Exam EE2S31 Signaalbewerking

June 30th, 2015

Answer each question on a **separate sheet**. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet. It is allowed to answer in Dutch or English.

Question 1 (10 points)

(1 p) (a) The input is WSS. The filter is LTI. The output is thus also

(1 p) (b) $Var[X[n]] = R_X[0] - E[X]^2 = 4 - 1 = 3.$

(1 p) (c) $E[Y[n]] = E[X[n]] \sum_n h[n] = 1.5.$

(3 p) (d) $R_Y[k] = 1.25R_X[k] + 0.5R_X[k+1] + 0.5R_X[k-1]$

(1 p) (e) When the process is ergodic.

(2 p) (f) $E[\bar{R}_X[k]] = (N-k)/N$. $\bar{R}_X[k]$ is biased as its expected value is not identical to $R_X[k]$.

(1 p) (g)

Question 2 (15 points)

(1 p) (a) This is an IIR filter as the output depends on previous outputs. If the input signal for this filter is white Gaussian noise, process $Y[n]$ is called an AR process.

(1 p) (b) $R_X[k]$ is a deltapulse at zero of σ_X^2.

(1 p) (c) The input is WSS and the filter is linear and time-invariant. As a consequence, the output is also WSS.

(2 p) (c) The system function is given by $H(z) = \frac{1}{1-1/2z^{-1}}$. The inverse Z-transform of this is given by $h[n] = \left(\frac{1}{2}\right)^n u[n]$.

(3 p) (d) $P_{XY}(z) = H(z)P_X(z) = \frac{\sigma_X^2}{1-1/2z^{-1}}$ leading to $\sigma_X^2 \frac{1}{2}^n u[n]$.

(3 p) (e) $f[k]] = \sum_n \left(\frac{1}{2}\right)^n u[n] \left(\frac{1}{2}\right)^{-(k-n)} u[-(k-n)]$ We can split this into a part for which $k \geq 0$ and a part for which $k < 0$.
For $k < 0$: $f[k]] = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{-(k-n)} = \left(\frac{1}{2}\right)^{-k} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} = \left(\frac{1}{2}\right)^{-k} \frac{1}{1-(\frac{1}{2})^2}$
For $k > 0$: $f[k]] = \sum_{n=k}^{\infty} \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{-(k-n)} = \left(\frac{1}{2}\right)^{-k} \sum_{n=k}^{\infty} \left(\frac{1}{2}\right)^{2n} = \left(\frac{1}{2}\right)^{-k} \frac{(\frac{1}{2})^{2k}}{1-(\frac{1}{2})^2} = \left(\frac{1}{2}\right)^{k} \frac{1}{1-(\frac{1}{2})^2}$
The two parts can be taken together resulting in $f[k] = \frac{4}{3}(\frac{1}{2})^{|k|}$.

(2 p) (f) $F(z) = \frac{1}{1-1/2z^{-1}} \frac{1}{1-1/2z^{1}}$

(2 p) (g) Using $f[k] = \frac{4}{3}(\frac{1}{2})^{|k|}$ we get $f[k] = \sigma_X^2 \frac{4}{3}(\frac{1}{2})^{|k|}$