ELECTRIC FIELD AND DIELECTRIC CONSTANT

1. OUTLINE

- ✓ Electric field
- ✓ Electric field generated by charges and charge densities
- ✓ Dielectric constant

2. ELECTRIC FIELD

The electric field at a certain position is the amount of electric force induced on a unit charge (1 C) if the charge was placed at that position. Therefore, for a particle with charge equal to q, the electric field can be expressed as the force induced on the particle normalized to the value of its charge. In other words:

$$\vec{E}(\vec{r}) = \vec{F}_{net}(\vec{r})\frac{1}{q} \tag{1}$$

where $\vec{F}_{net}(\vec{r})$ is the superposition of all the forces induced on the particle positioned at $\vec{r}$. Using the discussion on the electrostatic force in the previous lecture, the electric field generated by a point charge located at the origin, q, at a certain observation point indicated by $\vec{r}$, can be expressed as

$$\vec{E}(\vec{r}) = \frac{k_e q}{r^2}\hat{r} \tag{2}$$

Using the superposition principle, the electric field at $\vec{r}$, generated by multiple charges can be calculated as

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} \frac{k_e q_i}{|\vec{r} - \vec{r}_i|^2} \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|} \tag{3}$$

where q_i is the i-th charge positioned at $\vec{r}_i$ (Fig. 1), and N is the number of source charges present in the space.

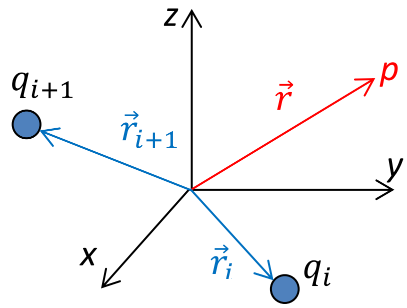

Figure 1. A system of several charges generating a field at an observation point, p.

The electric field is commonly illustrated by field lines. These lines indicate the vectorial direction of the field at each point in the space. Moreover, the spacing of lines describes the magnitude of the field; e.g., where the lines are closer, the field is stronger.

Starting from the equations of the electrostatic force in the previous lecture, for volumetric, surface, and line electric charge distributions the electric field can be written, respectively, as

$$\vec{E}(\vec{r}) = \iiint\limits_V \frac{k_e \rho_v(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2} \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\, d\vec{r}\,'$$

$$\vec{E}(\vec{r}) = \iint\limits_S \frac{k_e \rho_s(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2} \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\, d\vec{r}\,'$$

$$\vec{E}(\vec{r}) = \int\limits_L \frac{k_e \rho_l(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^2} \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\, d\vec{r}\,'$$

- *ELECTRIC DIPOLE*

The dipole configuration of charges refers to two particles with the same charge values and opposite signs which are placed close to each other, Fig. 2. The dipole moment is the product of the vector that indicates the distance between the two charges, $\vec{d}$, and the absolute value of the charges, q, i.e. :

$$\vec{p} = q\vec{d} \tag{4}$$

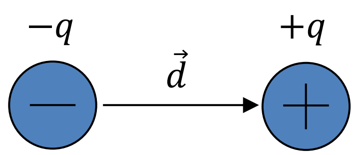

Figure 2. A dipole charge distribution.

By using Eq. (3), one can calculate the field generated by a dipole at a certain observation point, $\vec{E}_{di}(\vec{r})$. When this point is very far away, the following approximations on the equation are valid:

$$|\vec{r}-\vec{r}_i|^2 \simeq r^2 \tag{5}$$

$$\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|} \simeq \hat{r} \tag{6}$$

Therefore, $\vec{E}_{di}(\vec{r})$ can be approximated as

$$\lim_{r\to\infty} \vec{E}_{di}(\vec{r}) \simeq \frac{k_e q}{r^2}\hat{r} - \frac{k_e q}{r^2}\hat{r} = 0 \tag{7}$$

where q is the absolute value of the charges in the dipole, and $\vec{r} = r\hat{r}$ is the observation point vector. From an observation point very far away, the dipole configuration is seen as two opposite charges on top of each other. Such configuration yields zero net charge. Therefore, it is expected that the field generated by them equals to zero.

The field at an observation point at a finite large distance away from the dipole ($r \gg d$) can be calculated using the general expression below

$$\vec{E}_{di}(\vec{r}) = -\frac{k_e q}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1) + \frac{k_e q}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2) \tag{8}$$

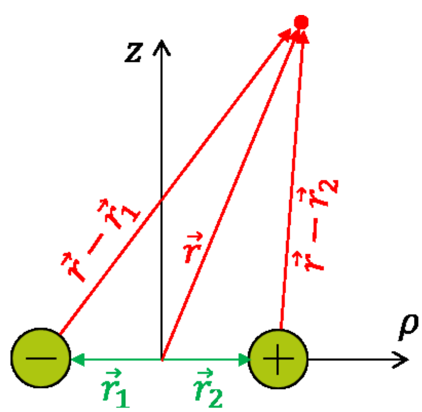

Figure 3. Configuration for calculating the electric field generated by a dipole at a certain finite distance.

where $\vec{\boldsymbol{r}}_1$ and $\vec{\boldsymbol{r}}_2$ indicate the position of the negative and positive charge in the space, respectively, (Fig. 3). One can represent $\vec{\boldsymbol{r}}_1$ and $\vec{\boldsymbol{r}}_2$ in terms of $\vec{\boldsymbol{d}}$ as

$$\vec{r}_1 = -\vec{d}/2$$

$$\vec{r}_2 = \vec{d}/2$$

Eq. (8) can then be rewritten as

$$\vec{E}_{di}(\vec{r}) = k_e q \left(\frac{\left(\vec{r} - \frac{\vec{d}}{2}\right)}{\left|\vec{r} - \frac{\vec{d}}{2}\right|^3} - \frac{\left(\vec{r} + \frac{\vec{d}}{2}\right)}{\left|\vec{r} + \frac{\vec{d}}{2}\right|^3} \right) \tag{9}$$

When $r \gg d$ one can use the following approximations

$$\begin{gathered} \frac{1}{\left|\vec{r} - \frac{\vec{d}}{2}\right|^3} \approx r^{-3}\left(1 + \frac{3\,\vec{r}\cdot\vec{d}}{2\,r^2}\right) \\ \frac{1}{\left|\vec{r} + \frac{\vec{d}}{2}\right|^3} \approx r^{-3}\left(1 - \frac{3\,\vec{r}\cdot\vec{d}}{2\,r^2}\right) \end{gathered} \tag{10}$$

Using the approximations in Eq. (10), Eq (9) can then be rewritten as

$$\vec{E}_{di}(\vec{r}) \simeq k_e q \left(\frac{\left(\vec{r} - \frac{\vec{d}}{2}\right)}{r^3}\left(1 + \frac{3\vec{r}\cdot\vec{d}}{2r^2}\right) - \frac{\left(\vec{r} + \frac{\vec{d}}{2}\right)}{r^3}\left(1 - \frac{3\vec{r}\cdot\vec{d}}{2r^2}\right) \right) = \frac{k_e}{r^3}\left((-q\vec{d}) + \frac{3\vec{r}\cdot q\vec{d}}{r^2}\vec{r} \right)$$

and using Eq. (4) one obtains the expression for the field of a dipole at finite large distance ($r \gg d$)

$$\vec{E}_{di}(\vec{r}) \simeq \frac{k_e}{r^3}\left(-\vec{p} + \frac{3\vec{r}\cdot\vec{p}}{r^2}\vec{r} \right) \tag{11}$$

3. DIELECTRICS

A medium consists of electrons and protons. In a dielectric medium the electrons are bound and cannot move freely as in a conductor medium. Moreover, a dielectric is electrically neutral when no external electric field is present. However, when such a field is present, the shape of the atoms are distorted and electrons and protons will be slightly separated. This separation leads to the presence of a small electric field inside the medium which opposes to the external one (Fig. 4). The total electric field can be represented as

$$\vec{E}_{tot} = \vec{E}_{ext} + \vec{E}_p \tag{10}$$

where $\vec{E}_{ext}$ is the external field, and $\vec{E}_p$ is the field generated by the distortion in the dielectric. For simplicity, one can assume that the external field is polarized along the x direction and caused by a certain charge, q, i.e.: $\vec{E}_{ext} = E_{ext}\hat{x} = \frac{q}{4\pi\epsilon_0 r^2}\hat{x}$.

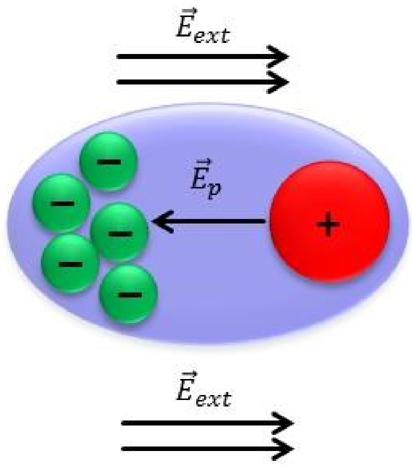

Figure 4. A distorted dielectric atom which represents a dipole distribution.

Since, the internal field is opposing to the external one, it is also possible to represent the averaged internal field polarized along x, i.e. $\vec{E}_p^{ave} = E_p^{ave}\hat{x}$. It is also worth noting that since $\hat{E}_p^{ave} = -\hat{E}_{ext}$, E_p^{ave} is a negative value. Using Eq. (10), one can calculate the average total field in the dielectric as

$$\vec{E}_{tot}^{ave} = \vec{E}_{ext} + \vec{E}_p^{ave} = E_{ext}\left(1 + \frac{E_p^{ave}}{E_{ext}}\right)\hat{x} = \frac{E_{ext}}{\epsilon_r}\hat{x} \tag{11}$$

where $\epsilon_r = \frac{E_{ext}}{E_{ext}+E_p^{ave}} > 1$ is the relative dielectric constant which represents the ratio between the averaged internal and external fields. Using the expression for $\vec{E}_{ext}$, Eq. (11) can reformulated as

$$\vec{E}_{tot}^{ave} = \frac{E_{ext}}{\epsilon_r}\hat{x} = \frac{q}{4\pi(\epsilon_0\epsilon_r)r^2}\hat{x} \tag{12}$$

Therefore, the dielectric materials can be characterized by ϵ_r. When the dielectric is homogeneously characterized by its relative dielectric constant ϵ_r, the Coulomb's law can also be used for the electric fields inside dielectrics, if the term $\frac{1}{4\pi\epsilon_0\epsilon_r}$ is used instead of $k_e = \frac{1}{4\pi\epsilon_0}$. This is only valid when the whole medium has a homogenized structure, i.e. ϵ_r is constant in respect to space coordinates.