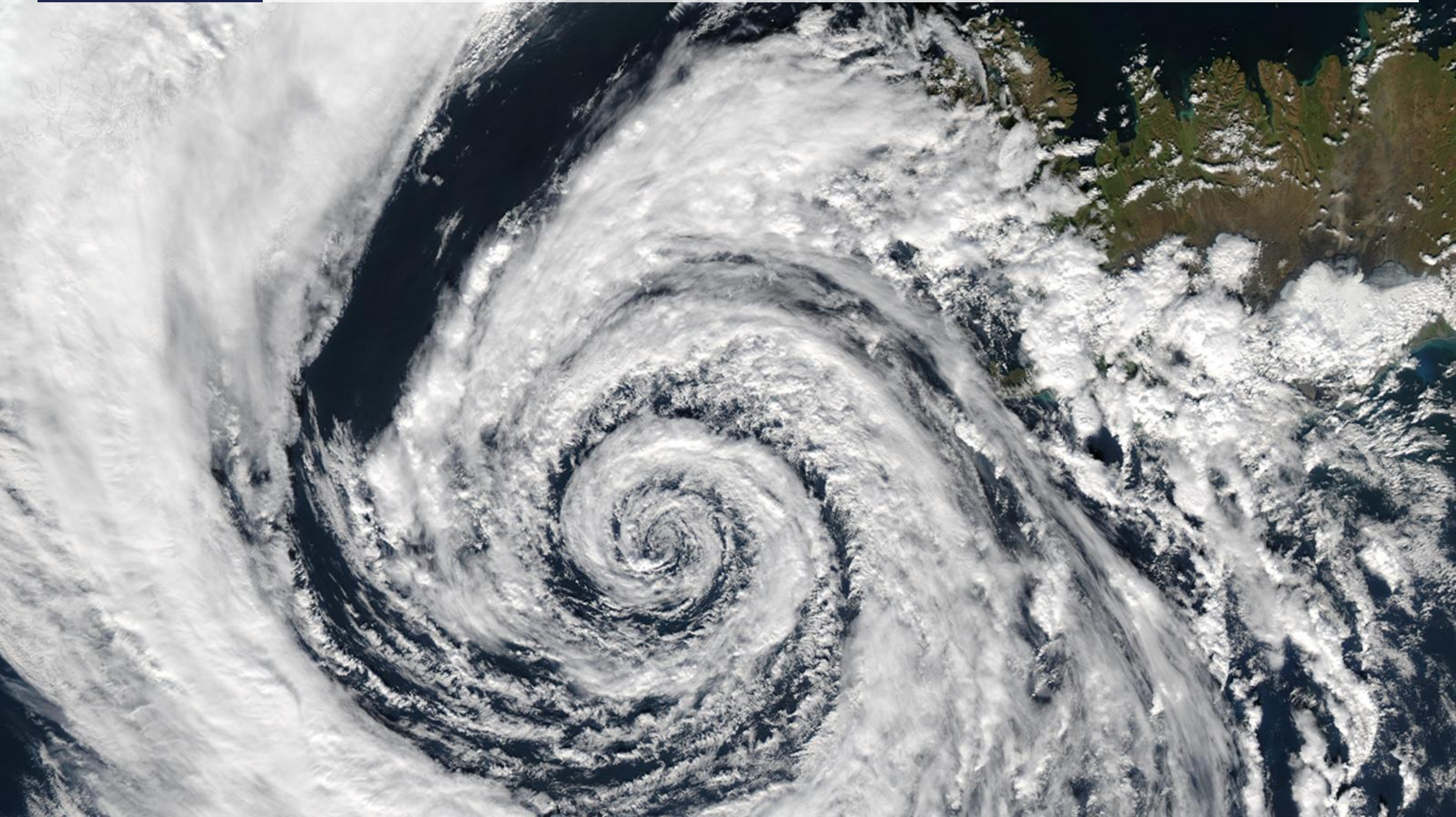
Statistics

Testing Hypothesis

Name teacher

TU Delft



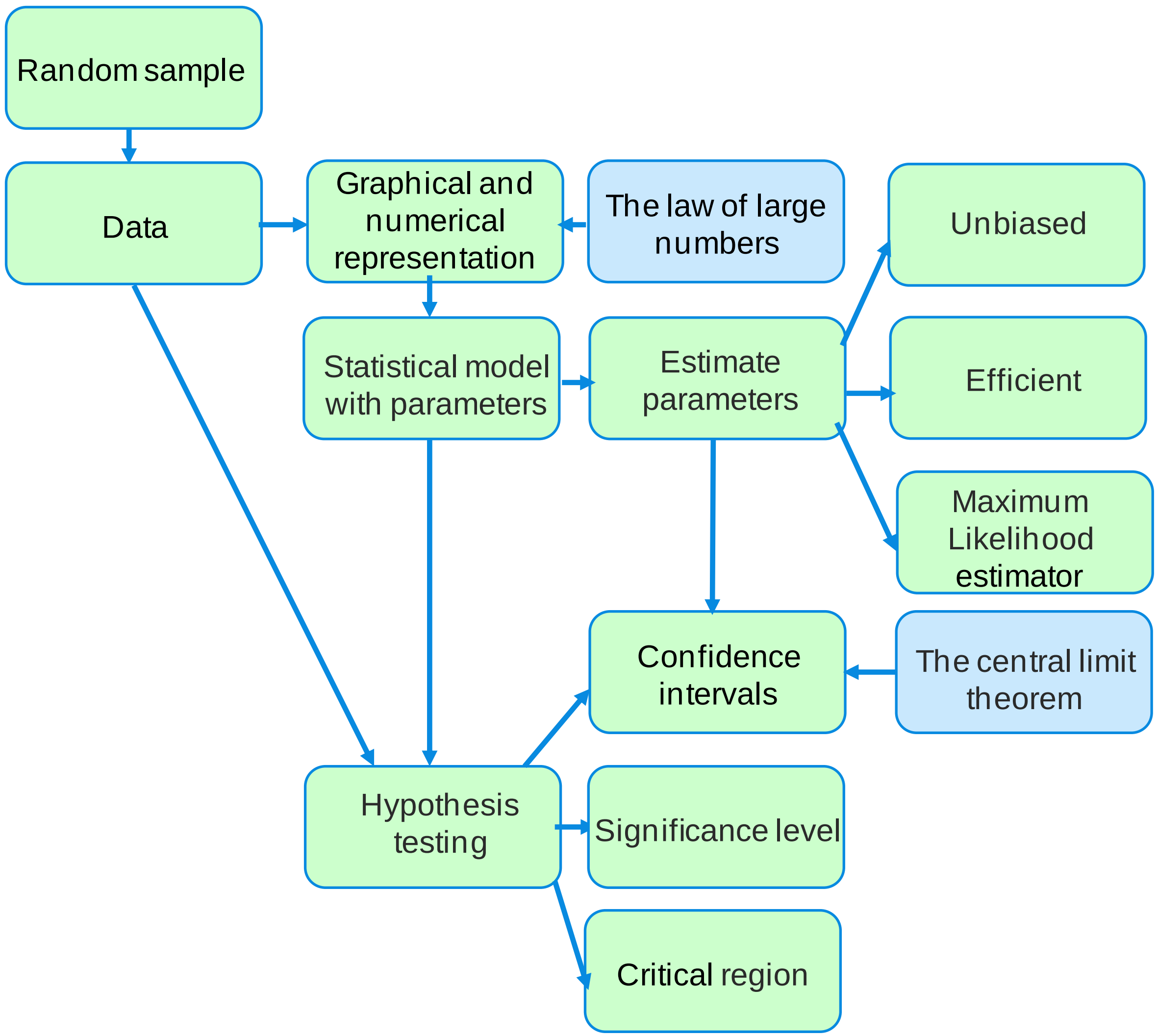
Learning objective

After this class you are able to

- **Set up and carry out statistical hypothesis tests**
- **Interpret the results of statistical hypothesis tests**

Book: Chapters 25 and 26

Statistics



Random sample
Data
Graphical and numerical representation
The law of large numbers
Unbiased
Statistical model with parameters
Estimate parameters
Efficient
Maximum Likelihood estimator
Confidence intervals
The central limit theorem
Hypothesis testing
Significance level
Critical region

Before this class (week 3.1 lesson 1):

Watch prelecture *'Hypothesis testing'*

Book: Section 25.1

Programme

- Test statistics
- Tail probabilities
- Significance level
- Practice
- Relation to confidence intervals

What is the H_1 the police uses?

Note: BAC = Blood Alcohol Content = promillage alcohol
Maximum allowed in the Netherlands for new drivers is 0.2

A) $H_1 : BAC < 0.2$

B) $H_1 : BAC = 0.2$

C) $H_1 : BAC > 0.2$

D) I have no clue

What type of error did the police make?

A) Type I

B) Type II

C) I have no clue

The three steps of hypothesis testing

1. Formulate H_0 and H_1
2. Do the experiment
3. Calculate whether results justify rejecting H_0

Result:	
Do not reject H_0	Reject H_0
Insufficient evidence to support H_1	H_1 true “beyond reasonable doubt”

Test Statistic

Definition:
Suppose the data are modelled as a realization of random variables X_i. A test statistic is any sample statistic

$$T = h(X_1, X_2, \ldots, X_n)$$

whose numerical value is used to decide whether we reject H_0.

Exercise

Suppose X_i are i.i.d. uniform on $[0, \theta]$ for some $\theta > 0$.

Our data are a realization of this sample, and we test

$$H_0 : \theta = 5$$

against

$$H_1 : \theta \neq 5$$

a. With $T_1 = \max\{X_1, X_2, \ldots, X_n\}$ as test statistic, what values of T_1 are in favor of H_0, and what values are in favor of H_1?

b. Same as a. but using $T_2 = |2\bar{X}_n - 5|$.

Tail probabilities

Definition:
Given a test statistic T, a <u>left tail probability</u> is $\mathrm{P}(T \leq t)$ for some t, a <u>right tail probability</u> is $\mathrm{P}(T \geq t)$.

Definition:
The <u>*p*-value</u> is the probability, given H_0 is true, of an event at least as extreme as the observations in the direction which provides evidence for H_1.

Suppose we test H_0 using the test statistic T. We observe the realization t. Suppose we reject H_0 if $\mathrm{P}(T \geq t) \leq 0.05$**. Let**

$$t = 3.15, \qquad P(T \geq 3.15) = 0.97$$

what is the p-value, and should we reject H_0?

A) $p = 0.03$, Do not reject H_0

B) $p = 0.03$, Reject H_0

C) $p = 0.97$, Do not reject H_0

D) $p = 0.97$, Reject H_0

Suppose we test H_0 using the test statistic T. We observe the realization t. Suppose we reject H_0 if $\mathrm{P}(T \geq t) \leq 0.05$**. Let**

$$t = 1.73, \qquad P(T \leq 1.73) = 0.11$$

what is the p-value, and should we reject H_0?

A) $p = 0.11$, Do not reject H_0

B) $p = 0.11$, Reject H_0

C) $p = 0.89$, Do not reject H_0

D) $p = 0.89$, Reject H_0

Suppose we test H_0 using the test statistic T. We observe the realization t. Suppose we reject H_0 if $\mathrm{P}(T \geq t) \leq 0.05$**. Let**

$$t = 0.94, \qquad P(T \leq 3.12) = 0.02$$

what is the p-value, and should we reject H_0?

A) $p > 0.02$, Do not reject H_0

B) $p < 0.02$, Reject H_0

C) $p > 0.98$, Do not reject H_0

D) $p > 0.89$, Reject H_0

Significance level and critical region

Definition:
The significance level α is the largest acceptable probability of committing a type I error.

Definition:
Suppose we test H_0 against H_1 by means of the test statistic T. The set of values for T for which we reject H_0 in favor of H_1 is called the critical region. Values on the boundary of this region are called critical values.

Exercise

Given a realization t of a random variable $T \sim N(\mu, 2)$. We test $H_0 : \mu = 0$ against $H_1 : \mu \neq 0$ using T as test statistic. We reject H_0 if $|t| \geq 2$.

a. Assuming H_0 is true, what is the probability of committing a type I error?

b. Suppose the real value is $\mu = 1$. What is the probability of committing a type II error?

c. Now we change our rule to reject H_0 if $|t| \geq 4$. What do the answers of a and b become then?

Relation to confidence intervals

Theorem:
Suppose we test $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$.

We reject H_0 at level α

if and only if

θ_0 is not in the $100(1-\alpha)\%$ one-sided confidence interval for θ.

For next class (week 3.9 lesson 1):

Complete MyStatlab assignments and book exercises 25.1, 26.3, 26.4, 26.7

Watch prelectures *'The t-test'*

Book: Section 27.1, 27.2 (only 'normal data')

After this class you can

- **Perform statistical hypothesis tests for the mean, under various assumptions**
- **Perform statistical hypothesis tests for the difference between the mean of two groups**

Statistics

Good luck!

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