Electricity and Magnetism

Overview Magnetism

- 28-5: Introduction, magnetism: field and force
- 1-6: Magnetism: Biot-Savart, Ampere
- 4-6: Electromagnetic induction
- 8-6: Electromagnetic induction
- 11-6: Maxwell’s equations and electromagnetic waves
- 15-6: Local laws for the magnetostatic field, local laws for the electromagnetic field, magnetic field intensity H
- 18-6: available for answering questions, exercises

Magnetism: force and field

- Learning objectives
- Magnetic field and magnetic forces
- Charged particles in magnetic fields
- Magnetic force on current
- Origin of the magnetic field
- Magnetic dipoles
- Magnetic matter
- Ampere’s law
- Concluding

In this lecture you'll learn to

- Describe magnetism in relation to electric charge
- Calculate magnetic forces on charges and currents
  - Describe the trajectories of charged particles in magnetic fields
- Explain the origin of magnetic fields
  - Calculate the magnetic fields of simple current distributions
- Describe the effects of magnetism in matter

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Clicker question 7

- Which of the following statements is TRUE?

A. A metal wire is bent into a square and carries a uniform current throughout it. The net magnetic field at the center of this square is zero.

B. The net magnetic field inside a conductor must be zero.

C. Two long, current-carrying wires run parallel to each other. If these wires tend to push away from each other, the currents in them must be going in opposite directions.

Clicker question 8

- A charged particle moving in a constant magnetic field

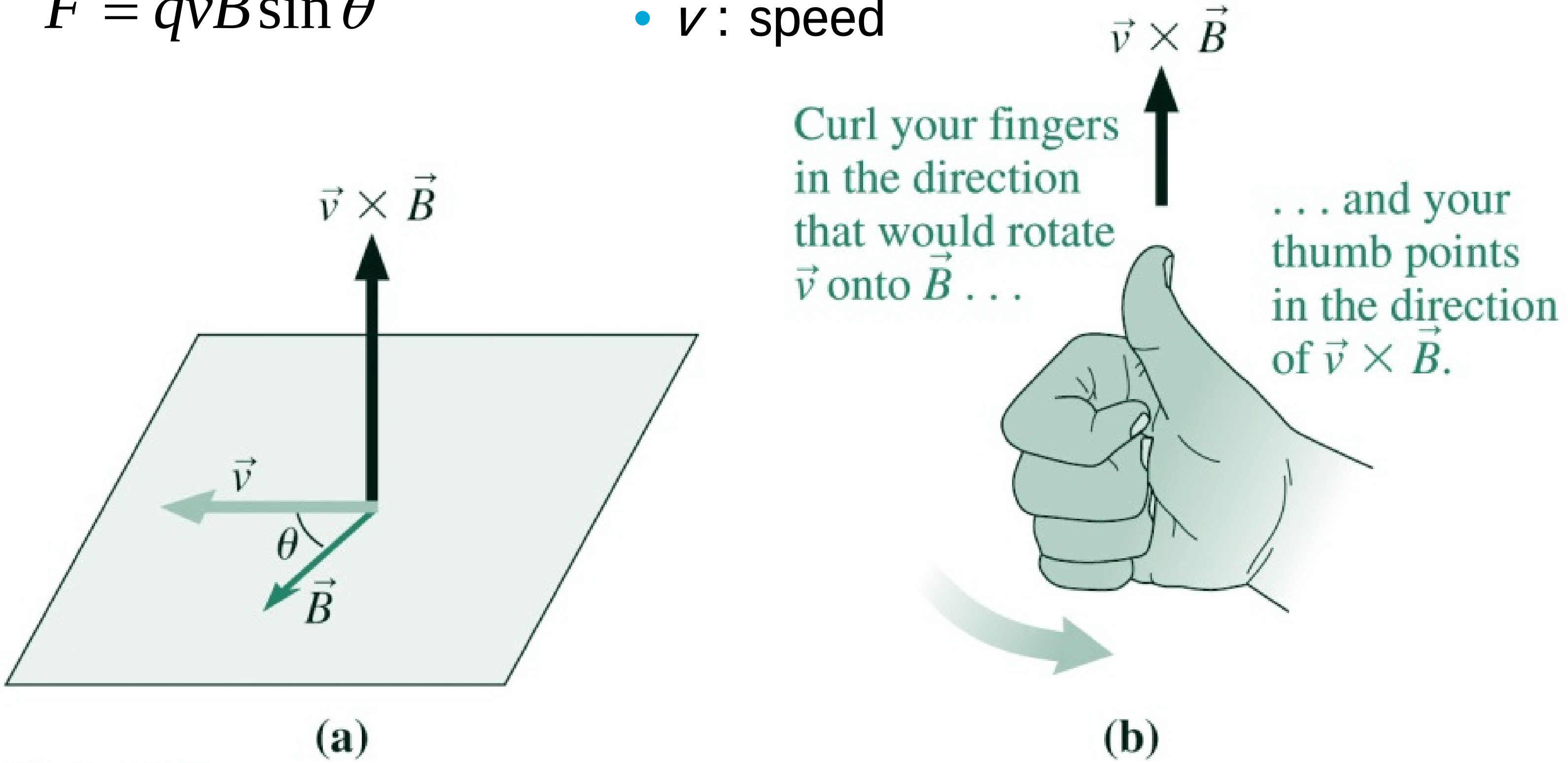
  A. always experiences a magnetic force, regardless of its direction of motion.
  B. may experience a magnetic force which will cause its speed to change.
  C. may experience a magnetic force, but its speed will not change.

Magnetic field and magnetic force

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB\sin\theta$$

- B : magnetic field, magnetic flux density
- q : charge in free space (vacuum)
- v : speed



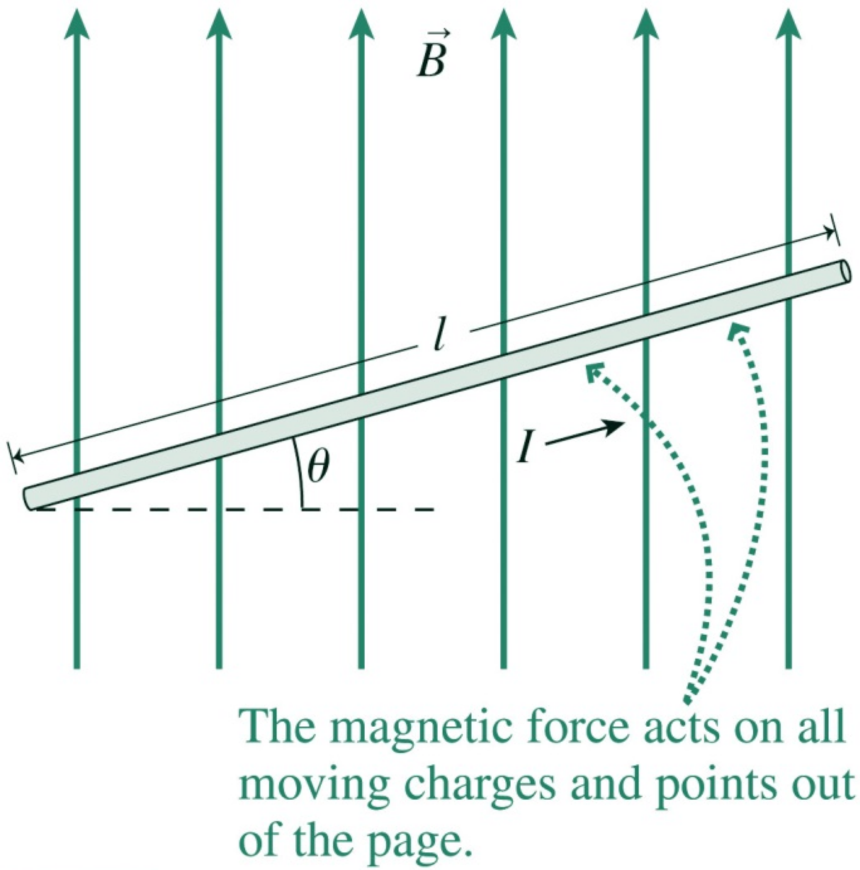
Magnetic force on current

$$\vec{F} = nALq\vec{v}_d \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$F = ILB\sin(\tfrac{1}{2}\pi - \theta)$$

- B : magnetic field
- q : charge
- v_d : drift velocity (average!)
- A : conductor cross section
- L : conductor length
- I : current
- n : charges per volume



Magnetism: force and field

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Origin of the magnetic field

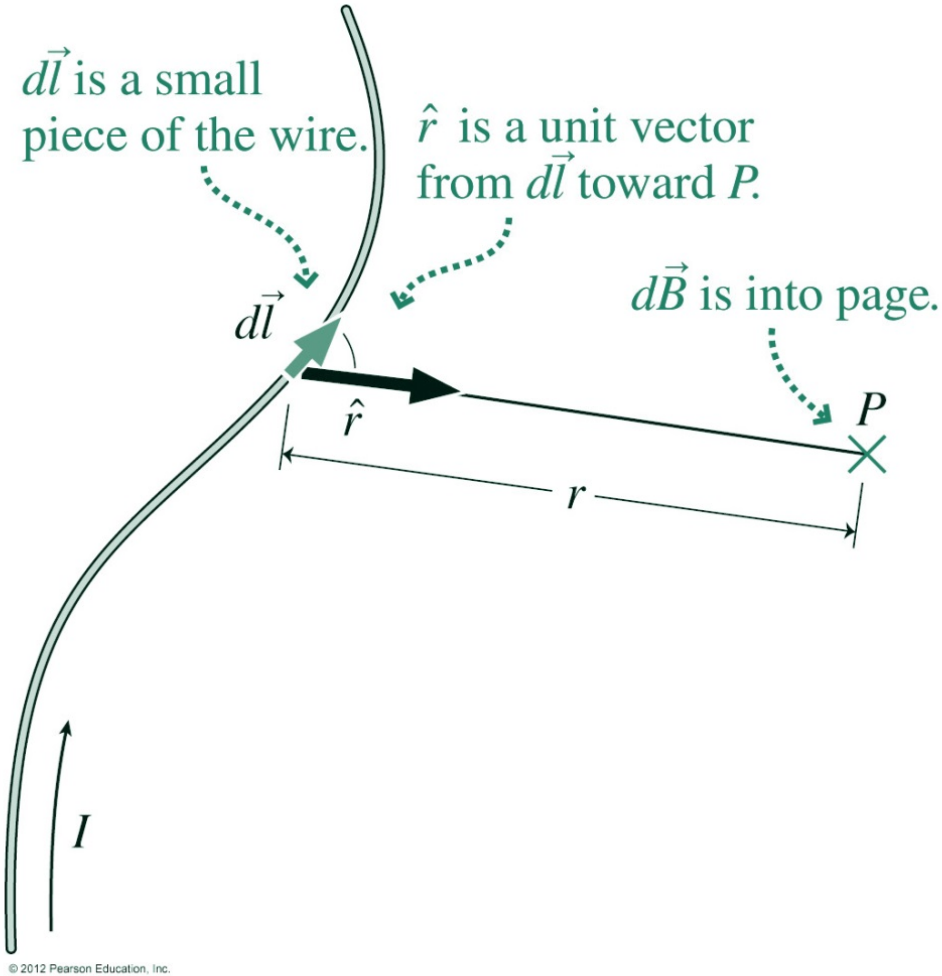
- Magnetic field
  - produces forces on moving electric charges,
  - arises from moving electric charge.
- Biot-Savart:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{L} \times \hat{r}}{r^2}$$

- Therefore:

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I \, d\vec{L} \times \hat{r}}{r^2}$$

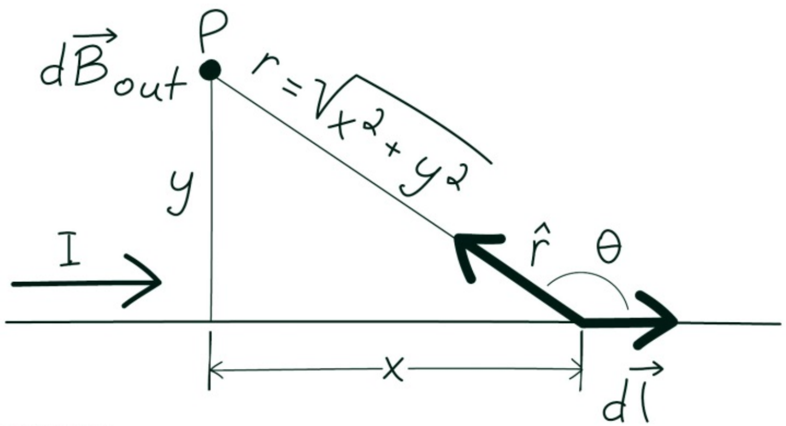
- $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 is the permeability constant.



Using Biot-Savart: a line current

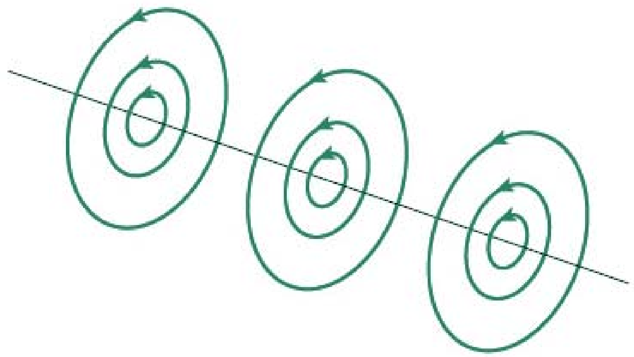
- The field contribution from a current element $I\mathrm{d}L = I\,\mathrm{d}x$ is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dL\sin\theta}{r^2} = \frac{\mu_0 I}{4\pi}\frac{y\,dx}{(x^2+y^2)^{3/2}}$$



- Integrating along an infinite line gives

$$B = \int dB = \frac{\mu_0 I y}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{(x^2+y^2)^{3/2}} = \frac{\mu_0 I}{2\pi y}$$

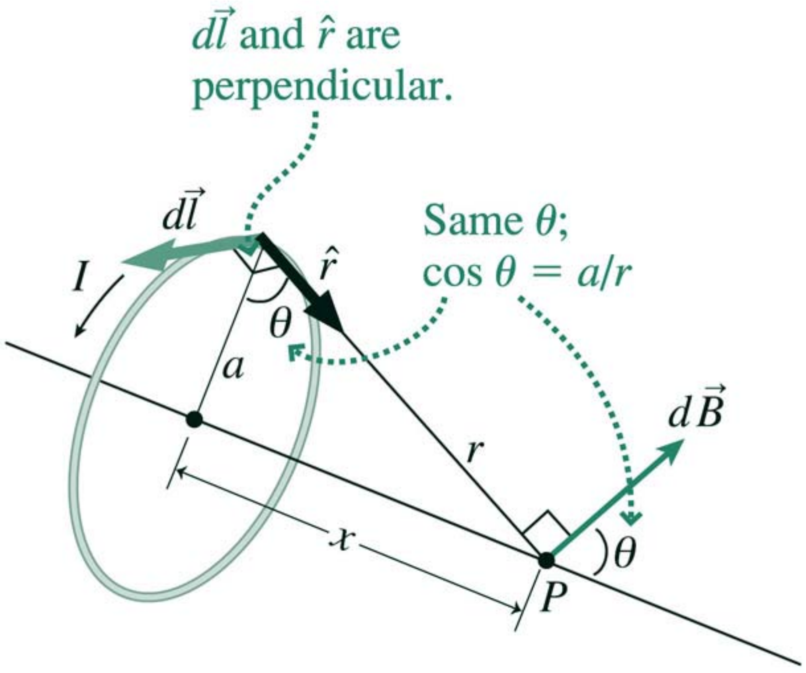


  - The field falls off as the inverse of the distance y from the wire.
  - The field encircles the current.

Using Biot-Savart: current loop

- The field contribution from a current element $I\mathrm{d}L = I\,\mathrm{d}x$ is

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{dL}{x^2+a^2}\frac{a}{\sqrt{x^2+a^2}}$$



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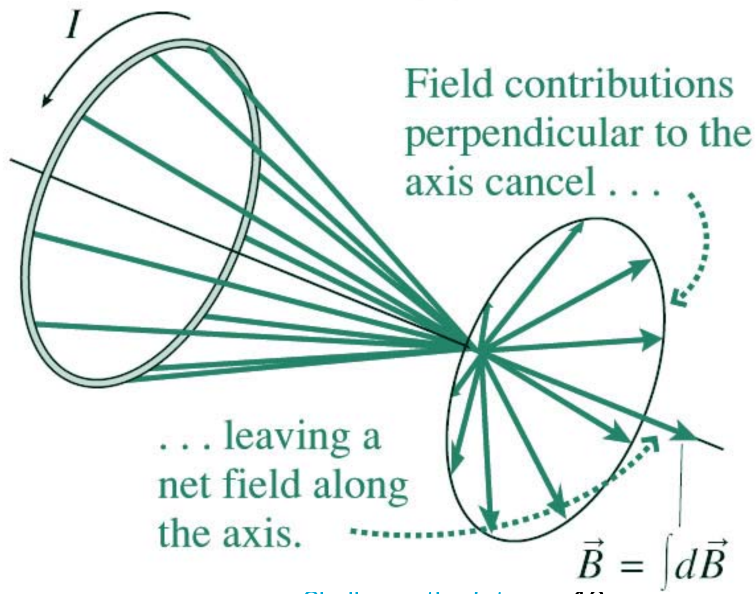


Using Biot-Savart: current loop

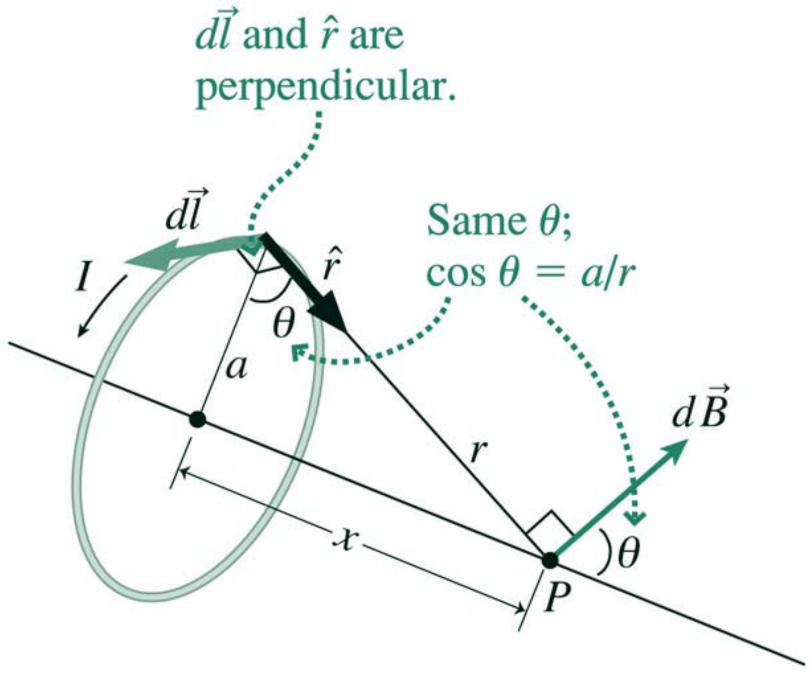
- Integrating the contributions along the circular current loop gives a field on the axis that depends on the distance *x* :

$$B = \int dB_x = \frac{\mu_0 I a}{4\pi(x^2 + a^2)^{3/2}} \int_{\text{loop}} dl$$

$$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

- For large distances ($x >> a$), this reduces to

$$B = \frac{\mu_0 I a^2}{2x^3}$$



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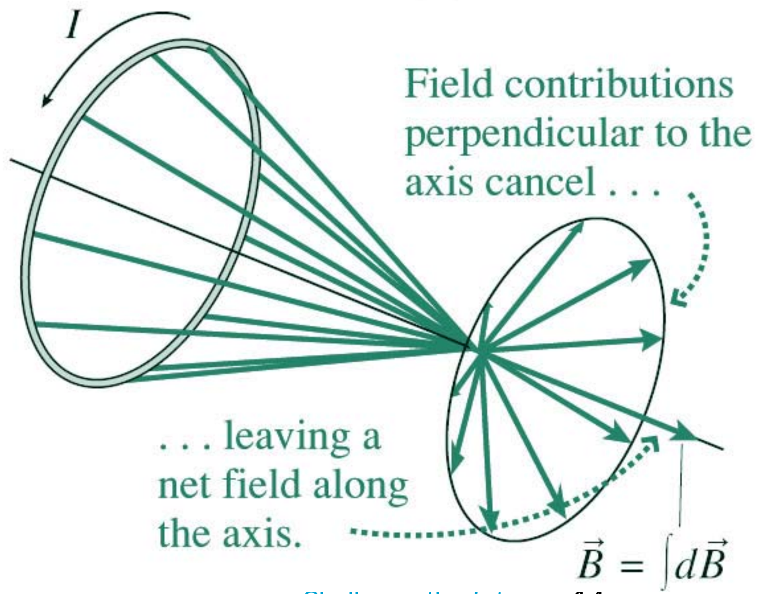

Magnetism: force and field

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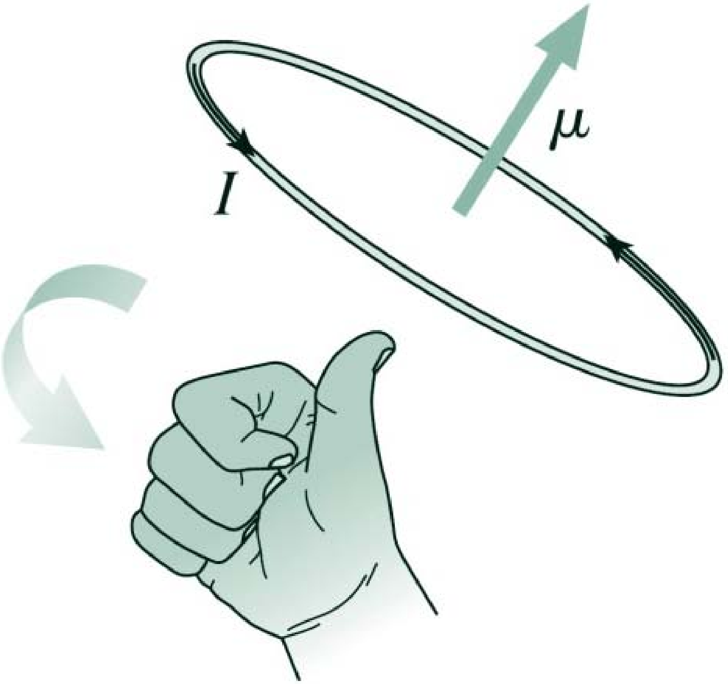
Magnetic dipoles and magnetic dipole moment

- The $1/x^3$ dependence of the current-loop's magnetic field is the same as for the electric field of an electric dipole.
- A current loop constitutes a magnetic dipole.
  - Magnetic dipole moment is $\vec{\mu} = I\vec{A}$ with A the loop area.
  - For an N-turn loop, $\vec{\mu} = NI\vec{A}$
  - Thus,

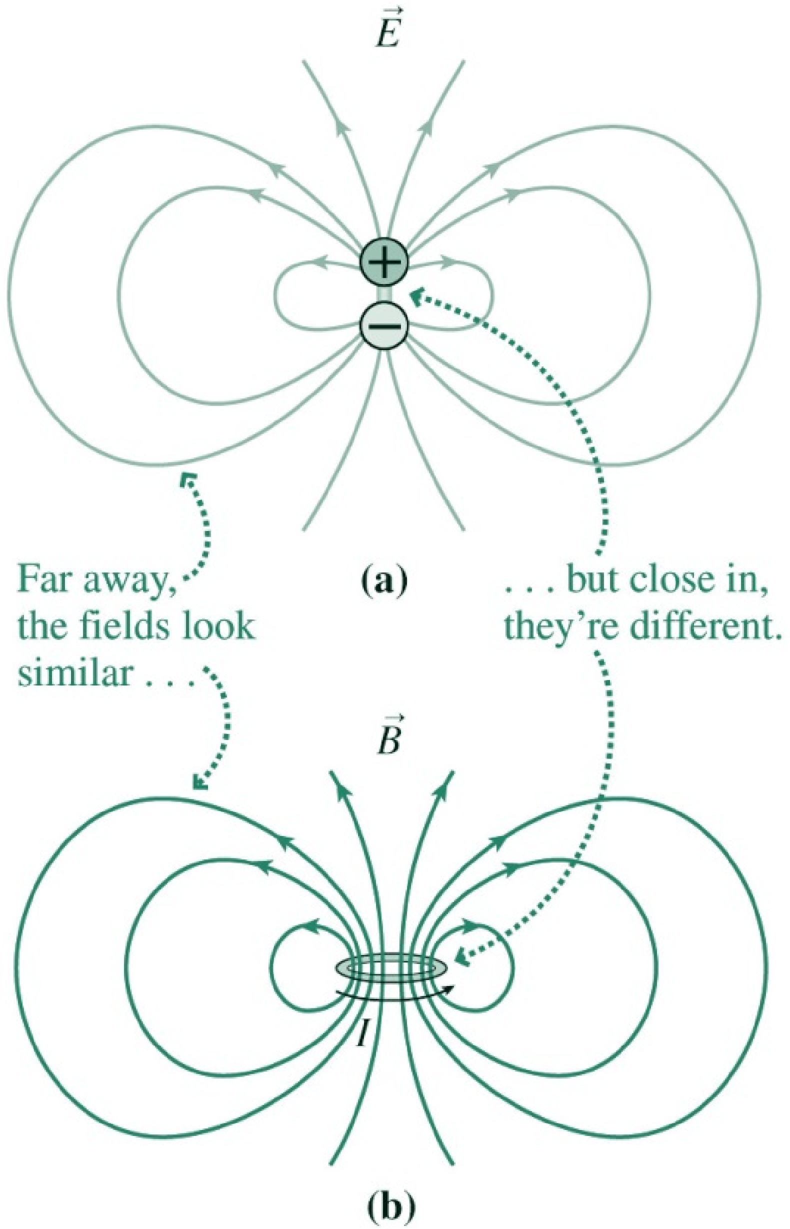
$$B = \frac{\mu_0}{2\pi}\frac{\mu}{x^3}$$

  - The direction of the dipole moment vector is perpendicular to the loop area.



Magnetic dipoles

- The $1/x^3$ dependence of the current-loop's magnetic field is the same as for the electric field of an electric dipole.
- A current loop constitutes a magnetic dipole.
  - The fields of electric and magnetic dipoles are similar far from their sources, but differ close to the sources.



Dipoles and monopoles: Gauss's law for magnetism

- There is no magnetic analog of electric charge.
- Such **magnetic monopoles**, if they existed, would be the source of radial magnetic field lines beginning on the monopoles, just as electric field lines begin on point charges.
- Instead, the dipole is the simplest magnetic configuration.
- The absence of magnetic monopoles is expressed in **Gauss's law for magnetism**:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

  - closed surface integral
  - one of the four fundamental laws of electromagnetism.
  - ensures that magnetic field lines have no beginnings or endings, but generally form closed loops.
- If monopoles are ever discovered, the right-hand side of Gauss's law for magnetism would be nonzero.

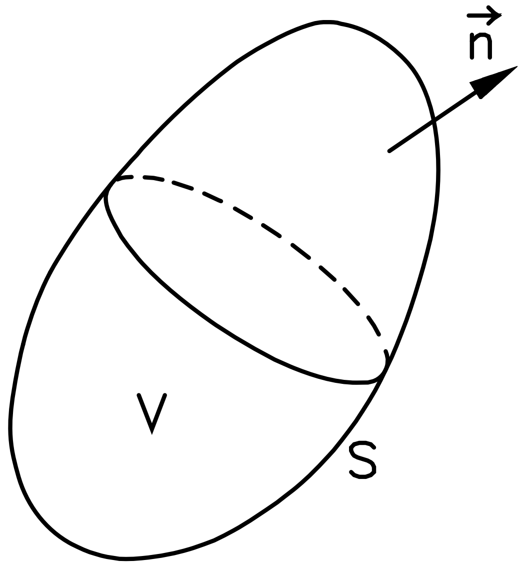
Gauss's law for magnetism

- Closed surface integral:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

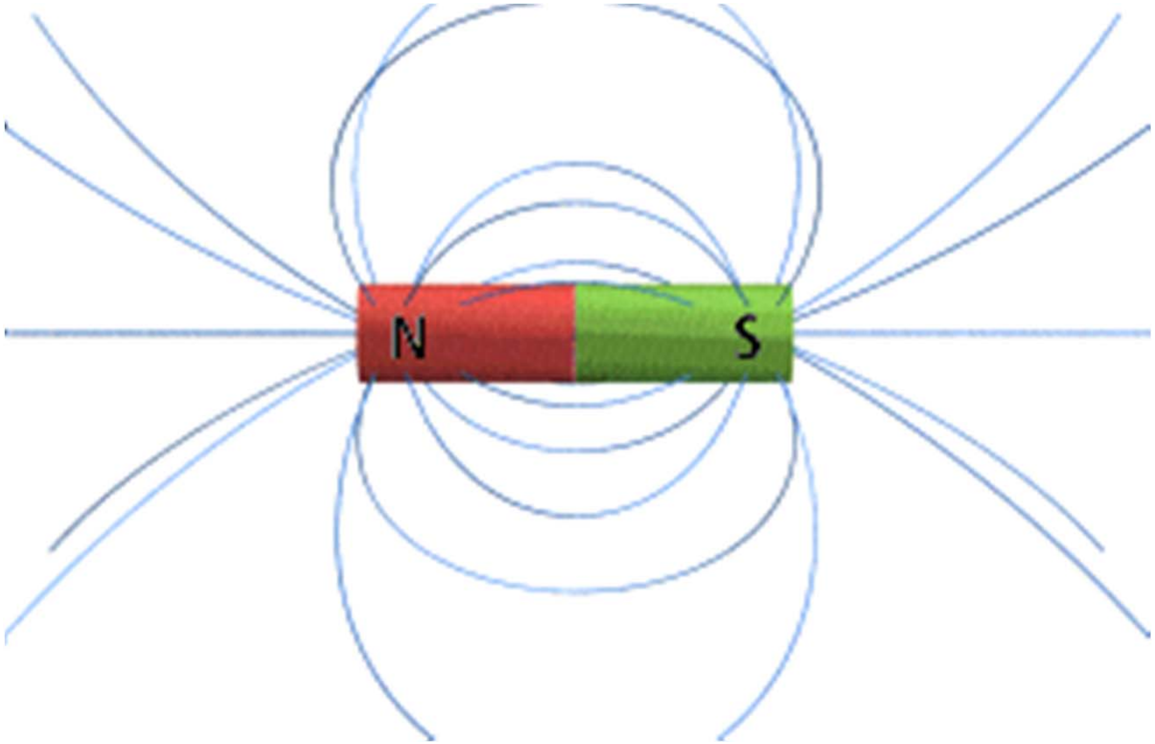
- This normally does not hold for an open surface integral:

$$\int \vec{B} \cdot d\vec{A} \neq 0$$



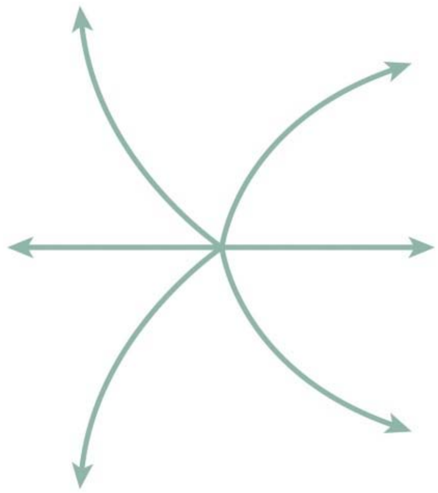
b.

Gauss's law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

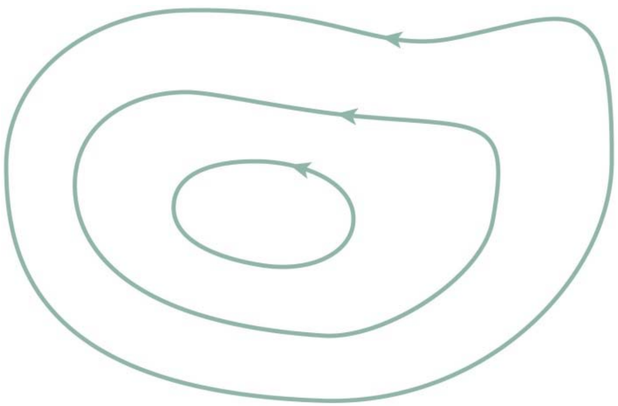


Clicker question 4

- The figure shows two sets of field lines. Which set could be a magnetic field?
  A. (a)
  B. (b)

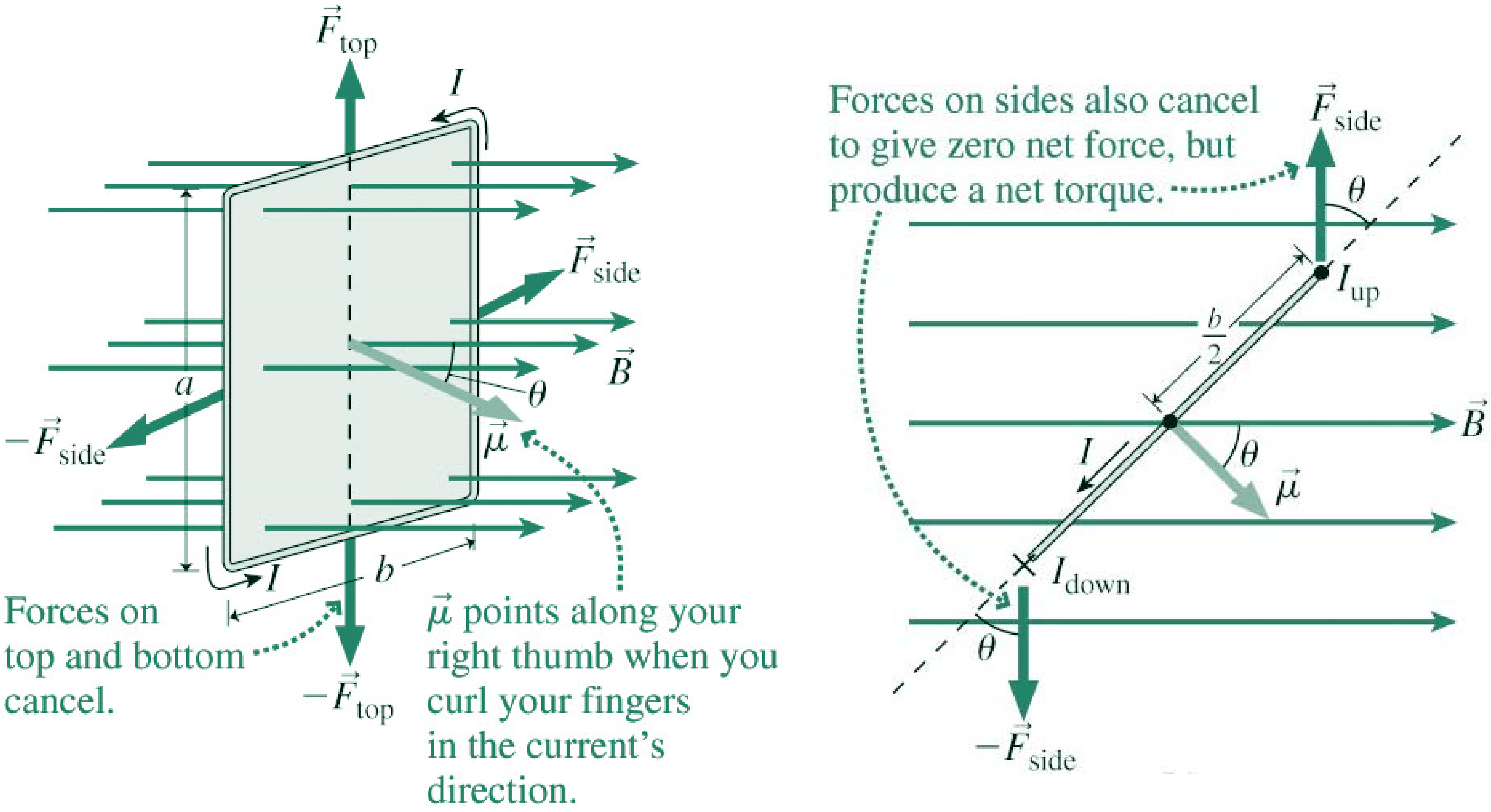


(a)



(b)

Torque on a current loop



Torque on a current loop

- A magnetic dipole experiences a torque in a magnetic field:

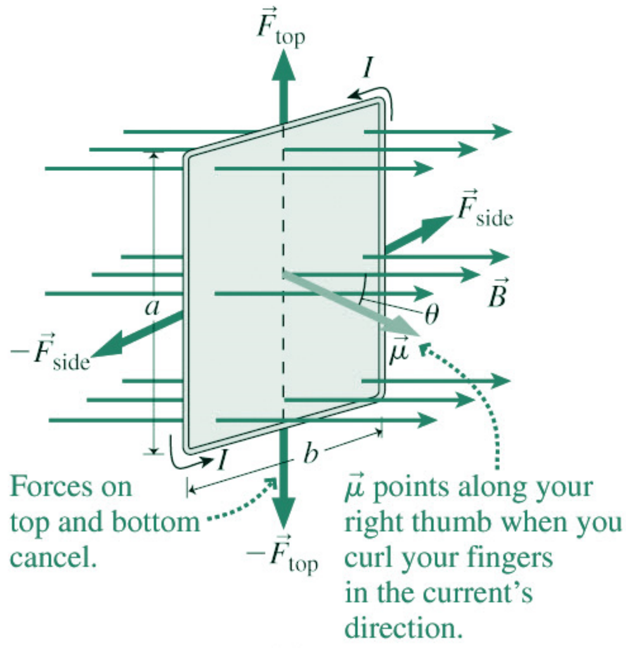
$$F_{\text{side}} = IaB$$

$$\tau_{\text{side}} = \tfrac{1}{2} bF_{\text{side}} \sin\theta = \tfrac{1}{2} IabB \sin\theta = \tfrac{1}{2} IAB \sin\theta$$

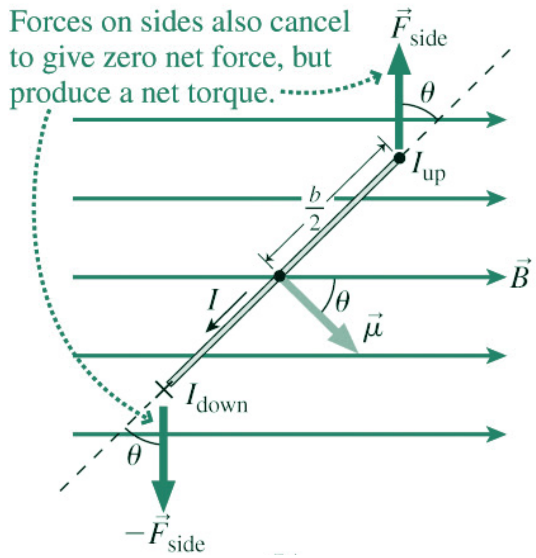
$$\tau = 2\tau_{\text{side}} = IAB \sin\theta = \mu B \sin\theta$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

- The dipole also experiences a net force if the field is non-uniform.

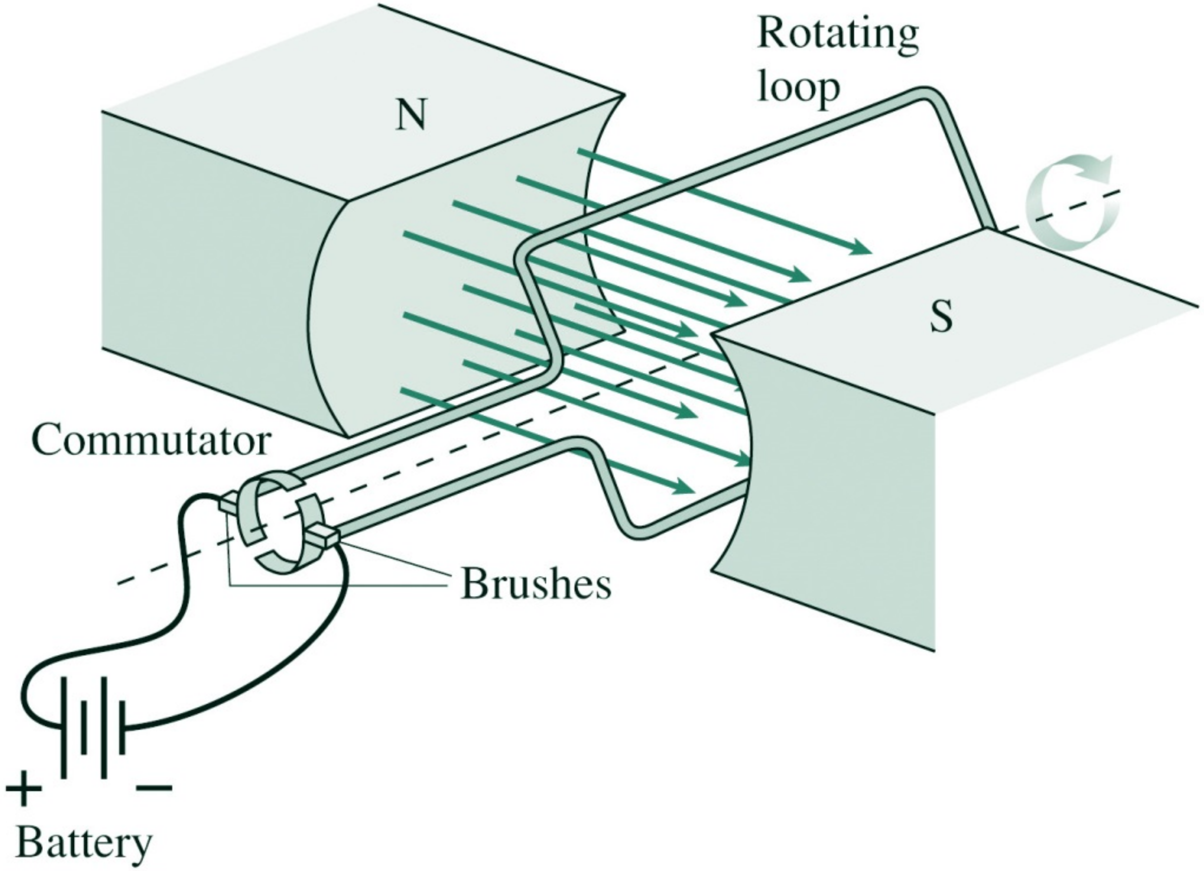


(a)



Electric motors

- Application of the torque on a current loop.
- A current loop spins between magnet poles.
- In a DC motor, the **commutator** keeps reversing the current direction to keep the loop spinning in the same direction.



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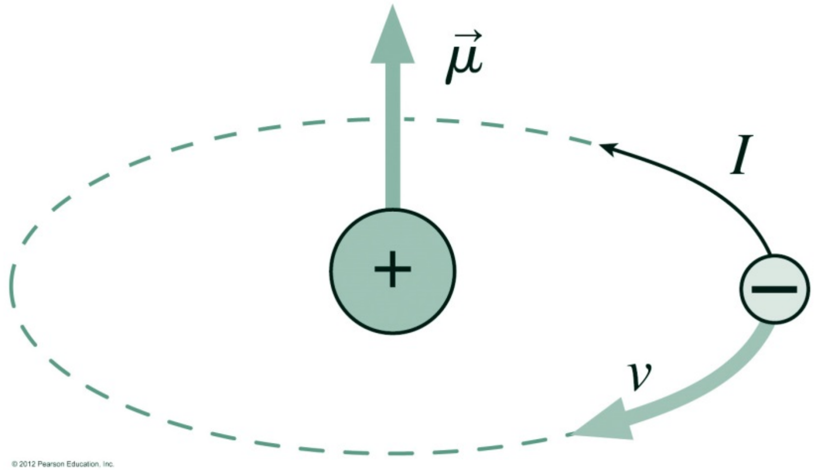
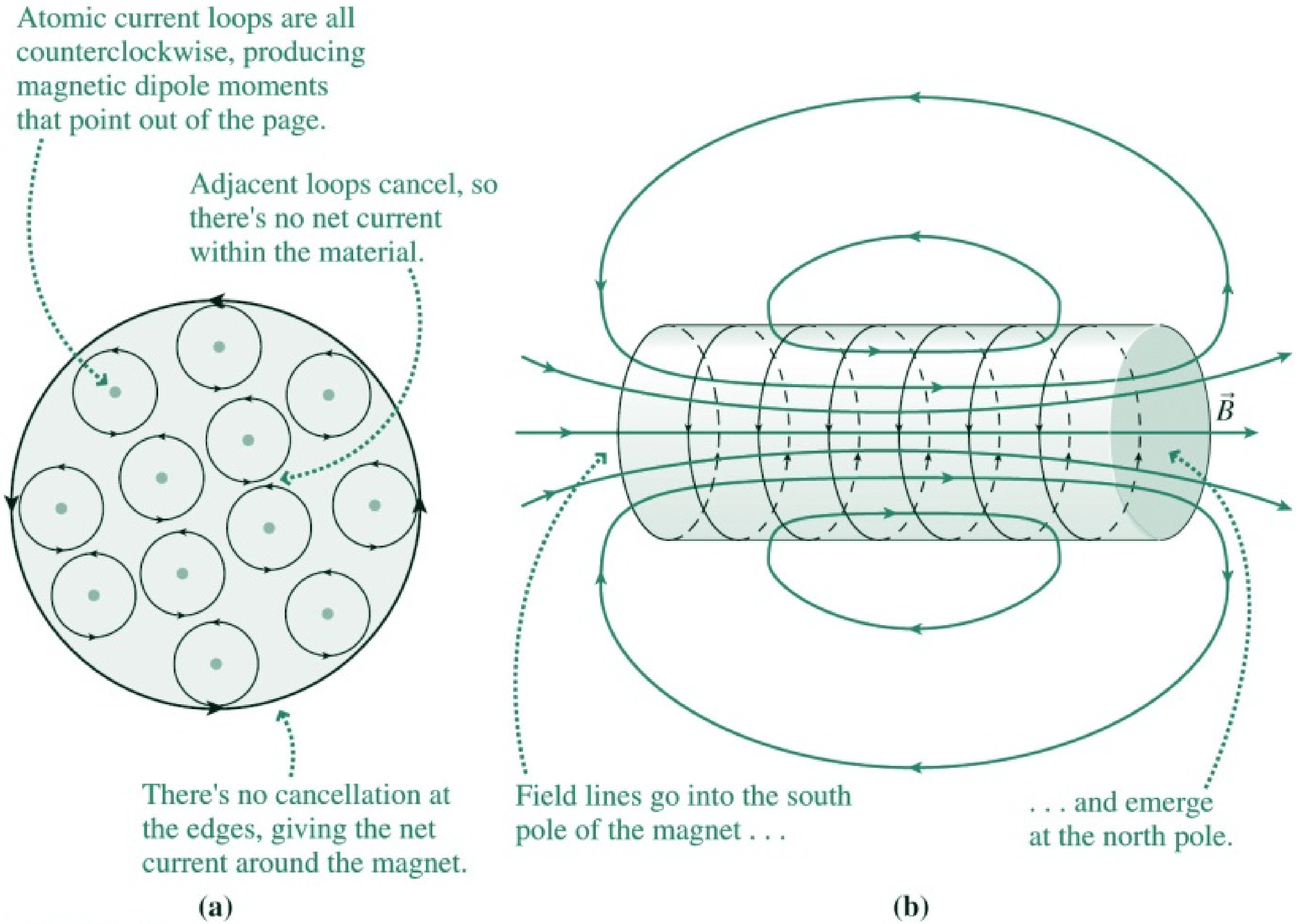
Magnetism in matter

- Magnetism in matter arises from atomic current loops associated with orbiting and spinning electrons.
- In **ferromagnetic** materials like iron, strong interactions among individual magnetic dipoles result in large-scale magnetic properties, including strong attraction to magnets.
- **Paramagnetic** materials exhibit much weaker magnetism.
- **Diamagnetic** materials respond oppositely, and are repelled by magnets
- https://www.youtube.com/watch?v=A1vyB-O5i6E



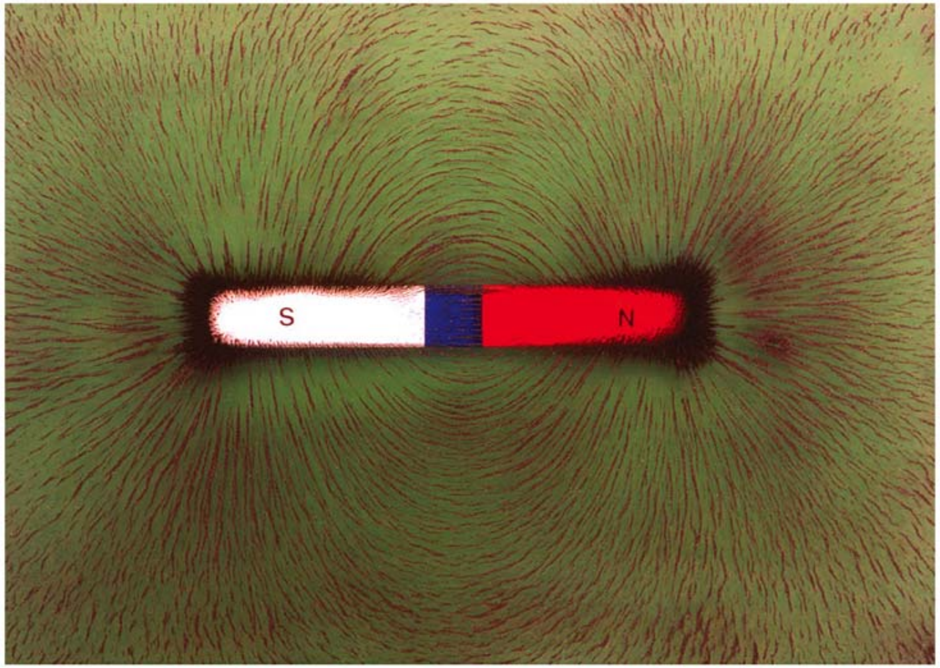
Classical picture of magnetic dipole moment arising from orbiting electron

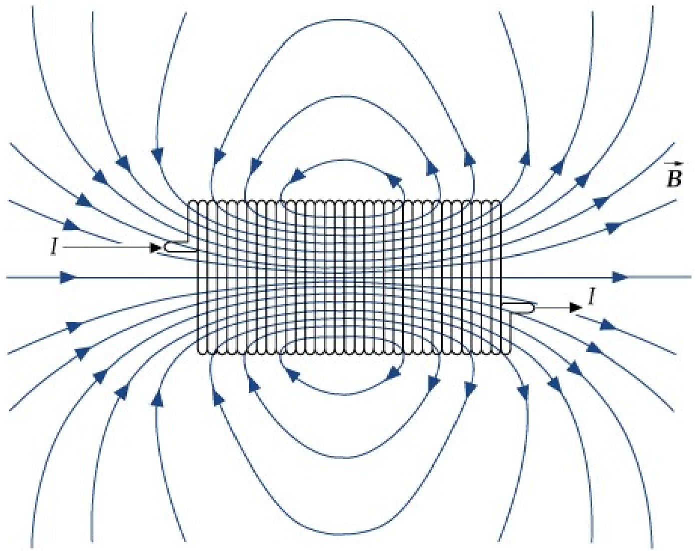
Permanent Magnetism in Matter



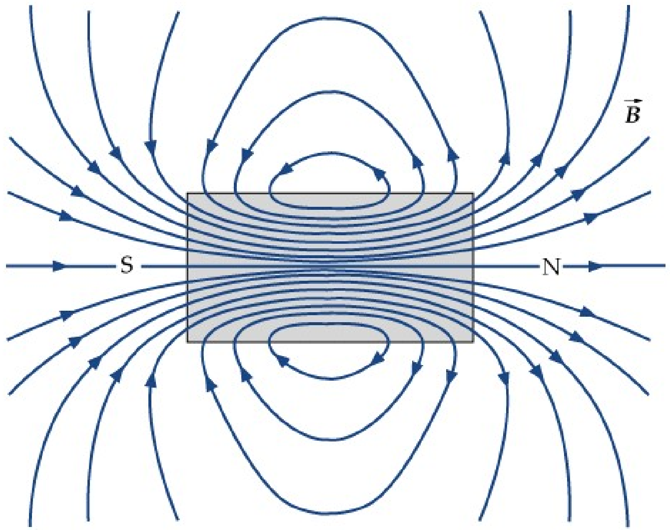
Magnetic dipole moment

- Magnetic dipole moments of a permanent magnet and of a coil have the same form





(a)



(b)

Magnetism in matter

- Why are ferromagnetic materials attracted by magnets?

Magnetism: force and field

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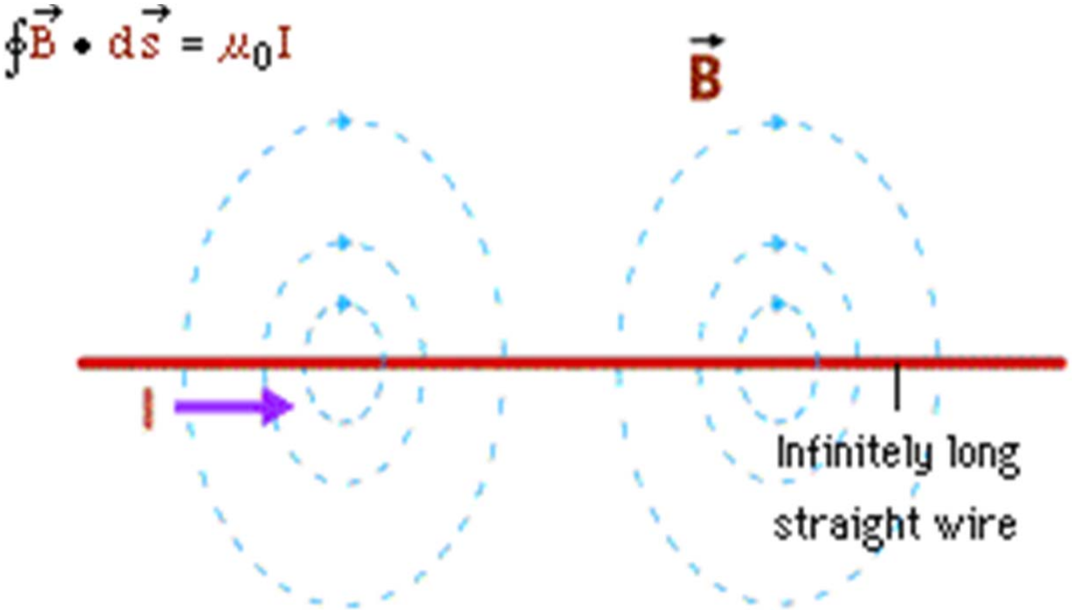
Ampère's law

- Gauss's law for electricity provides a global description of the electric field in relation to charge that is equivalent to Coulomb's law.
- Analogously, Ampère's law provides a global description of the magnetic field in relation to moving charge that is equivalent to the Biot-Savart law.
    - But where Gauss's law involves a **surface** integral over a closed surface, Ampère's law involves a **line** integral around a closed loop.
    - For steady currents, Ampère's law says $$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$$
    - where the integral is taken around *any* closed loop, and $I_{\mathrm{encircled}}$ is the current encircled *by that loop.*

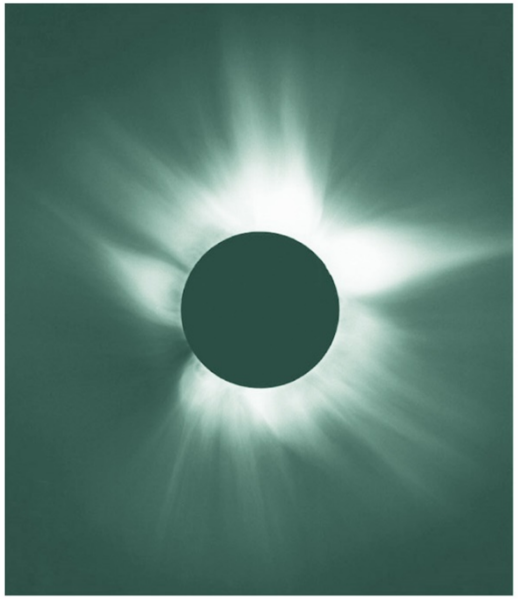
Ampère's law

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$$

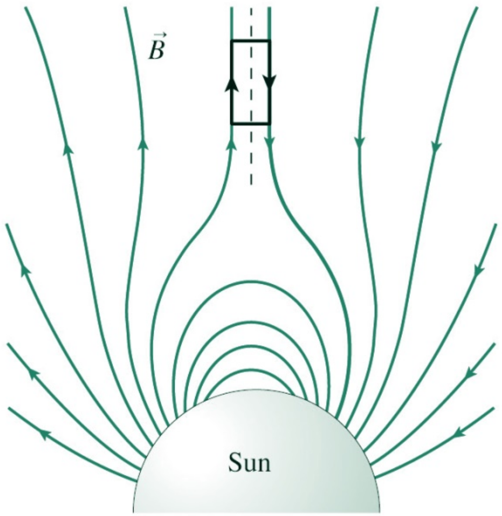


Ampère’s law and current

- Ampère’s law says wherever the integral of the magnetic field around a closed loop is nonzero, then there must be current flowing through the area bounded by the loop.
- The oppositely directed magnetic fields in these structures in the solar corona necessarily involve currents flowing perpendicular to the image, as application of Ampère’s law to the rectangular amperian loop shows.



(a)



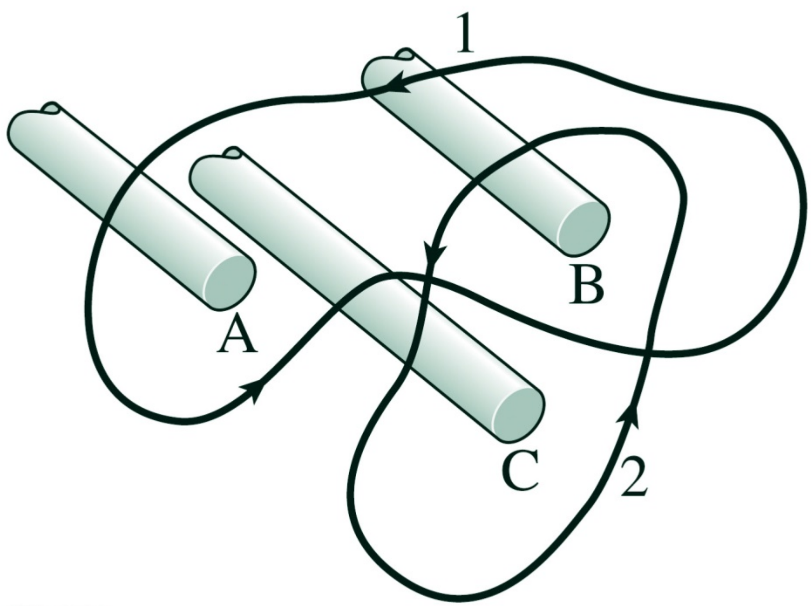
(b)

Clicker question 5

- The figure shows three parallel wires carrying current of the same magnitude I but in one of them the current direction is opposite that of the other two.
- Around loop 2, $\oint \vec{B} \cdot d\vec{r} \neq 0$
- Around loop 1, what is $\oint \vec{B} \cdot d\vec{r}$

A. $\oint \vec{B} \cdot d\vec{r} = 0$

B. $\oint \vec{B} \cdot d\vec{r} \neq 0$



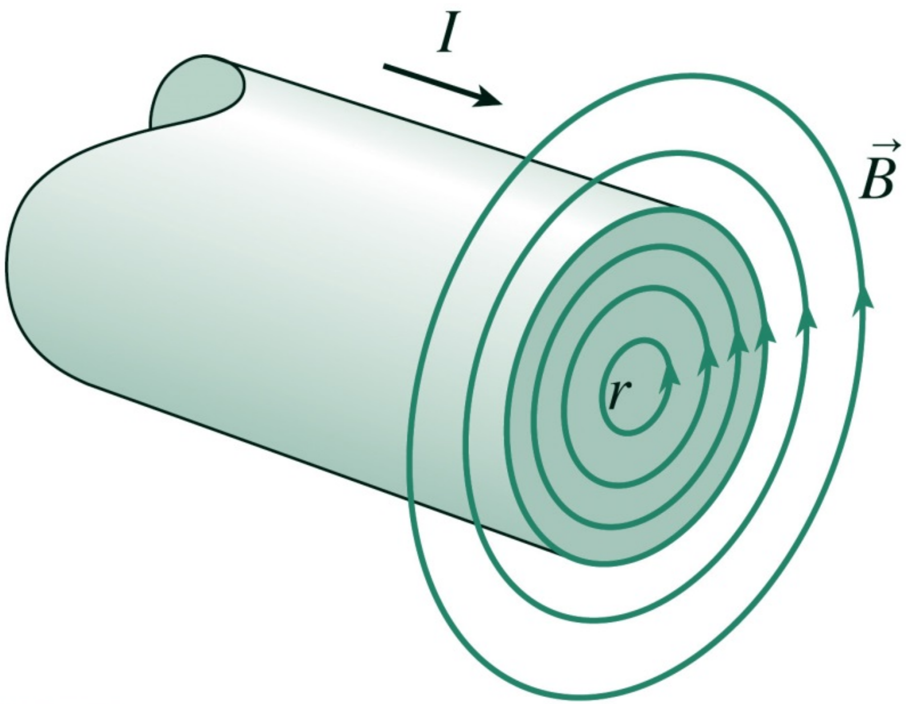
Using Ampère’s law

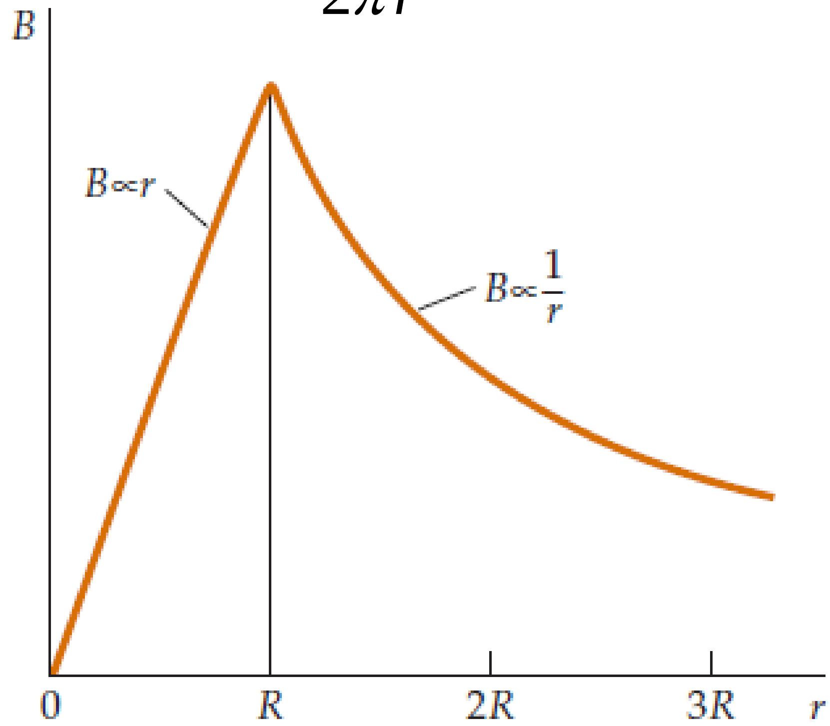
- Ampère’s law is always true, but it can be used to calculate magnetic fields only in cases with sufficient symmetry.
- Then it’s possible to choose an amperian loop around which Amperes law can be evaluated in terms of the unknown *B.*

Ampère's law: line current $\oint \vec{B}\cdot d\vec{r} = \mu_0 I_{encircled}$

$$r < R: \quad 2\pi r B = \mu_0 I_{encircled} \frac{r^2}{R^2} \quad \Rightarrow \quad B = \frac{\mu_0 I_{encircled}\, r}{2\pi R^2}$$

$$r > R: \quad 2\pi r B = \mu_0 I_{encircled} \quad \Rightarrow \quad B = \frac{\mu_0 I_{encircled}}{2\pi r}$$





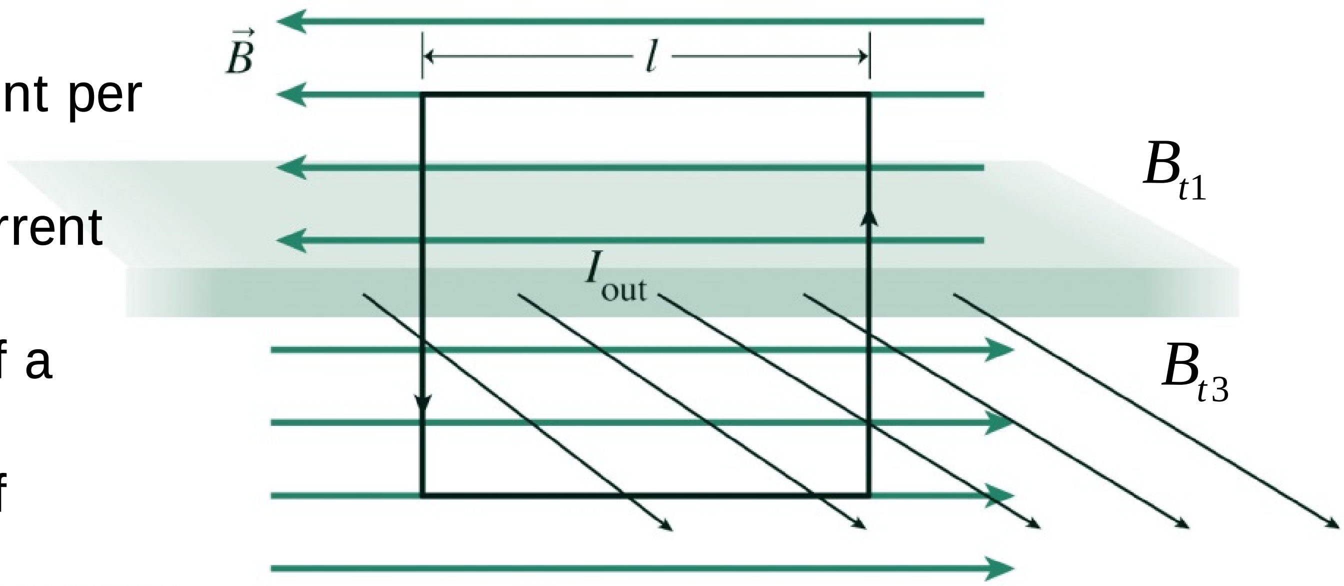
Ampère's law: current sheet

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$$

$$lB_{t1} + hB_{n2} + lB_{t3} + hB_{n4} = \mu_0 l J_s$$

$$B = \tfrac{1}{2} \mu_0 J_s$$

- J_s is the current per unit width.
- An infinite current sheet is an idealization of a wide, flat distribution of current.

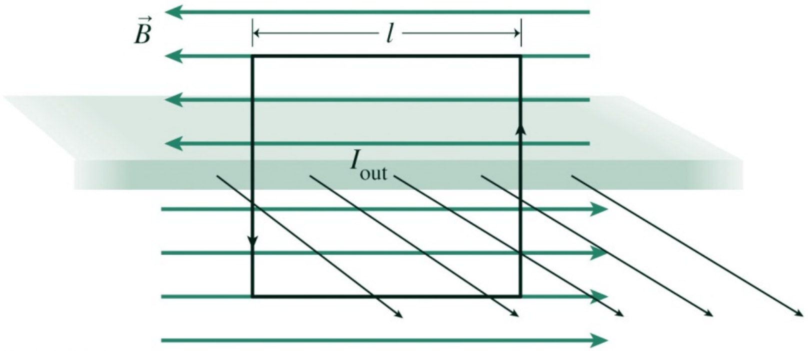


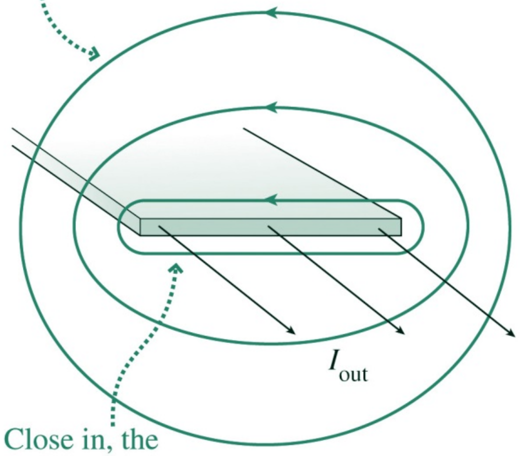
Ampère's law: current sheet

- Application of Ampère's law shows that the magnetic field outside the sheet is uniform and has magnitude

$$B = \tfrac{1}{2}\mu_0 J_s$$

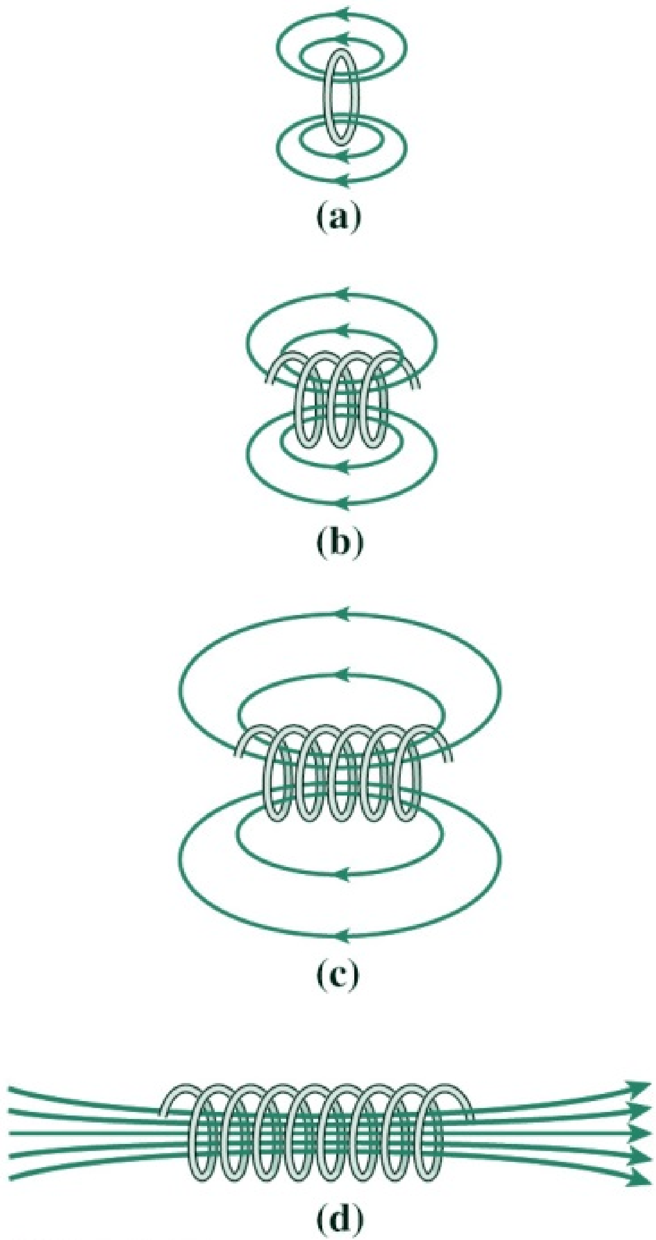
- J_s is the current per unit width.
- The field direction reverses across the current sheet.
- Far from a finite current sheet, the field resembles that of a line current.





Solenoids

- A solenoid is a long, tightly wound coil of wire. When a solenoid's length is much greater than its diameter, the magnetic field inside is nearly uniform except near the ends, and the field outside is very small.



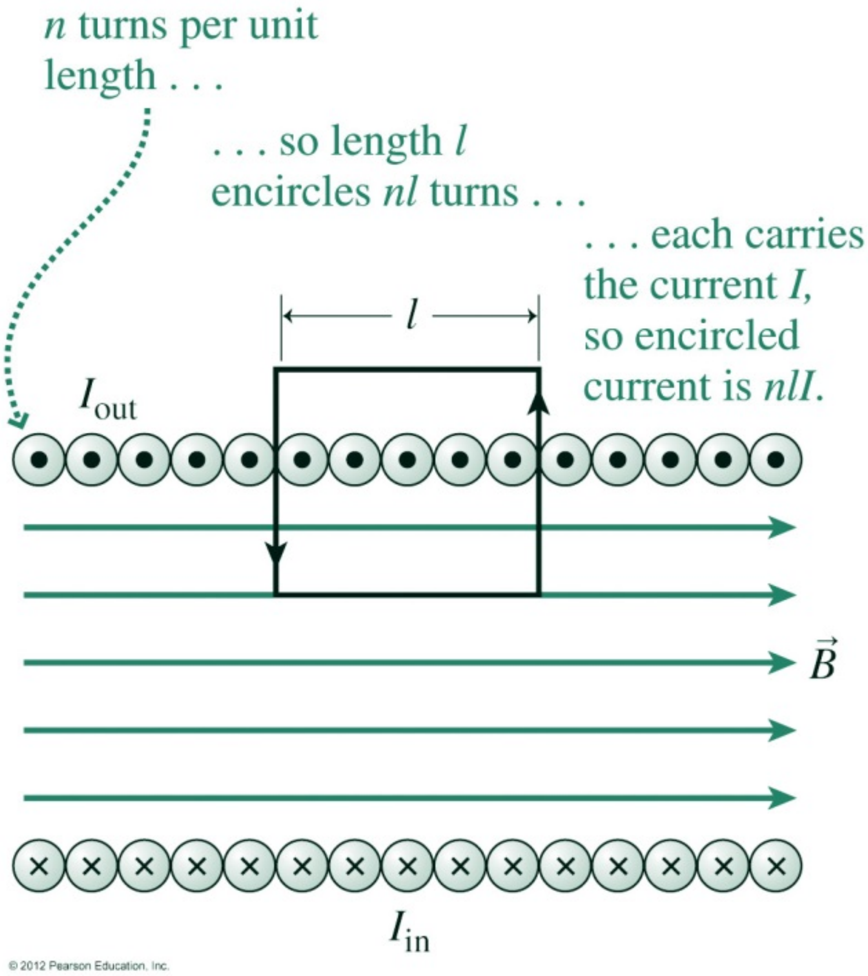
Solenoids

- In the ideal limit of an infinitely long solenoid, the field inside the solenoid is uniform everywhere, and the field outside is zero.

- Application of Ampère's law shows that the field of an infinite solenoid is

$$B = \mu_0 n I$$

where n is the number of turns per unit length.



Clicker question 9a

- A solenoid has length L, N windings, and radius b ($b \ll L$), with a current I flowing through the wire. If the length of the solenoid is doubled while all other quantities remain the same, the magnetic field inside the solenoid would

A. stay the same.
B. become twice as strong as the original.
C. become half as strong as the original.

Clicker question 9b

- A solenoid has length L, N windings, and radius b ($b << L$), with a current I flowing through the wire. If the number of turns of the solenoid is doubled while all other quantities remain the same, the magnetic field inside the solenoid would

A. stay the same.
B. become twice as strong as the original.
C. become half as strong as the original.

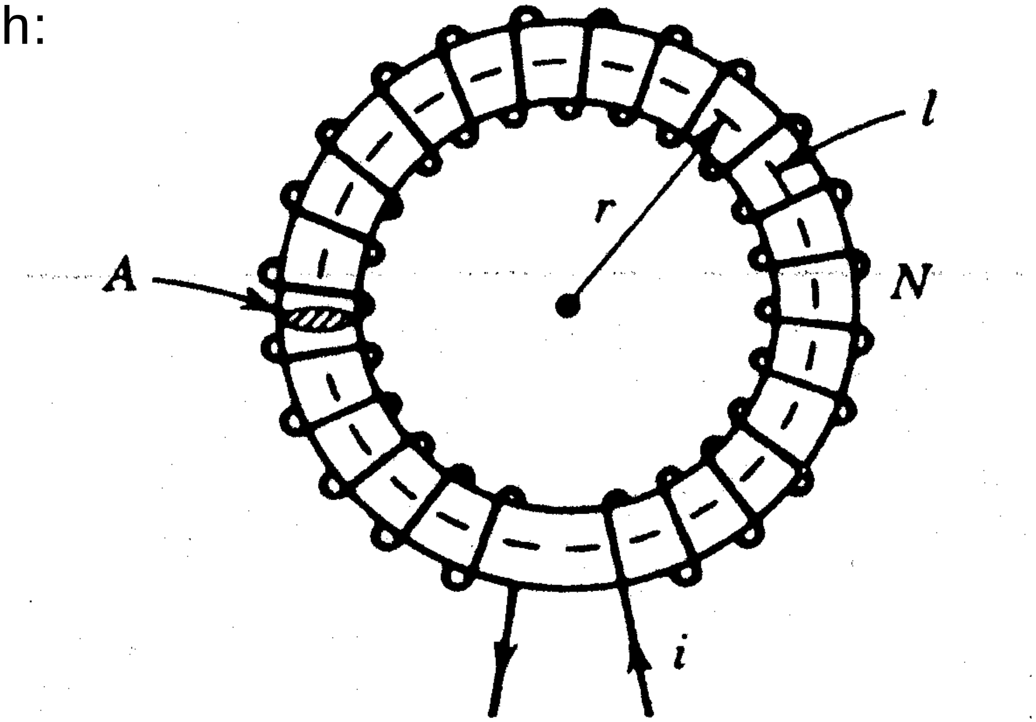
Toroidal coil

Contour follows magnetic path:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$$

$$2\pi r B = \mu_0 N i$$

$$B = \frac{\mu_0 N i}{2\pi r}$$



Assumption: symmetry

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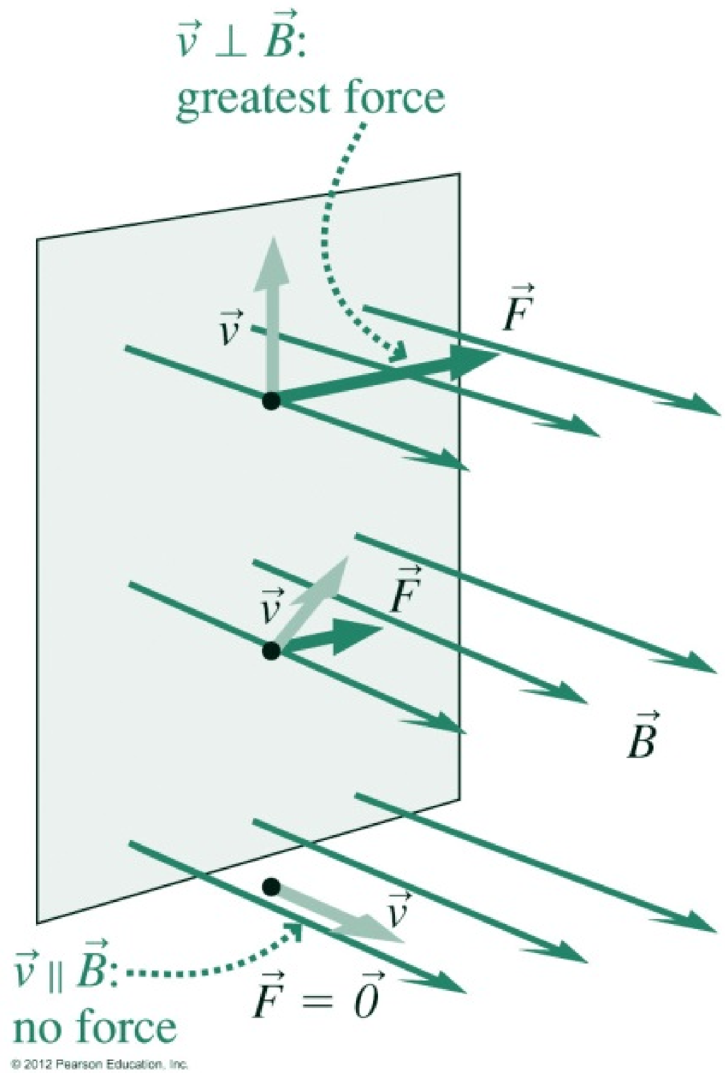
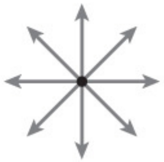
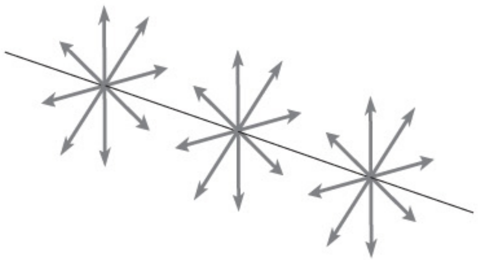
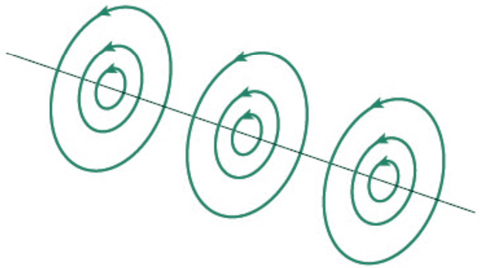
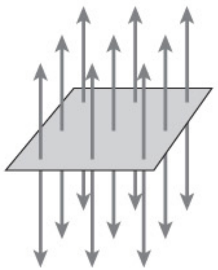
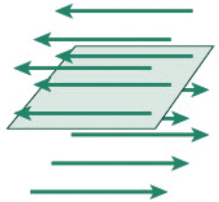
Does a uniform magnetic field exist?



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB\sin\theta$$

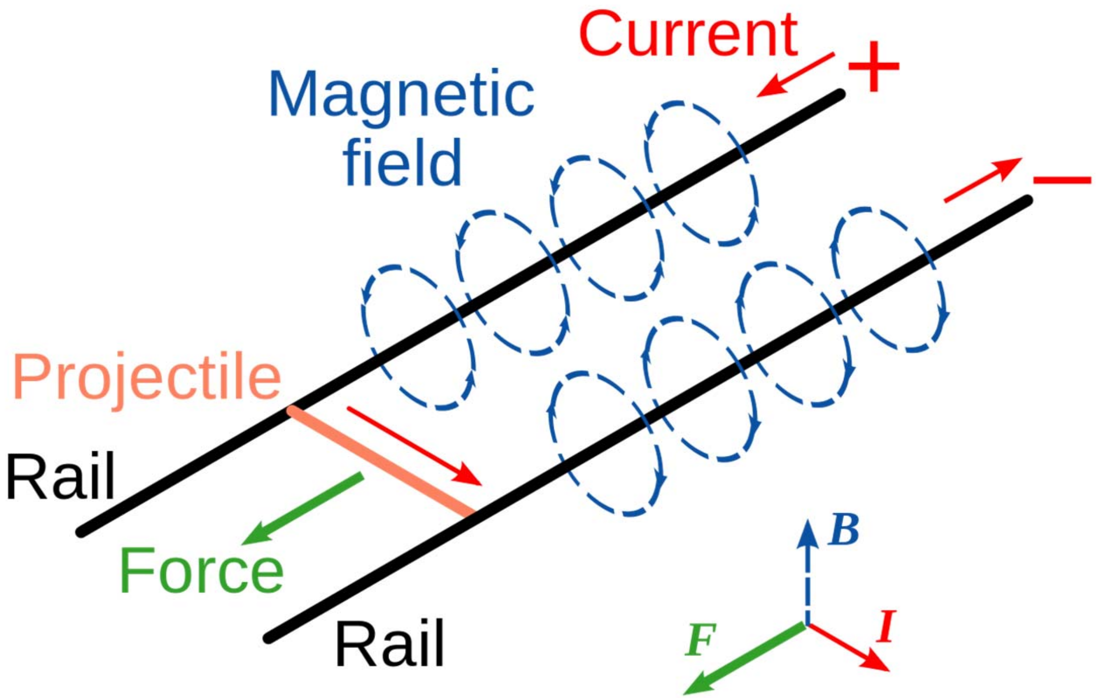
Electric and magnetic fields of common charge and current distributions

Field Dependence on Distance[a]	Charge Distribution	Electric Field	Current Distribution	Magnetic Field
$\frac{1}{r^3}$	Electric dipole		Magnetic dipole	
$\frac{1}{r^2}$	Point charge or spherically symmetric		Impossible for steady current	
$\frac{1}{r}$	Charge distribution with line symmetry		Current distribution with line symmetry	
Uniform field; no variation	Infinite flat sheet of charge		Current sheet	

Summary

- **Magnetism** involves *moving electric charge.*
- Magnetic fields exert forces on moving electric charges:
  - For a moving charge: $\vec{F} = q\vec{v} \times \vec{B}$
  - For a current: $\vec{F} = I\vec{L} \times \vec{B}$
- Magnetic fields arise from moving electric charge, as described by
  - Biot-Savart law: $\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$
  - Ampère's law: $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{encircled}$
- Magnetic fields encircle the currents and moving charges that are their sources.
  - Magnetic field lines don't begin or end.
  - This is expressed in Gauss's law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$

Railgun



- 2 rails r = 2 mm
- Distance d = 14 mm
- Current 1000 A
- The rails are very long

- Calculate
  - the magnetic flux density between the conducting rails
  - the magnetic force per meter of the rails on each other
  - the magnetic flux density in the middle of the projectile (Biot-Savart)
  - the magnetic force on the projectile
  - the acceleration of a 10 g projectile