

Figure 1: Example audio signal in both a) time and b) frequency domain.

Assignment 1:

Consider the audio signal depicted in Figure 1. Figure 1a) shows the time domain representations, where $x_a(t)$ (dashed line) denotes the original time-continuous signal and $x(n) = x_a(nT_s)$ the temporal sampled version of it. Figure 1b) shows the frequency domain representation of $x(n)$ expressed in both the angular frequency ω (dimensionless) and the frequency f expressed in cycles/sec (or, equivalently, Hertz (Hz)).

- a) What is the sampling frequency f_s at which audio signal $x_a(t)$ has been sampled? Motivate your answer. (Answer: $f_s = 48$ kHz since the spectrum is f_s -periodic.)

Suppose we are going to decimate the discrete-time signal x by a factor two, see Figure 2a), after which we expand the signal by a factor two (inserting zeros in between every two samples), see Figure 2b).



Figure 2: a) Decimation by a factor two and b) expanding by a factor two.

- b) Give a sketch of both the time and frequency domain representation of the signals $y(n)$ and $z(n)$ in Figure 3 and Figure 4, respectively, with the frequency representation expressed in both f and ω .

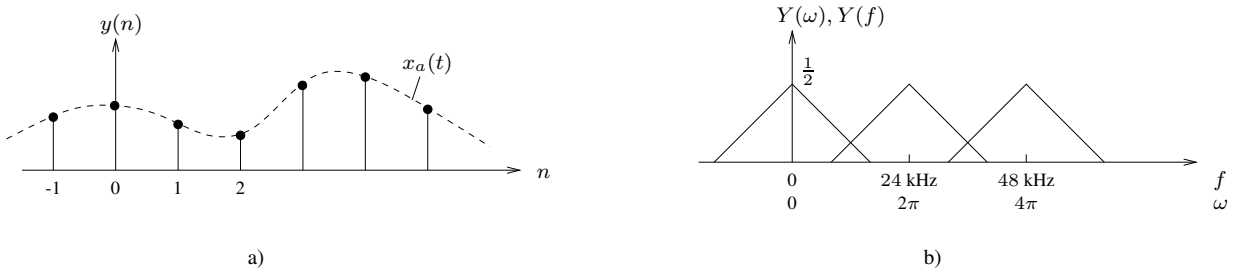


Figure 3: Sketch of a) time and b) frequency domain representation of $y(n)$.

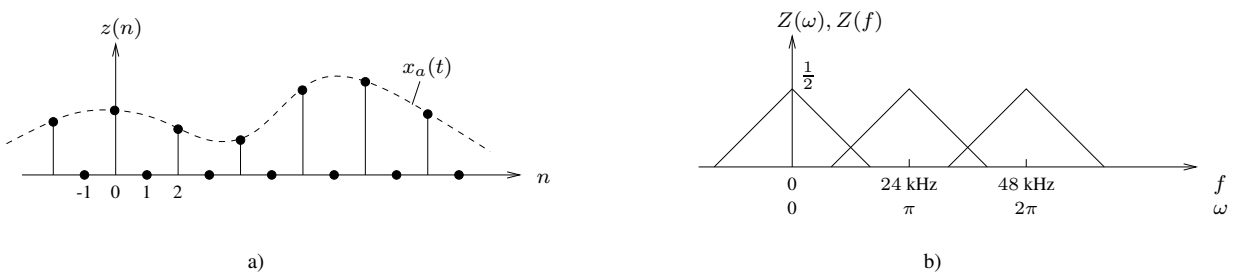


Figure 4: Sketch of a) time and b) frequency domain representation of $z(n)$.

After decimation and expanding, we would like to reconstruct the analog signal $\hat{x}_a(t)$ by passing the output $z(n)$ of the expander through an ideal D/A convertor.

- c) Explain how the ideal D/A convertor looks like and sketch the analog signal $\hat{x}_a(t)$ (both in time and frequency) in the figure below. Do we have perfect reconstruction? That is, do we have $\hat{x}_a(t) = x_a(t)$ for all $t \in \mathbb{R}$? (Answer: We first interpolate $z(n)$ with a digital filter having bandwidth $B_z = 12 \text{ kHz}$ to remove the images at $(2k - 1)24 \text{ kHz}$. The analog signal is obtained by ideal (analog) brickwall filtering of the signal thus obtained with bandwidth $B = \frac{1}{2}f_s = 24 \text{ kHz}$, which corresponds to sinc interpolation in the time domain.)

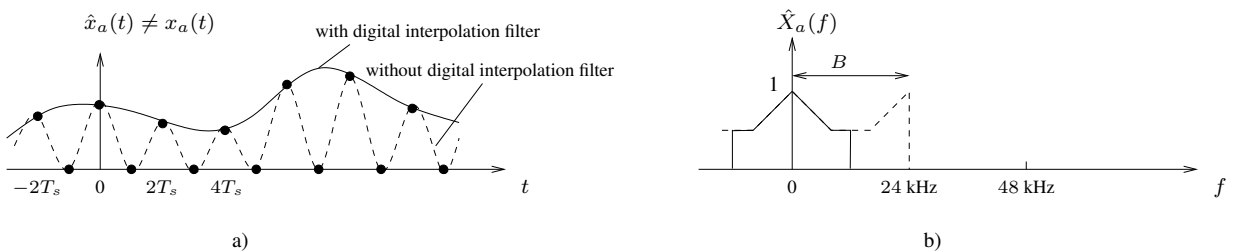


Figure 5: Sketch of the reconstructed analog signal $\hat{x}_a(t)$ in both a) time and b) frequency domain.