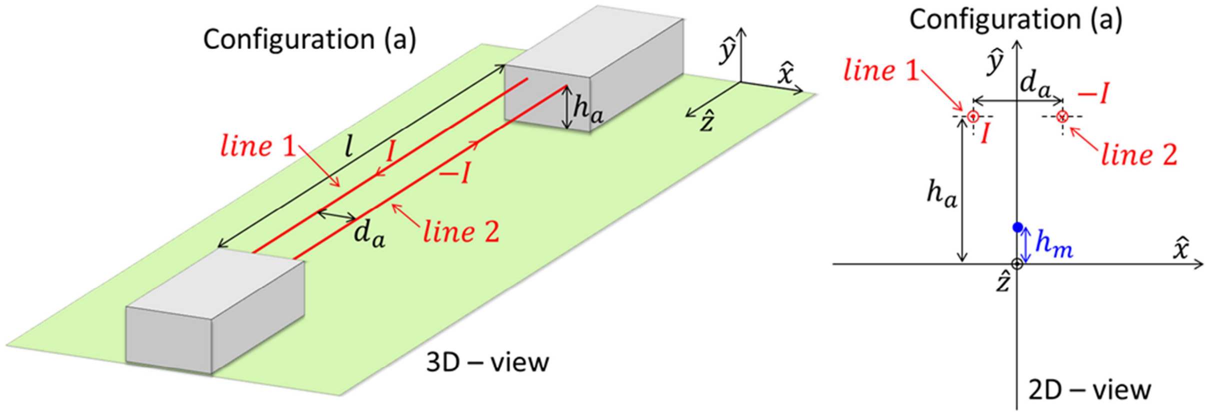


Fig. 1.: Configuration (a). Power lines in air above the terrain surface. The images are not in scale.

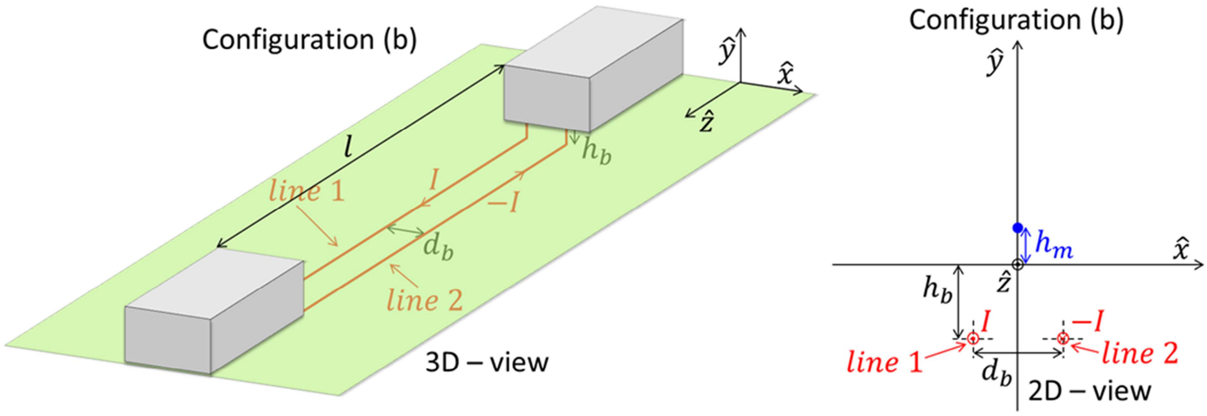


Fig. 2.: Configuration (b). Power lines in soil below the terrain surface. The images are not in scale.

In the next questions, we just consider Configuration (a).

e) Give the expressions for the magnetic field between the two lines in the plane containing them for Configuration (a).

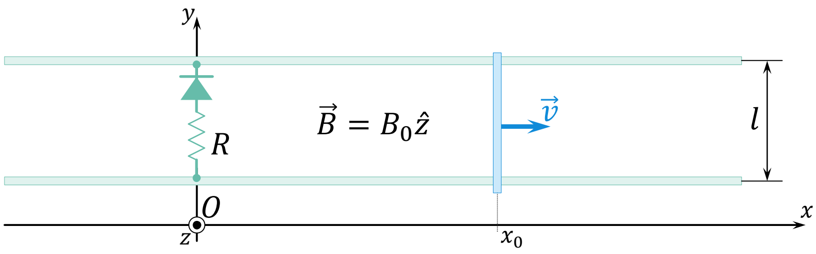
f) Calculate the flux of the magnetic field through the area (with unit normal $\hat{y}$) between the two wires for Configuration (a). (Do not consider the flux passing through the wires; for Configuration (a) consider the area of sides l and $d_a - 2R$.

g) Assuming that the two wires form a rectangular loop closing them at their ends, and neglecting the field contributions originating by their junction, calculate the value of the self-inductance for the electric power-line for Configuration (a).

h) Calculate the total energy stored in the magnetic field.

Question 2 (4+4+4+4+4+4+4=28 points)

The configuration in the figure below consists of two parallel, infinitely long, perfectly conducting rails of negligible cross-sections, spaced at the distance l and located in the $z = 0$ plane. The rails are joined at $x = 0$ by a bridge consisting of a perfect diode, with zero turn-on voltage, and a resistor of resistance R. A perfectly conducting bar of negligible cross-section and mass m_b can glide along the rails without friction. The configuration is immersed in a uniform magnetic field $\vec{B} = B_0\hat{z}$.



I. Assume that the bar's initial position is at $x = x_0$, with $x_0 > 0$, and that its initial velocity is $\vec{v}_1 = v_0\hat{x}$.

a) Calculate the initial value of the current through the resistance R.

b) Determine the initial magnetic force $\overrightarrow{F_m}$ exerted on the bar.

c) Describe the bar's subsequent motion: uniform, accelerated or decelerated (a very brief justification of the choice is mandatory).

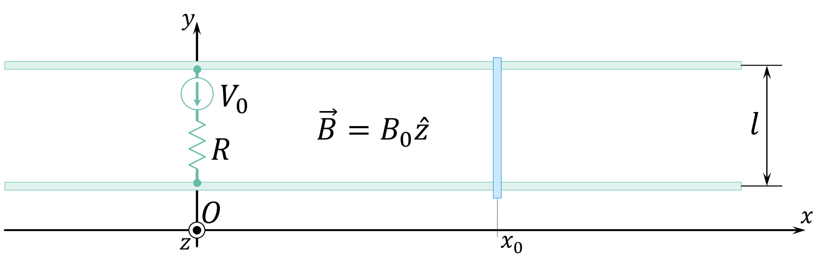
II. Assume now that the bar's initial velocity is $\vec{v}_2 = -v_0\hat{x}$.

d) Determine the new magnetic force $\overrightarrow{F_m}$ exerted on the bar.

e) Describe the bar's subsequent motion: uniform, accelerated or decelerated (a very brief justification of the choice is mandatory).

f) After passing left of the $x = 0$ plane, the bar's motion will be: uniform, accelerated or decelerated (a very brief justification of the choice is mandatory).

III. The diode is now replaced by an ideal voltage source that generates a voltage V_0 (see the figure below). Assume that the bar's initial position is at $x = x_0$, with $x_0 > 0$, and that it is initially at rest.



g) Calculate the value of the current through the resistance R ***when the bar reaches its terminal velocity***.

Question 3 (4+4+4+4+4+4+4=28 points)

A (idealised) radar system produces pulses consisting of 200 full cycles of a sinusoidal 77 GHz ($77 \cdot 10^9$ Hz) electromagnetic wave. The average power while the transmitter produces these pulses is 1 MW. The electromagnetic waves are confined to a circular beam 3 m in diameter.

a) Calculate the intensity during a pulse.
b) Calculate the peak electric field.
c) Calculate the peak magnetic field.
d) Calculate the wavelength.
e) Calculate the total energy in a pulse.
f) Calculate the total momentum in a pulse.
g) If the transmitter produces 500 pulses of 200 cycles per second, what is its average power?

Form with equations for Electricity and Magnetism

Speed of light: $c = 3.00 \cdot 10^8$ m/s

Magnetic permeability of vacuum: $\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2

Electrical permittivity of vacuum: $\varepsilon_0 = \dfrac{1}{\mu_0 c^2} = 8.854 \cdot 10^{-12} \dfrac{\mathrm{C}^2}{\mathrm{Nm}^2}$

Charge of an electron: $e = 1.6 \cdot 10^{-19}$ C

Coulomb's law: $\vec{F} = k \dfrac{q_1 q_2}{r_{1,2}^2} \hat{r}_{1,2}$ with $k = \dfrac{1}{4\pi\varepsilon_0} = 8.99 \cdot 10^9 \ \mathrm{Nm}^2/\mathrm{C}^2$

Force on a charge in an electric field $\vec{F} = q\vec{E}$

Torque on an electric dipole: $\vec{\tau} = \vec{p} \times \vec{E}$ with the electric dipole moment: $\vec{p} = q\vec{d}$

Potential energy of an electric dipole: $U = -\vec{p} \cdot \vec{E}$

Potential difference: $\Delta V_{AB} = V_B - V_A = -\int_{r_a}^{r_b} \vec{E} \cdot \mathrm{d}\vec{r}$

Potential of a charge distribution: $V = \int_V \dfrac{k}{r} \mathrm{d}q$

Electric field of a change in potential: $\vec{E} = -\vec{\nabla} V$

Force on a charge in a magnetic field: $\vec{F} = q\vec{v} \times \vec{B}$

Lorenz's law: $\vec{F} = I\vec{L} \times \vec{B}$

Torque on a magnetic dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$ with the magnetic dipole moment: $\vec{\mu} = NI\vec{A}$

Potential energy of a magnetic dipole: $U = -\vec{\mu} \cdot \vec{B}$

Biot-Savart's law: $\vec{B} = \int \mathrm{d}\vec{B} = \int \dfrac{\mu_0}{4\pi} \dfrac{I \, \mathrm{d}\vec{L} \times \hat{r}}{r^2}$

Electrical current $I = \dfrac{\Delta Q}{\Delta t} = nqA\left|\vec{v}_d\right|$ with $\vec{v}_d$ the drift velocity.

Current density $\vec{J} = nq\vec{v}_d = \sigma\vec{E}$ with σ the conductivity.

Resistance: $R = \dfrac{\rho l}{A} = \dfrac{l}{\sigma A}$ with ρ the resistivity.

Ohm's law: $V = IR$.

Energy density of the electric field: $u_E = \dfrac{1}{2} \varepsilon_0 \varepsilon_r E^2$

Capacity: $C = \dfrac{Q}{V}$

Energy stored in a capacitor: $U = \dfrac{1}{2} C V^2$

Energy density of the magnetic field: $u_B = \dfrac{B^2}{2\mu_0 \mu_r}$

Inductance: $L = \dfrac{\Phi_B}{I}$

Energy stored in an inductance: $U = \dfrac{1}{2} L I^2$

The electric flux $\Phi_E = \int \vec{E} \cdot \mathrm{d}\vec{A}$

Gauss's law for the electric field:

- From book: $\oint \vec{E} \cdot \mathrm{d}\vec{A} = \int \vec{\nabla} \cdot \vec{E} \mathrm{d}V = \frac{1}{\varepsilon_0} \int \rho \mathrm{d}V$
- Including material properties: $\oint \vec{D} \cdot \mathrm{d}\vec{A} = \int \vec{\nabla} \cdot \vec{D} \mathrm{d}V = \int \rho_f \mathrm{d}V$

The magnetic flux $\Phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A}$

Gauss's law for the magnetic field: $\oint \vec{B} \cdot \mathrm{d}\vec{A} = \int \vec{\nabla} \cdot \vec{B} \mathrm{d}V = 0$

Faraday's law: $\oint \vec{E} \cdot \mathrm{d}\vec{r} = \int (\vec{\nabla} \times \vec{E}) \cdot \mathrm{d}\vec{A} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{B} \cdot \mathrm{d}\vec{A}$

Ampère's law:

- From book: $\oint \vec{B} \cdot \mathrm{d}\vec{r} = \int (\vec{\nabla} \times \vec{B}) \cdot \mathrm{d}\vec{A} = \mu_0 \int \vec{J} \cdot \mathrm{d}\vec{A} + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{E} \cdot \mathrm{d}\vec{A}$
- Including material properties: $\oint \vec{H} \cdot \mathrm{d}\vec{r} = \int (\vec{\nabla} \times \vec{H}) \cdot \mathrm{d}\vec{A} = \int \vec{J}_f \cdot \mathrm{d}\vec{A} + \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{D} \cdot \mathrm{d}\vec{A}$

The constitutive equations:

- For dielectric materials in general: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$
- For linear dielectric materials: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 \kappa \vec{E} = \varepsilon_0 (1 + \chi_e) \vec{E}$
- For magnetic materials in general: $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$
- For linear magnetic materials: $\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H}$

The Poynting vector: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

Momentum of an electromagnetic wave: $p = \frac{U}{c}$

For an electromagnetic wave in vacuum: $E = cB$ and $f\lambda = c$.

Answer question 1 (4+4+12+4+6+6+4+4=44 points)

a) Calculate the magnitude of the magnetic field B within a conductor.

Using Ampere:

$\oint \vec{B}\cdot d\vec{r} = \mu_0 \int \vec{J}\cdot d\vec{A}$

Within the conductor with radius R=10 mm, the enclosed current is proportional to $\frac{r^2}{R^2}$. Therefore, $2\pi r B = \frac{\mu_0 I r^2}{R^2} \quad \Rightarrow \quad B = \frac{\mu_0 I r}{2\pi R^2}$

b) Calculate the magnitude of the magnetic field B outside a conductor.

Outside the conductor with radius R=10 mm, the complete current is enclosed:

$2\pi r B = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$

c) Assuming the lines to be infinite, what is the magnetic field $\vec{B}$ in the middle of the lines at a height $h_m = 1\text{m}$ from the terrain for Configuration (a) and Configuration (b), respectively? (Vectorial quantities with the relevant $\hat{x}, \hat{y}, \hat{z}$, components are expected for both Configuration (a) and Configuration (b).)

For Configuration (a)

$$\vec{B}_a(h_m) = \vec{B}_{1a}(h_m) + \vec{B}_{2a}(h_m)$$

Defining

$$r_{1a} = \sqrt{(h_a - h_m)^2 + \left(\frac{d_a}{2}\right)^2}$$

And

$$\alpha_{1a} = \text{acos}\left(\frac{h_a - h_m}{r_{1a}}\right)$$

The magnetic field due to Line 1 (Fig.1) can be expressed as

$$\vec{B}_{1a}(h_m) = \frac{\mu_0 I}{2\pi r_{1a}}[\cos\alpha_{1a}\,\hat{x} + \sin\alpha_{1a}\,\hat{y}]$$

Analogously, for Line 2 (Fig.1)

$$r_{2a} = \sqrt{(h_a - h_m)^2 + \left(\frac{d_a}{2}\right)^2} = r_{1a}$$

$$\alpha_{2a} = \text{acos}\left(\frac{h_a - h_m}{r_{2a}}\right) = \alpha_{1a}$$

$$\vec{B}_{2a}(h_m) = \frac{\mu_0 I}{2\pi r_{1a}}[-\cos\alpha_{1a}\,\hat{x} + \sin\alpha_{1a}\,\hat{y}]$$

Therefore,

$$\vec{B}_a(h_m) = \frac{\mu_0 I}{2\pi r_{1a}}[\cos\alpha_{1a}\,\hat{x} + \sin\alpha_{1a}\,\hat{y}] + \frac{\mu_0 I}{2\pi r_{1a}}[-\cos\alpha_{1a}\,\hat{x} + \sin\alpha_{1a}\,\hat{y}]$$

$$= \frac{\mu_0 I}{\pi r_{1a}}\sin\alpha_{1a}\,\hat{y}$$

Using the given numbers

$$r_{1a} = 39.05\text{m}$$

$$\alpha_{1a} = 0.0512\text{rad}$$

$$\vec{B}_a(h_m) = \frac{\mu_0 I}{\pi r_{1a}}\sin\alpha_{1a}\,\hat{y} = \frac{\mu_0 I}{\pi r_{1a}}\frac{d_a}{2r_{1a}} = 0.2623\cdot 10^{-6}\text{T}$$

For Configuration (b)

$$\vec{B}_b(h_m) = \vec{B}_{1b}(h_m) + \vec{B}_{2b}(h_m)$$

Defining

$$r_{1b} = \sqrt{(h_m - h_b)^2 + \left(\frac{d_b}{2}\right)^2}$$

And

$$\alpha_{1b} = \mathrm{acos}\left(\frac{h_m - h_b}{r_{1b}}\right)$$

The magnetic field due to Line 1 (Fig.2) can be expressed as

$$\vec{B}_{1b}(h_m) = \frac{\mu_0 I}{2\pi r_{1b}}[-\cos\alpha_{1b}\,\hat{x} + \sin\alpha_{1b}\,\hat{y}]$$

Analogously, for Line 2 (Fig.2)

$$\vec{B}_{2b}(h_m) = \frac{\mu_0 I}{2\pi r_{1b}}[\cos\alpha_{1b}\,\hat{x} + \sin\alpha_{1b}\,\hat{y}]$$

Therefore,

$$\vec{B}_b(h_m) = \frac{\mu_0 I}{\pi r_{1b}}\sin\alpha_{1b}\,\hat{y}$$

Using the given numbers

$$r_{1b} = 2.236\mathrm{m}$$
$$\alpha_{1b} = 0.4636\mathrm{rad}$$
$$\vec{B}_b(h_m) = \frac{\mu_0 I}{\pi r_{1b}}\sin\alpha_{1b}\,\hat{y} = \frac{\mu_0 I}{\pi r_{1b}}\frac{d_b}{2r_{1b}} = 40.00 \cdot 10^{-6}\mathrm{T}$$

d) What are the magnetic field energy densities at a height $h_m = 1\mathrm{m}$ from the terrain for Configuration (a) and for Configuration (b), respectively?
For Configuration (a)

$$u_{\vec{B}_a}(h_m) = \frac{1}{2}\frac{\left|\vec{B}_a(h_m)\right|^2}{\mu_0} = 27.37 \cdot 10^{-9}\mathrm{J/m^3}$$

For Configuration (b)

$$u_{\vec{B}_b}(h_m) = \frac{1}{2}\frac{\left|\vec{B}_b(h_m)\right|^2}{\mu_0} = 636.6 \cdot 10^{-6}\mathrm{J/m^3}$$

e) Give the expressions for the magnetic field between the two lines in the plane containing them, for Configuration (a)

$$\vec{B}_a(x, h_a) = \vec{B}_{1a}(x, h_a) + \vec{B}_{2a}(x, h_a) = \frac{\mu_0 I}{2\pi\left|x + \frac{d_a}{2}\right|}\hat{y} + \frac{\mu_0 I}{2\pi\left|x - \frac{d_a}{2}\right|}\hat{y}$$

For $x \in \left(-\frac{d_a}{2}, \frac{d_a}{2}\right)$

$$\vec{B}_a(x, h_a) = \frac{\mu_0 I}{2\pi\left(x + \frac{d_a}{2}\right)}\hat{y} + \frac{\mu_0 I}{2\pi\left(\frac{d_a}{2} - x\right)}\hat{y}$$
$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x + \frac{d_a}{2}} - \frac{1}{x - \frac{d_a}{2}}\right]\hat{y}$$

f) Calculate the flux of the magnetic field through the area (with unit normal $\hat{y}$) between the two wires for Configuration (a). (Do not consider the flux passing through the wires; for Configuration (a) consider the area of sides l and $d_a - 2R$.

$$\Phi_a = l\int_{-\frac{d_a}{2}+R}^{\frac{d_a}{2}-R} \vec{B}_a(x,h_a)\cdot\hat{y}\,dx = \frac{\mu_0 Il}{2\pi}\int_{-\frac{d_a}{2}+R}^{\frac{d_a}{2}-R}\left[\frac{1}{x+\frac{d_a}{2}}+\frac{1}{\frac{d_a}{2}-x}\right]dx$$

$$= \frac{\mu_0 Il}{2\pi}\left[\log\left(x+\frac{d_a}{2}\right)-\log\left(\frac{d_a}{2}-x\right)\right]_{-\frac{d_a}{2}+R}^{\frac{d_a}{2}-R}$$

$$= \frac{\mu_0 Il}{2\pi}\left[\log\left(\frac{d_a}{2}-R+\frac{d_a}{2}\right)-\log\left(-\frac{d_a}{2}+R+\frac{d_a}{2}\right)-\log\left(\frac{d_a}{2}-\frac{d_a}{2}+R\right)+\log\left(\frac{d_a}{2}+\frac{d_a}{2}-R\right)\right]$$

$$= \frac{\mu_0 Il}{2\pi}[\log(d_a-R)-\log(R)-\log(R)+\log(d_a-R)]$$

$$= \frac{\mu_0 Il}{\pi}\log\frac{d_a-R}{R}$$

$$\Phi_a = \frac{\mu_0 Il}{\pi}\log\frac{d_a-R}{R} = 5.989\mathrm{Tm}^2$$

g) Assuming that the two wires form a rectangular loop closing them at their ends, and neglecting the field contributions originating by their junction, calculate the value of the self-inductance for the electric power-line for both Configuration (a).

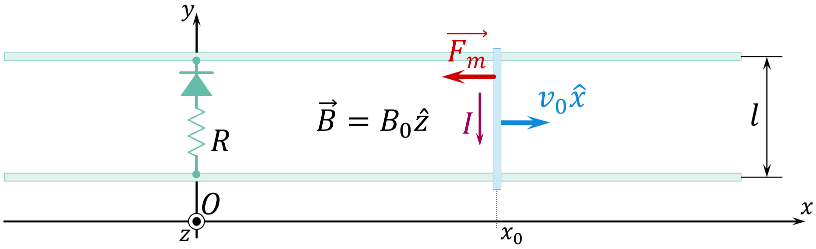
$$L_a = \frac{\Phi_a}{I} = \frac{\mu_0 l}{\pi}\log\frac{d_a-R}{R} = 12.0\mathrm{mH}$$

a) Assuming that the two wires form a rectangular loop closing them at their ends, and neglecting the field contributions originating by their junction, calculate the value of the self-inductance for the electric power-line for both Configuration (a).

$$U = \frac{1}{2}L_a I^2 = 1500\mathrm{J}$$

Answer question 2 (4+4+4+4+4+4+4=28 points)

a) Due to the bar moving with $\vec{v}_1 = v_0\hat{x}$, the surface intercepting the uniform magnetic field $\vec{B} = B_0\hat{z}$ will increase, yielding an induced electromotive force in the circuit formed by the bar, the rails and the bridge at $x = 0$. According to Lenz's law, the current has the direction in the figure below. The induced voltage polarises directly the ideal diode that, hence, has 0 resistance.



The initial induced current I has then the value

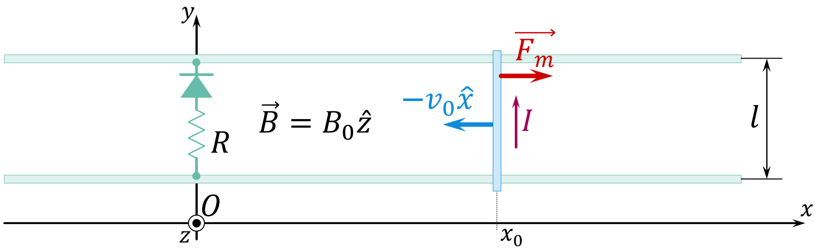
$$|I| = \frac{|E|}{R} = \frac{B_0 l}{R}\frac{dx}{dt} = \frac{B_0 lv}{R} \quad (A).$$

b) The current I entails a magnetic force $\vec{F_m}$ exerted on the bar (see the figure)

$$\overrightarrow{F_m} = I\,\vec{l} \times \vec{B} = -I\,B_0\,(\hat{y} \times \hat{z}) = -I\,B_0\,\hat{x} = -\frac{l^2\,B_0^2\,v}{R}\,\hat{x} \quad (N).$$

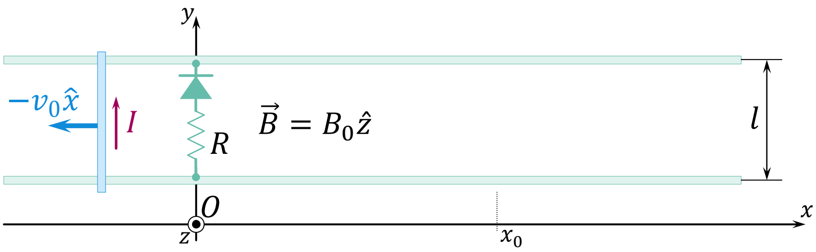
c) The force $\overrightarrow{F_m}$ exerted on the bar is oriented towards $-\hat{x}$ and, hence, will result in an acceleration oriented towards $-\hat{x}$ that will decelerate the bar of mass m_b. Eventually, the bar's speed will become zero.

d) Due to the bar moving with $\vec{v}_1 = -v_0\hat{x}$, the surface intercepting the uniform magnetic field $\vec{B} = B_0\hat{z}$ will decrease, yielding an induced electromotive force in the circuit formed by the bar, the rails and the bridge at $x = 0$. According to Lenz's law, the current has the direction in the figure below. The induced voltage polarises inversely the ideal diode that, hence, acts as an open circuit. Consequently, the induced current will be zero and, hence, the magnetic force $\overrightarrow{F_m}$ exerted on the bar will be $\overrightarrow{F_m} = \vec{0}$ (N).

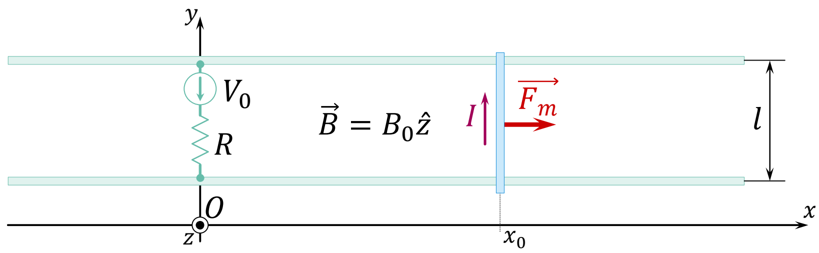


e) Since the magnetic force exerted on the bar is zero, the bar's motion will, in this case, be uniform

f) After passing left of the $x = 0$ plane, the induced current will have, according to Lenz's law, the direction in the figure below. The induced voltage polarises inversely the ideal diode that, hence, acts as an open circuit and, thus, the bar's motion will be again uniform.



g) The ideal voltage source produces a current in the circuit with the orientation given in the figure below. This current, in turn, produces a magnetic force $\overrightarrow{F_m}$ that moves the bar in the $\hat{x}$ direction, thus enlarging the surface intercepting the uniform magnetic field $\vec{B} = B_0\hat{z}$. The induced electromotive force produces a current with the direction at point (a) that, in turn, produces a magnetic force that opposes the one moving the bar to the right. The terminal velocity is reached when the two forces balance each other out, implying that the absolute value of the induced electromotive force equals V_0. At that moment, the net current through the resistance R becomes 0.



Answer question 3 (4+4+4+4+4+4+4=28 points)

a) This problem involves characterizing an electromagnetic wave given the relevant parameters. The average intensity of a pulse is the average power during a pulse divided by the beam area:

$$\bar{S} = \frac{P}{\pi R^2} = \frac{(1\cdot 10^6\ \mathrm{W})}{\pi(1.50\,\mathrm{m})^2} = 141\,\mathrm{kW/m^2}$$

b) From Equation 29.20b of the book, the peak electric field can be calculated as

$$E_p = \sqrt{2\mu_0 c\bar{S}} = \sqrt{\frac{2\mu_0 cP}{\pi R^2}} = \sqrt{\frac{2(4\pi 10^{-7}\ \mathrm{H/m})(3\cdot 10^8\ \mathrm{m/s})(1\cdot 10^6\ \mathrm{W})}{\pi(1.5\,\mathrm{m})^2}} = 10.3\,\mathrm{kV/m}$$

c) $$B_p = \frac{E_p}{c} = \frac{30.0\,\mathrm{MV/m}}{3.00\cdot 10^8\ \mathrm{m/s}} = 34.4\,\mu\mathrm{T}$$

d) The wavelength may be found using Equation 29.16c, $c = f\lambda$.

The wavelength is $\lambda = \frac{c}{f} = \left(3.00\times 10^8\,\mathrm{m/s}\right)/\left(77\ \mathrm{GHz}\right) = 3.90\ \mathrm{mm}$.

e) To find the energy in a pulse, use $U = \bar{P}_{\mathrm{pulse}}\Delta t$, where $\Delta t = NT = N/f$, with $N = 200$ and $f = 77$ GHz. The total energy in a pulse is

$U = \bar{P}_{\mathrm{pulse}}N/f = \left(1\,\mathrm{MW}\right)\left(200\right)\left(1/77\ \mathrm{GHz}\right) = 2.60$ mJ.

f) The momentum per pulse is given by

$p = U/c = \left(2.6\ \mathrm{mJ}\right)/\left(3.00\times 10^8\,\mathrm{m/s}\right) = 8.66\times 10^{-12}\ \mathrm{kg\cdot m/s}.$

g) To find the average power output, calculate the power in a pulse, multiply by 500 because there are 500 pulses per second, and divide by 1 s to get the power (energy per unit time). Every pulse carries 50 mJ, and there are 500 per second, so the average power is

$\bar{P} = \left(2.6\ \mathrm{mJ}\right)\left(500\right)/\left(1.0\ \mathrm{s}\right) = 1.3\ \mathrm{W}$.

The average power in the beam is much less than the power per pulse because the duty cycle is

$$\text{duty cycle} = \frac{t_{\mathrm{on}}}{t_{\mathrm{tot}}} = \frac{\left(500\right)\left(200\right)}{\left(77\ \mathrm{GHz}\right)\left(1\,\mathrm{s}\right)} = 1.30\cdot 10^{-6}$$

which explains the six-order-of-magnitude difference between $\bar{P}_{pulse}$ and $\bar{P}$.