Mid-term Exam **Signal Processing** (EE2S31)

May 27, 2015
(14:00 - 16:00)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1:

The joint probability density function of two variables X and Y is given by:

$$f_{X,Y}(x,y) = \begin{cases} ce^{-x}ye^{-y^2} & \text{for} \quad -1 \leq x \leq \infty \text{ and } 1 \leq y \leq \infty \\ 0 & \text{otherwise.} \end{cases}$$

(1p) a) Draw for this probability density function the region of non-zero probability.

(2p) b) Show that the cumulative distribution function is given by

$$F_{X,Y}(x,y) = \frac{c}{2}\left(e^{1} - e^{-x}\right)\left(e^{-1} - e^{-y^2}\right),$$

Do this by integrating the above given probability density function.

(2p) c) Calculate the value of constant c. (*Hint: This can be done either using the pdf or the cdf.*)

(2p) d) Proof that

$$F_X(x) = F_{X,Y}(x,\infty).$$

(2p) e) Calculate the probability $P(Y > 10)$.

(2p) f) Calculate the marginal probability density function $f_X(x)$.

(1p) g) Are random variables X and Y dependent? Give an argumentation.

Assignment 2:

Consider a causal linear time-invariant system, having zeros at $z = 0$ and $z = \frac{1}{2}r$ and poles at $z = re^{j\theta}$ and $z = re^{-j\theta}$.

(1p) a) Give the corresponding pole-zero plot.

(1p) b) Determine the corresponding system function. What is the region of convergence? Is this function unique?

(1p) c) What are the conditions for r and θ such that the system is BIBO stable? Motivate your answer.

(3p) d) Determine and sketch the magnitude response of the system.

(3p) e) Compute the inverse $\mathcal{Z}$-transform of the system function $H(z)$ in case the region of convergence is $|z| > r$.

Consider the following (suddenly applied) input signal

$$x(n) = e^{j\frac{\pi}{3}n}u(n),$$

and assume the system is initially in rest.

(3p) f) Compute the response of the system. What is the steady-state response and what is the transient response?

Assignment 3:

Consider the following discrete-time signal

$$x(n) = \begin{cases} 1, & n = 0, \ldots, N-1 \\ 0, & \text{otherwise} \end{cases}$$

as depicted in Figure 1.

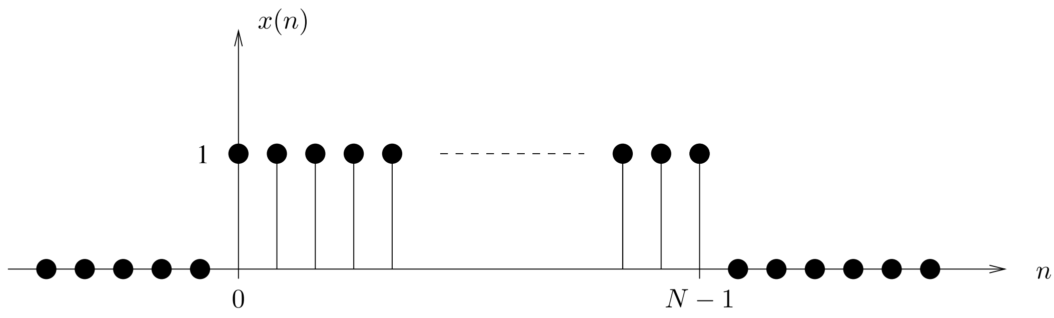

Figure 1: Discrete-time signal $x(n)$.

(3p) a) Show that the spectrum (Fourier transform) of x is given by

$$X(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}.$$

Give a sketch of the spectrum. Is the spectrum continuous? Is it periodic? Motivate your answer.

In order to efficiently filter $x(n)$, we want to implement the filter operation (convolution) in the frequency domain (point-wise multiplication) using a DSP. In order to do so, we have to sample the spectrum $X(e^{j\omega})$.

(3p) b) What is the minimum number of samples, say M, we need to take from $X(e^{j\omega})$ in order to be able to perfectly reconstruct $X(e^{j\omega})$ out of its samples? Motivate your answer.

(3p) c) Suppose we sample $X(e^{j\omega})$ with $M = 3N$ samples. Determine and sketch the corresponding time-domain signal. Is this time-domain signal time-continuous? Is it periodic? Motivate your answer.

(3p) d) How can we recover the original time-domain signal, as depicted in Figure 1, out of the M samples of $X(e^{j\omega})$? Give the reconstruction formula for $X(e^{j\omega})$ out of its M samples $X\left(e^{j\frac{2\pi}{M}k}\right)$. What happens if we take M too small?