



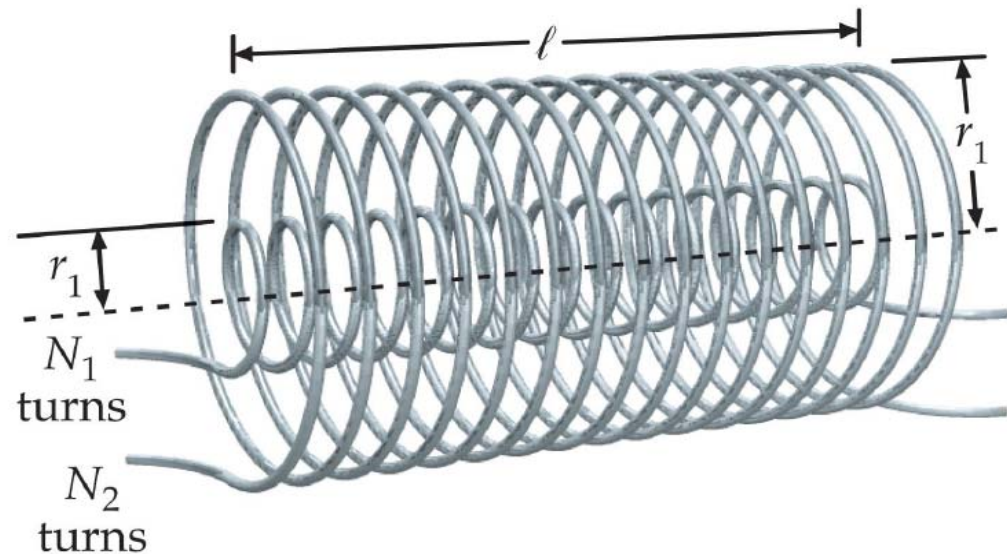
Electricity and Magnetism

Overview Magnetism

- 28-5: Introduction, magnetism: field and force
- 1-6: Magnetism: Biot-Savart, Ampere
- 4-6: Electromagnetic induction
- 8-6: Electromagnetic induction
- 11-6: Maxwell's equations and electromagnetic waves
- 15-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 18-6: available for answering questions, exercises

Concentric solenoids

- Two concentric solenoids:
- N_1 turns and radius r_1
- N_2 turns and radius r_2
- The current in coil 2 is i_2 .



- Calculate
- The flux density
- The flux linkage of coil 2
- The voltage induced in coil 2
- The self inductance of coil 2
- The flux linkage of coil 1
- The voltage induced in coil 1
- The mutual inductance between coil 1 and 2

Flux density, flux linkage, voltage and self inductance

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

$$Bl = \mu_0 N_2 i_2 \quad \Rightarrow \quad B = \frac{\mu_0 N_2 i_2}{l}$$

$$\Phi_2 = \int \vec{B} \cdot d\vec{A} = \pi r_2^2 B N_2 = \frac{\mu_0 \pi r_2^2 N_2^2}{l} i_2$$

$$u_2 = R_2 i_2 + \frac{d\Phi_2}{dt} = R_2 i_2 + L_2 \frac{di_2}{dt}$$

$$L_2 = \frac{\Phi_2}{i_2} = \frac{\mu_0 \pi r_2^2 N_2^2}{l}$$

Mutual inductance, voltage equations

$$\Phi_1 = \int \vec{B} \cdot d\vec{A} = \pi r_1^2 B N_1 = \frac{\mu_0 \pi r_1^2 N_2 N_1}{l} i_2$$

$$u_1 = \frac{d\Phi_1}{dt} = M \frac{di_2}{dt}$$

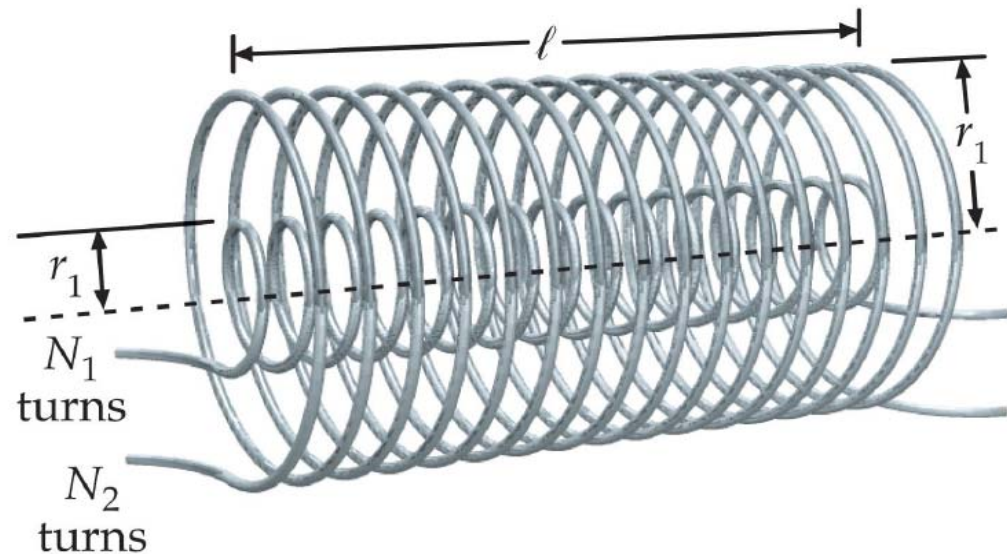
$$M = \frac{\Phi_1}{i_2} = \frac{\mu_0 \pi r_1^2 N_1 N_2}{l}$$

$$u_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

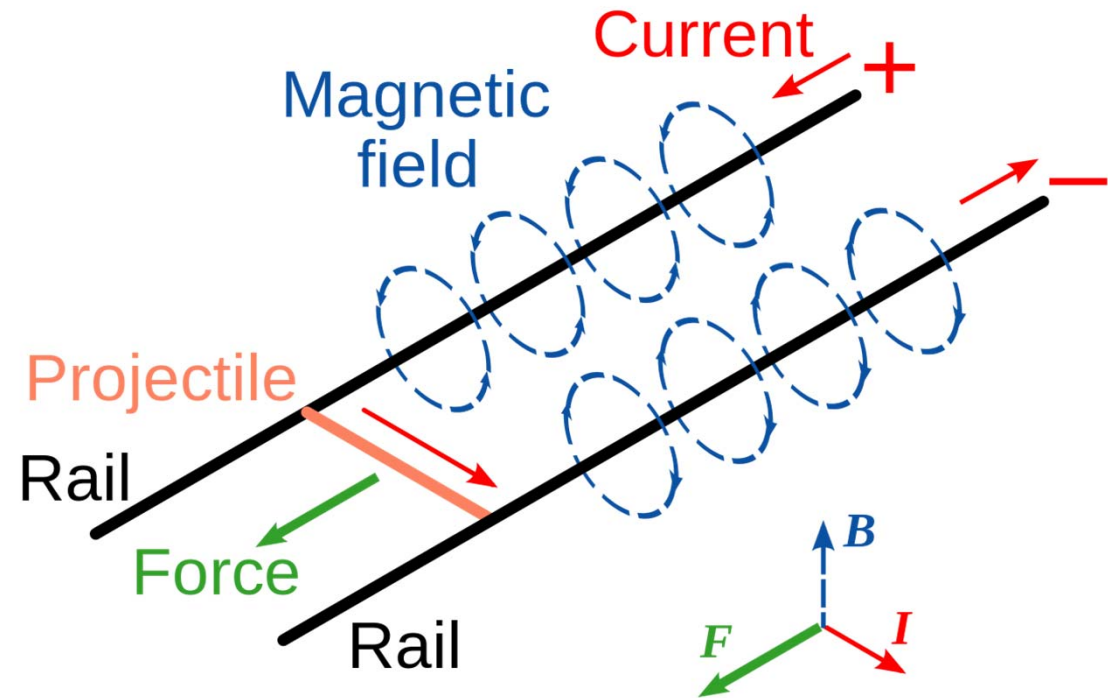
Concentric solenoids

- Two concentric solenoids:
- N_1 turns and radius r_1
- N_2 turns and radius r_2
- The current in coil 2 is i_2 .
- $r_1 = r_2$; $N_1 = N_2$
- Coil 1 is short-circuited and has resistance 0
- Calculate the voltage over coil 2
- Calculate the flux density inside the coil



Railgun

- 2 rails $r = 2 \text{ mm}$
- Distance $d = 14 \text{ mm}$
- Current 1000 A
- The rails are very long



- Calculate
 - The inductance per meter
 - Energy stored in the magnetic field per meter
 - Voltage induced in the loop
 - Energy supplied by the source
 - Energy taken by the projectile

Calculate flux density

- Flux density $2\pi rB = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$
- Force on rails $\frac{F}{l} = \frac{\mu_0 I^2}{2\pi r} = 14.29 \text{ N/m}$

Flux density and force

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I d\vec{L} \times \hat{r}}{r^2} \quad r = \sqrt{x^2 + (d/2)^2}$$

$$\begin{aligned} B &= 2 \int_{-\infty}^{\infty} \frac{\mu_0 I}{4\pi} \frac{d/2}{r^3} dx = \frac{\mu_0 I}{2\pi} \int_{-\infty}^{\infty} \frac{d/2}{(x^2 + (d/2)^2)^{3/2}} dx \\ &= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{4x}{d \sqrt{x^2 + (d/2)^2}} dx = \frac{\mu_0 I}{\pi d} = 28.57 \text{ mT} \end{aligned}$$

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$$F = ILB = 0.2857 \text{ N}$$

$$a = F / m = 28.57 \text{ m/s}^2$$

Flux and inductance

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int_0^l \int_0^d B dr dl = l \int_0^d B dr$$

$$\begin{aligned} \frac{\Phi_B}{l} &= 2 \int_0^d B dr = 2 \int_0^{r_{Cu}} B dr + 2 \int_{r_{Cu}}^d B dr = 2 \int_0^{r_{Cu}} \frac{\mu_0 I r}{2\pi r_{Cu}^2} dr + 2 \int_{r_{Cu}}^d \frac{\mu_0 I}{2\pi r} dr \\ &= \frac{\mu_0 I}{2\pi} + \frac{\mu_0 I}{\pi} \ln\left(\frac{d}{r_{Cu}}\right) = 978 \mu \text{ Wb/ m} \end{aligned}$$

$$\frac{L}{l} = \frac{\Phi_B}{lI} = \frac{\mu_0}{\pi} \left(\frac{1}{2} + \ln\left(\frac{d}{r_{Cu}}\right) \right) = 978 \mu \text{ H/ m}$$

Energy, force from power balance

$$\frac{W_m}{l} = \frac{1}{2} \frac{L}{l} I^2 = \frac{\mu_0 I^2}{2\pi} \left(\frac{1}{2} + \ln \left(\frac{d}{r_{Cu}} \right) \right) = 0.489 \text{ J/m}$$

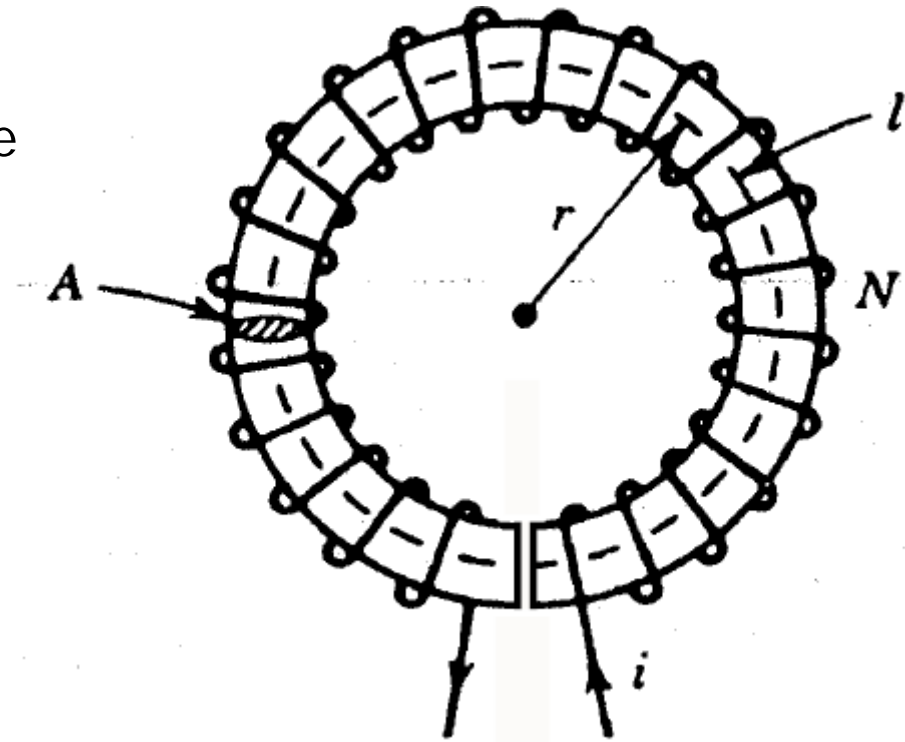
$$\begin{aligned} E &= \frac{d}{dt} \int \vec{B} \cdot d\vec{A} = \frac{dLI}{dt} = I \frac{dx}{dt} \frac{dL}{dx} \\ &= Iv \frac{\mu_0}{\pi} \left(\frac{1}{2} + \ln \left(\frac{d}{r_{Cu}} \right) \right) = v 978 \mu \text{ Vs/m} \end{aligned}$$

$$P_{el} = IE = \frac{\mu_0 I^2 v}{\pi} \left(\frac{1}{2} + \ln \left(\frac{d}{r_{Cu}} \right) \right) = v 978 \text{ mWs/m}$$

$$F = \frac{P_{mech}}{v} = \frac{1}{v} \left(P_{el} - \frac{dW_m}{dt} \right) = 0.489 \text{ N}$$

Toroid with air gap

- Calculate the magnetic flux density in the core and in the gap
- Calculate the flux linked by the coil
- Calculate the inductance



Magnetic field around a conductor

- A conductor carries a current I . This conductor lies in a flat surface that forms the separation between a half space with air and a half space with a linear magnetic material with permeability μ_r . We assume the magnetic field lines to form circles around the conductor.
- Sketch the situation
- Calculate the magnetic flux density
- Does this satisfy the boundary conditions for the magnetic field?