Electricity and Magnetism

Overview Magnetism

- 28-5: Introduction, magnetism: field and force
- 1-6: Magnetism: Biot-Savart, Ampere
- 4-6: Electromagnetic induction
- 8-6: Electromagnetic induction
- 11-6: Maxwell’s equations and electromagnetic waves
- 15-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 18-6: available for answering questions, exercises

Electromagnetic induction

- Learning objectives
- Induced currents
- Faraday’s law
- Induction and energy
- Inductance
- Magnetic energy
- Concluding

In this lecture you'll learn

- To explain the phenomenon of electromagnetic induction
- To calculate induced emfs and currents
  - To use energy conservation to find the direction of induced effects
- To describe important technological applications of induction
- To explain inductance
  - And describe the role of inductance in simple circuits
- That magnetic fields store energy
  - And how to calculate that energy
- To recognize Faraday's law as one of the four fundamental laws of electromagnetism
  - And to calculate induced electric fields
- Time varying effects, not completely stationary!!!

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Faraday's law

- **Faraday's law** describes induction by relating the emf induced in a circuit to the rate of change of magnetic flux through the circuit:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

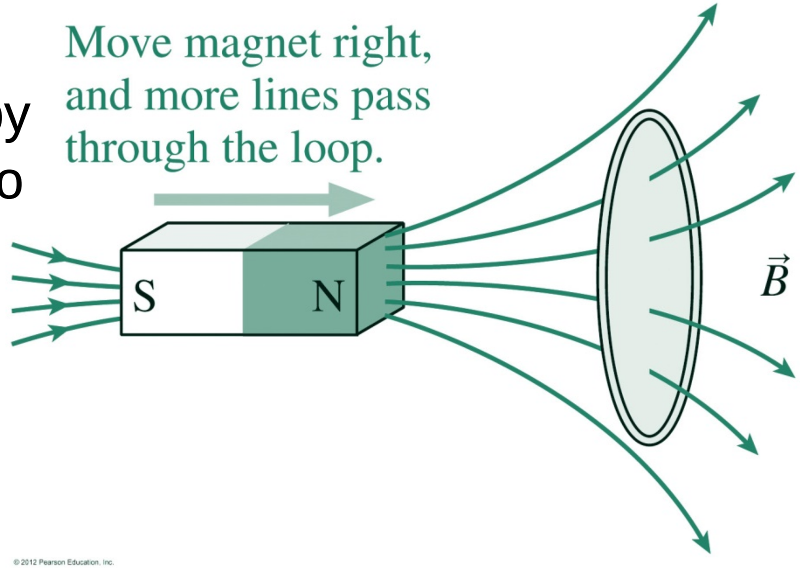
- where the **magnetic flux** is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- With a flat area and uniform field, this becomes

$$\Phi_B = BA\cos\theta$$

- The flux can change by changing the field *B*, the area *A*, or the orientation θ.



Moving a magnet near a wire loop increases the flux through the loop. The result is an induced emf given by Faraday's law. The induced emf drives an induced current in the loop.

Clicker question 7

- An emf can be induced in a loop of wire by changing

A. the magnetic field.
B. the area of the loop in the field.
C. the loop's orientation with respect to the field.
D. All of the above.

Clicker question 6

- A magnet, with its north pole pointed downwards, is falling down directly above a wire loop. As the magnet approaches the wire loop (before it passes through the loop),

A. an induced current flows through the wire loop in a clockwise orientation (as seen from above).
B. an induced current flows through the wire loop in a counter-clockwise orientation (as seen from above).
C. no current flows through the wire loop.

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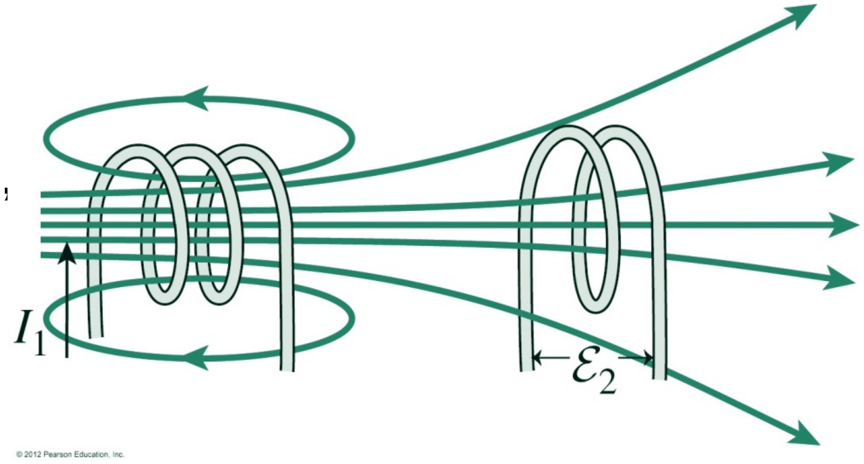
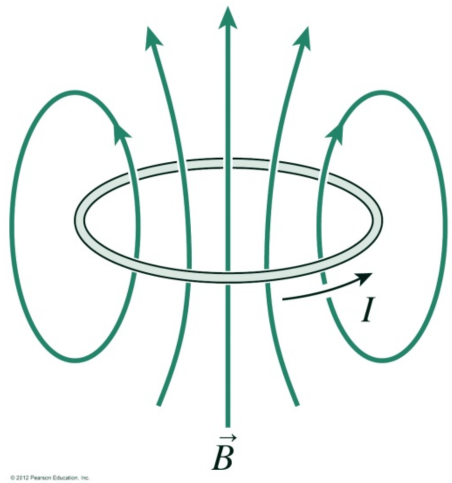
Inductance

- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit.
  - Some of the magnetic flux produced by one circuit passes through the other circuit.
- **Self-inductance** occurs when a changing current in a circuit results in an induced emf that opposes the change in the circuit itself.
  - The magnetic flux produced in a circuit passes through that same circuit.





Applications

- Mutual inductance
  - Transformers
  - Contactless energy transfer
  - Electrical machines

- Self inductance
  - Filters
  - Parasitic inductance

Mutual induction

- https://www.youtube.com/watch?v=djjh5yxJaQY

Self-inductance

- The self-inductance *L* of a circuit is defined as the ratio of the magnetic flux through the circuit to the current in the circuit:

$$L = \Phi_B / I$$

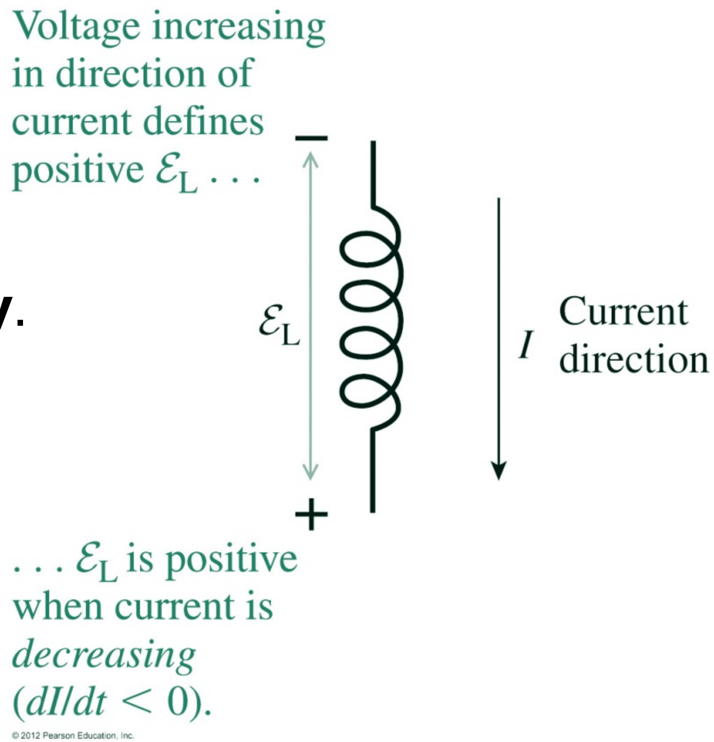
- In differential form:

$$\frac{d\Phi_B}{dt} = L\frac{dI}{dt}$$

- The SI units of *L* are $\mathrm{T \cdot m^2/A}$, or **Henry**.
- By Faraday's law, the emf across an inductor is

$$\varepsilon_L = -L\frac{dI}{dt}$$

- The minus sign shows that the direction of the inductor emf is such as to *oppose* the change in the inductor current.



Self-inductance of a solenoid

- Consider a long solenoid of cross-sectional area A, length l, with n turns per unit length.
- Magnetic field inside the solenoid:

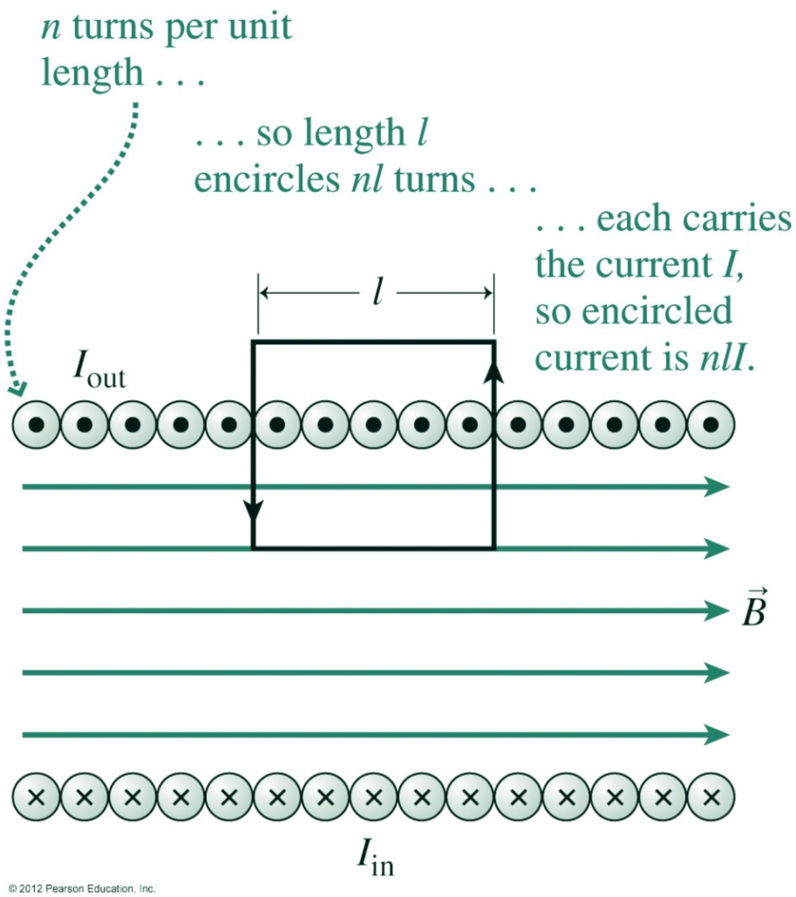
$$B = \mu_0 n I$$

- With n turns per unit length, the solenoid contains a total of nl turns, so the flux through all the turns is

$$\Phi_B = nlBA = nl(\mu_0 n I)A = \mu_0 n^2 IAl$$

- The self-inductance of the solenoid is

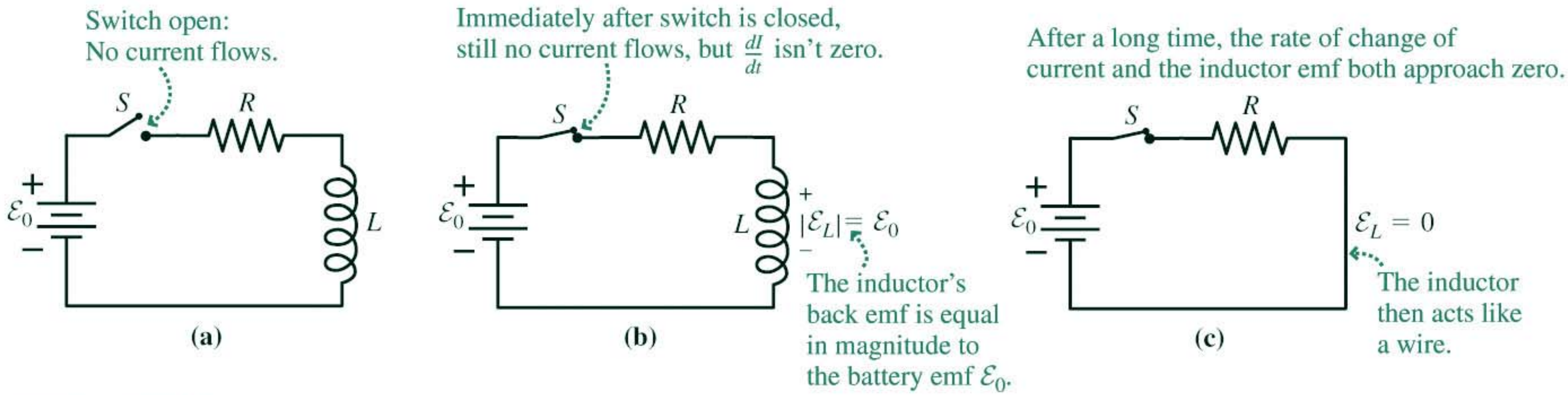
$$L = \frac{\Phi_B}{I} = \mu_0 n^2 Al$$



Inductors in circuits

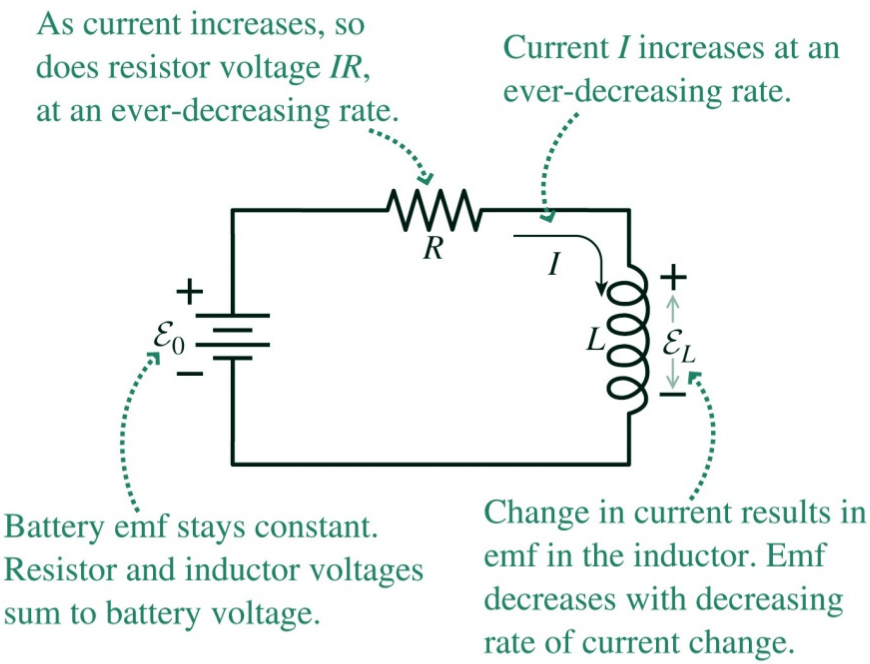
- **The current through an inductor can't change instantaneously.**
- Otherwise an impossible infinite emf would develop.
- Rapid changes in current result in large, possibly dangerous emfs.
- The buildup of current in an *RL* circuit occurs gradually.

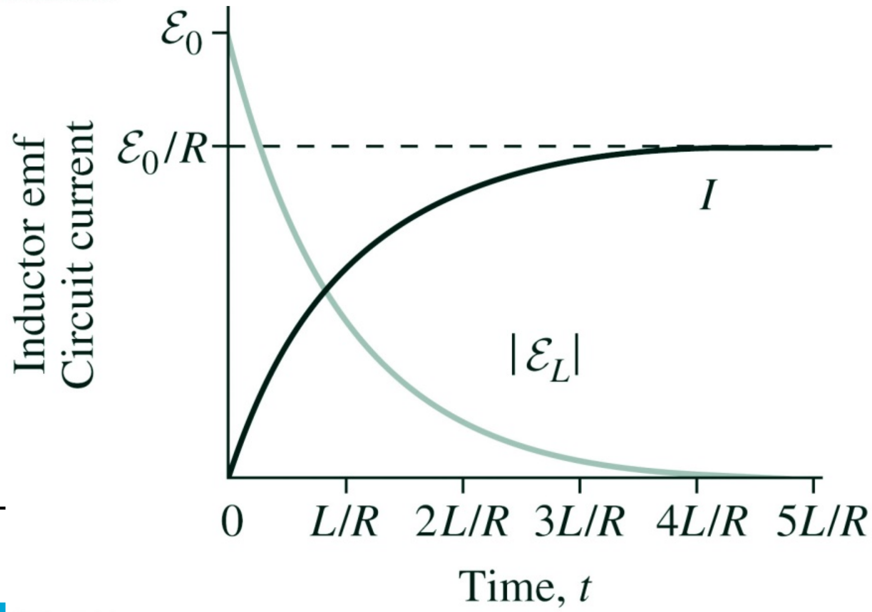
$$\varepsilon_L = -L\frac{dI}{dt}$$



The inductive time constant

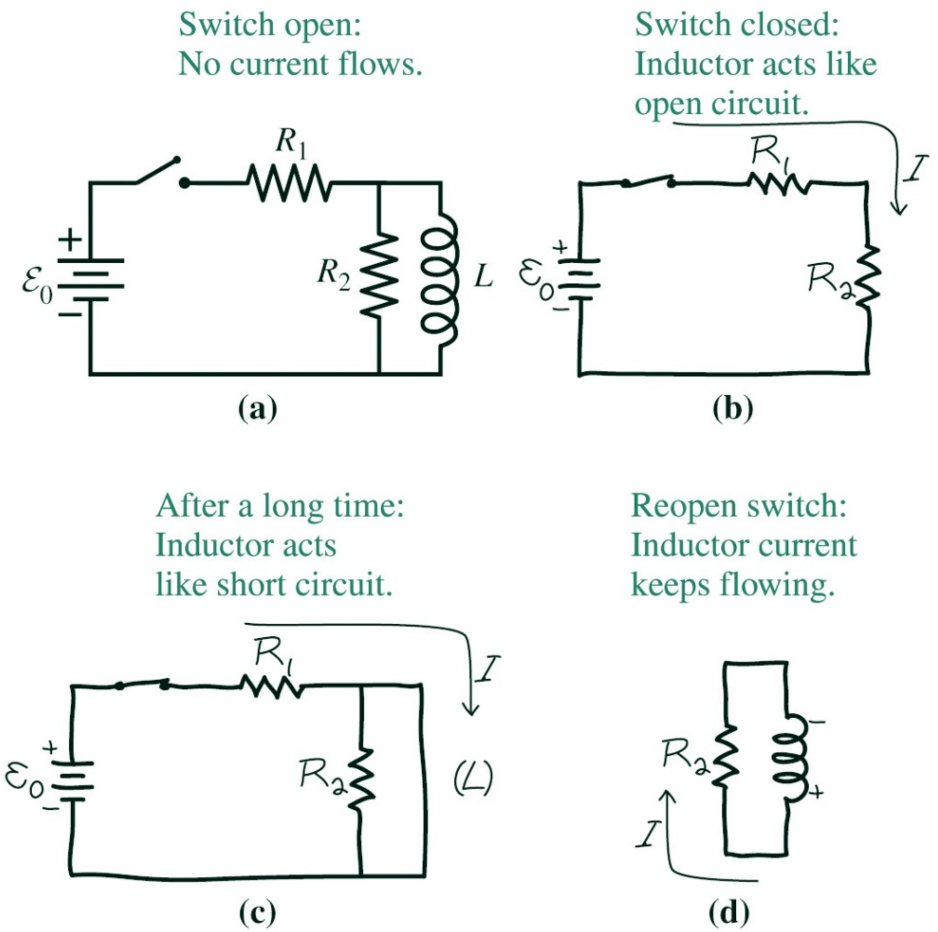
- The loop law: $\varepsilon_0 - IR + \varepsilon_L = 0$
- The solution to the differential equation is $$I = \frac{\varepsilon_0}{R}\left(1 - e^{-Rt/L}\right)$$
- The inductor current starts at zero and builds up with time constant *L*/*R*.
  - As the current increases, its rate of change decreases.
  - The inductor emf therefore decays exponentially to zero. This decay has the same time constant *L*/*R*.





Short- and long-term behavior of inductors

- Since current can't change instantaneously, an inductor in an *RL* circuit with no current through it acts instantaneously like an open circuit.
- If there's current flowing, it keeps flowing momentarily despite changes in the circuit.
- After a long time, the inductor current stops changing.
  - The inductor emf is zero.
  - The inductor acts like an ordinary wire.



Magnetic energy

- As current builds up in an inductor, the inductor absorbs energy from the circuit. That energy is stored in the inductor's magnetic field.
- The rate at which the inductor stores energy is $$P = IL\frac{dI}{dt}$$
- For an inductor, the stored energy is
$$U_B = \int P dt = \int_0^I LIdI = \frac{1}{2}LI^2$$
- Considering the uniform magnetic field inside a solenoid shows that the **magnetic energy density** is

$$L = \mu_0 n^2 Al \qquad B = \mu_0 nI \qquad U_B = Al\frac{B^2}{2\mu_0} \qquad u_B = \frac{B^2}{2\mu_0}$$

- This is a universal expression

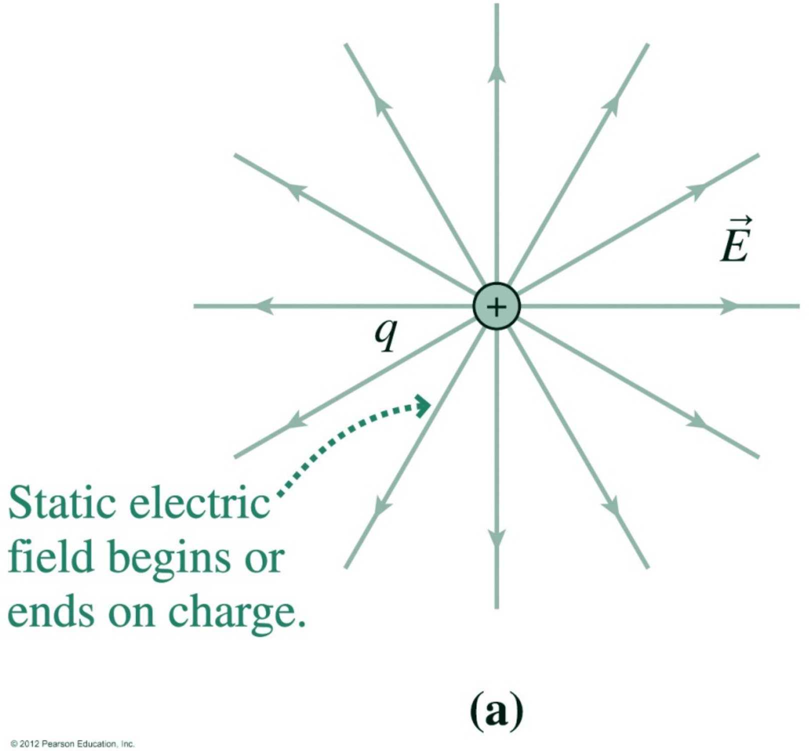
Induced electric fields

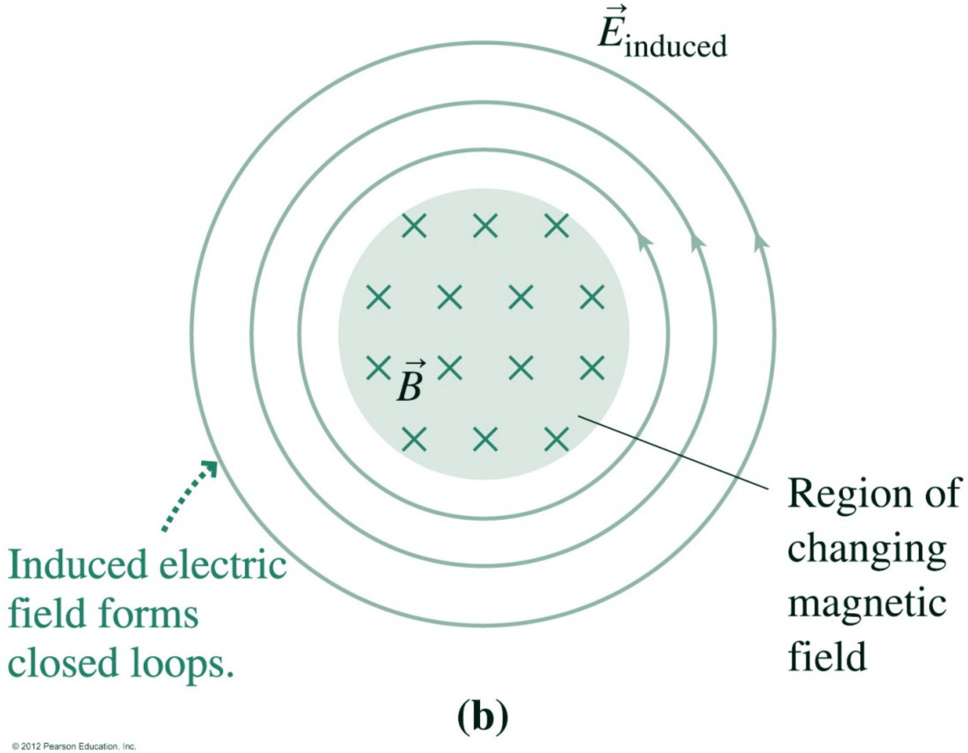
- The induced emf in a circuit subject to changing magnetic flux results from an **induced electric field.**
- Induced electric fields result from changing magnetic flux.
  - This is described by the full form of Faraday's law, one of the four fundamental laws of electromagnetism:
  - $$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

    where the integral is taken around any closed loop, and where the flux is through any area bounded by the loop.
- The equation states that **a changing magnetic field produces an electric field.**
  - Thus not only charges but also changing magnetic fields are sources of electric field.
  - Unlike the electric field of a static charge distribution, the induced electric field is *not conservative.*

Static and induced electric fields

- Static electric fields begin and end on charges.
- Induced electric fields generally form closed loops.





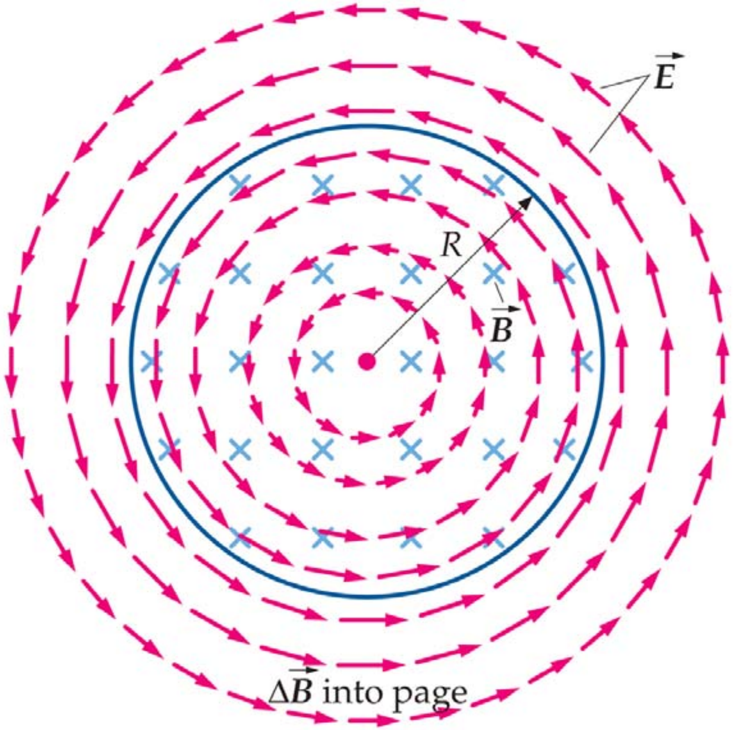
Induced electric field

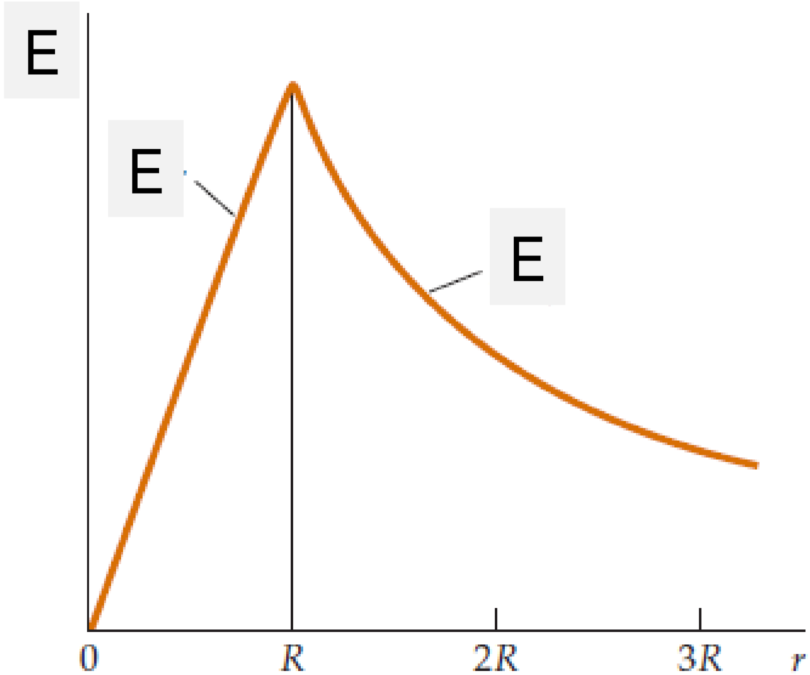
$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

$$r < R: \quad 2\pi r E = -\frac{r^2}{R^2}\frac{d\Phi_B}{dt} \quad \Rightarrow \quad E = -\frac{r}{2\pi R^2}\frac{d\Phi_B}{dt}$$

$$r > R: \quad 2\pi r E = -\frac{d\Phi_B}{dt} \Rightarrow \quad E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt}$$

Uniform B!

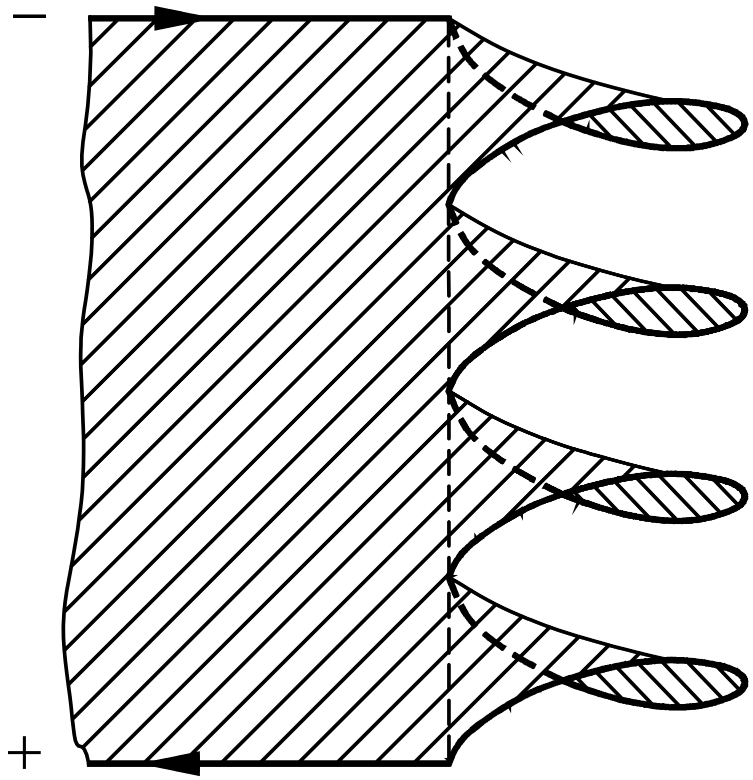




Faraday’s law

$$\oint_{C_e} \vec{E} \cdot \mathrm{d}\vec{s} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{S_e} \vec{B} \cdot \mathrm{d}\vec{A} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

If we want to calculate voltages induced in coils, the contour is chosen in the electric path, in the wire!



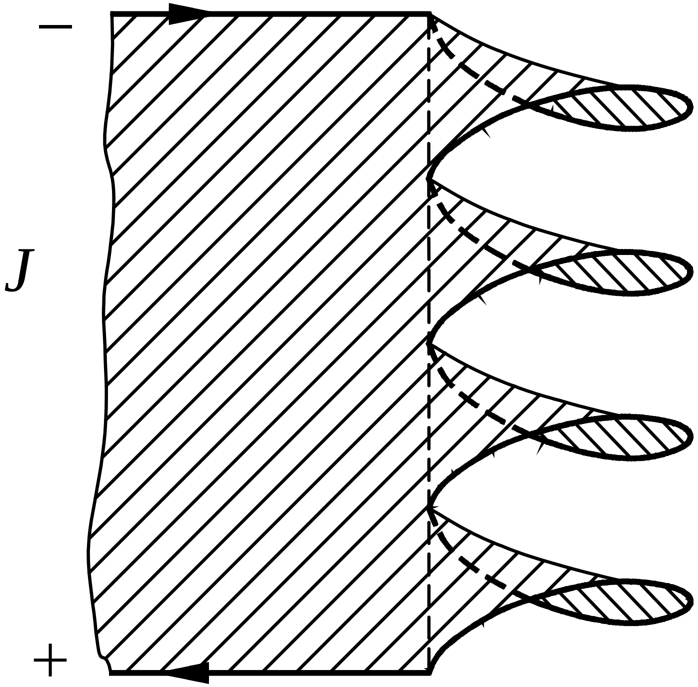
Faraday's law

$$\oint_{C_e} \vec{E}\cdot \mathrm{d}\vec{s} = \int_{+term}^{-term} \vec{E}\cdot \mathrm{d}\vec{s} + \int_{-term}^{+term} \vec{E}\cdot \mathrm{d}\vec{s}$$

$$\oint_{C_e} \vec{E}\cdot \mathrm{d}\vec{s} = V + \int_{-term}^{+term} \rho_{Cu}\vec{J}\cdot \mathrm{d}\vec{s} = V + l_{Cu}\rho_{Cu}J$$

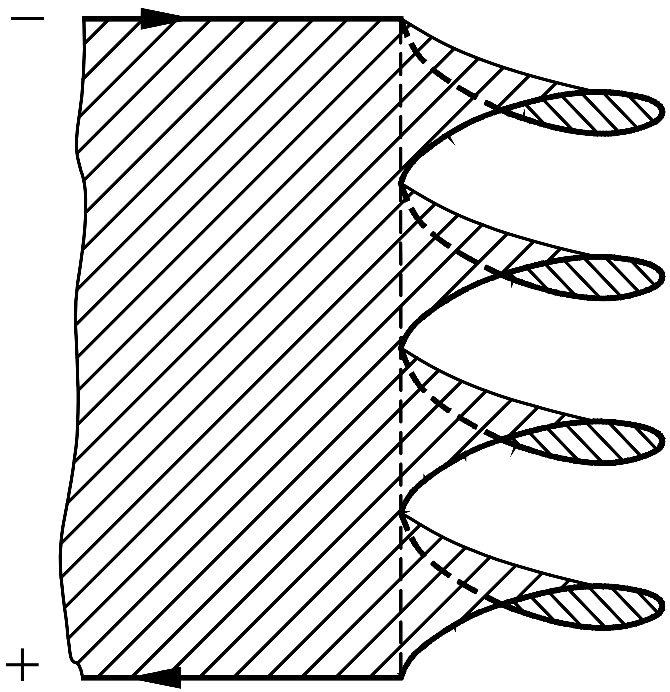
$$J = \frac{I}{A_{Cu}}$$

$$\oint_{C_e} \vec{E}\cdot \mathrm{d}\vec{s} = V + \frac{\rho_{Cu}l_{Cu}}{A_{Cu}} I = V + RI$$



Faraday's law

$$\oint_{C_e} \vec{E} \cdot \mathrm{d}\vec{s} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_e} \vec{B} \cdot \mathrm{d}\vec{A} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$



Therefore,

$$V = -RI - \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -RI - \varepsilon$$

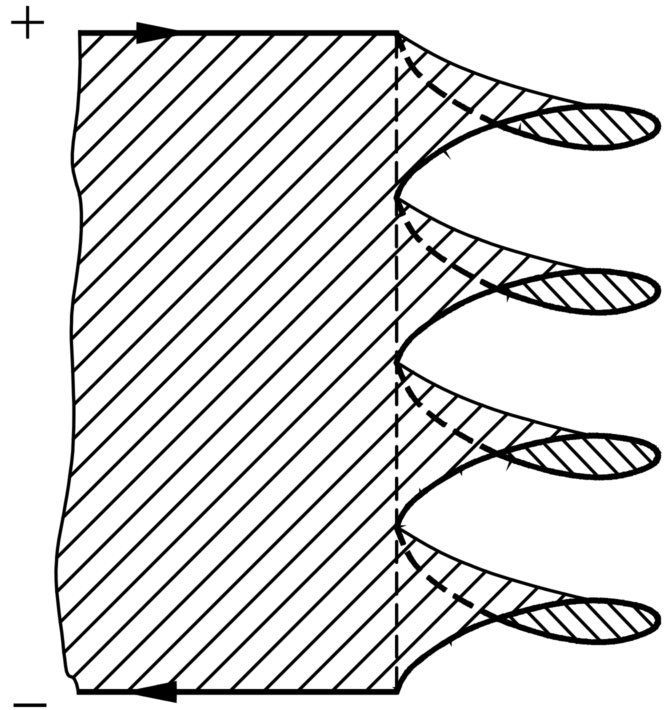
Faraday's law

$$\oint_{C_e} \vec{E} \cdot \mathrm{d}\vec{s} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{S_e} \vec{B} \cdot \mathrm{d}\vec{A} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

Therefore,

$$V = RI + \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = RI + \varepsilon$$

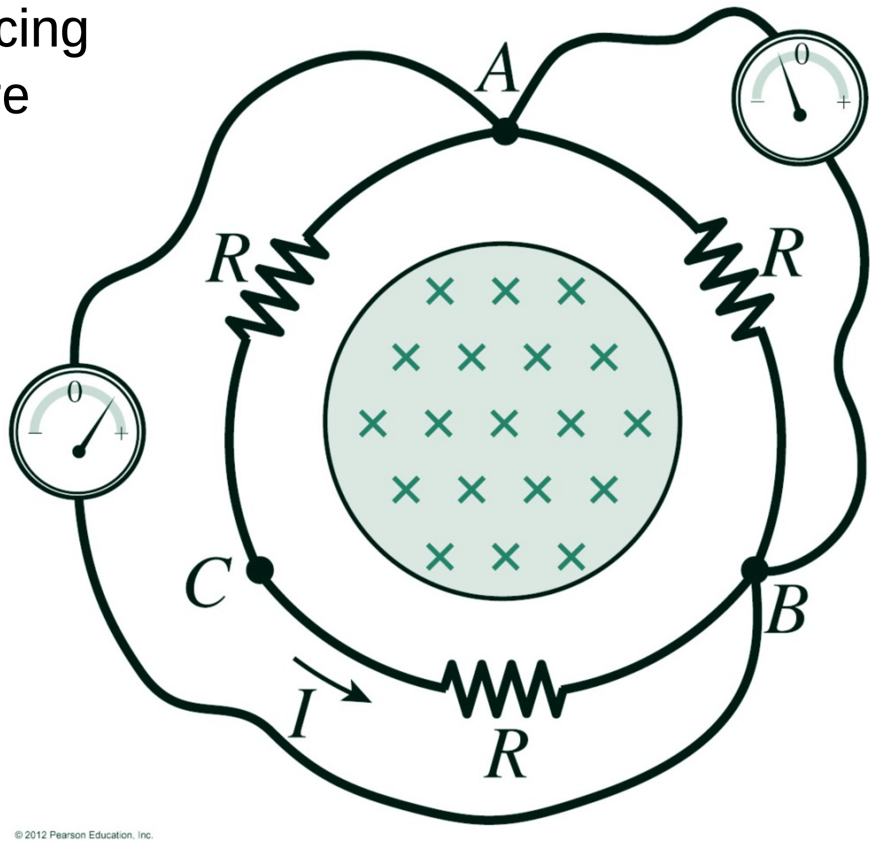
The sign depends on the definition of the positive current direction and the positive voltage terminal!



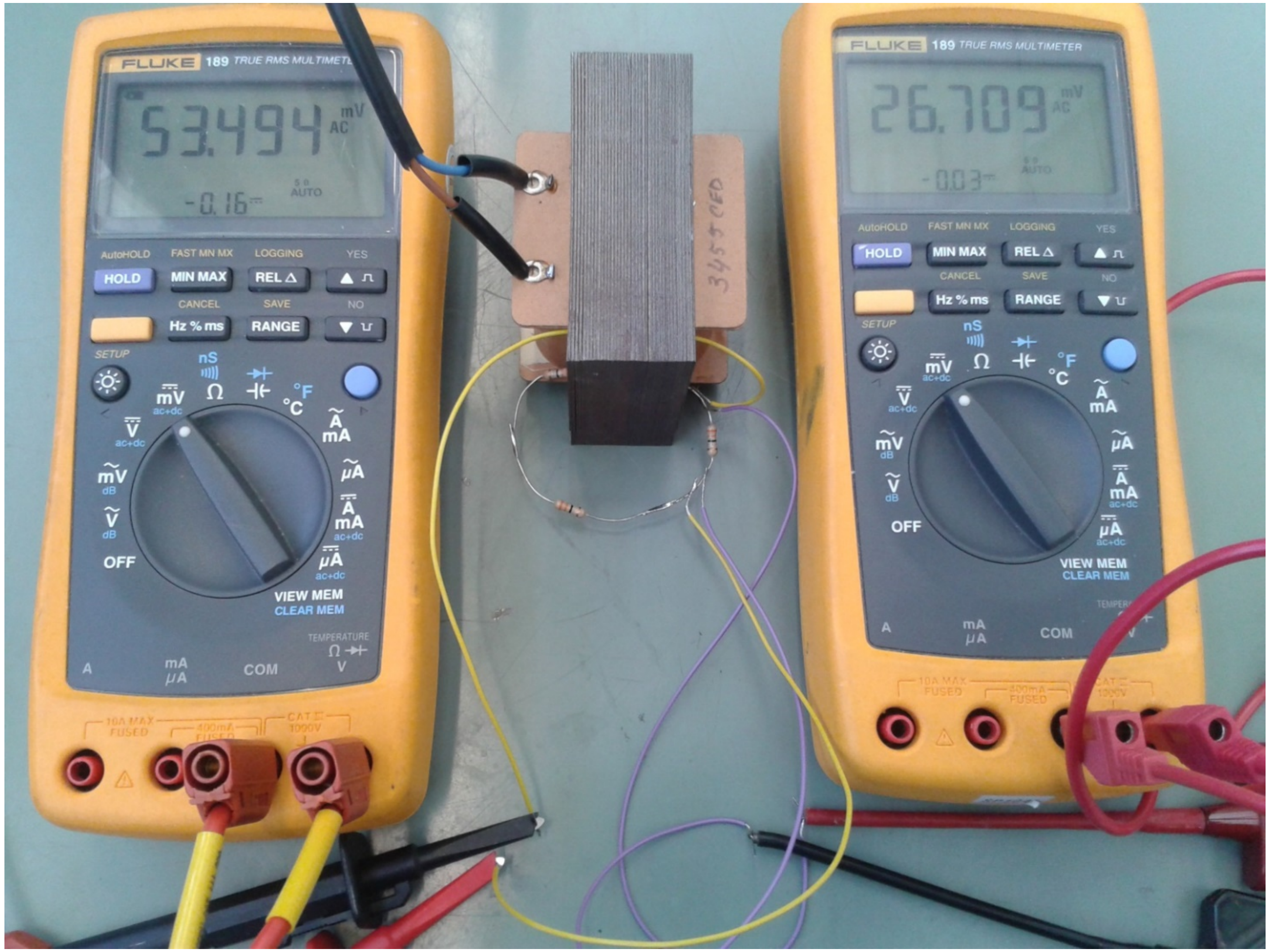
Question

- Three resistors are connected around a solenoid with an increasing magnetic field inducing a current I. Two voltmeters are connected to points A and B. What does each indicate?

- A both V=0V
- B both V=RI
- C both V=2RI
- D right V=RI, left V=2RI
- E something else



Demonstration



- https://www.youtube.com/watch?v=eqjl-qRy71w
- https://www.youtube.com/watch?v=1bUWcy8HwpM

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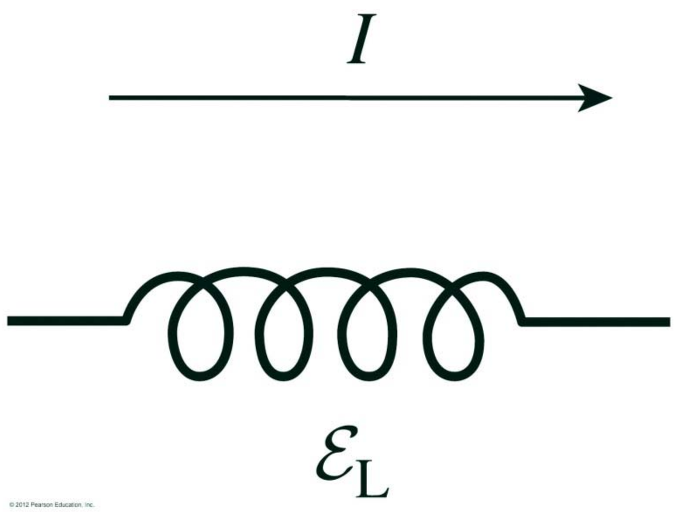
Clicker question 5

- Current flows from left to right through the inductor as shown. A voltmeter connected across the inductor gives a constant reading, and shows that the left end is positive. Is the current in the inductor changing, and if so, how?

A. The current is increasing.
B. The current is decreasing.
C. The current is constant.



Clicker question 8

- Which of the following statements is FALSE?

A. A changing magnetic field can produce an electric current.
B. An emf is induced in a wire by moving the wire near a magnet.
C. An emf is induced in a wire by keeping a stationary magnet near the wire.
D. An emf is induced in a wire by changing the current in that wire.

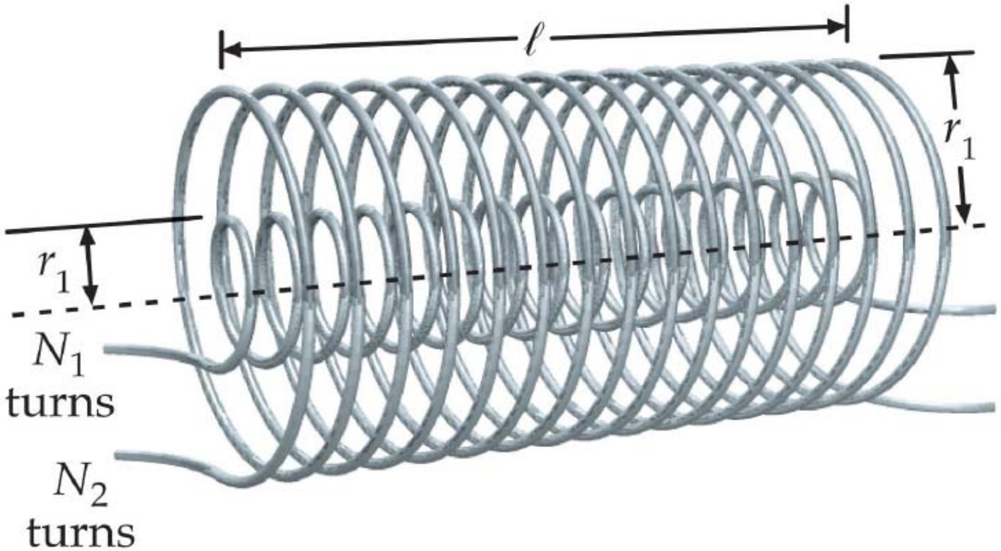
Summary

- **Faraday's law** describes electromagnetic induction, most fundamentally the phenomenon whereby **a changing magnetic field produces an electric field:**

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- This induced electric field is nonconservative and its field lines have no beginnings or endings.
- In the presence of a circuit, the induced electric field gives rise to an induced emf and an induced current.
  - Lenz's law states that the direction of the induced current is such that the magnetic field it produces acts to *oppose* the change that gives rise to it.
  - Self-inductance is a circuit property whereby changing current in a circuit results in an induced emf that opposes the change.
- Consideration of current buildup in an inductor shows that all magnetic fields store energy, with energy density $B^2/2\mu_0$.

Concentric solenoids



- Two concentric solenoids:
- N1 turns and radius r1
- N2 turns and radius r2
- The current in coil 2 is i2.

- Calculate
- The flux density
- The flux linkage of coil 2
- The voltage induced in coil 2
- The self inductance of coil 2
- The flux linkage of coil 1
- The voltage induced in coil 1
- The mutual inductance between coil 1 and 2

Flux density, flux linkage, voltage and self inductance

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

$$Bl = \mu_0 N_2 i_2 \quad \Rightarrow \quad B = \frac{\mu_0 N_2 i_2}{l}$$

$$\Phi_2 = \int \vec{B} \cdot d\vec{A} = \pi r_2^2 B N_2 = \frac{\mu_0 \pi r_2^2 N_2^2}{l} i_2$$

$$u_2 = R_2 i_2 + \frac{d\Phi_2}{dt} = R_2 i_2 + L_2 \frac{di_2}{dt}$$

$$L_2 = \frac{\Phi_2}{i_2} = \frac{\mu_0 \pi r_2^2 N_2^2}{l}$$

Mutual inductance, voltage equations

$$\Phi_1 = \int \vec{B} \cdot d\vec{A} = \pi r_1^2 B N_1 = \frac{\mu_0 \pi r_1^2 N_2 N_1}{l} i_2$$

$$u_1 = \frac{d\Phi_1}{dt} = M \frac{di_2}{dt}$$

$$M = \frac{\Phi_1}{i_2} = \frac{\mu_0 \pi r_1^2 N_1 N_2}{l}$$

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$