Electricity and Magnetism

Overview Magnetism

- 28-5: Introduction, magnetism: field and force
- 1-6: Magnetism: Biot-Savart, Ampere
- 4-6: Electromagnetic induction
- 8-6: Electromagnetic induction
- 11-6: Maxwell's equations and electromagnetic waves
- 15-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 18-6: available for answering questions, exercises

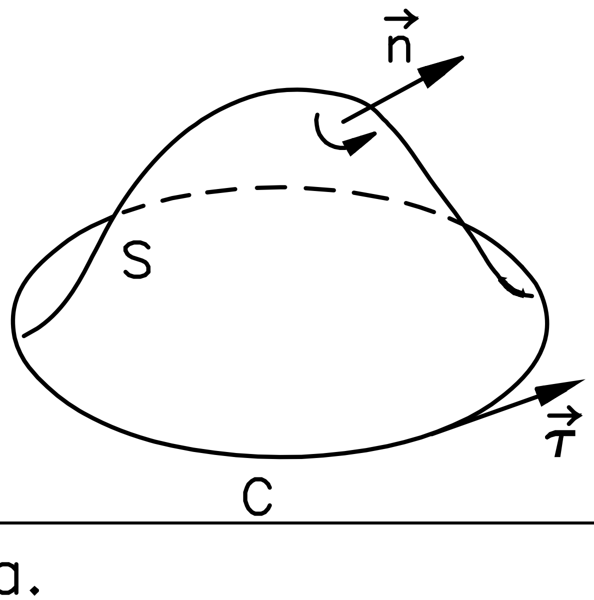
Summary magnetism force and field

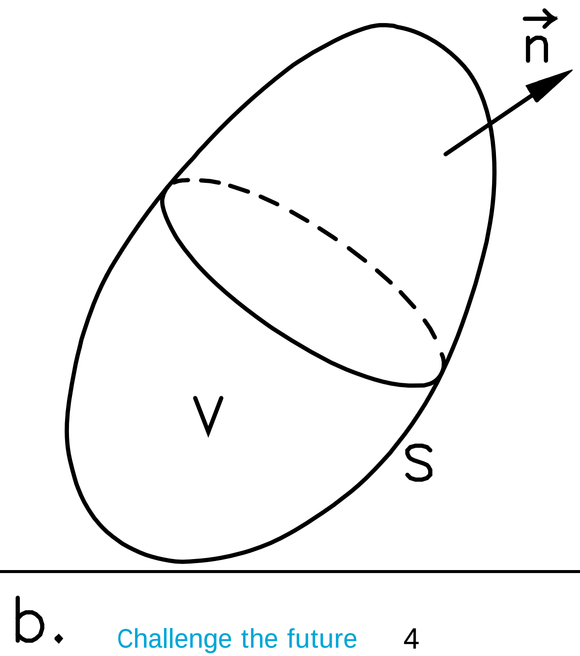
- **Magnetism** involves *moving electric charge.*
- Magnetic fields exert forces on moving electric charges:
  - For a moving charge: $\vec{F} = q\vec{v} \times \vec{B}$
  - For a current: $\vec{F} = I\vec{L} \times \vec{B}$
- Magnetic fields arise from moving electric charge, as described by
  - Biot-Savart law: $\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$
  - Ampère's law: $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$
- Magnetic fields encircle the currents and moving charges that are their sources.
  - Magnetic field lines don't begin or end.
  - This is expressed in Gauss's law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$

Ampère's law $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$

- What is the direction of the vector $d\vec{r}$

A. In the same direction as the contour
B. Normal to the surface that has the contour as a boundary





Gauss's law $\oint \vec{B} \cdot d\vec{A} = 0$

- What is the direction of the vector $d\vec{A}$

A. In the same direction as the contour
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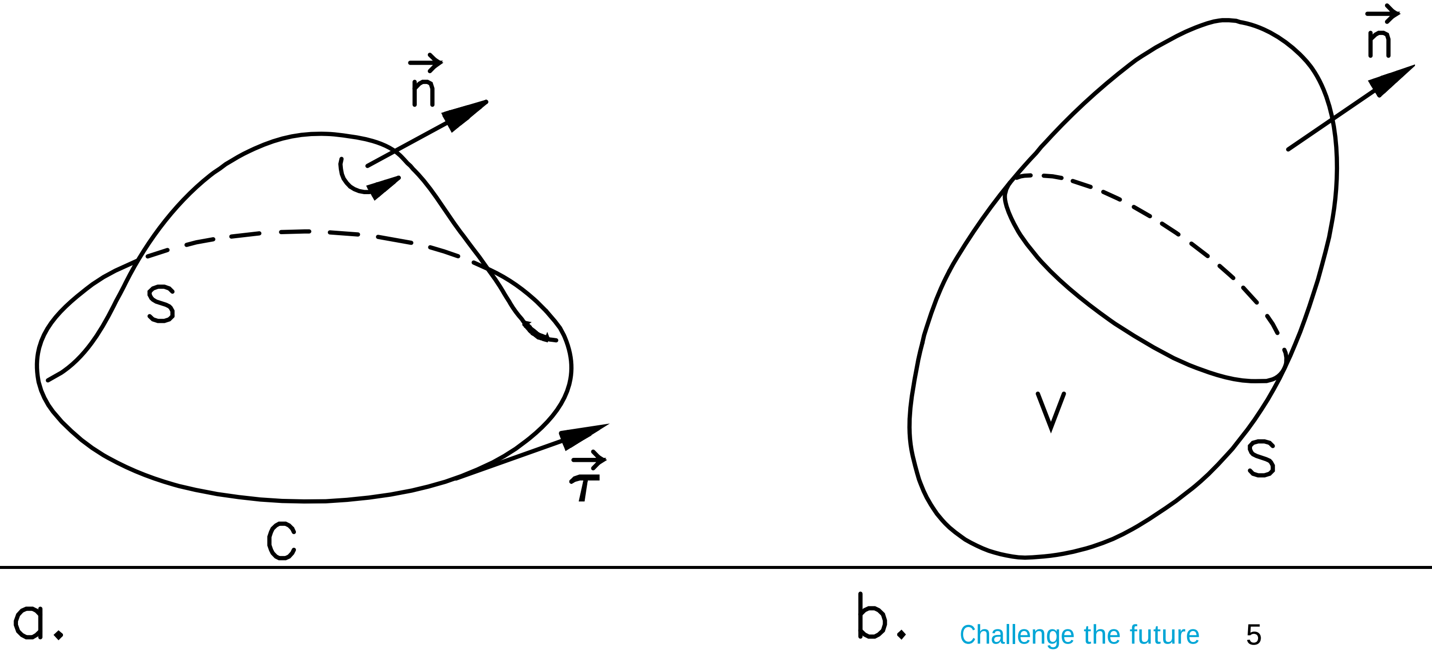
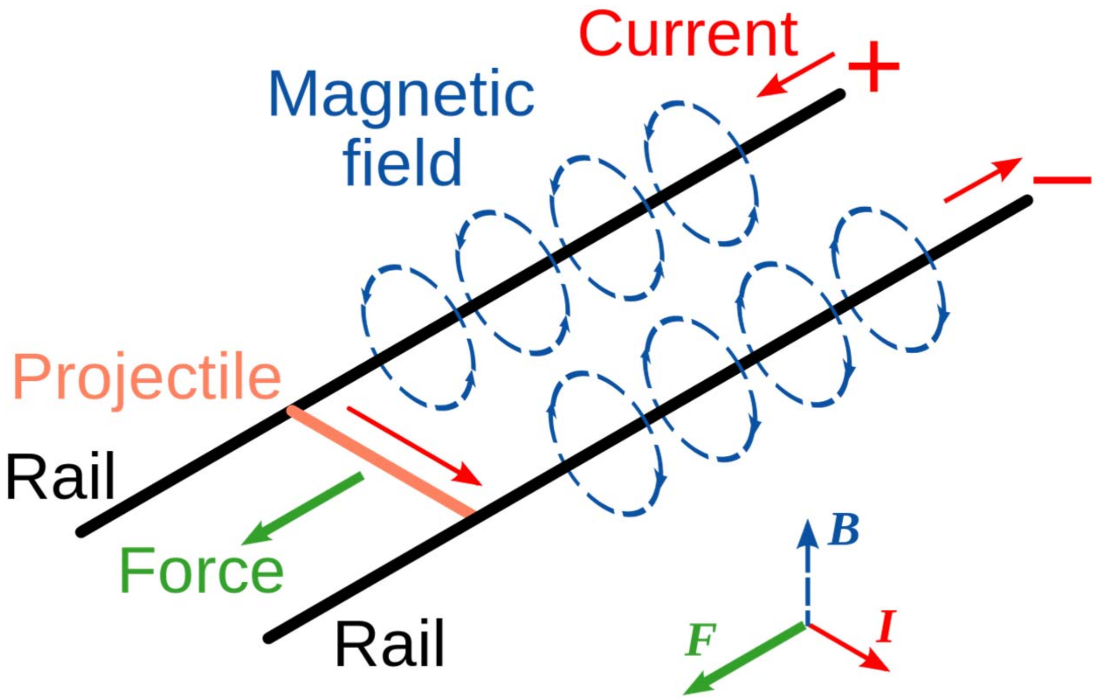


Railgun



- 2 rails r = 2 mm
- Distance d = 14 mm
- Current 1000 A
- The rails are very long

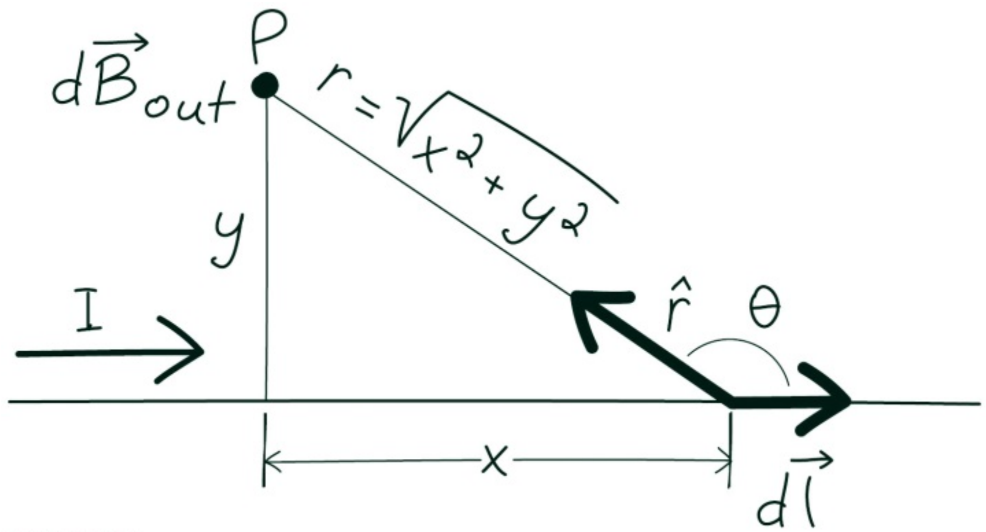
- Calculate
  - the magnetic flux density between the conducting rails
  - the magnetic force per meter of the rails on each other
  - the magnetic flux density in the middle of the projectile (Biot-Savart)
  - the magnetic force on the projectile
  - the acceleration of a 10 g projectile

Calculate flux density

- Flux density $$2\pi r B = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$
- Force on rails $$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi r} = \frac{\mu_0 I^2}{2\pi d} = 14.29\,\mathrm{N/\,m}$$

Flux density of one rail

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$$



$$r = \sqrt{x^2 + y^2}$$

$$B = \int_0^\infty \frac{\mu_0 I}{4\pi} \frac{y}{r^3} dx = \frac{\mu_0 I y}{4\pi} \int_0^\infty \frac{1}{(x^2 + y^2)^{3/2}} dx$$

$$= \frac{\mu_0 I y}{4\pi} \left. \frac{x}{y^2 \sqrt{x^2 + y^2}} \right|_0^\infty = \frac{\mu_0 I}{4\pi y}$$

Force and acceleration

- Flux density due to one rail (Ampere)

$$B = \frac{\mu_0 I}{4\pi y} = \frac{\mu_0 I}{2\pi d} = 14.29\,\mathrm{mT}$$

- Flux density due to both rails

$$B = \frac{\mu_0 I}{2\pi y} = \frac{\mu_0 I}{\pi d} = 28.57\,\mathrm{mT}$$

- Force and acceleration - inaccurate because flux density is not constant over the width of the projectile!!

$$F = ILB = 0.2857\,\mathrm{N} \qquad a = F/m = 28.57\,\mathrm{m/s^2}$$

Electromagnetic induction

- Learning objectives
- Induced currents
- Faraday’s law
- Induction and energy
- Inductance
- Magnetic energy
- Concluding

In this lecture you'll learn

- To explain the phenomenon of electromagnetic induction
- To calculate induced emfs and currents
  - To use energy conservation to find the direction of induced effects
- To describe important technological applications of induction
- To explain inductance
  - And describe the role of inductance in simple circuits
- That magnetic fields store energy
  - And how to calculate that energy
- To recognize Faraday's law as one of the four fundamental laws of electromagnetism
  - And to calculate induced electric fields
- Time varying effects, not completely stationary!!!

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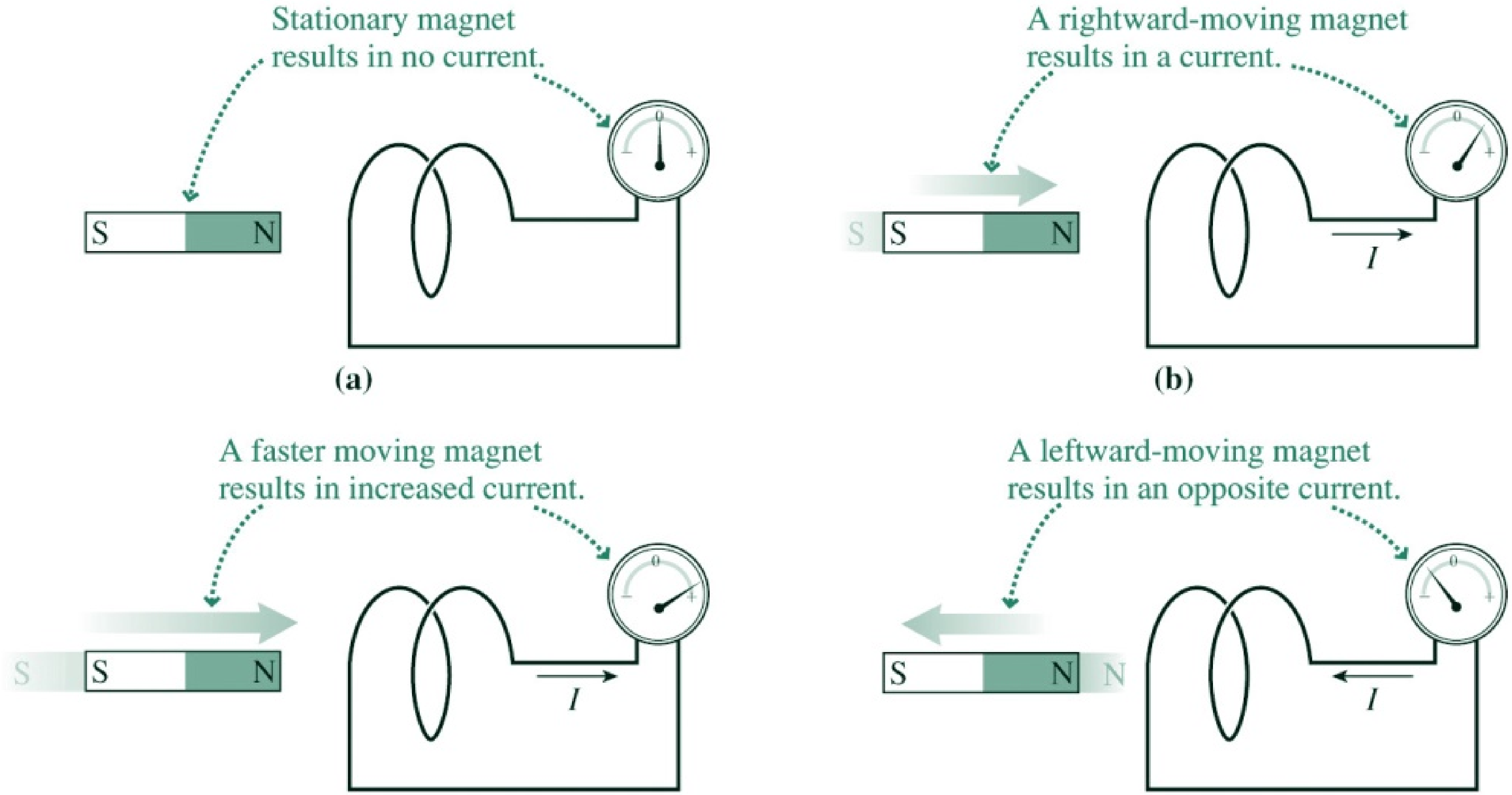
Electromagnetic induction

- https://www.youtube.com/watch?v=vwIdZjjd8fo

Electromagnetic induction

- **Electromagnetic induction** involves electrical effects due to *changing magnetic fields.*
- Simple experiments that result in induced current:
- Move a magnet near a circuit:



Electromagnetic induction

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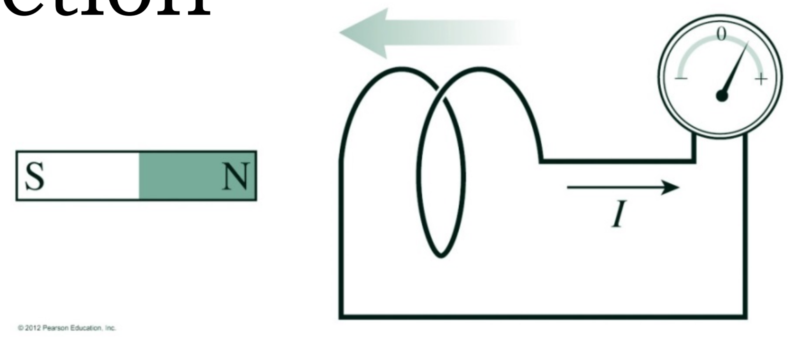
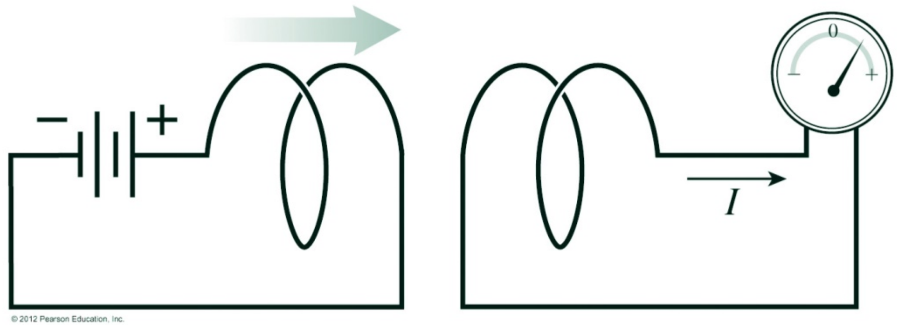
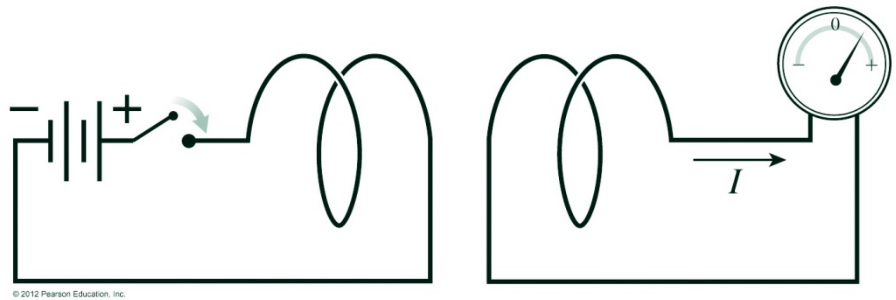


- Simple experiments:
- Move a circuit near a magnet
- Energize one coil to make it an electromagnet; move it near a circuit, or hold it stationary and move a circuit near it



- Change the current in one circuit, and thus the magnetic field it produces



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Faraday's law

- **Faraday's law** describes induction by relating the emf induced in a circuit to the rate of change of magnetic flux through the circuit:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

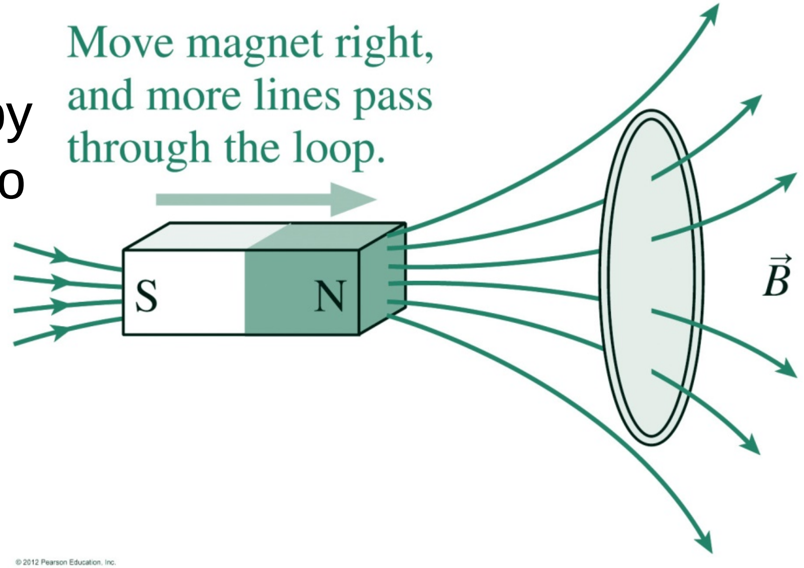
- where the **magnetic flux** is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- With a flat area and uniform field, this becomes

$$\Phi_B = BA\cos\theta$$

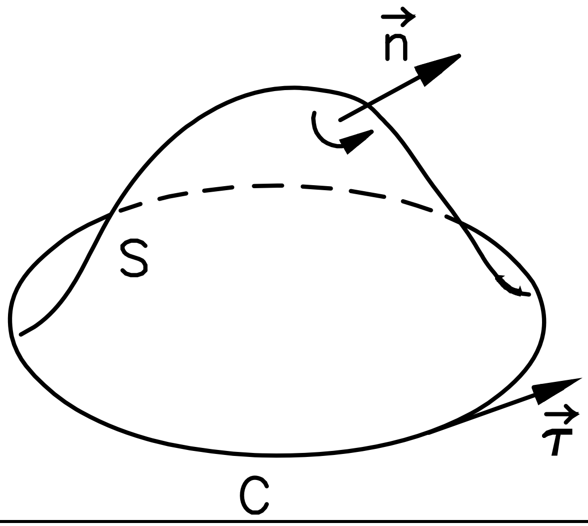
- The flux can change by changing the field *B*, the area *A*, or the orientation θ.

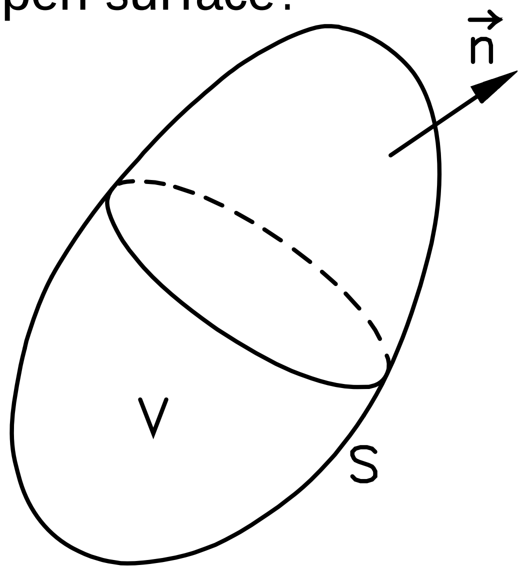


Moving a magnet near a wire loop increases the flux through the loop. The result is an induced emf given by Faraday's law. The induced emf drives an induced current in the loop.

Magnetic flux

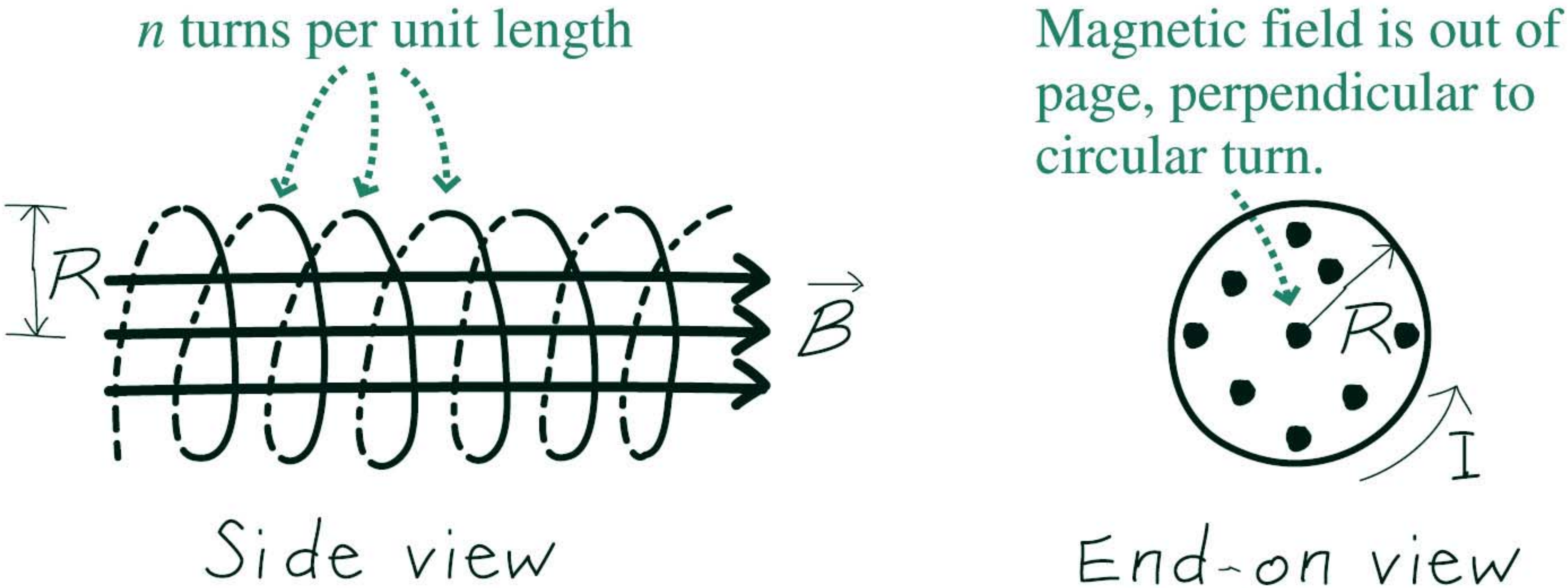
- Flux: open surface integral: $\Phi_B = \int \vec{B} \cdot d\vec{A}$
- Closed surface integral (Gauss): $\oint \vec{B} \cdot d\vec{A} = 0$
- Unit flux: 1 Wb =1 Tm^2
- Does it make a difference how we choose the open surface?





Magnetic flux per turn

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \pi R^2 B$$



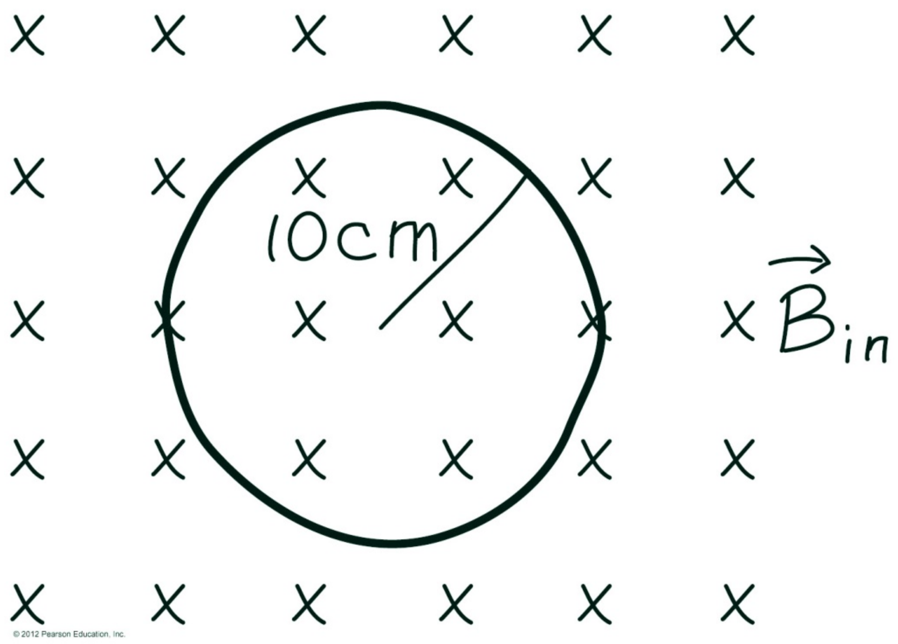
Example changing field

- The loop has radius *r*, resistance *R*, and is in a magnetic field changing at the rate *dB*/*dt*. The induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -\pi r^2\frac{dB}{dt}$$

- and the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\pi r^2}{R}\frac{dB}{dt}.$$



Example changing field

- The bar slides on the conducting rails, increasing the circuit area at a rate

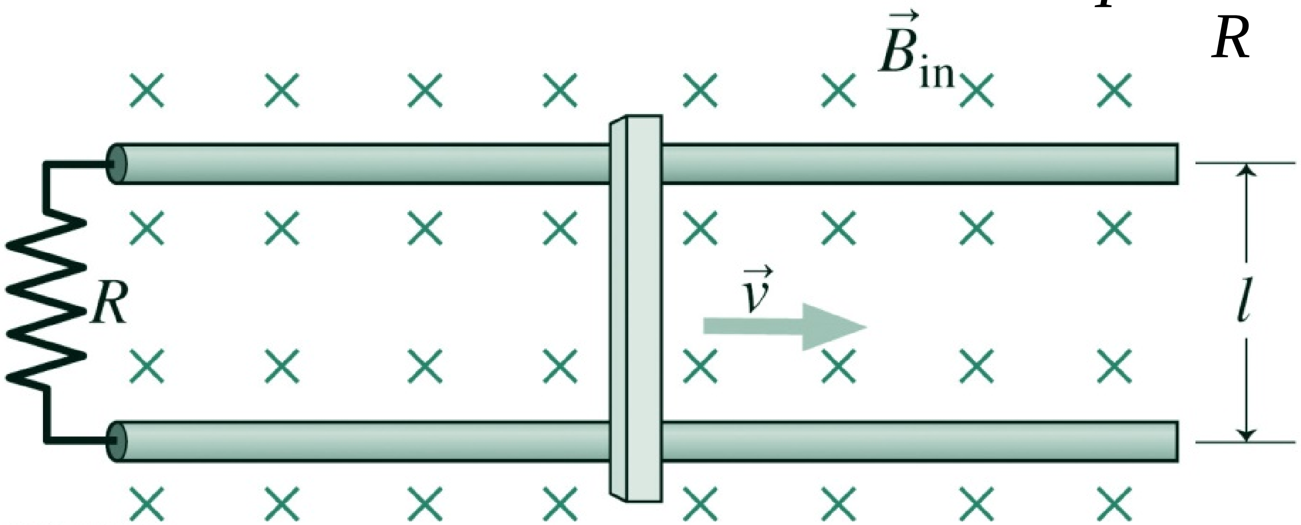
$$\frac{dA}{dt} = \frac{d(lx)}{dt} = l\frac{dx}{dt} = lv$$

- The induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -Blv$$

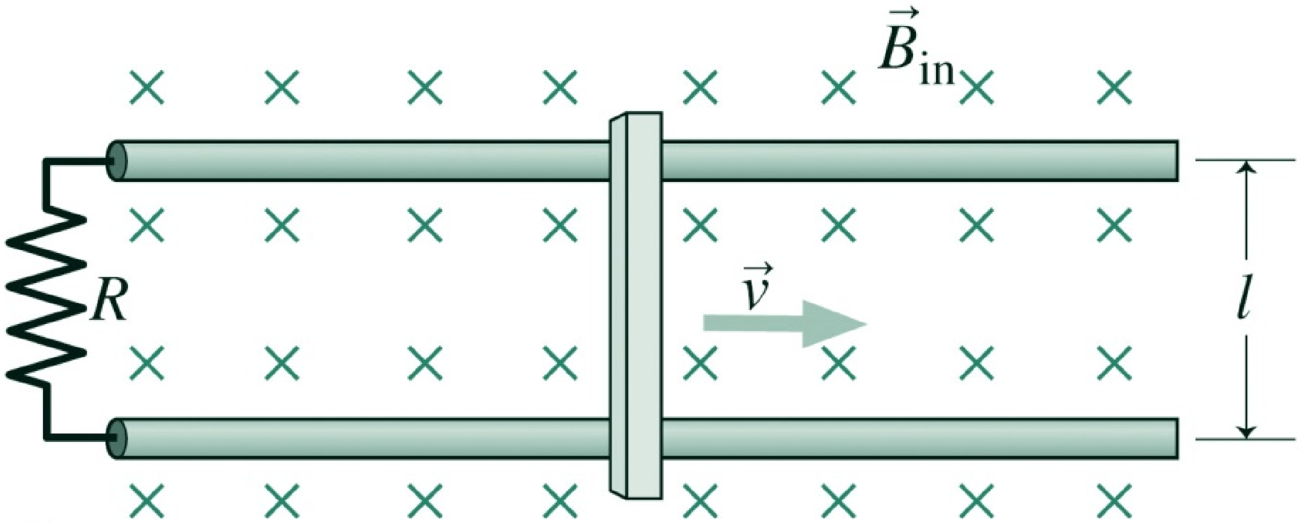
- The induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\mathrm{Blv}}{R}$$



Example changing field

- The induced current is $$I = \frac{|\varepsilon|}{R} = \frac{\mathrm{Blv}}{R}$$
- What do you think is the direction of the current? Why?



Electromagnetic induction

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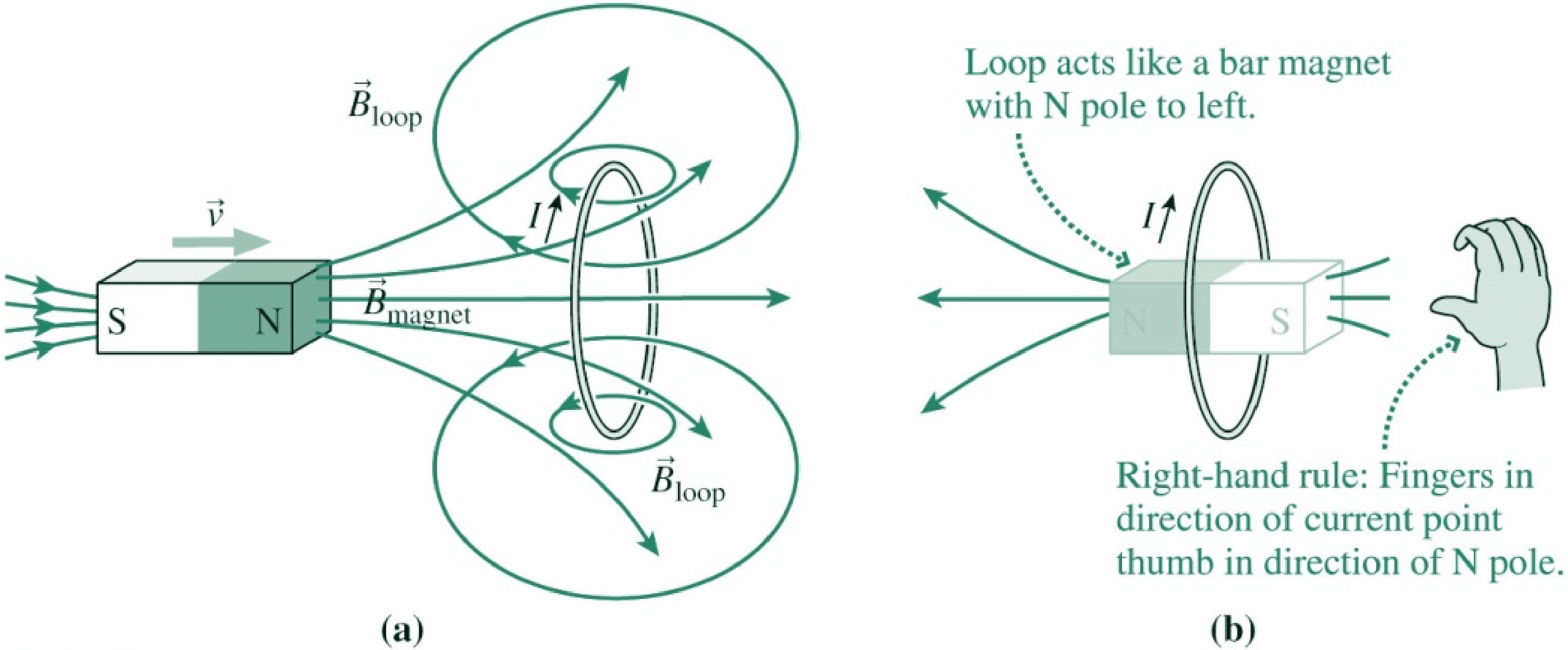
Direction of the induced current: Lenz’s law

- The direction of the induced emf and current is described by the minus sign in Faraday’s law, but it’s easier to get the direction from conservation of energy.

- **Lenz’s law**: The direction of the induced current must be such as to oppose that change that gives rise to it.
  - Otherwise we could produce energy without doing any work!

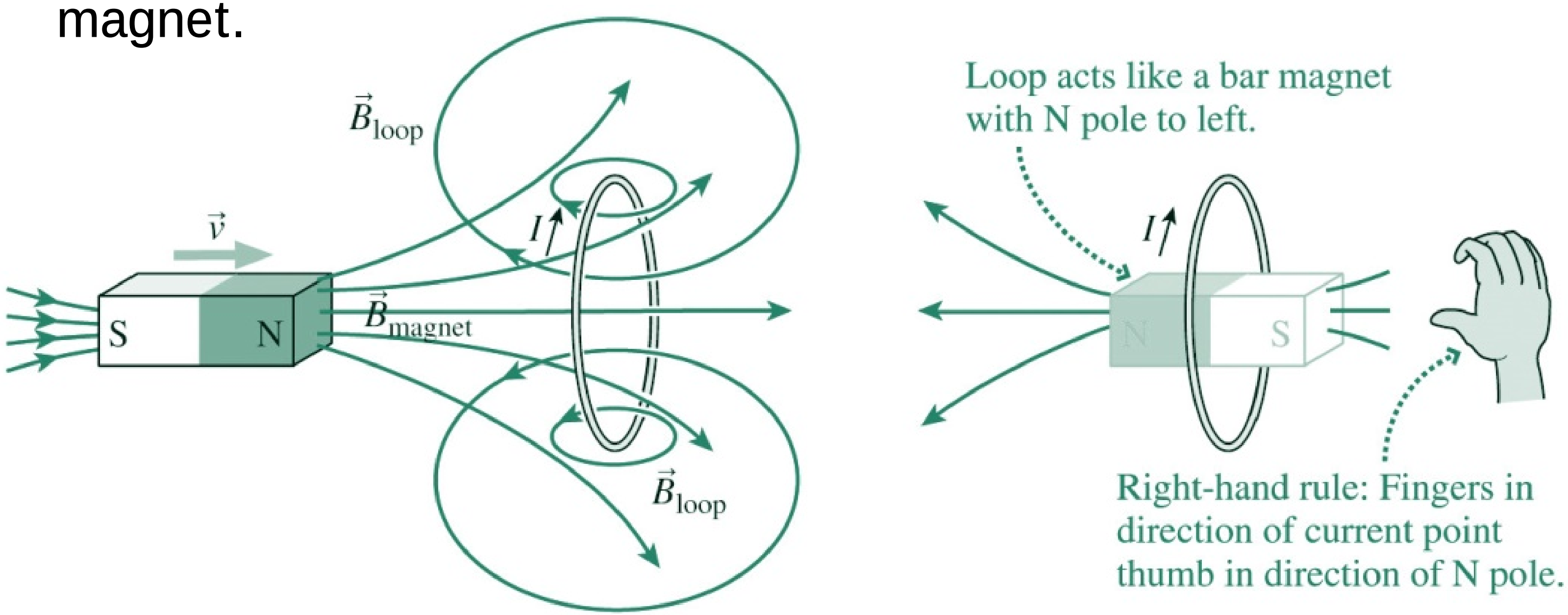
Direction of the induced current: Lenz's law

- Example: Here the north pole of the magnet approaches the loop. So the induced current makes the loop a bar magnet with north to the left, opposing the approaching magnet.



It’s the ***change*** that matters

- Lenz’s law says that induced effects oppose the *changes* that give rise to them.
- Now the induced current flows the opposite way, making the loop’s south pole to the left and opposing the withdrawal of the magnet.

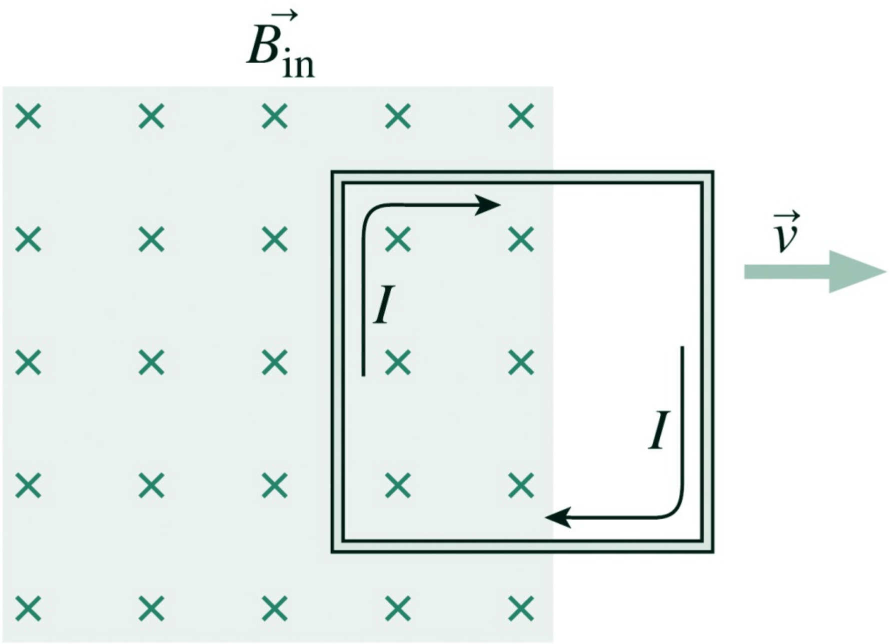


(a) (b)

Clicker question 1a

- What will be the direction of the current in the loop when it first enters the field shown, coming into the field from the left side?

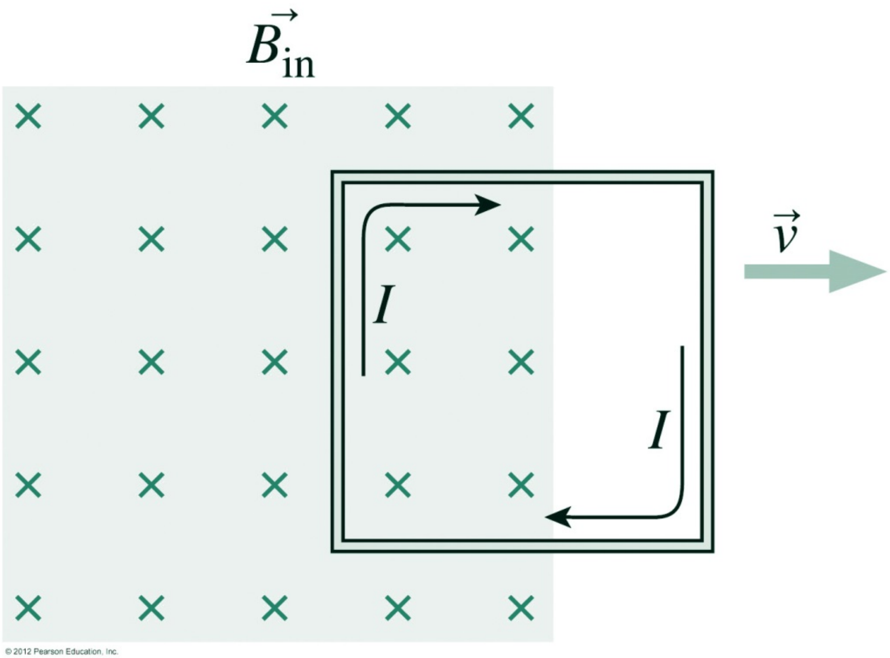
A. Clockwise
B. Counterclockwise



Clicker question 1b

- What will be the direction of the force on the current in the loop when it first enters the field shown, coming into the field from the left side?

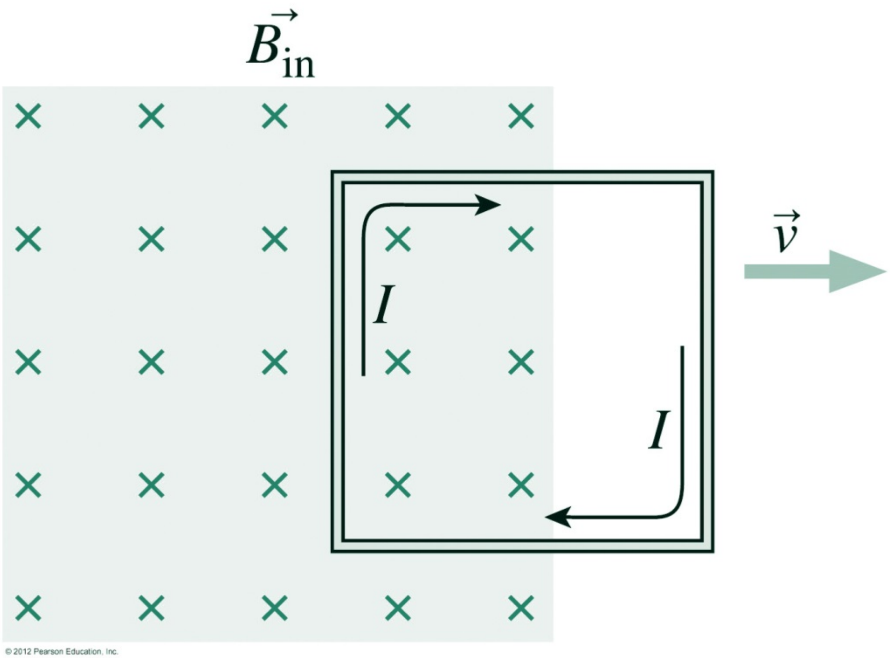
A. To the left
B. To the right



Clicker question 1c

- What will be the direction of the current in the loop when it first leaves the field shown, leaving the field on the right side?

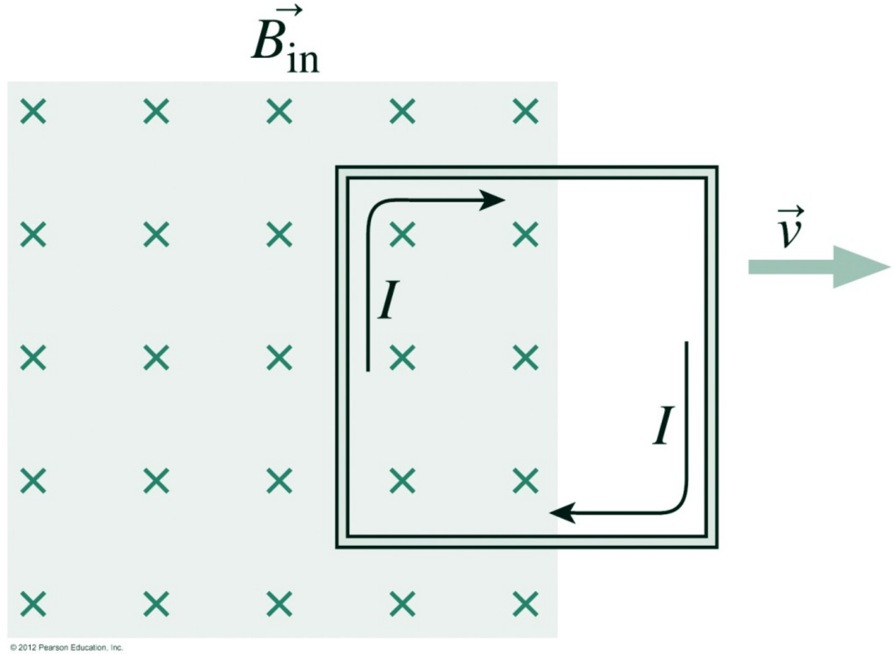
A. Clockwise
B. Counterclockwise



Clicker question 1d

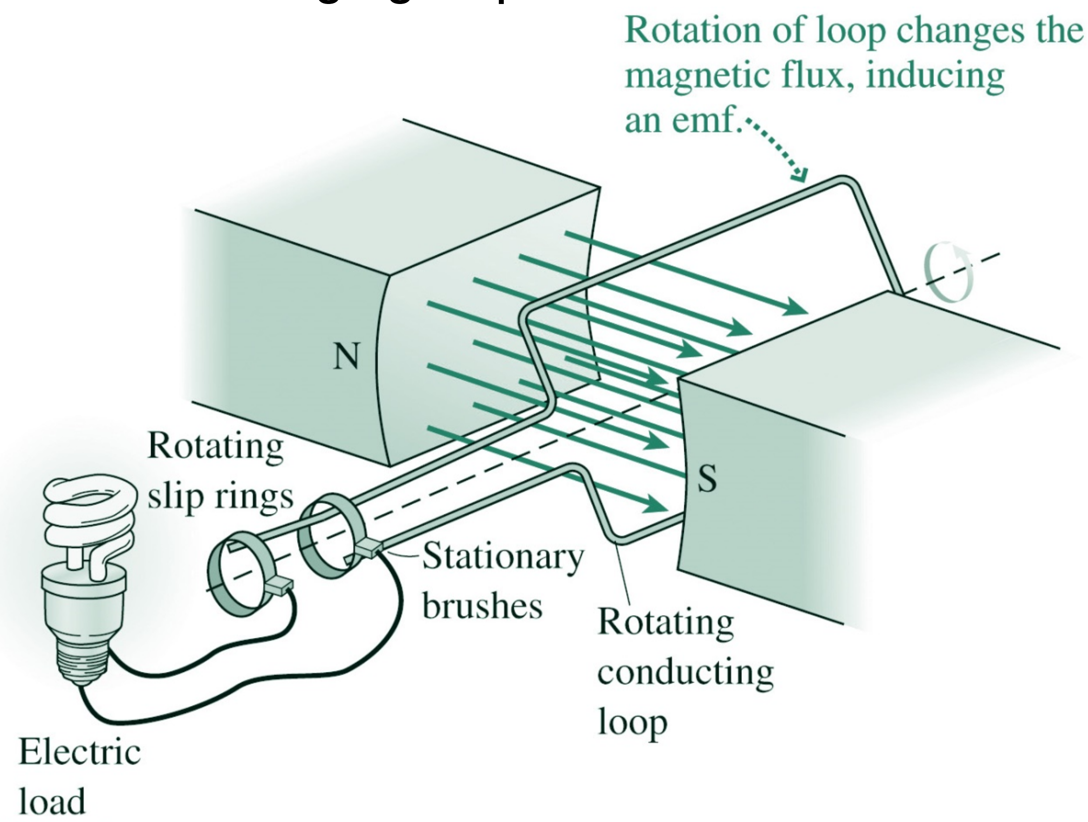
- What will be the direction of the force on the current in the loop when it leaves the field shown, leaving the field on the right side?

A. To the left
B. To the right



Electric generators

- Electric generators use a rotating coil in a magnetic field to convert mechanical to electrical energy.
- Here it's the orientation θ that's changing to produce the changing magnetic flux.
- Lenz's law makes it hard to turn a generator that's supplying electrical energy.
- That's why we have to burn fuels or use the energies of water or wind to generate electricity.



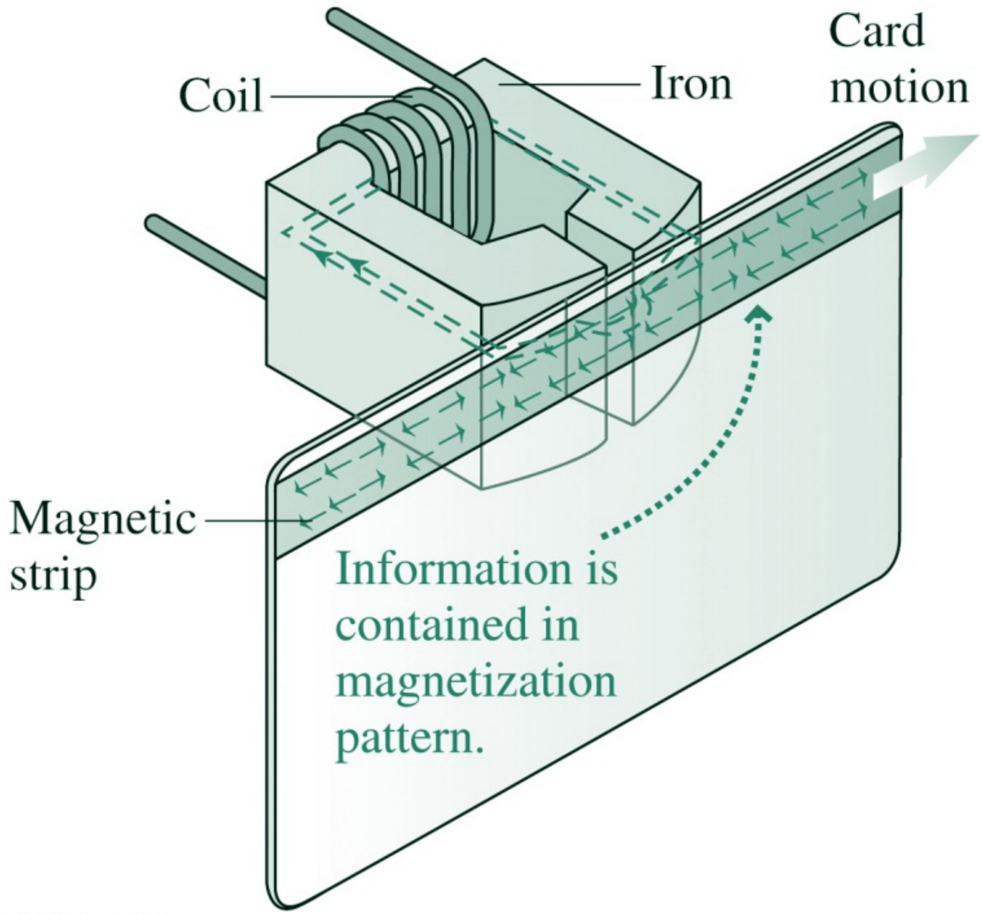
Clicker question 2

- If you lower the electrical resistance connected across a generator while turning the generator at constant speed, how will turning the generator change?

A. It will get harder to turn the generator.
B. It will get easier to turn the generator.
C. You will not notice any change in turning the generator.

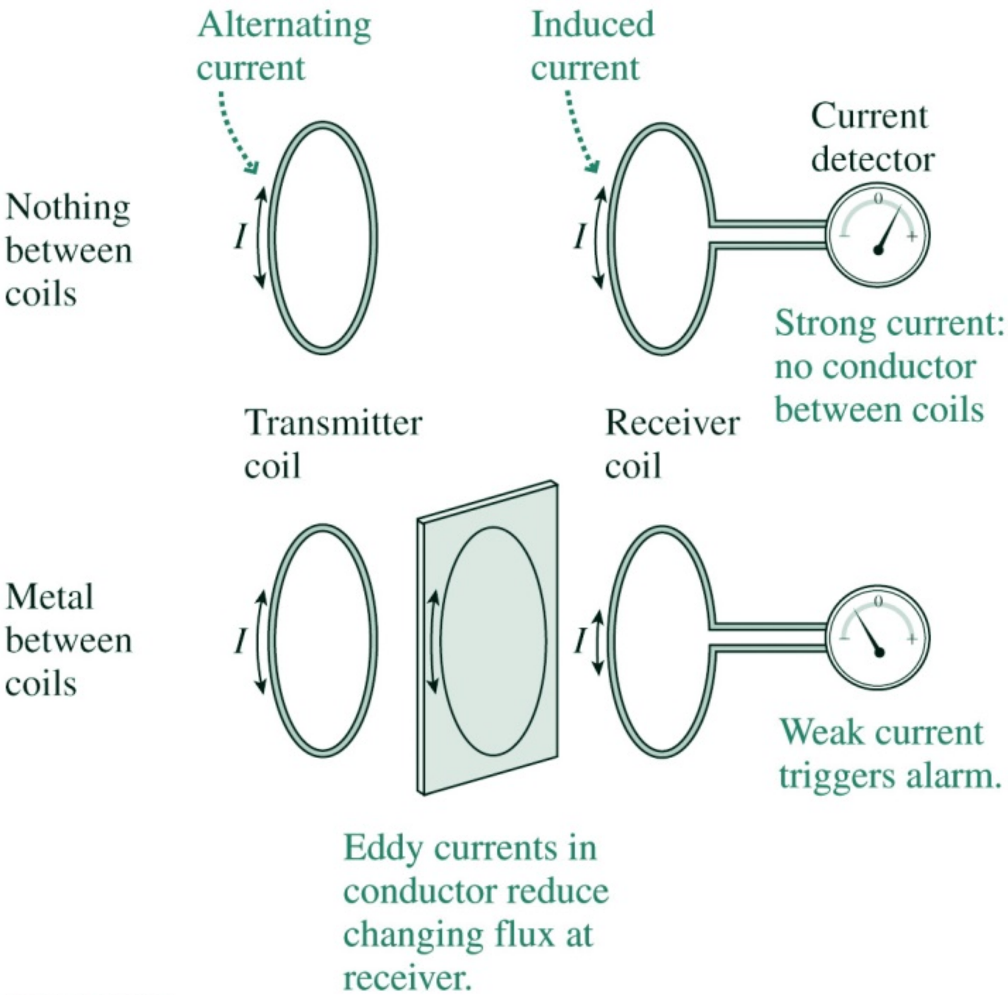
Other uses of induction

- Electromagnetic induction is used to retrieve information stored magnetically on audio and video tapes and credit cards.



Other uses of induction

- **Eddy currents** produced by induction in moving conductors act as a kind of electromagnetic friction.
  - This may be a nuisance, dissipating energy.
  - It can be used for electromagnetic braking of spinning machinery.
  - Eddy currents play an important role in metal detectors.



Clicker question 3

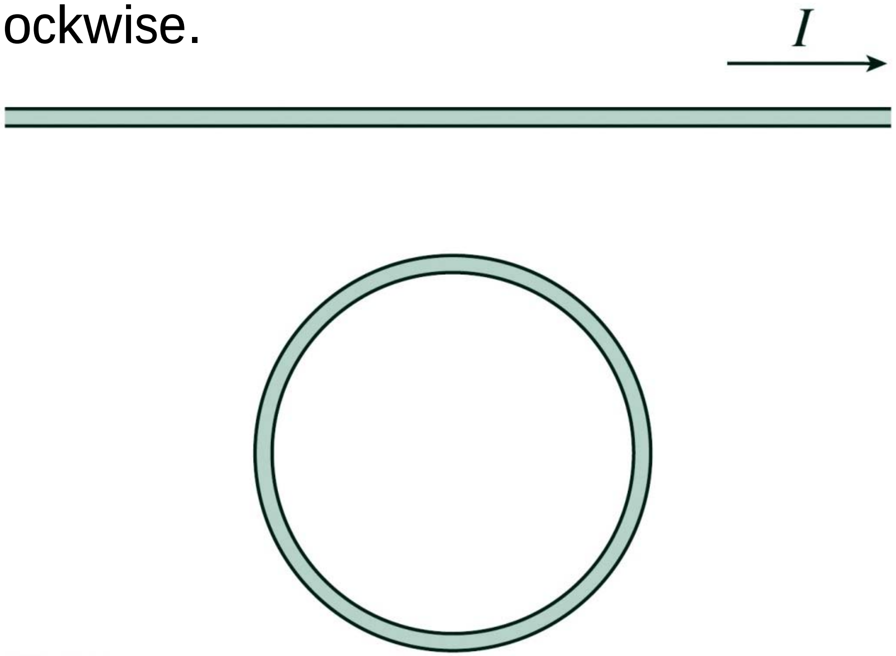
- A copper penny falls on a path that takes it between the poles of a magnet. Does the penny hit the ground going faster or slower than if the magnet were not present?

A. The penny will hit the ground going faster.
B. The penny will hit the ground going slower.
C. The penny will hit the ground going the same speed either way.

Clicker question 4

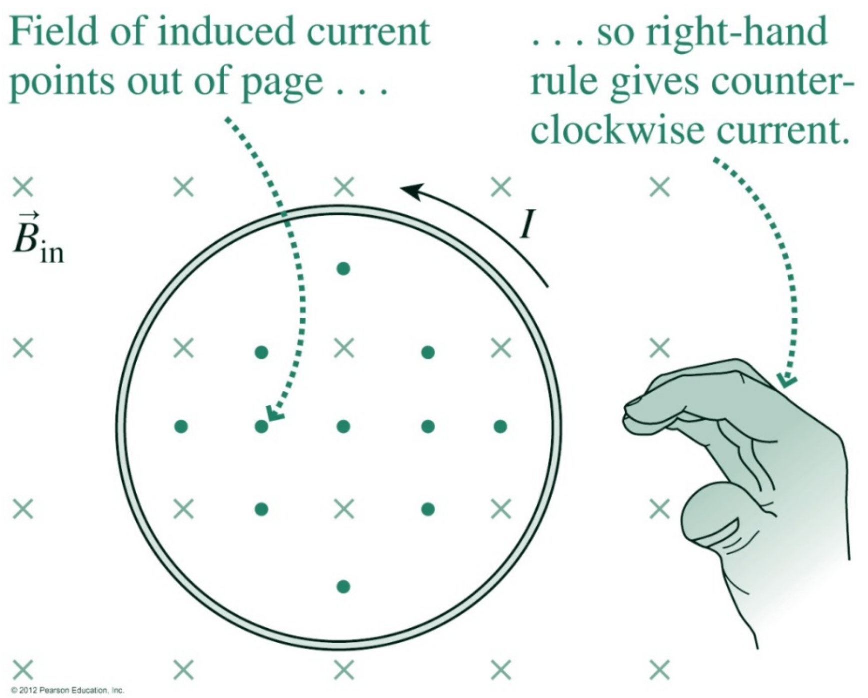
- A long wire carries a current I as shown. What is the direction of the current in the circular conducting loop when I is decreasing?

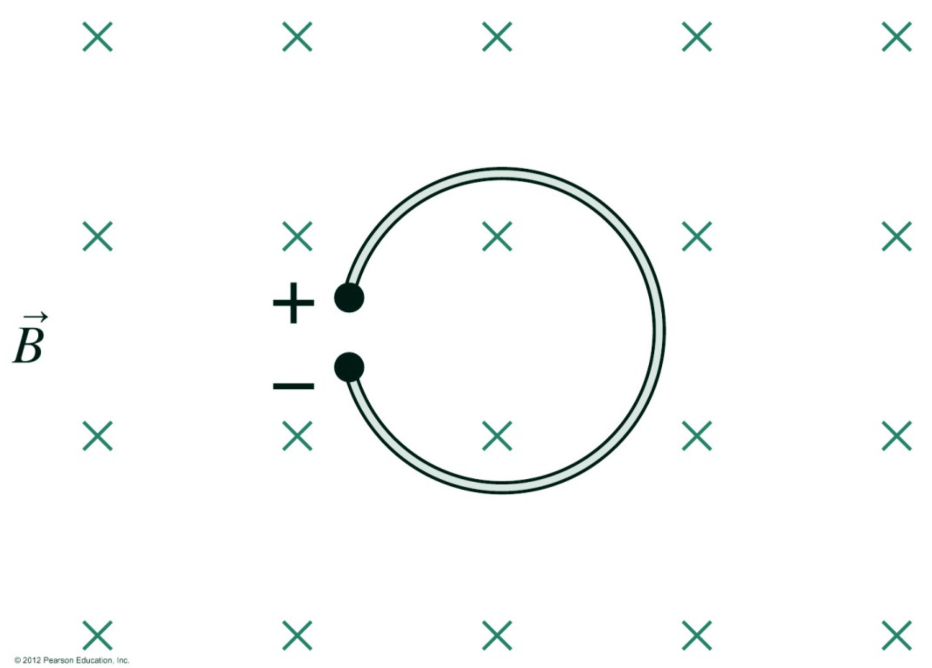
A. The current flows counterclockwise.
B. The current flows clockwise.



Closed circuits and open circuits; emf

- The field into the screen is increasing
- If the circuit is open, the emf can be measured





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