Probability

Lecture 2.1: Frequently used discrete random variables

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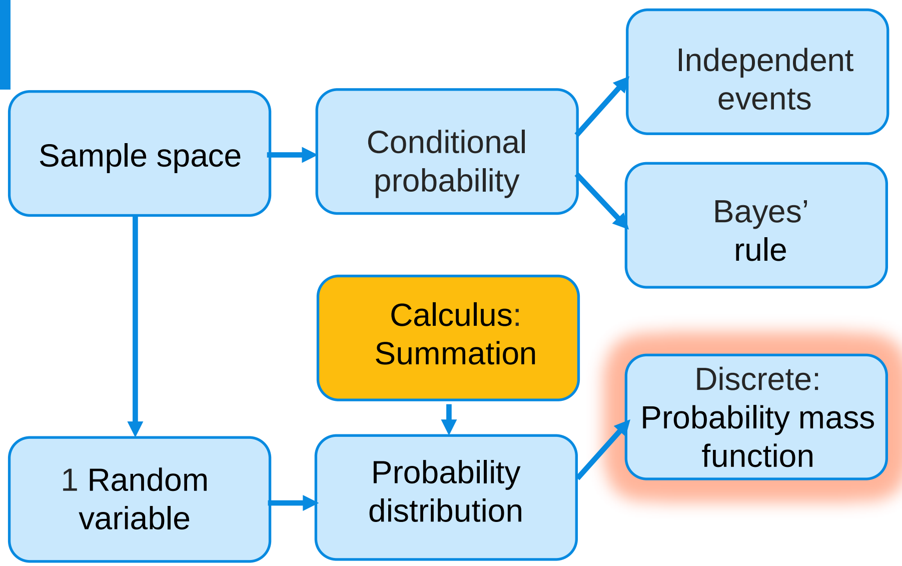
Learning objective

After this class you

- **know five standard discrete distributions**
- **recognize the contexts in which they occur**

Book: Sections 4.3, 4.4

Probability



Before this class (week 3.2 lesson 1):

Watch the prelecture ‘*Binomial coefficients*’

Book: section 4.3; chapter 12, from page 169 till Quick Exercise 12.1

Programme

- Distributions: Bernoulli, Binomial, Geometric
- Exercises
- Negative Binomial Distribution
- Exercises
- Poisson distribution
- Conclusion

Which equality is correct:

(1) $$\binom{12}{3} = \frac{12!}{3!9!}$$

(2) $$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1}$$

A) Both are correct

B) Only (1) is correct

C) Only (2) is correct

D) None of the two is correct

Carla, Christa and Cathy, three ET-students, can choose a date from twenty-five (male) fellow students to go to the ETV gala night. How many 'couplings' are possible?

A) $25 \cdot 24 \cdot 23$

B) $\binom{25}{3}$

C) $28 \cdot 27 \cdot 26$

D) $\binom{28}{3}$

Bernoulli distribution

Definition:
A random variable X has a **Bernoulli distribution**
if X only takes on the values 0 and 1, with probabilities

$$\mathrm{P}(X = 1) = p,$$
$$\mathrm{P}(X = 0) = 1 - p.$$

Notation: $X \sim Ber(p)$

Binomial distribution

Definition:

A random variable X has a binomial distribution with parameters ***n*** and ***p*** if X can take on the values $k = 0, 1, \ldots, n$ with probabilities

$$\mathrm{P}\left(X = k\right) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Notation: $X \sim Bin(n, p)$

Geometric distribution

Definition:

A random variable X has a **geometric distribution** with parameter $\boldsymbol{p}$ if X can take on the values $k = 1, 2, 3, \ldots,$ with probabilities

$$\mathrm{P}\,(X = k) = p \cdot (1 - p)^{k-1}$$

Notation: $X \sim Geo(p)$

Exercise

You toss n coins, each showing heads with probability p, independently of the other tosses. Each coin that shows tails is tossed again once. Let X be the number of heads.

a) What type of distribution does X have? Specify its parameter(s).
b) What is the probability mass function of the total number of heads.

A shop receives a batch of 1000 cheap lamps. The probability that a lamp is defective is 0.1%. Let X be the number of defective lamps in the batch.

a) What type of distribution does X have? Specify its parameter(s).
b) What is the probability that the batch contains no defective lamps? Exactly one defective lamp? More than two defective ones?

Exercises

You decide to play monthly in two different lotteries, and you stop playing as soon as you win a prize in one (or both) lotteries of at least one million euros. Suppose that every time you participate in these lotteries, the probability to win one million (or more) euros is p_1 for one of the lotteries and p_2 for the other. Let M be the number of times you participate in these lotteries until winning at least one prize. What kind of distribution does M have, and what is its parameter?

You and a friend want to go to a concert, but unfortunately only one ticket is still available. The man who sells the tickets decides to toss a coin until heads appears. In each toss heads appears with probability p, where $0 < p < 1$, independent of each of the previous tosses. If the number of tosses needed is odd, your friend is allowed to buy the ticket; otherwise you can buy it. Would you agree to this arrangement?

Negative binomial distribution

Throw a die until a 5 comes up <u>for the third time</u>. Denote the number of throws until the third 5 by X.

a) Find $\mathrm{P}(X=2)$, $\mathrm{P}(X=3)$, $\mathrm{P}(X=4)$, $\mathrm{P}(X=10)$.
b) Give a general formula for $\mathrm{P}(X=k)$ for $k \geq 3$.

X has a **negative binomial distribution** with parameters $p = 1/6$ and $r = 3$.

Negative binomial distribution

Definition:

A random variable X has a **negative binomial distribution** with parameters $\boldsymbol{r}$ and $\boldsymbol{p}$ if X can take on the values $k = r, r+1, r+2, \dots,$ with probabilities

$$P(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$$

Notation: $X \sim NB(r, p)$

Put the cards of a well shuffled deck (52 cards) one by one on the table until the first ace appears. The number of cards until this first ace is called X. The distribution of X is of type:

A) Bernoulli

B) Binomial

C) Geometric

D) Negative binomial

E) Other

The first ten cards of a well shuffled deck are put on the table. The number H of cards that are hearts has a distribution of type:

A) Bernoulli

B) Binomial

C) Geometric

D) Negative binomial

E) Other

Poisson distribution

Definition:
A random variable X has a **Poisson distribution** with parameter μ if X can take on the values $k = 0, 1, 2, \ldots,$ with probabilities

$$\mathrm{P}(X = k) = \frac{\mu^k}{k!} e^{-\mu}$$

Notation: $X \sim Pois(\mu)$

Exercises

Let X have a $Pois(2)$ distribution. What is $P(X \leq 1)$?

Have a second look at the exercise below. Now find the (approximation of) the answer using the Poisson approximation. Compare with the answer you found earlier.
A shop receives a batch of 1000 cheap lamps. The probability that a lamp is defective is 0.1%. Let X be the number of defective lamps in the batch.

a) What type of distribution does X have? Specify its parameter(s).
b) What is the probability that the batch contains no defective lamps? Exactly one defective lamp? More than two defective ones?

For next class (week 3.2 lesson 2):

Complete MyStatlab assignments and book exercises

Watch prelecture *'Continuous random variable'*

Book: Section 5.1

After this class you can

- **manage continuous RVs via the probability density function and distribution function**
- **know four standard continuous distributions recognize the contexts in which they occur**

Probability

Good luck!