Graded homework 2

May 16, 2016

Notes:

Your assignment must be handed as hardcopy, *at the beginning* of the lecture starting at 10:45 on Monday, May 23, 2016. In the case when you are not able to attend the lecture, you are allowed to submit a *good quality* scanned copy of your solution by e-mail, to the address i.e.lager@tudelft.nl.

Solutions submitted after Monday, May 23, 10:45 will not be considered!

Each solution must be handed in on a separate page. Please indicate on each page your name, your study number and the exercise number.

Please indicate in all cases the relevant measure units.

Exercise 1 – 4.5 points

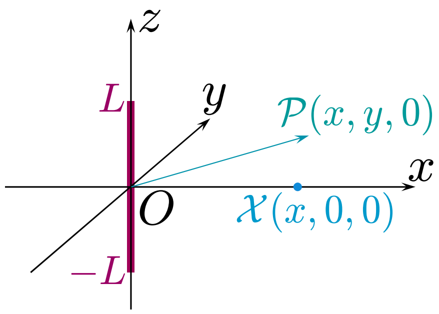

Figure 1: The nonuniform charged line configuration.

Let the configuration in Fig. 1, concerning a nonuniform linear charge distribution along the z-axis, the charge density having the expression

$$\lambda(z) = \lambda_0 |z/L| \,(\mathrm{C/m}) \text{ for } -L \leqslant z \leqslant L$$

with λ_0 being a constant.

a) By making use of Coulomb's constant $k = 9 \cdot 10^9\,\mathrm{Nm^2/C^2}$, determine the potential $V(x, 0, 0)$ at points $\mathcal{X}(x, 0, 0,)$, with $x \geqslant 0$. Please specify explicitly the value of the potential $V(0, 0, 0)$ at the origin. (2 points)

b) Show mathematically that the potential $V(x, 0, 0)$ at points $\mathcal{X}(x, 0, 0,)$, with $x \gg L$, is similar with that given by a point charge and indicate the value of that charge. (1.5 points)

   Hint: Please note that L/x is, in this case, a small quantity and, thus, it is amenable to being used in a Maclaurin series representation.

c) Use the results from (a) and (b) for commenting on the limits of the modelling of the nonuniform charge distribution as a point charge. (1 point)

Exercise 2 – 5.5 points

Let the case of a parallel-place capacitor with square plates of size $2a \times 2a$, with $a > 0$ a constant, spaced at $a/100$ (see Fig. 2). The capacitor has a dielectric filling consisting of Polytetrafluoroethylene (PTFE) with the relative permittivity $\varepsilon_{\mathrm{r}} = 2$.

Fringing effects are neglected throughout this exercise. You do not have to fill in the numerical value of ε_0 in your solution.

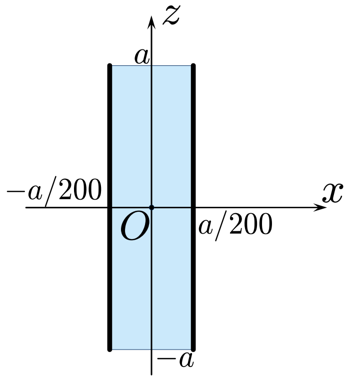

Figure 2: Configuration concerning the dielectrically filled capacitor. The figure shows the configuration's cross-section in the xOz-plane, the configuration being symmetric with respect to this plane.

a) Determine the capacitance C_1 of the parallel-plate capacitor in Fig. 2. (1 point)

b) This capacitor is connected to a DC voltage source generating the voltage V_0 V, with the $\oplus$ being connected to the plate located at $x = a/200$. After electrostatic equilibrium was established, the DC source is disconnected. Determine the total charge Q_{f} stored on the plate located at $x = a/200$ – *both value and sign*. Determine the electrostatic energy W_1 stored in the capacitor. (1 point)

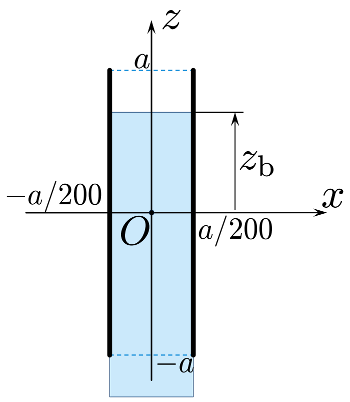

Figure 3: Configuration concerning the capacitor with displaced dielectric filling.

c) *With the source disconnected*, the PTFE filling is now slowly pulled downwards until its top is at $0 \leqslant z_{\mathrm{b}} < a$. The remaining upper part of the parallel-plate capacitor is empty (see Fig. 3). Determine the capacitance $C_2(z_{\mathrm{b}})$ of the new configuration and the corresponding stored electrostatic energy $W_2(z_{\mathrm{b}})$. (1.5 points)

d) Assume now that the PTFE block has the mass m and that it can move frictionlessly between the capacitor's plates. Determine the value V_0 of the DC voltage that should have been applied at point (b) such that the block stays in equilibrium at $z_{\mathrm{b}} = 0$. (2 points)

Hint: Please consult your recent Mastering Physics solutions.