

Electricity and Magnetism

Overview Magnetism

- 26-5: Introduction, magnetism: field and force
- 30-5: Magnetism: Biot-Savart, Ampere
- 2-6: Electromagnetic induction
- 6-6: Electromagnetic induction
- 9-6: Maxwell's equations and electromagnetic waves
- 13-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 16-6: available for answering questions, exercises

Electromagnetic induction

- Learning objectives
- Induced currents
- Faraday's law
- Induction and energy
- Self- and mutual inductance
- Magnetic energy
- Induced electric fields
- Concluding

In this lecture you'll learn

- To explain the phenomenon of electromagnetic induction
 - To calculate induced emfs and currents
 - To use energy conservation to find the direction of induced effects
 - To describe important technological applications of induction
 - To explain inductance
 - And describe the role of inductance in simple circuits
 - That magnetic fields store energy
 - And how to calculate that energy
 - To recognize Faraday's law as one of the four fundamental laws of electromagnetism
 - And to calculate induced electric fields
- ⇒ Time varying effects, not completely stationary!!!

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Faraday's law

- **Faraday's law** describes induction by relating the emf induced in a circuit to the rate of change of magnetic flux through the circuit:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

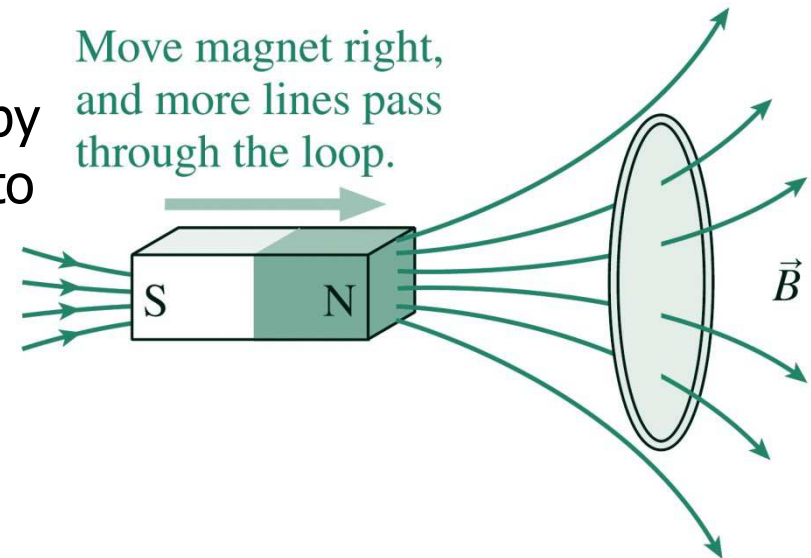
- where the **magnetic flux** is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- With a flat area and uniform field, this becomes

$$\Phi_B = BA \cos \theta$$

- The flux can change by changing the field B , the area A , or the orientation θ .



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Moving a magnet near a wire loop increases the flux through the loop. The result is an induced emf given by Faraday's law. The induced emf drives an induced current in the loop.



Clicker question 7

- An emf can be induced in a loop of wire by changing
 - A. the magnetic field.
 - B. the area of the loop in the field.
 - C. the loop's orientation with respect to the field.
 - D. All of the above.



Clicker question 6

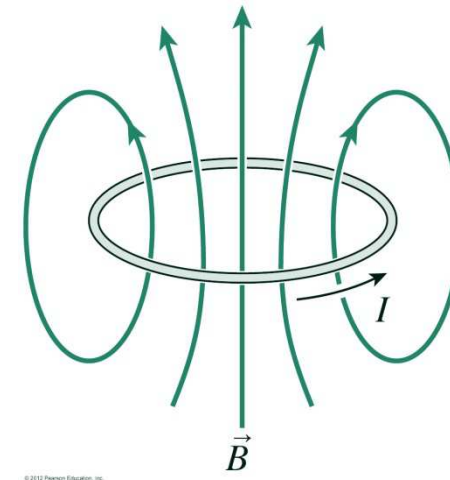
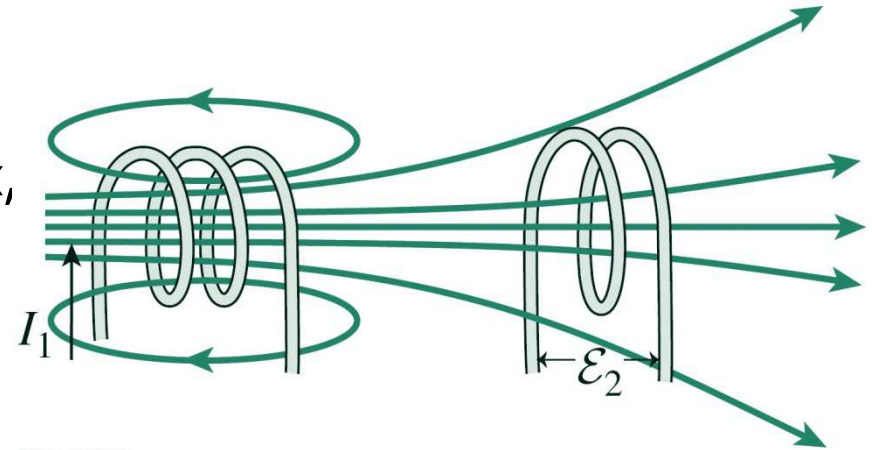
- A magnet, with its north pole pointed downwards, is falling down directly above a wire loop. As the magnet approaches the wire loop (before it passes through the loop),
 - A. an induced current flows through the wire loop in a clockwise orientation (as seen from above).
 - B. an induced current flows through the wire loop in a counter-clockwise orientation (as seen from above).
 - C. no current flows through the wire loop.

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Inductance

- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit.
 - Some of the magnetic flux produced by one circuit passes through the other circuit.
- **Self-inductance** occurs when a changing current in a circuit results in an induced emf that opposes the change in the circuit itself.
 - The magnetic flux produced in a circuit passes through that same circuit.



Applications

- Mutual inductance
 - Transformers
 - Contactless energy transfer
 - Electrical machines
- Self inductance
 - Filters
 - Parasitic inductance

Mutual induction

- <https://www.youtube.com/watch?v=djjh5yxJaQY>

Self-inductance

- The self-inductance L of a circuit is defined as the ratio of the magnetic flux through the circuit to the current in the circuit:

$$L = \Phi_B / I$$

- In differential form:

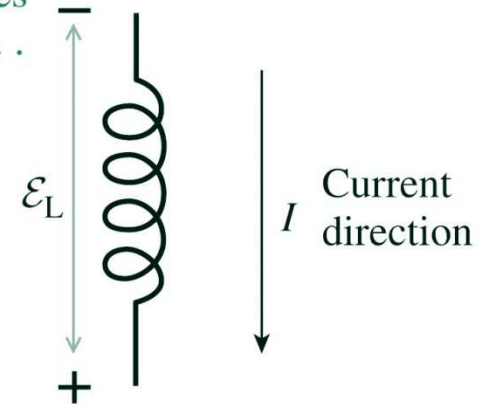
$$\frac{d\Phi_B}{dt} = L \frac{dI}{dt}$$

- The SI units of L are $\text{T} \cdot \text{m}^2/\text{A}$, or **Henry**.
- By Faraday's law, the emf across an inductor is

$$\mathcal{E}_L = -L \frac{dI}{dt}$$

- The minus sign shows that the direction of the inductor emf is such as to *oppose* the change in the inductor current.

Voltage increasing
in direction of
current defines
positive $\mathcal{E}_L \dots$



$\dots \mathcal{E}_L$ is positive
when current is
decreasing
($dI/dt < 0$).

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Self-inductance of a solenoid

- Consider a long solenoid of cross-sectional area A , length l , with n turns per unit length.
- Magnetic field inside the solenoid:

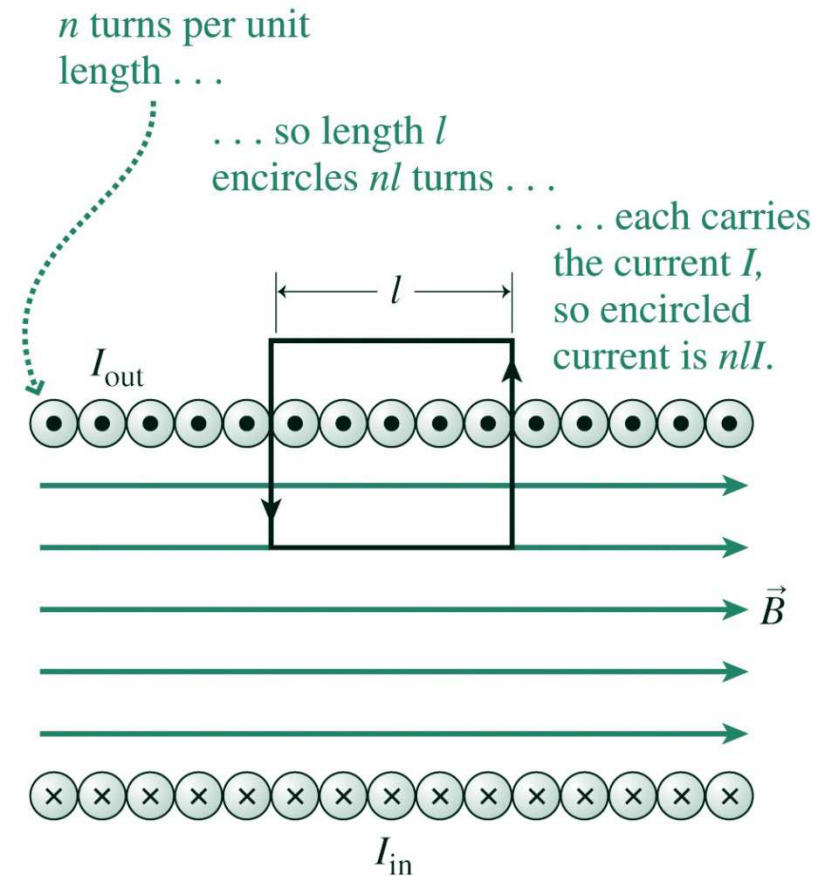
$$B = \mu_0 n I$$

- With n turns per unit length, the solenoid contains a total of $n l$ turns, so the flux through all the turns is

$$\Phi_B = n l B A = n l (\mu_0 n I) A = \mu_0 n^2 I A l$$

- The self-inductance of the solenoid is

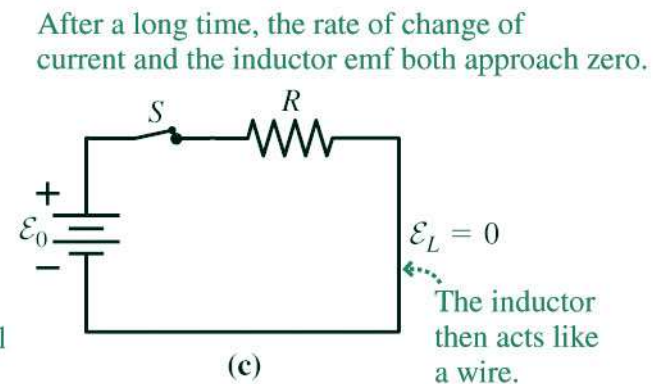
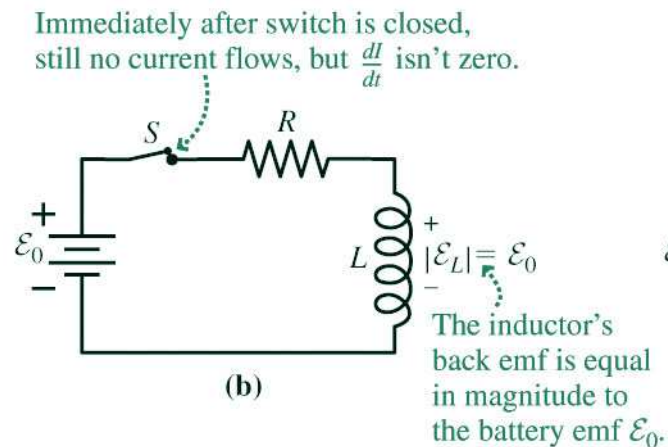
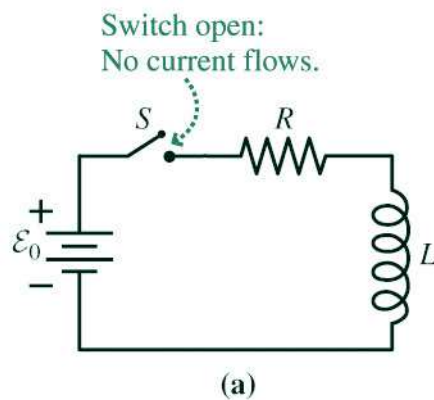
$$L = \frac{\Phi_B}{I} = \mu_0 n^2 A l$$



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Inductors in circuits

- **The current through an inductor can't change instantaneously.** $\mathcal{E}_L = -L \frac{dI}{dt}$
- Otherwise an impossible infinite emf would develop.
- Rapid changes in current result in large, possibly dangerous emfs.
- The buildup of current in an RL circuit occurs gradually.

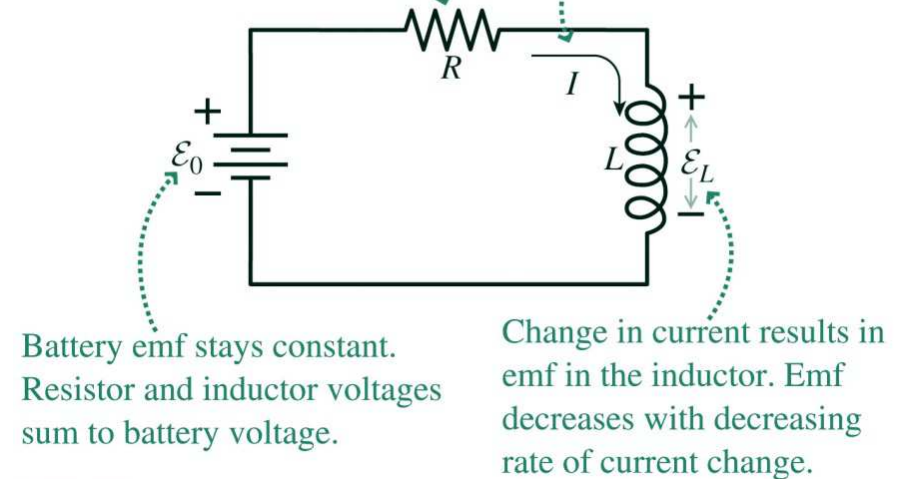


The inductive time constant

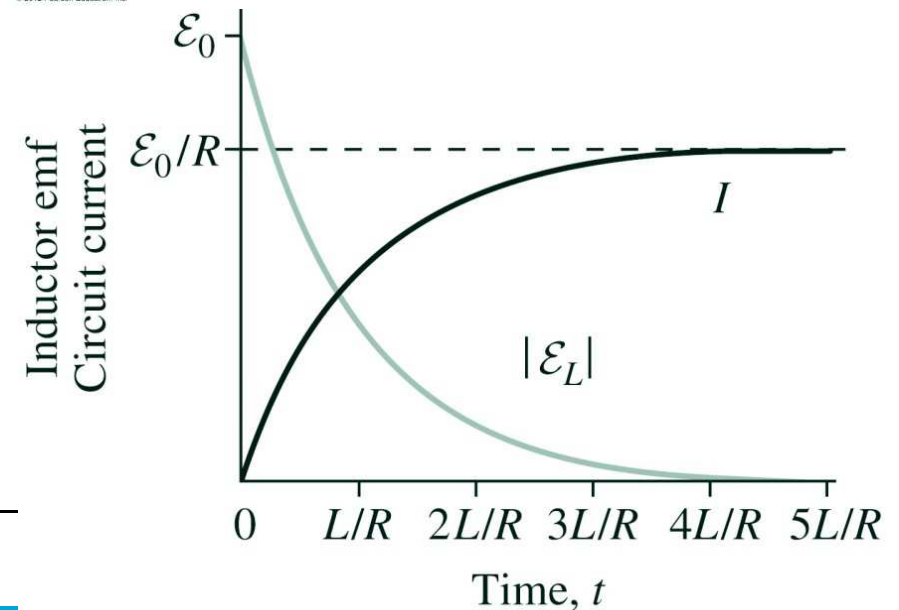
- The loop law: $\mathcal{E}_0 - IR + \mathcal{E}_L = 0$
- The solution to the differential equation is
$$I = \frac{\mathcal{E}_0}{R} \left(1 - e^{-Rt/L} \right)$$
- The inductor current starts at zero and builds up with time constant L/R .
 - As the current increases, its rate of change decreases.
 - The inductor emf therefore decays exponentially to zero. This decay has the same time constant L/R .

As current increases, so does resistor voltage IR , at an ever-decreasing rate.

Current I increases at an ever-decreasing rate.



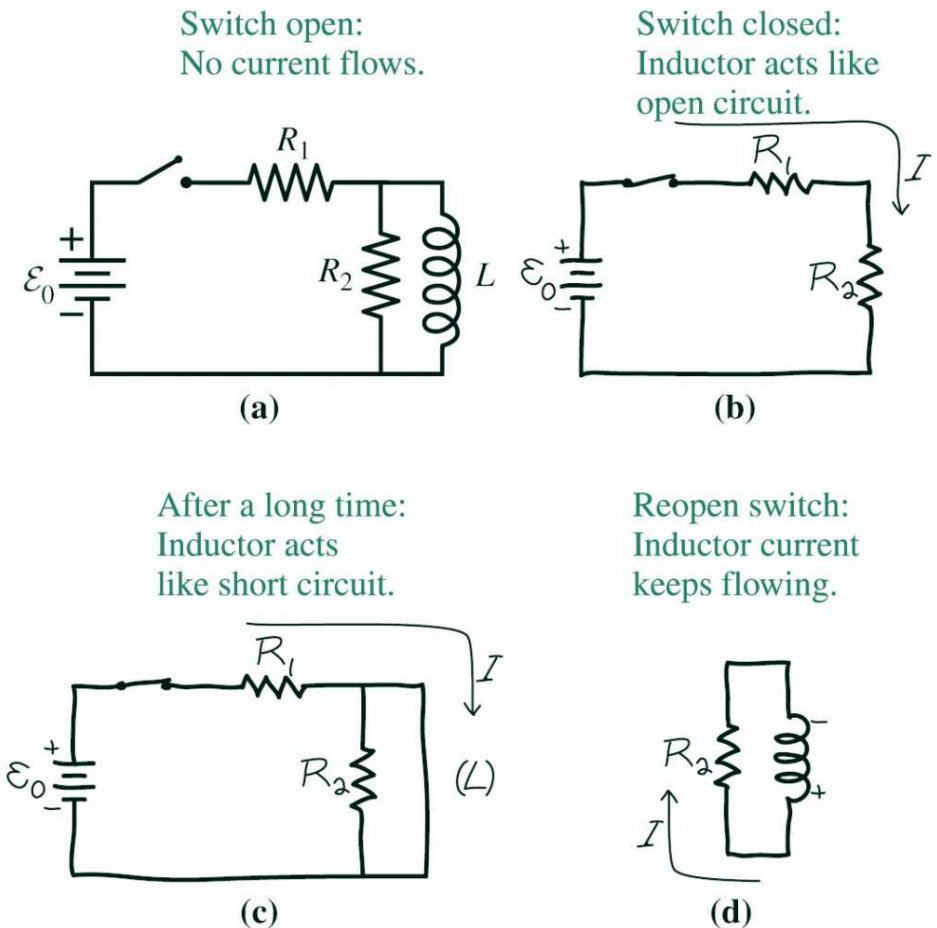
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Short- and long-term behavior of inductors

- Since current can't change instantaneously, an inductor in an RL circuit with no current through it acts instantaneously like an open circuit.
- If there's current flowing, it keeps flowing momentarily despite changes in the circuit.
- After a long time, the inductor current stops changing.
 - The inductor emf is zero.
 - The inductor acts like an ordinary wire.



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Magnetic energy

- As current builds up in an inductor, the inductor absorbs energy from the circuit. That energy is stored in the inductor's magnetic field.

- The rate at which the inductor stores energy is $P = IL \frac{dI}{dt}$

- For an inductor, the stored energy is

$$U_B = \int P dt = \int_0^I LI dI = \frac{1}{2} LI^2$$

- Considering the uniform magnetic field inside a solenoid shows that the **magnetic energy density** is

$$L = \mu_0 n^2 Al \quad B = \mu_0 nI \quad U_B = Al \frac{B^2}{2\mu_0} \quad u_B = \frac{B^2}{2\mu_0}$$

- This is a universal expression

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Induced electric fields

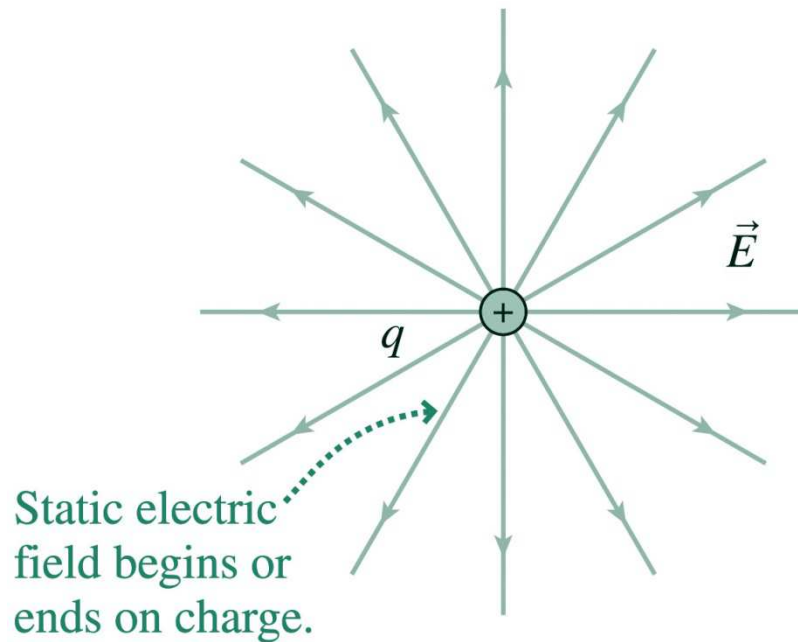
- The induced emf in a circuit subject to changing magnetic flux results from an **induced electric field**.
- Induced electric fields result from changing magnetic flux.
 - This is described by the full form of Faraday's law, one of the four fundamental laws of electromagnetism:

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

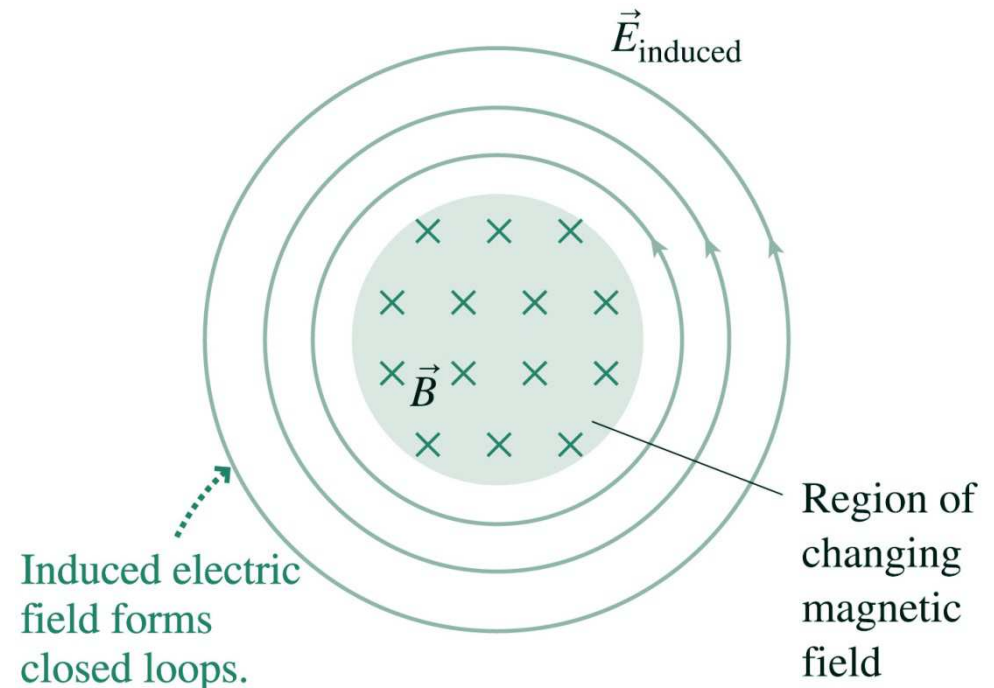
- where the integral is taken around any closed loop, and where the flux is through any area bounded by the loop.
- The equation states that **a changing magnetic field produces an electric field**.
 - Thus not only charges but also changing magnetic fields are sources of electric field.
 - Unlike the electric field of a static charge distribution, the induced electric field is *not conservative*.

Static and induced electric fields

- Static electric fields begin and end on charges.
- Induced electric fields generally form closed loops.



(a)



(b)

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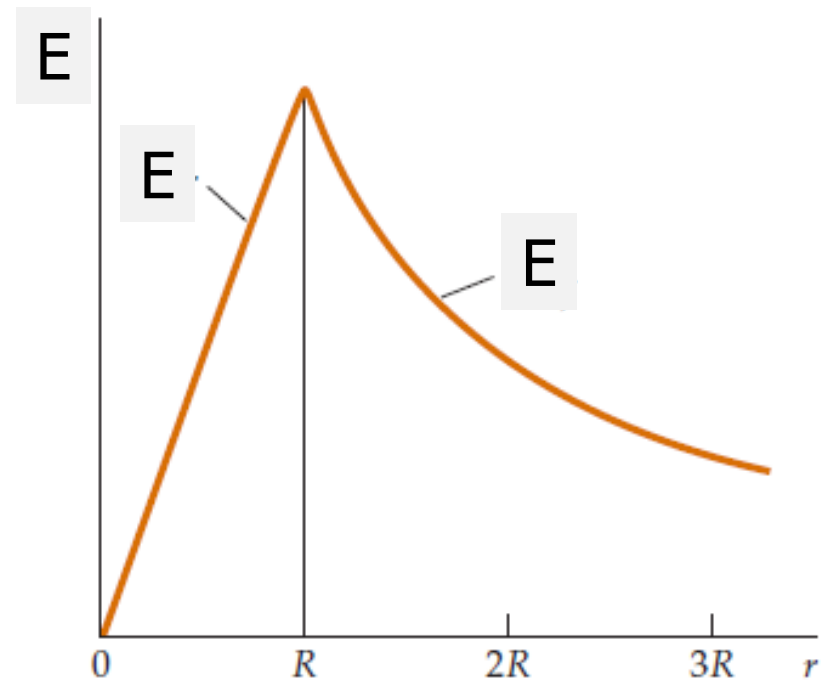
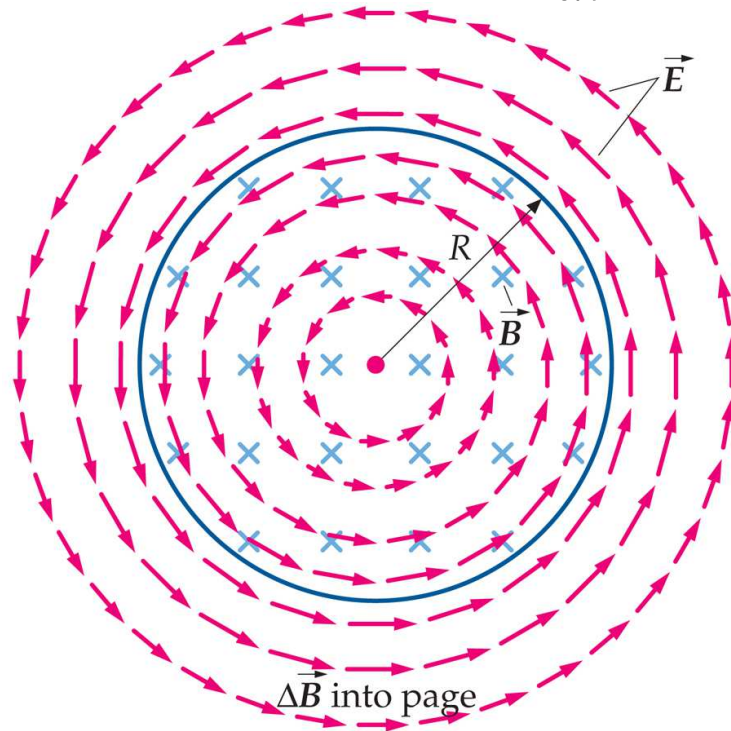
Induced electric field

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

$$r < R: \quad 2\pi r E = -\frac{r^2}{R^2} \frac{d\Phi_B}{dt} \Rightarrow E = -\frac{r}{2\pi R^2} \frac{d\Phi_B}{dt}$$

$$r > R: \quad 2\pi r E = -\frac{d\Phi_B}{dt} \Rightarrow E = -\frac{1}{2\pi r} \frac{d\Phi_B}{dt}$$

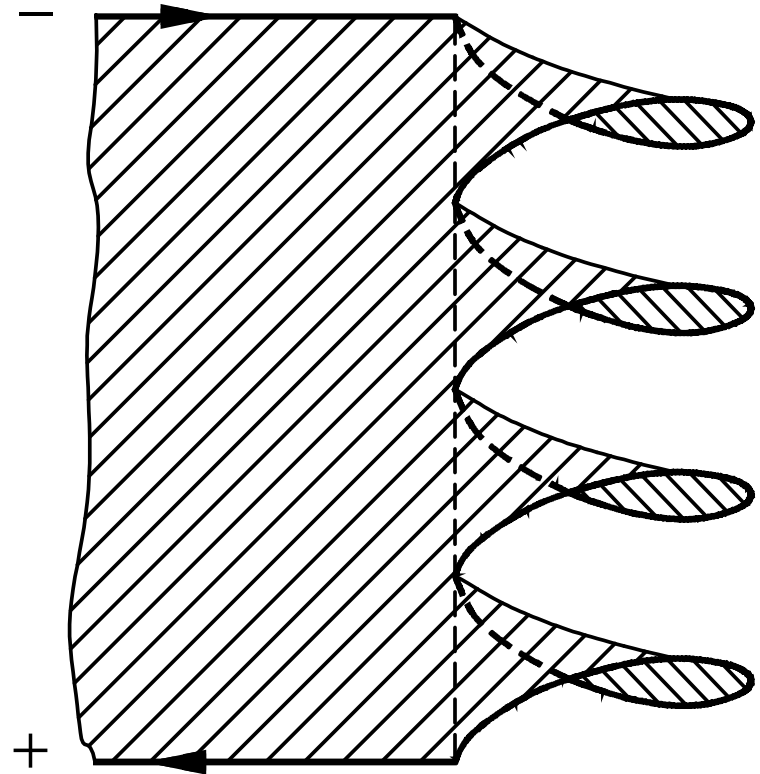
Uniform B!



Faraday's law

$$\oint_{C_e} \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_{S_e} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

If we want to calculate voltages induced in coils, the contour is chosen in the electric path, in the wire!



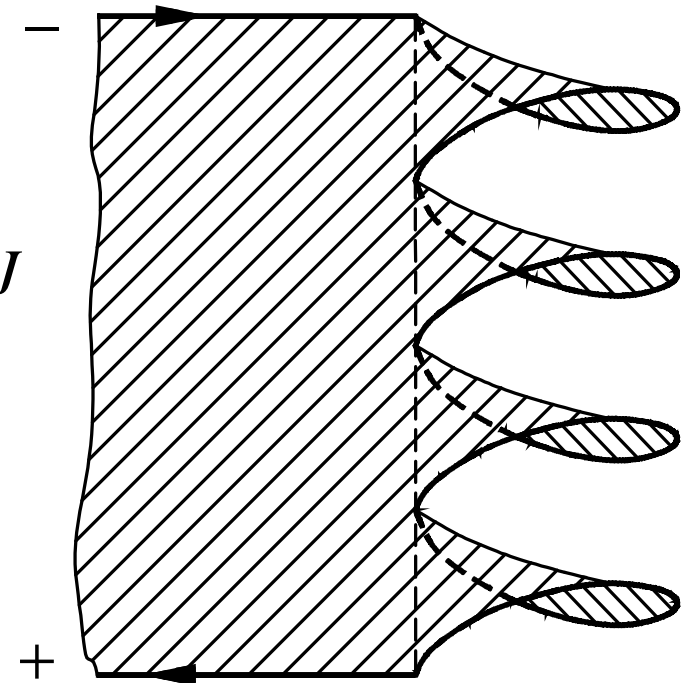
Faraday's law

$$\oint_{C_e} \vec{E} \cdot d\vec{s} = \int_{+term}^{term} \vec{E} \cdot d\vec{s} + \int_{-term}^{term} \vec{E} \cdot d\vec{s}$$

$$\oint_{C_e} \vec{E} \cdot d\vec{s} = V + \int_{-term}^{term} \rho_{Cu} \vec{J} \cdot d\vec{s} = V + l_{Cu} \rho_{Cu} J$$

$$J = \frac{I}{A_{Cu}}$$

$$\oint_{C_e} \vec{E} \cdot d\vec{s} = V + \frac{\rho_{Cu} l_{Cu}}{A_{Cu}} I = V + RI$$

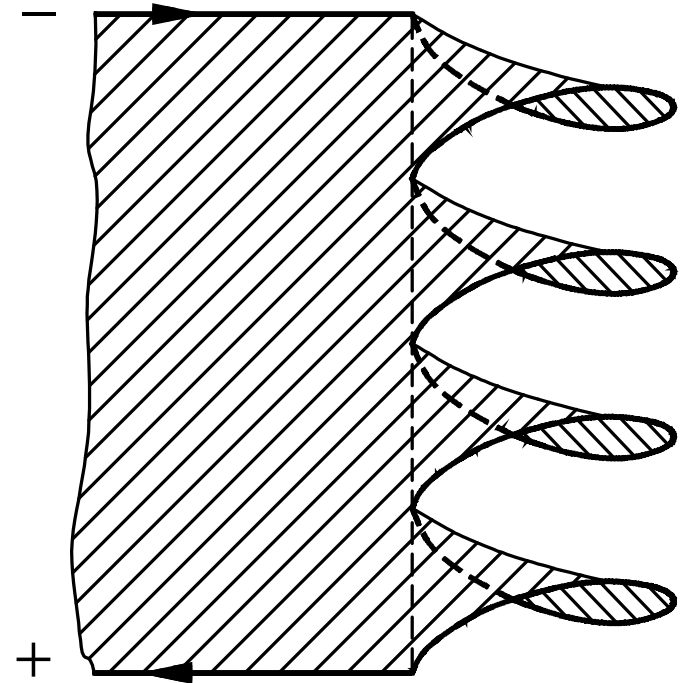


Faraday's law

$$\oint_{C_e} \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_{S_e} \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$$

Therefore,

$$V = -RI - \frac{d\Phi_B}{dt} = -RI - \mathcal{E}$$



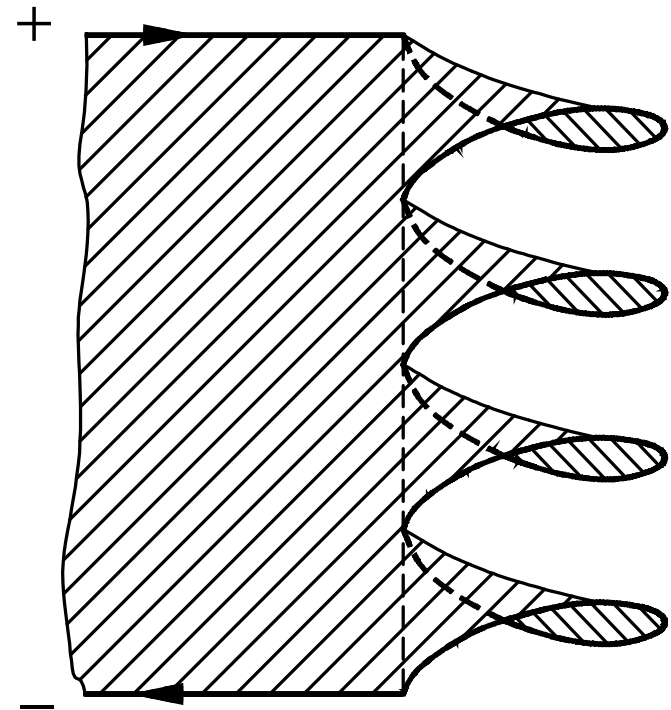
Faraday's law

$$\oint_{C_e} \vec{E} \cdot d\vec{s} = \frac{d}{dt} \int_{S_e} \vec{B} \cdot d\vec{A} = \frac{d\Phi_B}{dt}$$

Therefore,

$$V = RI + \frac{d\Phi_B}{dt} = RI + \varepsilon$$

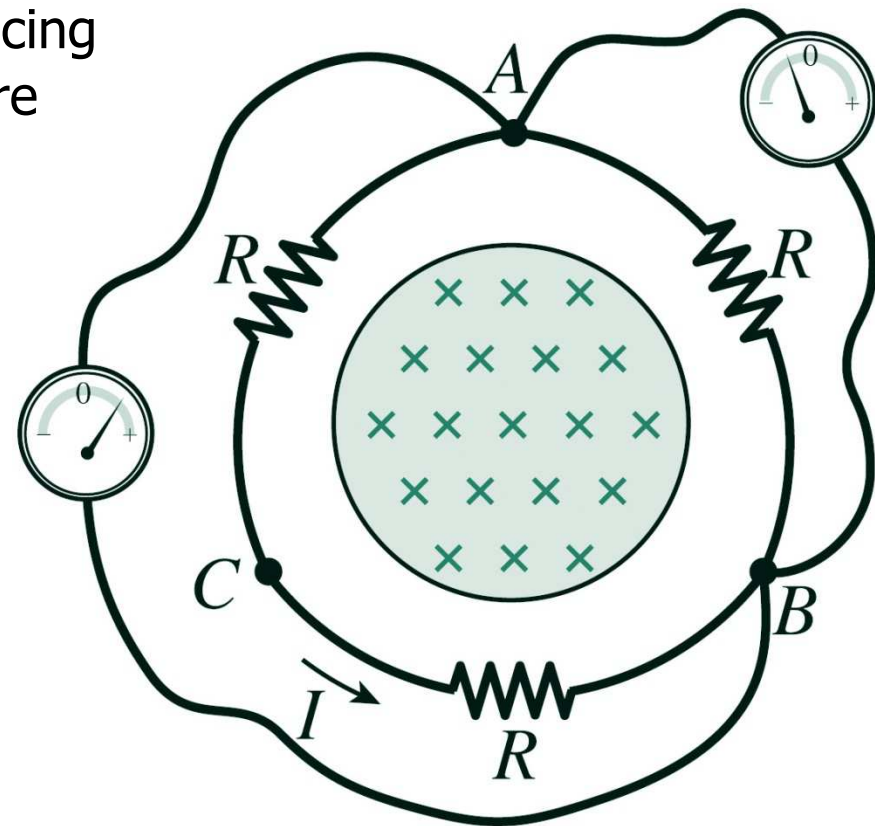
The sign depends on the definition of the positive current direction and the positive voltage terminal!



Question

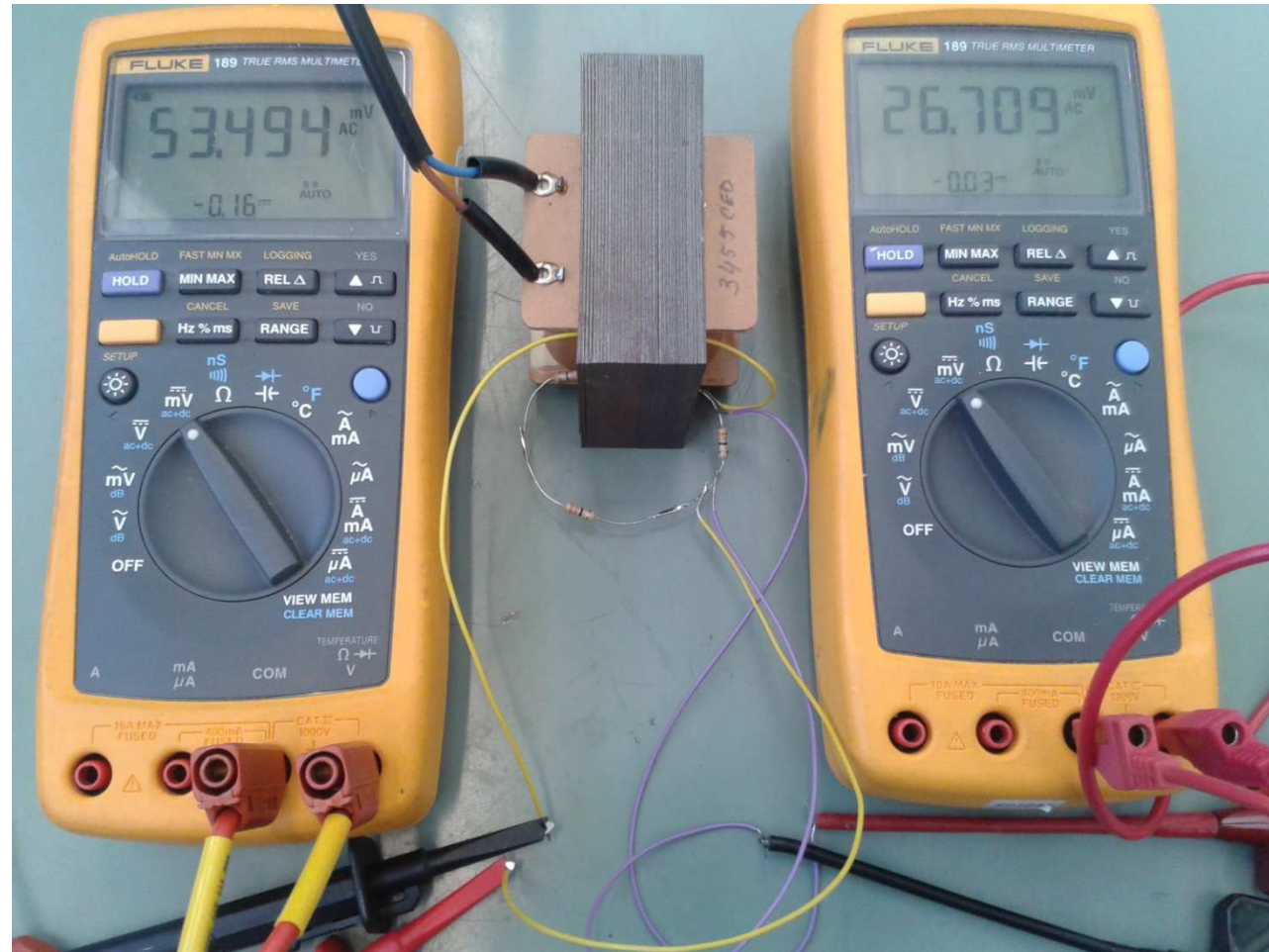
- Three resistors are connected around a solenoid with an increasing magnetic field inducing a current I . Two voltmeters are connected to points A and B. What does each indicate?

- A both $V=0V$
- B both $V=RI$
- C both $V=2RI$
- D right $V=RI$, left $V=2RI$
- E something else



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Demonstration



- <https://www.youtube.com/watch?v=eqjl-qRy71w>
- <https://www.youtube.com/watch?v=1bUWcy8HwpM>

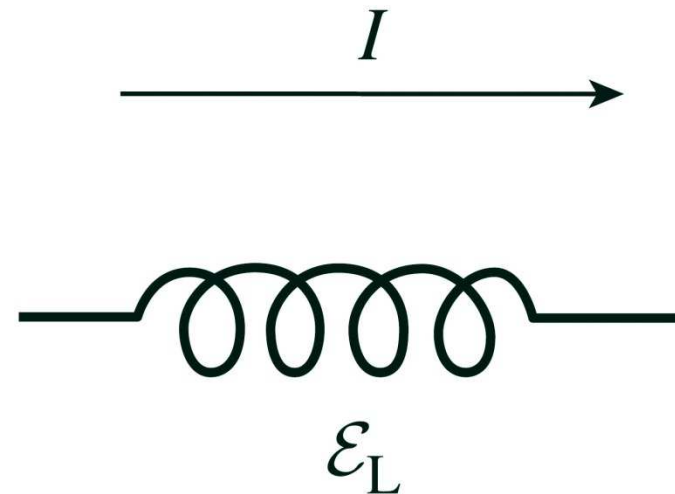
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Clicker question 5

- Current flows from left to right through the inductor as shown. A voltmeter connected across the inductor gives a constant reading, and shows that the left end is positive. Is the current in the inductor changing, and if so, how?
- A. The current is increasing.
B. The current is decreasing.
C. The current is constant.



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Clicker question 8

- Which of the following statements is FALSE?
 - A. A changing magnetic field can produce an electric current.
 - B. An emf is induced in a wire by moving the wire near a magnet.
 - C. An emf is induced in a wire by keeping a stationary magnet near the wire.
 - D. An emf is induced in a wire by changing the current in that wire.

Summary

- **Faraday's law** describes electromagnetic induction, most fundamentally the phenomenon whereby **a changing magnetic field produces an electric field**:

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

- This induced electric field is nonconservative and its field lines have no beginnings or endings.
- In the presence of a circuit, the induced electric field gives rise to an induced emf and an induced current.
 - Lenz's law states that the direction of the induced current is such that the magnetic field it produces acts to *oppose* the change that gives rise to it.
 - Self-inductance is a circuit property whereby changing current in a circuit results in an induced emf that opposes the change.
- Consideration of current buildup in an inductor shows that all magnetic fields store energy, with energy density $B^2/2\mu_0$.

Concentric solenoids

- Two concentric solenoids:
 - N_1 turns and radius r_1
 - N_2 turns and radius r_2
 - The current in coil 2 is i_2 .
-
- Calculate
 - The flux density
 - The flux linkage of coil 2
 - The voltage induced in coil 2
 - The self inductance of coil 2
 - The flux linkage of coil 1
 - The voltage induced in coil 1
 - The mutual inductance between coil 1 and 2
 - The current in coil 1 if coil 1 is short-circuited with resistance 0
 - The induced voltage in coil 2 if $n_1=n_2$, $r_1=r_2$, and $R_1=R_2=0$

