Probability

Lecture 3.2.2: Continuous random variables

Name teacher

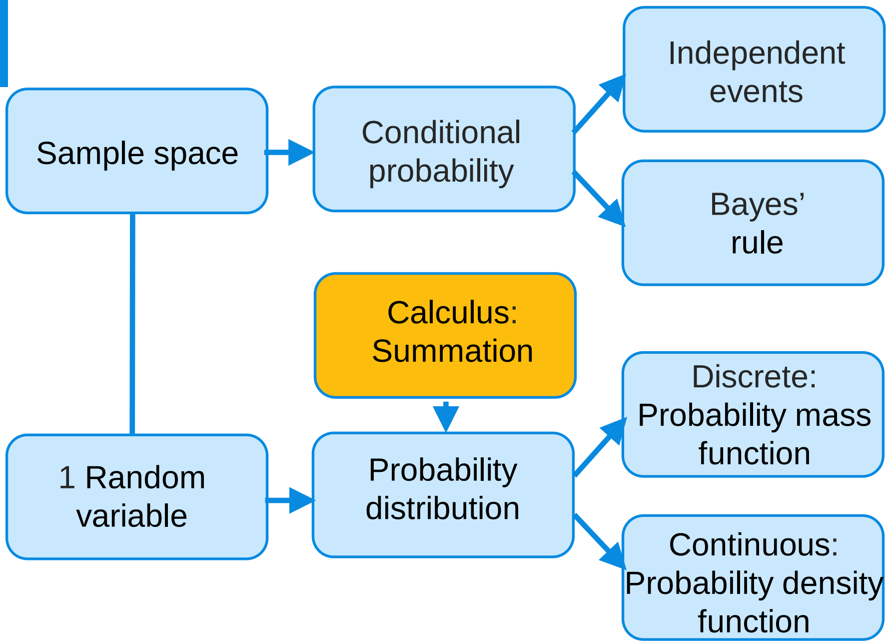
TU Delft

After this class you can

- **manage continuous RVs via the probability density function and distribution function**
- **know four standard continuous distributions recognize the contexts in which they occur**

Book: Sections 5.1, 5.2, 5.3, 5.4, 5.5

Probability



Before this class (week 3.2 lesson 2):

Watch prelecture *‘Continuous random variables’*

Book: Section 5.1

Context: Different experiments are described by different RVs

Four continuous distributions

Exercises

Describing the experiments

Let X be the result of the perfect random number generator from the pre-lecture. The density function of X is

A) $f(x) = 1 - x$

B) $f(x) = x$

C) $f(x) = \frac{1}{x}$

D) $f(x) = 1$

Let X be the result of the perfect random number generator from the pre-lecture. The distribution function of X on the interval (0,1) is

A) $F(x) = x - \frac{1}{2}x^2$

B) $F(x) = x$

C) $F(x) = \ln x$

D) $F(x) = 1$

Context

Context

Context

Signal with noise

Suppose we send a signal voltage as follows: $\begin{cases} -1V & \text{if signal is } 0 \\ 1V & \text{if signal is } 1 \end{cases}$

While sending this signal noise may occur. Noise is denoted with *n*. Suppose we send a signal *x*, then the receiver will have signal $y = x + n$.

How can you model this RV?

The uniform distribution

Definition:
A continuous random variable X has a uniform distribution on the interval $[\alpha, \beta]$ if its probability density function f is given by

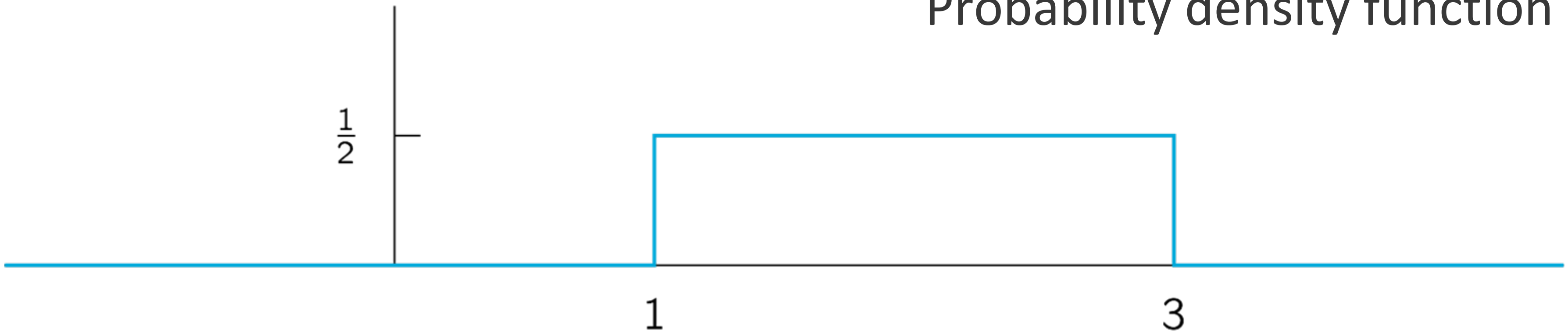
$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{for } x \in [\alpha, \beta] \\ 0 & \text{for } x \text{ not in } [\alpha, \beta] \end{cases}$$

Notation:

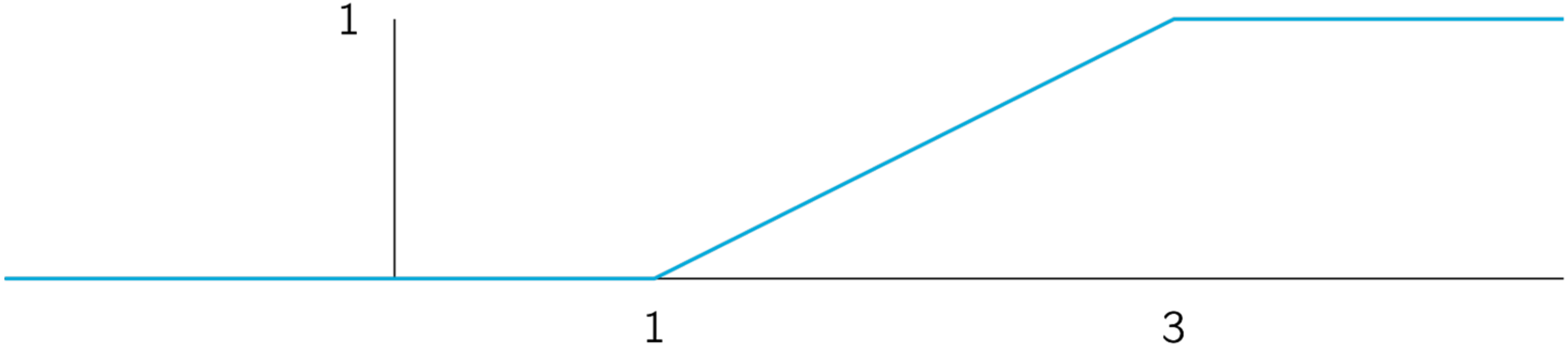
$$X \sim U(\alpha, \beta)$$

Uniform distribution: U(1,3)

Probability density function



Distribution function



The distribution function of a uniformly distributed RV on the interval (a, b) is

A) $$F(x) = \frac{x}{\beta - \alpha}$$

B) $$F(x) = \frac{x - \alpha}{\beta - \alpha}$$

C) $$F(x) = \frac{x - \beta}{\beta - \alpha}$$

D) $$F(x) = x$$

The exponential distribution

Definition:
A continuous random variable X has an <u>exponential distribution with parameter λ</u> if its probability density function f is given by

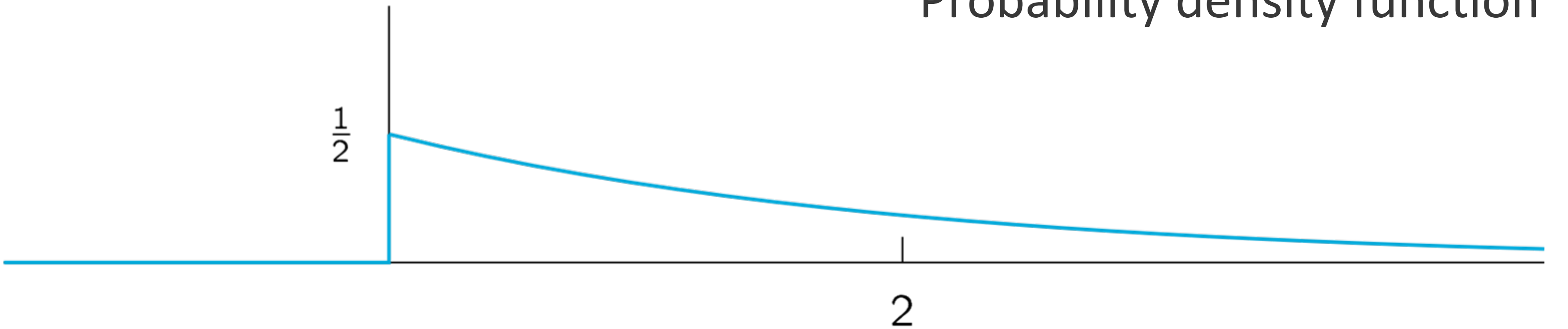
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

Notation:

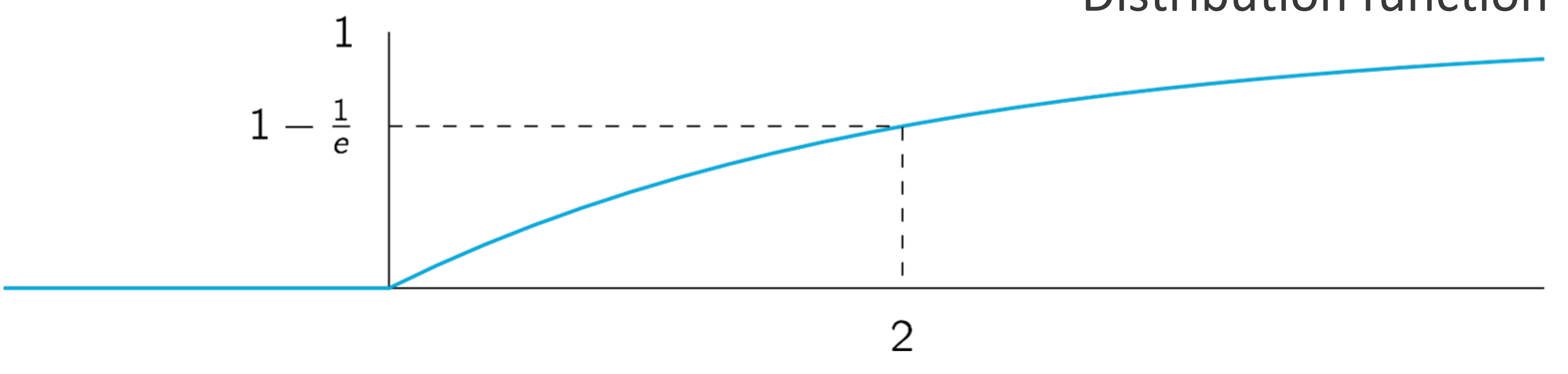
$$X \sim \text{Exp}(\lambda)$$

Exponential distribution: Exp(0.5)

Probability density function



Distribution function



Exercise

Let X be exponentially distributed with parameter 0.3 .
Compute $P(X > 1)$.

The Pareto distribution

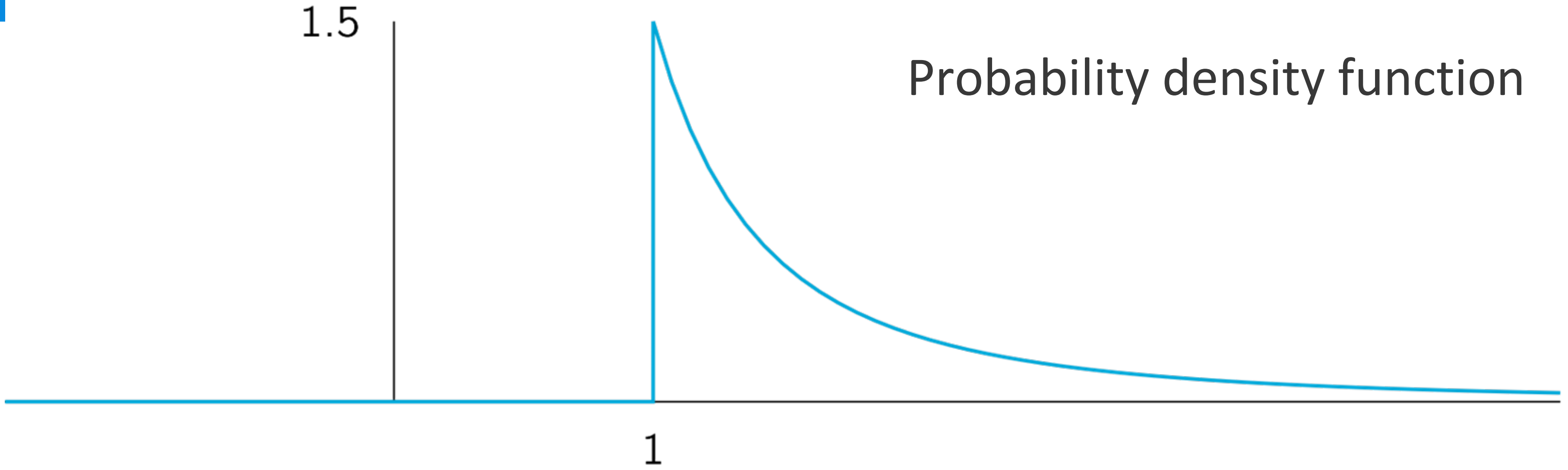
Definition:
A continuous random variable X has a Pareto distribution with parameter $\alpha > 0$ if its probability density function f is given by

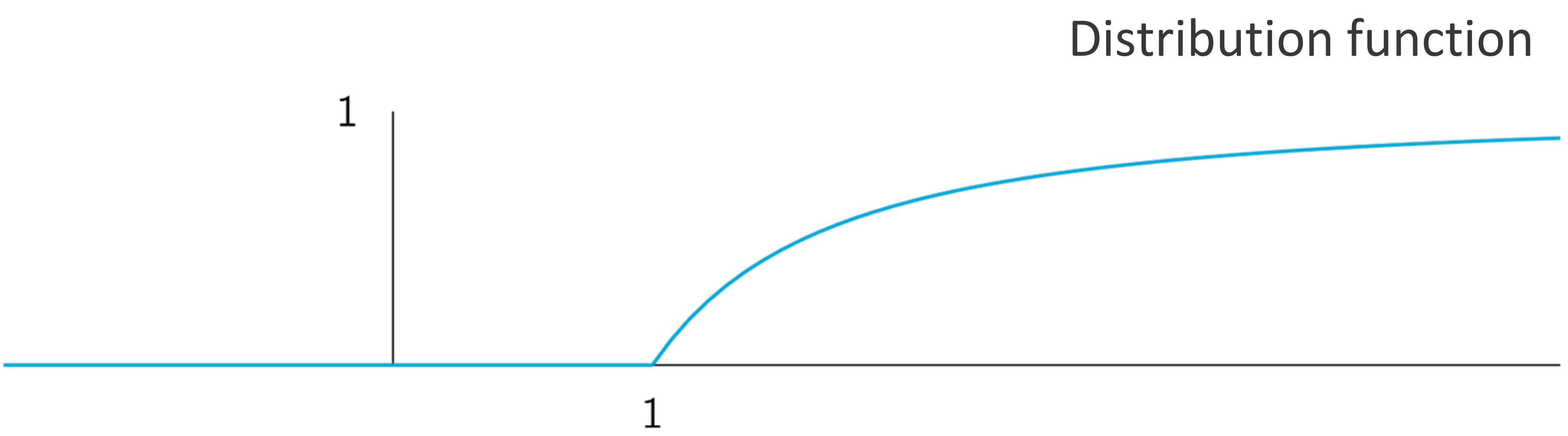
$$f(x) = \begin{cases} \frac{\alpha}{x^{\alpha+1}} & \text{for } x \geq 1 \\ 0 & \text{for } x < 1 \end{cases}$$

Notation:

$$X \sim \mathrm{Par}(\alpha)$$

Pareto distribution: Par(1.5)





The distribution function of a Pareto distributed RV with parameter α is

A) $F(x) = 1 - \alpha x^{-\alpha}$

B) $F(x) = x^{-\alpha}$

C) $F(x) = -\alpha x^{-\alpha}$

D) $F(x) = 1 - x^{-\alpha}$

The normal distribution

Definition:
A continuous random variable X has a <u>normal distribution with parameters μ and σ^2</u> if its probability density function f is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

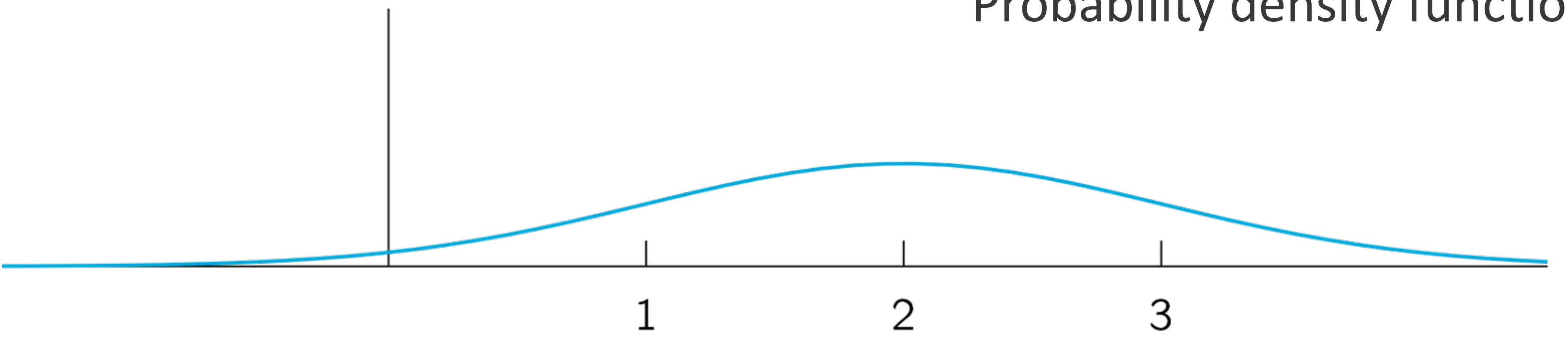
Notation:

$$X \sim N(\mu, \sigma^2)$$

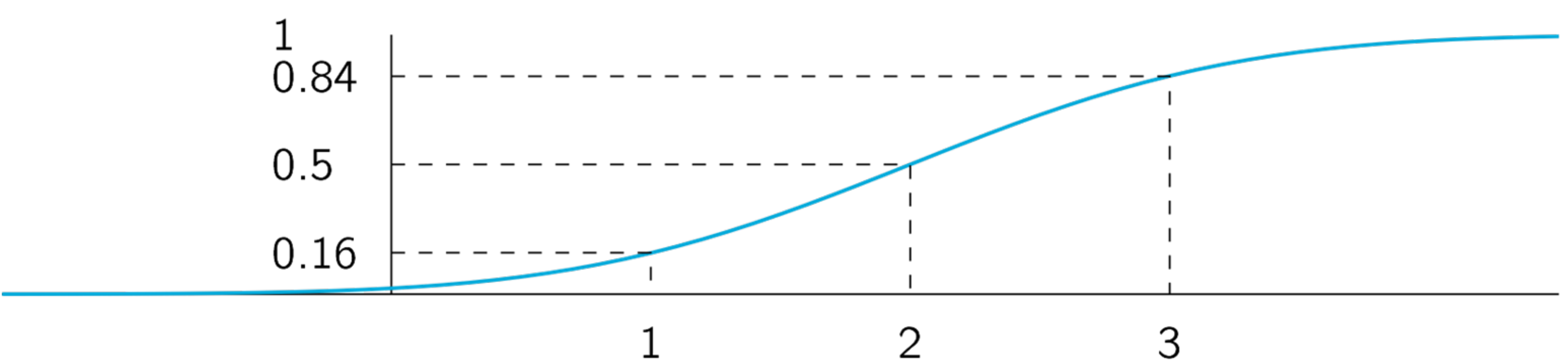
Property: Symmetric around μ

Normal distribution: N(2, 1)

Probability density function



Distribution function



The standard normal distribution

If $\mu = 0$ and $\sigma^2 = 1$, the distribution $N(0, 1)$ is called the standard normal distribution.

In table B.1 of the book, right tail probabilities are given for the standard normal distribution:

$$P(Z \geq a)$$

Let Z be a standard normal RV. Compute $P(Z > 0)$.

A) 0.5

B) 1

C) 0.25

D) 0

Let Z be a standard normal RV. Compute $P(Z > 1.07)$

A) 0.2206

B) 0.2389

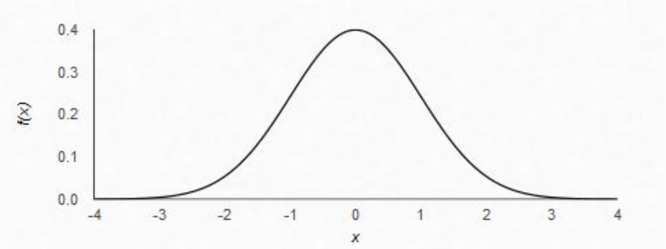
C) 0.1423

D) 0.4721

Signal with noise

Suppose we send a signal voltage as follows; $\begin{cases} -1V & \text{if signal is 0} \\ 1V & \text{if signal is 1} \end{cases}$

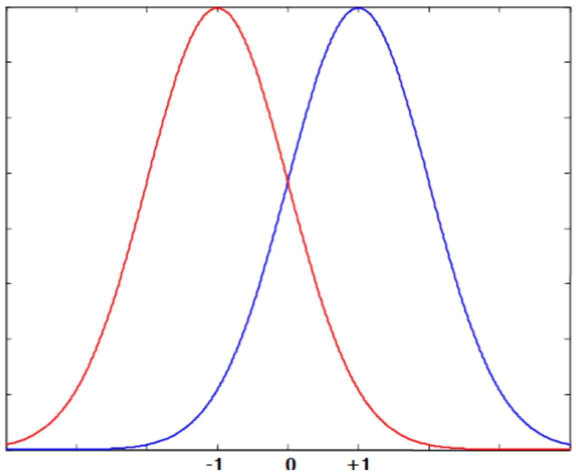
While sending this signal noise may occur. Noise is denoted with *n*. Suppose we send a signal *x,* then the receiver will have signal $y = x + n$. This noise is called Additive White Gaussian Noise (AWGN). The distribution is as follows:



Presume we send the following signals:

x	y	Interpretation
-1V	+0.2V	+1V
-1V	-0.9V	-1V
+1V	+0.3V	+1V

Signal with noise



We can conclude that the first signal is received wrongly.

Question 1: What is the probability that a zero signal is received as a one signal?

Question 2: What is the probability that a signal is wrongly received?

Exercise

Let Z be a standard normal RV.
Compute $P(Z \leq -2.03)$.

Exercise

Suppose we choose arbitrarily a point from the square with corners at (2,1), (3,1), (2,2) and (3,2). The random variable A is the area of the triangle with its corners at (2,1), (3,1) and the chosen point.

(a) What is the largest area A that can occur, and what is the set of points for which $A \leq 1/4$?

(b) Determine the distribution function of A.

(c) Determine the density function of A.

For next class (week 3.3 lesson 1):

Complete MyStatlab assignments and book exercises

Watch prelecture *'Expectation'*

Book: Section 7.1

After this class you can

- **Compute the expectation and variance of discrete and continuous RVs**
- **Apply the change-of-variables formula**
- **Apply the change-of-units formula**

Probability

Good luck!