Electricity and Magnetism

Overview Magnetism

- 28-5: Introduction, magnetism: field and force
- 1-6: Magnetism: Biot-Savart, Ampere
- 4-6: Electromagnetic induction
- 8-6: Electromagnetic induction
- 11-6: Maxwell's equations and electromagnetic waves
- 15-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 18-6: available for answering questions, exercises

Summary magnetism force and field

- **Magnetism** involves *moving electric charge.*
- Magnetic fields exert forces on moving electric charges:
  - For a moving charge: $\vec{F} = q\vec{v} \times \vec{B}$
  - For a current: $\vec{F} = I\vec{L} \times \vec{B}$
- Magnetic fields arise from moving electric charge, as described by
  - Biot-Savart law: $\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$
  - Ampère's law: $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$
- Magnetic fields encircle the currents and moving charges that are their sources.
  - Magnetic field lines don't begin or end.
  - This is expressed in Gauss's law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$

Summary electromagnetic induction

- **Faraday's law** describes electromagnetic induction, most fundamentally the phenomenon whereby **a changing magnetic field produces an electric field:**

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- This induced electric field is nonconservative and its field lines have no beginnings or endings.
- In the presence of a circuit, the induced electric field gives rise to an induced emf and an induced current.
  - Lenz's law states that the direction of the induced current is such that the magnetic field it produces acts to *oppose* the change that gives rise to it.
  - Self-inductance is a circuit property whereby changing current in a circuit results in an induced emf that opposes the change.
- Consideration of current buildup in an inductor shows that all magnetic fields store energy, with energy density $B^2/2\mu_0$.

Maxwell’s equations and electromagnetic waves

- Learning objectives
- Maxwell’s equations
  - The four laws of electromagnetism
  - Ambiguity in Ampere’s law
  - Maxwell’s equations
- Electromagnetic waves
- The electromagnetic spectrum
- Producing electromagnetic waves
- Energy and momentum in electromagnetic waves
- Polarization of electromagnetic waves
- Summarizing

In this lecture you'll learn

- To explain the four fundamental equations that describe all electromagnetic phenomena
- To describe electromagnetic waves in terms of
    - Frequency
    - Wavelength
    - Amplitude
    - Speed
- To describe the electromagnetic spectrum
- To explain how electromagnetic waves are produced
- To determine energy and momentum in electromagnetic waves

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Equations of electromagnetism

- The four fundamental equations of electromagnetism
  - Incomplete: Ampère's law holds only for steady currents
- Gauss for the electric field E
  - Charge produces electric field
  - Field lines begin and end on charges or close

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- Gauss for the magnetic field B
  - No magnetic charges
  - Field lines close

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Faraday
  - Changing magnetic flux produces electric field

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

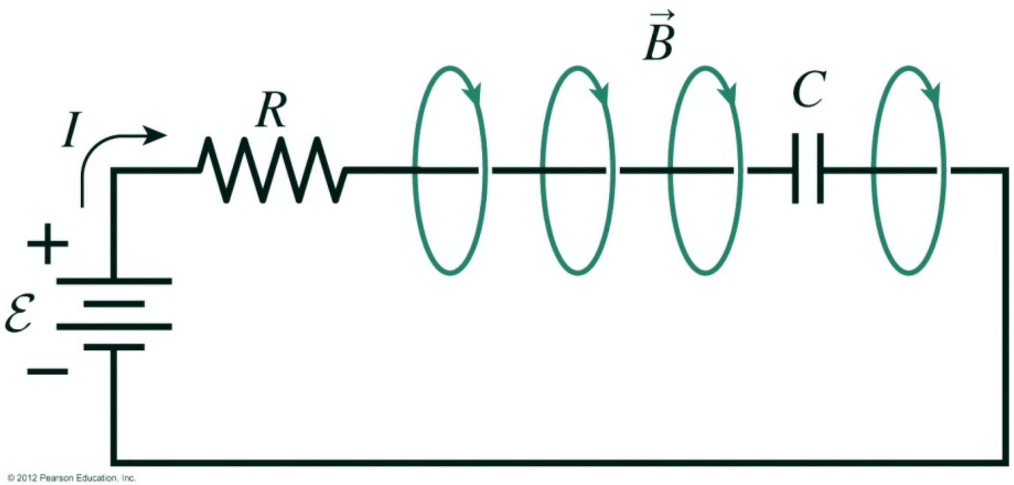
- Ampere
  - Electric currents produces magnetic field

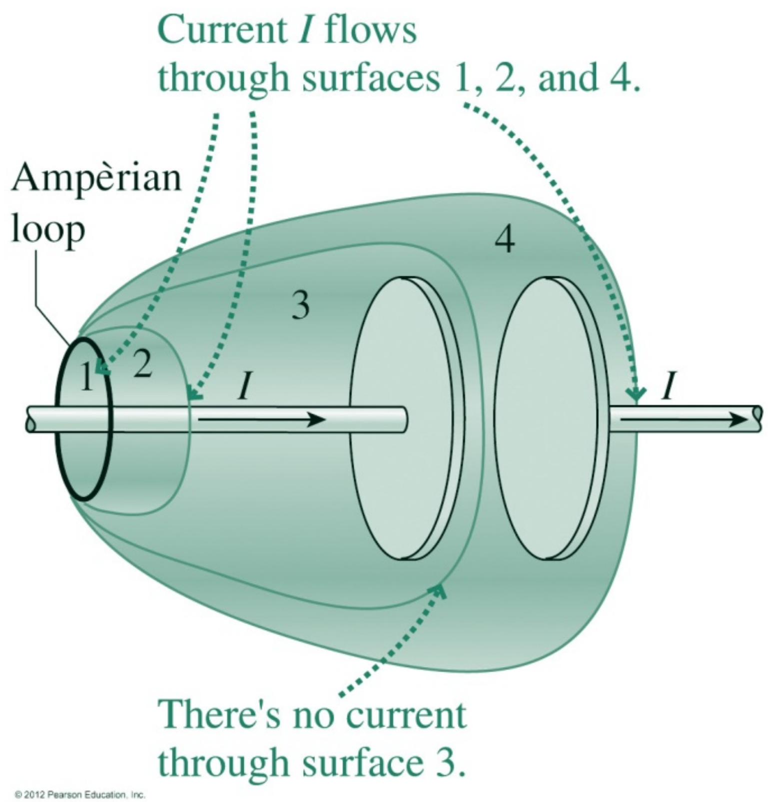
$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

Maxwell’s insight

- Applying Ampère’s law to a circuit with a changing current results in an ambiguity.
  - The result depends on which surface is used to determine the encircled current.
  - Ampere:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$





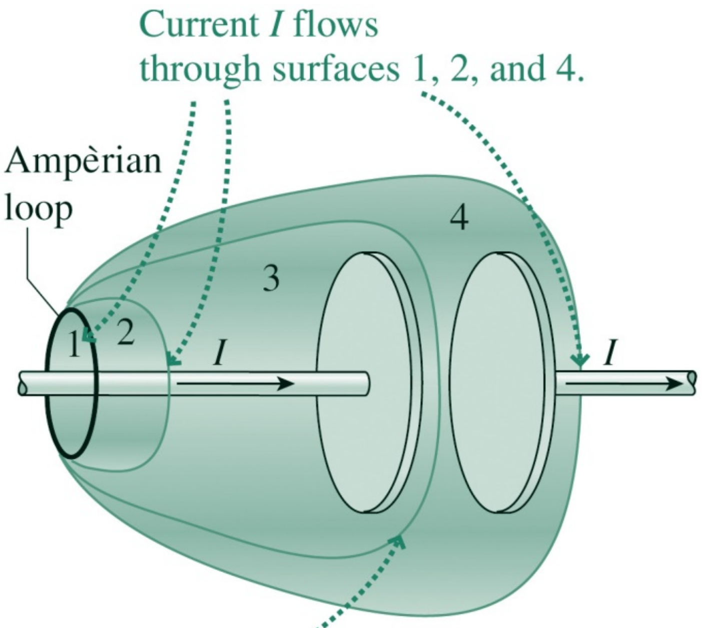
Maxwell’s insight

- Maxwell used this ambiguity, along with symmetry considerations, to conclude that a changing electric field, in addition to current, should be a source of magnetic field.
- Maxwell therefore modified Ampère’s law to read

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- For a capacitor

$$I_d = \varepsilon_0 \frac{dEA}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$



Equations of electromagnetism

- The four complete laws of electromagnetism are collectively called **Maxwell's equations.** They describe all electromagnetic fields in the universe, outside the realm of quantum physics.
- Gauss for the electric field E
  - Charge produces electric field
  - Field lines begin and end on charges or close

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- Gauss for the magnetic field B
  - No magnetic charges
  - Magnetic field lines close

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- Faraday
  - Changing magnetic flux produces electric field

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- Ampere
  - Electric currents and changing electric flux produce magnetic field

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

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Maxwell's equations in vacuum (air)

- In vacuum there's no electric charge and therefore no electric current.
  - Then there's complete symmetry between electric and magnetic fields.
  - Now the only source of either type of field is change in the other type of field.

$$\oint \vec{E} \cdot d\vec{A} = 0 \quad (\text{Gauss}, \vec{E})$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad (\text{Gauss}, \vec{B})$$

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday})$$

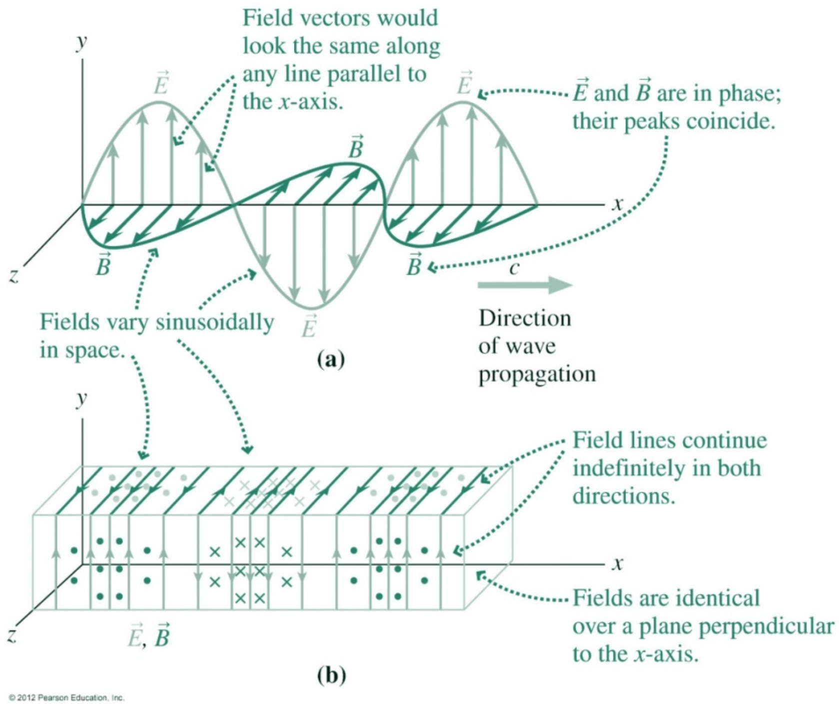
$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad (\text{Ampere})$$

Plane electromagnetic waves

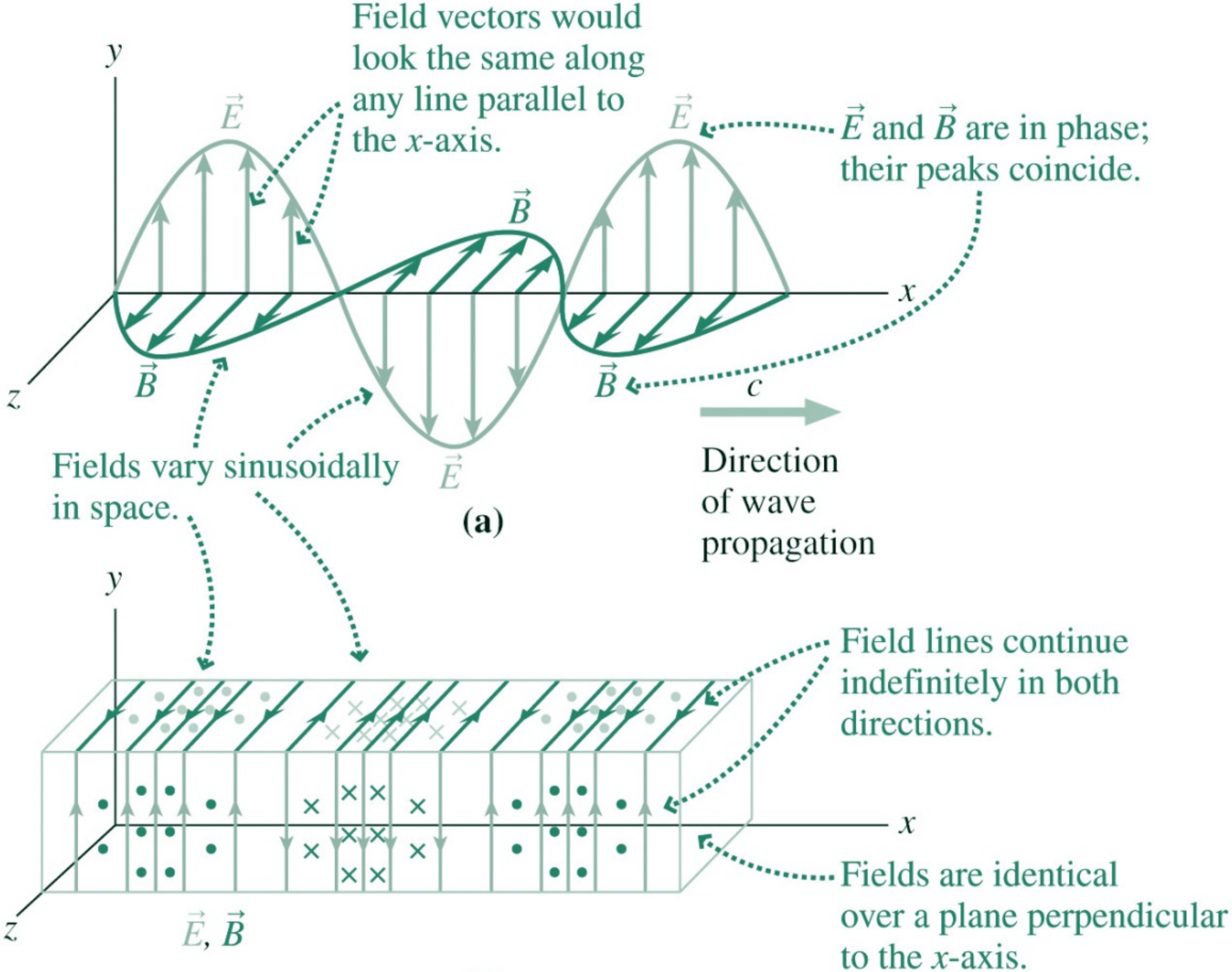
- A plane electromagnetic wave consists of electric and magnetic fields that vary in space only in the direction of the wave propagation.
- The fields are perpendicular to each other and to the direction of propagation.
- Mathematically, the wave fields shown have the form

$$\vec{E}(x,t) = E_p \sin(kx - \omega t)\,\hat{j}$$

$$\vec{B}(x,t) = B_p \sin(kx - \omega t)\,\hat{k}$$



Plane electromagnetic waves



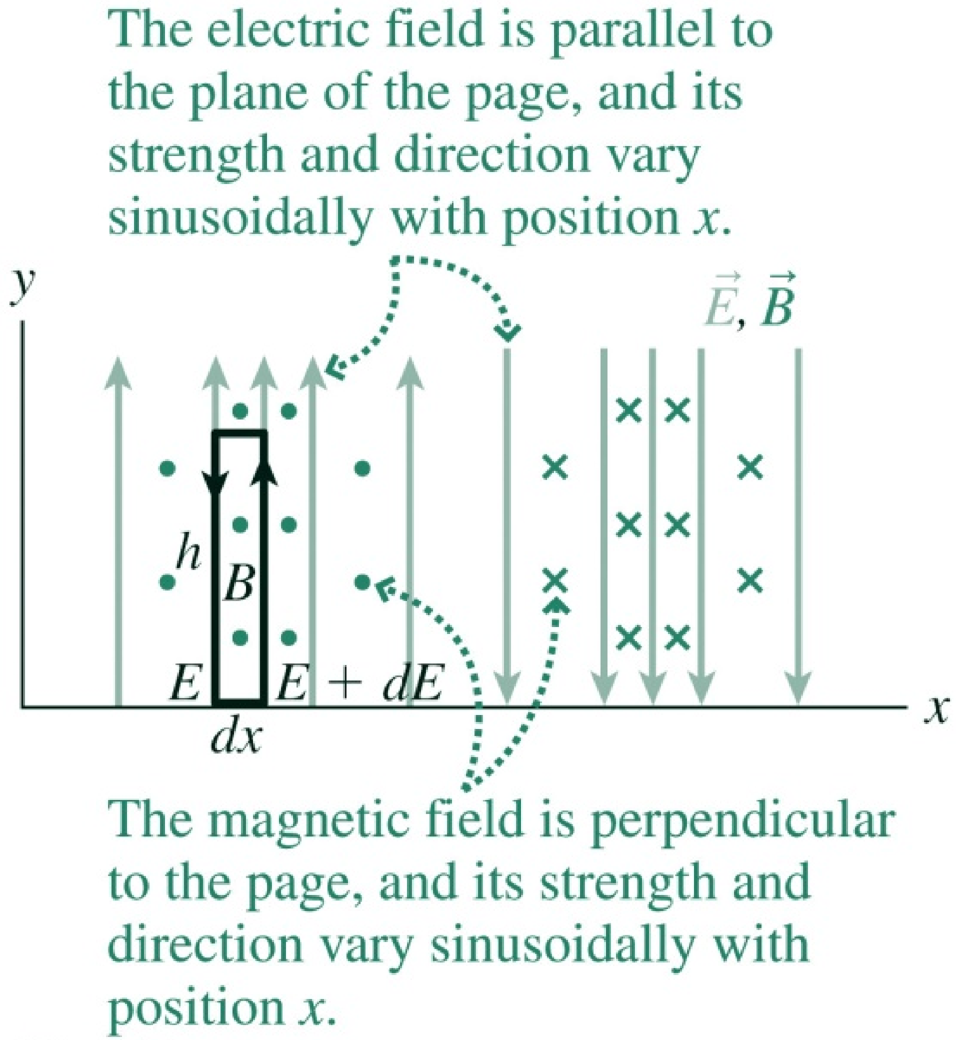
Satisfying Maxwell's equations

- Since the field lines of the EM wave extend to infinity, they have no beginnings or endings and thus satisfy both Gauss's law for electricity and Gauss's law for magnetism.
- Applying Faraday's law in the *x-y* plane gives

$$\oint \vec{E} \cdot d\vec{r} = -Eh + (E + dE)h = hdE$$

$$\frac{d\Phi_B}{dt} = hdx \frac{dB}{dt}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$



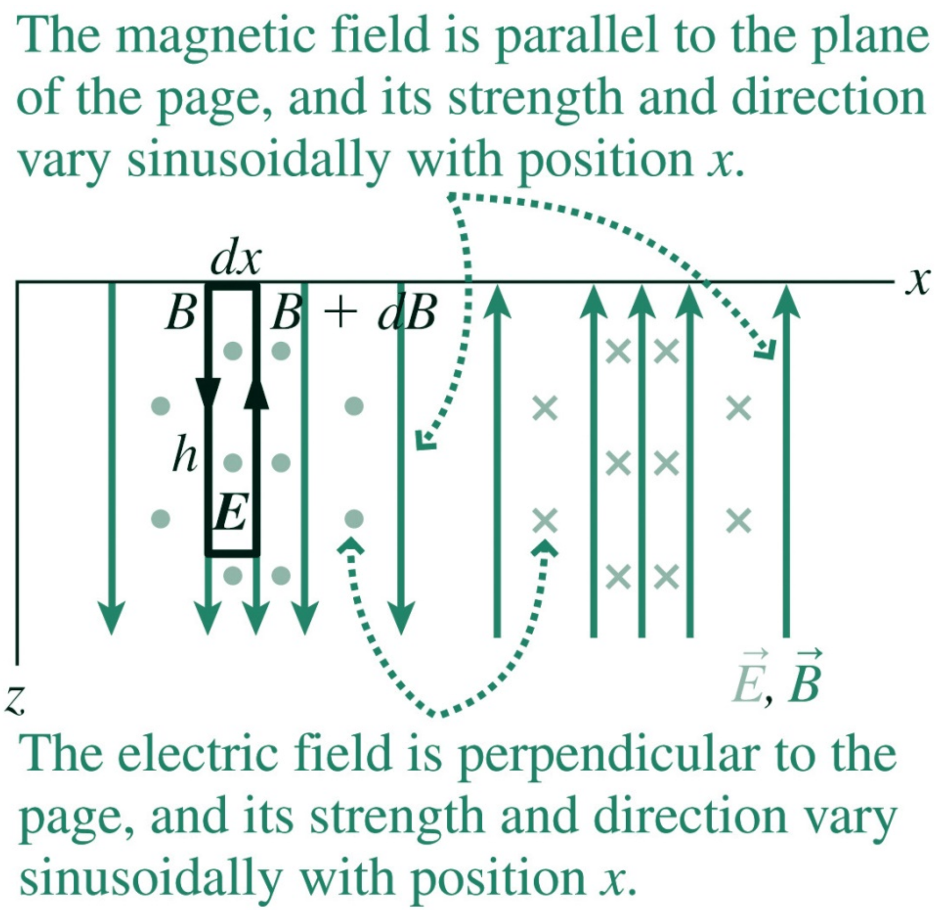
Satisfying Maxwell's equations

- Since the field lines of the EM wave extend to infinity, they have no beginnings or endings and thus satisfy both Gauss's law for electricity and Gauss's law for magnetism.
- Applying Ampère's law in the *x*-*z* plane gives

$$\oint \vec{B}\cdot d\vec{r} = Bh-(B+dB)h = -hdB$$

$$\frac{d\Phi_E}{dt} = hdx\frac{dE}{dt}$$

$$\frac{\partial B}{\partial x} = -\varepsilon_0\mu_0\frac{\partial E}{\partial t}$$



Resulting equations

- Combining the two equations results in

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}\right) = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \frac{\partial}{\partial x}\left(-\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}\right) = -\varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial x}\right) = -\varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right) = \varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

More elegant derivation in vacuum

$$\oint \vec{E}\cdot d\vec{A} = \int \nabla\cdot\vec{E}dV = \frac{1}{\varepsilon_0}\int \rho_f dV \qquad \nabla\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\oint \vec{B}\cdot d\vec{A} = \int \nabla\cdot\vec{B}dV = 0 \qquad \nabla\cdot\vec{B} = 0$$

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} \qquad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\oint \vec{B}\cdot d\vec{r} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} = \mu_0\varepsilon_0\frac{d}{dt}\int \vec{E}\cdot d\vec{A} \qquad \nabla\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\nabla\times\vec{E} = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\nabla^2\vec{E} = -\frac{\partial\nabla\times\vec{B}}{\partial t} = -\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\nabla\times\nabla\times\vec{B} = \nabla(\nabla\cdot\vec{B}) - \nabla^2\vec{B} = -\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial\nabla\times\vec{E}}{\partial t} = -\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

Electromagnetic waves

- Combining the two equations results in

$$\frac{\partial^2 E}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

- Substitution of

$$\vec{E}(x,t) = E_p \sin(kx - \omega t)\,\hat{j}$$

$$\vec{B}(x,t) = B_p \sin(kx - \omega t)\,\hat{k}$$

- Gives

$$k^2 = \varepsilon_0 \mu_0 \omega^2$$

- Therefore

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

Electromagnetic waves

- Both *E* and *B* obey the wave equation with a wave speed given by

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c = 3.00 \cdot 10^8 \text{ m/ s}$$

  - This is precisely the speed of light,
  - Light is an electromagnetic wave!
- Faraday and Ampère also require that $E = cB$.
  - Any wave amplitude is possible if *E* and *B* are related in this way.
  - Any frequency or wavelength is possible, provided

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

$$f\lambda = c$$

Clicker Question 1

- At a particular point, the electric field of an electromagnetic wave points in the $+y$ direction, while the magnetic field points in the $-z$ direction. Which of the following describes the propagation direction?

A. $+x$
B. $-x$
C. Either $+x$ or $-x$, but you can't tell which
D. $-y$

Clicker Question 4

- Which of the following statements is TRUE?

  A. All electromagnetic waves travel at the same speed in vacuum.
  B. Light speeds up when it moves from air into water.
  C. High-frequency electromagnetic waves travel at higher speed in vacuum than do low-frequency electromagnetic waves.
  D. Electrons are a type of electromagnetic wave.

Clicker Question 5

- Two electromagnetic waves are traveling at the same speed. The wave with the higher wavelength

A. has a higher frequency than the other wave.
B. has a lower frequency than the other wave.
C. is traveling faster than the other wave.
D. is traveling slower than the other wave.

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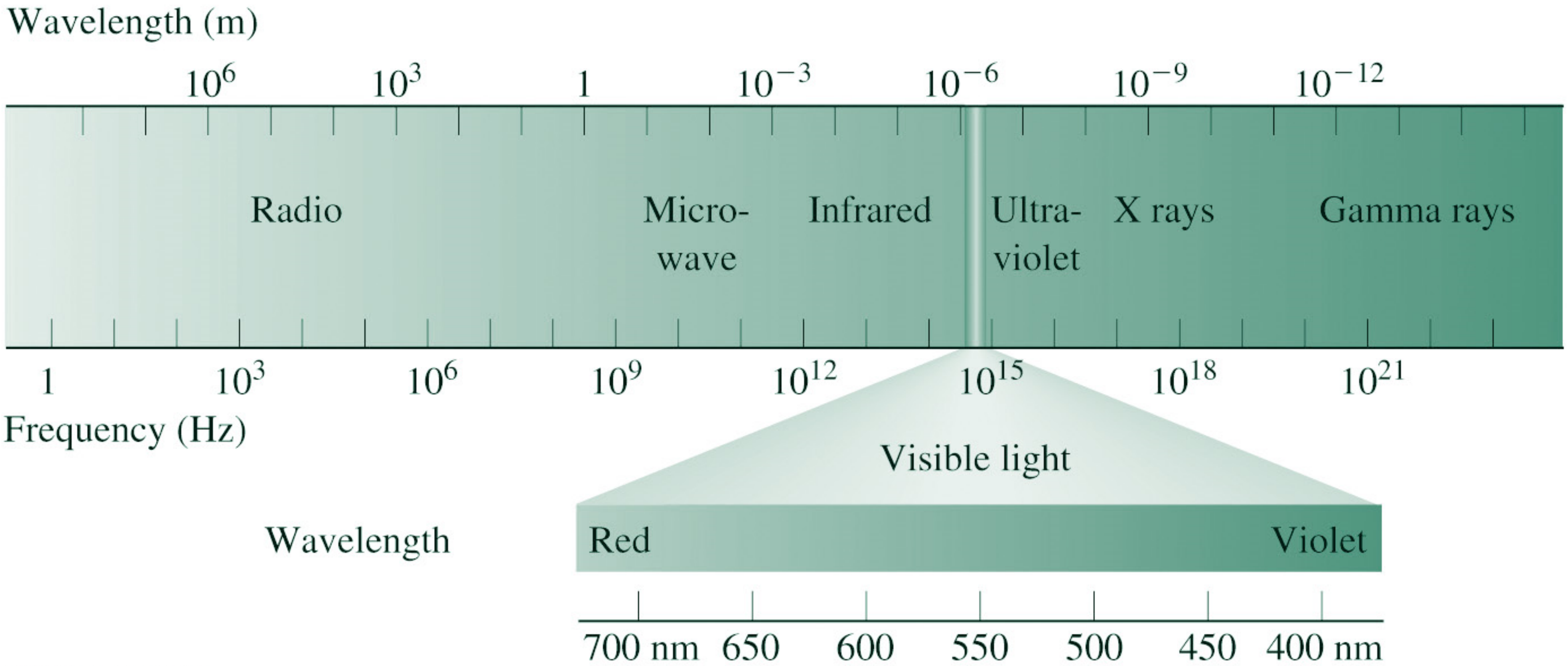
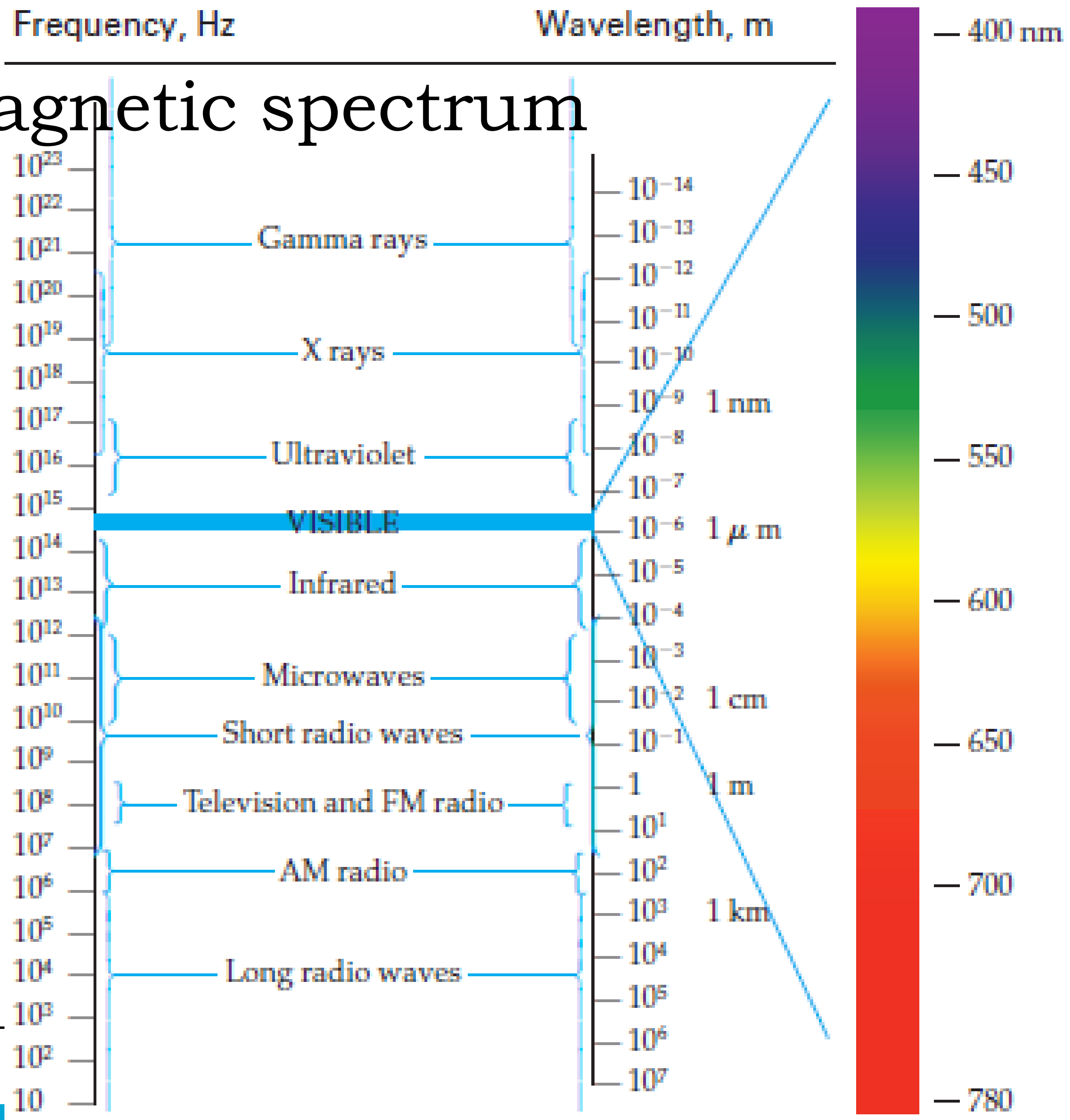
The electromagnetic spectrum

- Electromagnetic waves come in a vast range of frequencies and wavelengths, from low-frequency, long-wavelength radio waves through very high-frequency, short-wavelength gamma rays.
  - Visible light: wavelength range from 380 nm to 780 nm.



Electromagnetic spectrum



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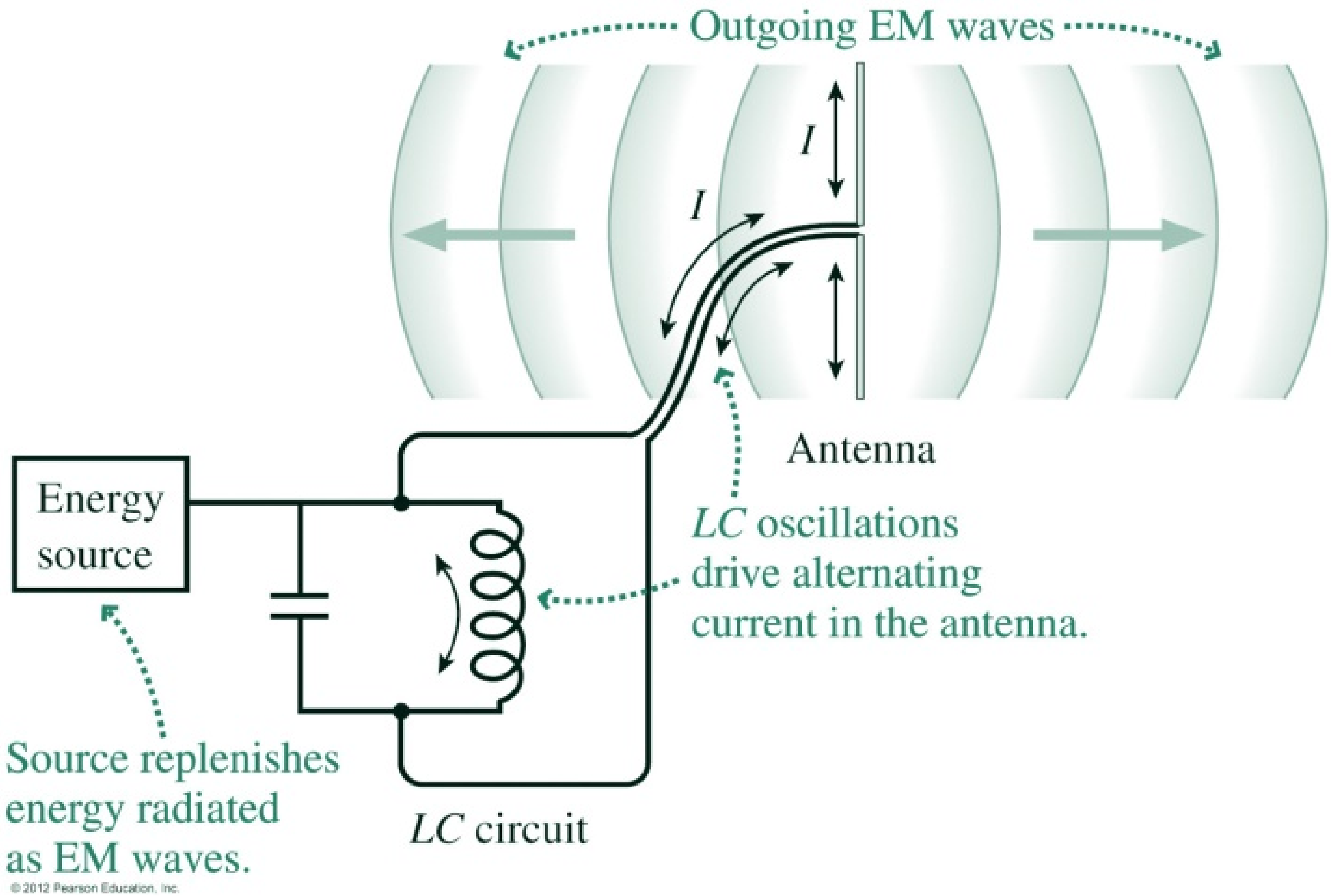
Producing electromagnetic waves

- Electromagnetic waves are generated ultimately by *accelerated electric charge.*
- Details of emitting systems depend on wavelength, with most efficient emitters being roughly a wavelength in size.
    - Radio waves are generated by alternating currents in metal antennas.
    - Molecular vibration and rotation produce infrared waves.
    - Visible light arises largely from atomic-scale processes.
    - X rays are produced in the rapid deceleration of electric charge.
    - Gamma rays result from nuclear processes.

A radio transmitter and antenna



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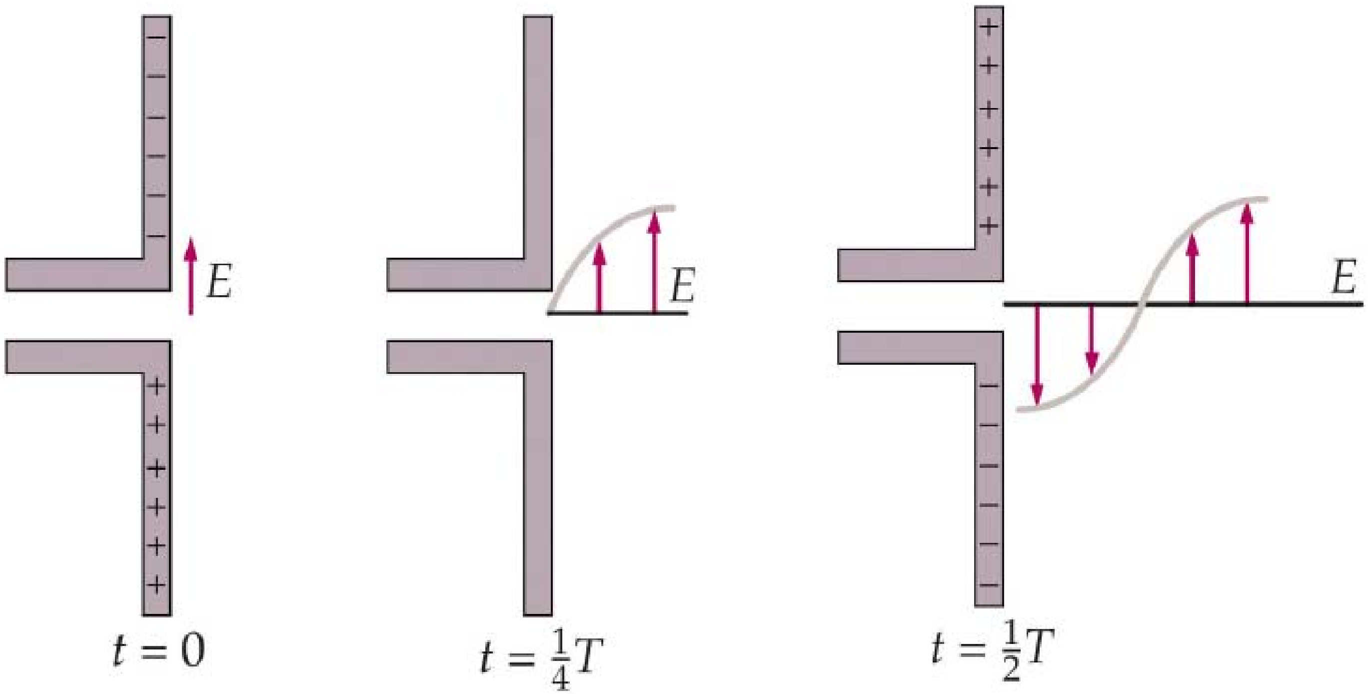
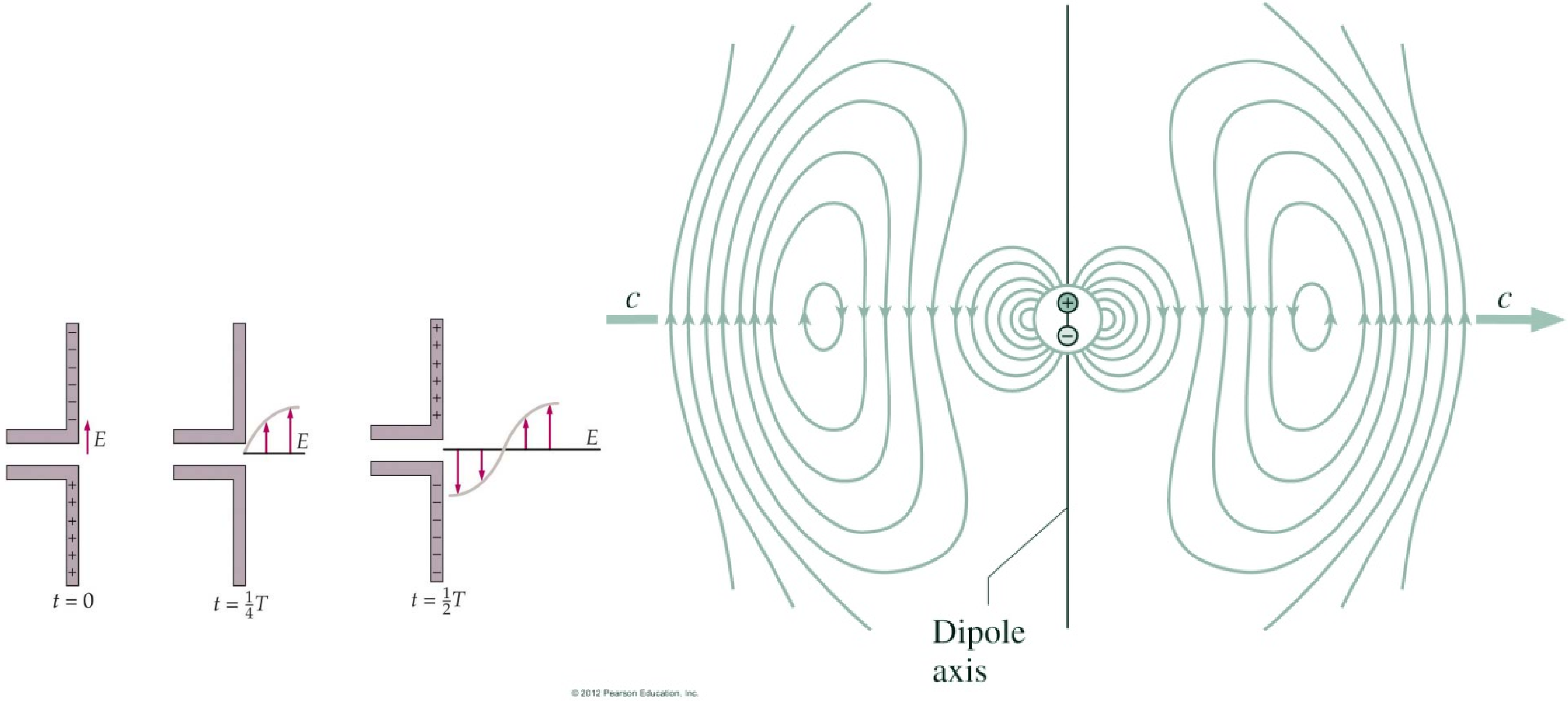
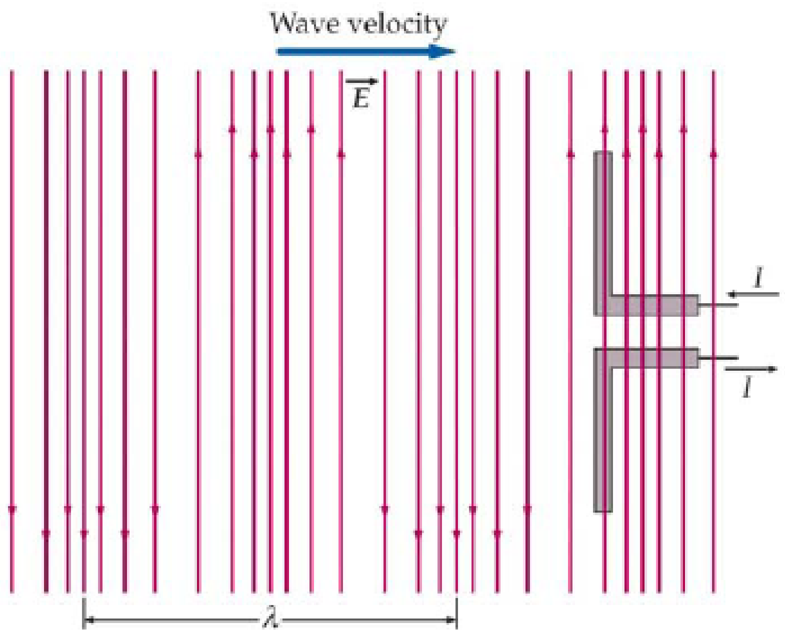
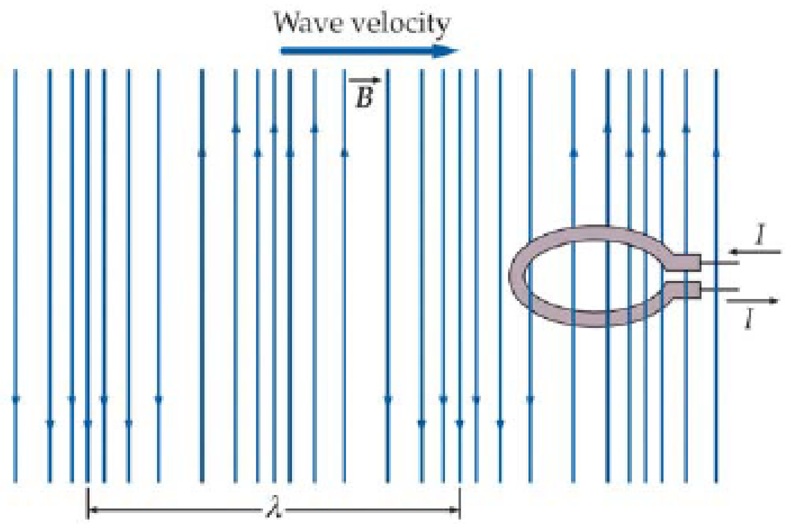


Electric fields of an oscillating electric dipole



Electromagnetic fields and receiving antennas





Clicker Question 6

- An electromagnetic wave is radiated by a straight wire antenna that is oriented vertically. Such a wave could be best detected by

  A. a loop antenna oriented with the plane of the loop horizontal.
  B. a loop antenna oriented with the plane of the loop vertical and parallel to the velocity vector of the wave.
  C. a loop antenna oriented with the plane of the loop perpendicular to the velocity vector of the wave.
  D. a straight wire antenna placed in a horizontal plane.

Clicker Question 7

- Which of the following scenarios does NOT result in emission of electromagnetic radiation?

  A. A charged particle moving in a circle at a constant speed
  B. A charged particle moving in a straight line at constant speed
  C. A stationary solid sphere with its total charge varying in time

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Energy in electromagnetic waves

- Electromagnetic waves carry the energy of their electric and magnetic fields.
- Energy density (J/m^3)

$$u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

- Intensity: energy flow per unit of area (W/m^2)

$$S = c(u_E + u_B) = c\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}\right)$$

- Using

$$E = cB \quad ; \quad \varepsilon_0 \mu_0 c^2 = 1$$

$$S = \frac{c^2 \varepsilon_0 \mu_0 EB + EB}{2\mu_0} = \frac{EB}{\mu_0}$$

Energy in electromagnetic waves

- Electromagnetic waves carry the energy of their electric and magnetic fields.
- The **Poynting vector** describes rate of energy flow per unit area (W/m^2 in SI):

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

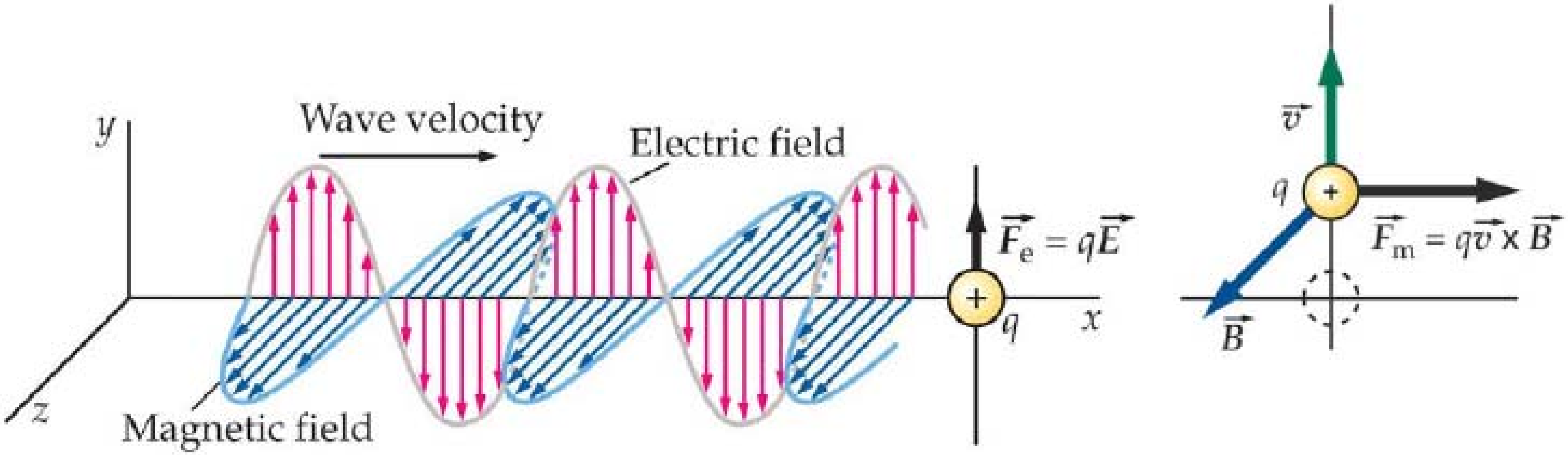
- Averaging over the time variations of the oscillating fields gives the average value, also called the **average intensity**:

$$\overline{S} = \frac{E_p B_p}{2\mu_0}$$

- The intensity of waves from a localized source drops off as the inverse square of the distance from the source, as wave energy spreads over ever-larger areas, where *P* is the total power emitted.

$$S = \frac{P}{4\pi r^2}$$

Momentum in electromagnetic waves



Momentum in electromagnetic waves

- Electromagnetic waves also carry momentum.
  - The wave energy U and momentum p are related by

$$p = U / c$$

  - The wave intensity $\bar{S}$ is the average rate at which the wave carries energy per unit area.
    - The wave carries momentum per unit area at the rate $\bar{S}/c$.

  - This results in **radiation pressure**

$$P_{\text{rad}} = \bar{S}/c$$

on an opaque object, and twice this on a reflective object.

Clicker Question 3

- Lasers 1 and 2 emit light of the same color. The electric field in the beam from laser 1 is twice as strong as the electric field in the beam from laser 2. How do the intensities of the two laser beams compare?

A. $S_1 = S_2$

B. $S_1 = 2S_2$

C. $S_1 = 4S_2$

D. $S_1 = \sqrt{S_2}$

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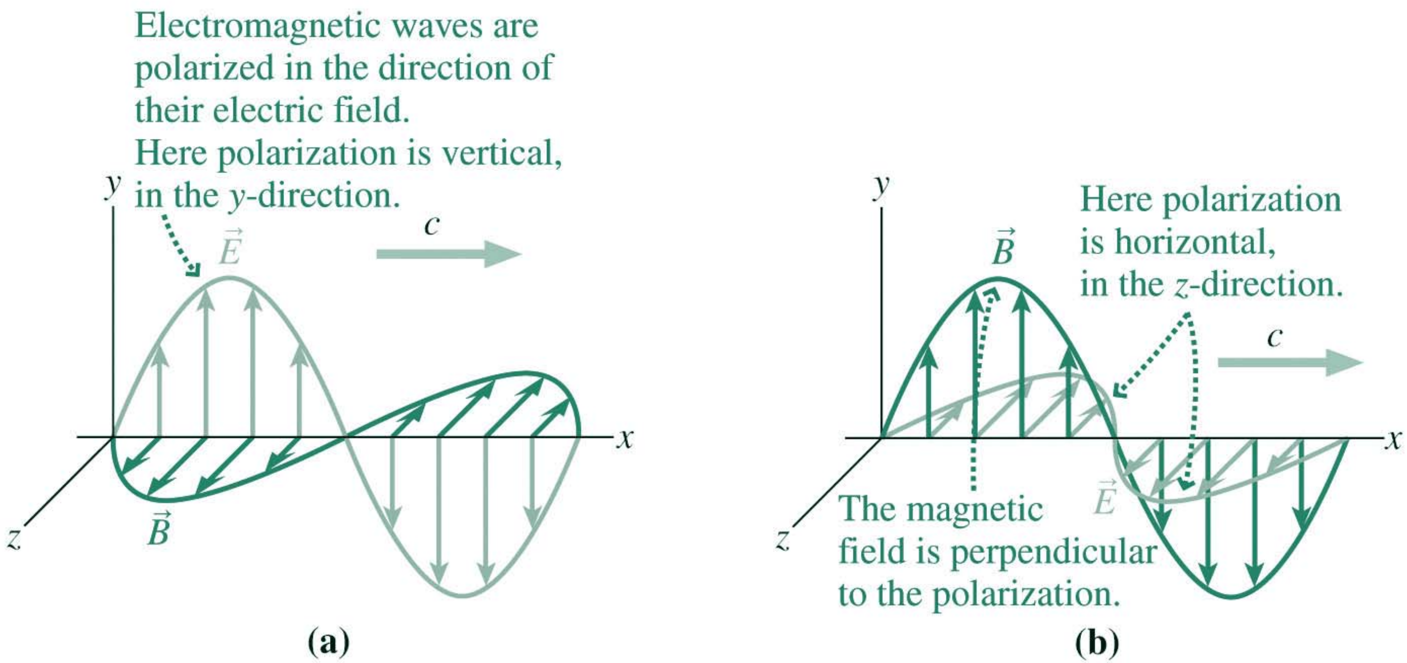
Polarization

- The fields of a vacuum electromagnetic wave are perpendicular to each other and to the direction of propagation.
  - But any orientation in the plane perpendicular to the propagation direction is allowed.
  - The direction of the electric field defines the direction of the wave's **polarization.**

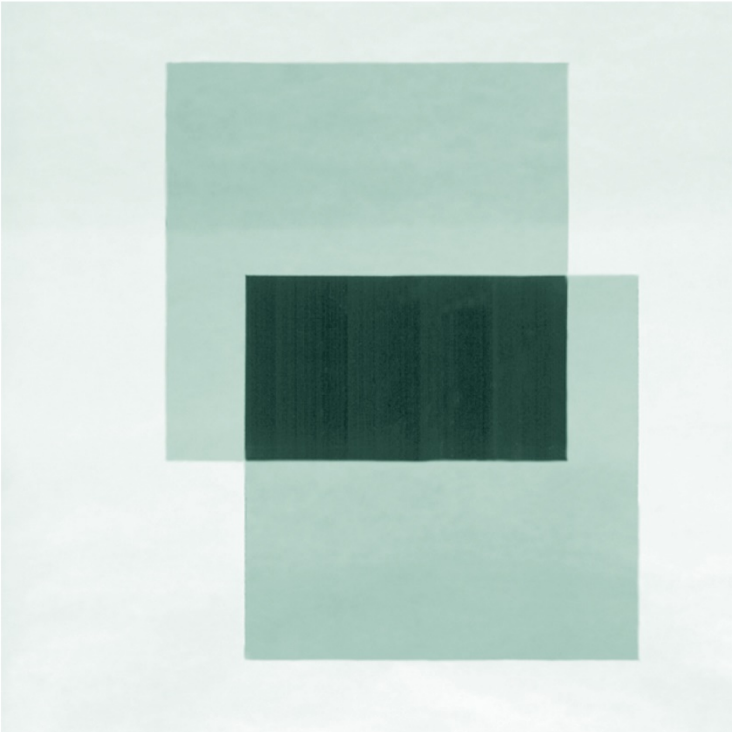


Law of Malus

- Typical sources of light emit a mix of polarizations.
  - Special polarizing materials pass only one polarization.
  - Crossing two polarizers results in no light transmission.
- A wave of intensity S_0 emerges from a polarizer with intensity given by the **Law of Malus**:

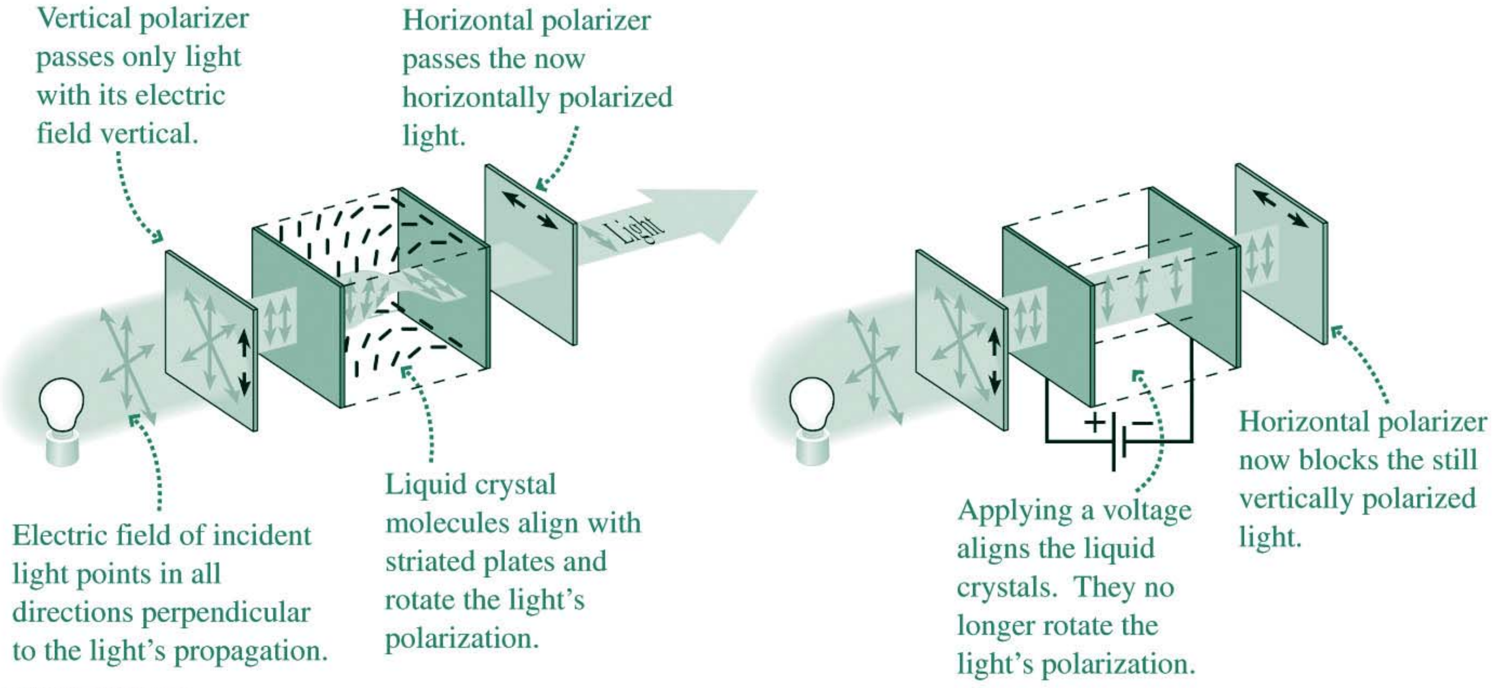
$$S = S_0 \cos^2 \theta$$

  - Polarization has many uses in science and technology.
  - Measuring polarization tells us about sources of electromagnetic waves and about materials through which they propagate.



Liquid crystal displays

- The liquid crystal displays (LCDs) found in computers, televisions, cell phones, iPods, and numerous other devices use liquid crystals to rotate the polarization of light, either blocking the light or allowing it to pass.
- Many individual pixels make up a typical liquid crystal display.



Clicker Question 2

- Two polarizers are oriented at right angles so no light gets through the combination. A third polarizer is inserted between the two, with its preferred direction at 45° to the others. How will this "sandwich" of polarizers affect the intensity of a beam of initially unpolarized light?

A. All of the initial light will be blocked.
B. 7/8 of the initial light will be blocked.
C. 3/4 of the initial light will be blocked.
D. 1/2 of the initial light will be blocked.
E. 1/4 of the initial light will be blocked.
F. None of the initial light will be blocked.

Summary

- Maxwell’s equations describe all electromagnetic fields outside the realm of quantum physics.
  - They include Maxwell’s modification of Ampère’s law to show that a changing electric field produces a magnetic field.
- Maxwell’s equations show that electromagnetic waves are possible; in vacuum plane EM waves have these properties:
  - Wave speed is the speed of light c = 300 Mm/s
  - Wave amplitude $E = cB$
  - Electric and magnetic fields perpendicular to each other and to the direction of propagation
  - Any frequency and wavelength, subject to $f\lambda = c$
  - The range of frequencies/wavelengths comprises the **electromagnetic spectrum.**
  - Electromagnetic waves originate in accelerated charge.
  - Electromagnetic waves carry both energy and momentum.