Telecommunicatie A (EE2T11)

Lecture 7 overview:

Transmission of digital signals in baseband
* Line codes
* Power spectra
* Transmission quality: eye-pattern, regenerative repeater
* Spectral efficiency

Inter-symbol interference (ISI)
* Pulse shaping (Nyquist criterion)
* Sinc-pulses vs. Raised Cosine pulses

EE2T11 Telecommunicatie A

Dr.ir. Gerard J.M. Janssen

February 1, 2016

Colleges en Werkcolleges Telecommunicatie A

Colleges:

Dinsdag	29-3	7^e en 8^e uur, EWI-Pi

Werkcolleges:

Maandag	4-4	5^e en 6^e uur, EWI-Pi

Review digital signaling (1)

A waveform of N symbols (N dimensions) of a digital signal can be written as a sequence of N orthogonal terms:

$$w(t) = \sum_{k=1}^{N} w_k \varphi_k(t) \qquad 0 \le t \le T_0$$

w_k represents the kth symbol value and contains the digital information.

$\varphi_k(t)$ are orthogonal functions (in general analog waveforms), like a time slot, frequency, phase or a code, with:

$$\int \varphi_k(t)\varphi_l^*(t)dt = \begin{cases} 1 & \text{for } k = l \\ 0 & \text{for } k \ne l \end{cases} \qquad k = 1, 2, \ldots \ldots N$$

Review digital signaling (2)

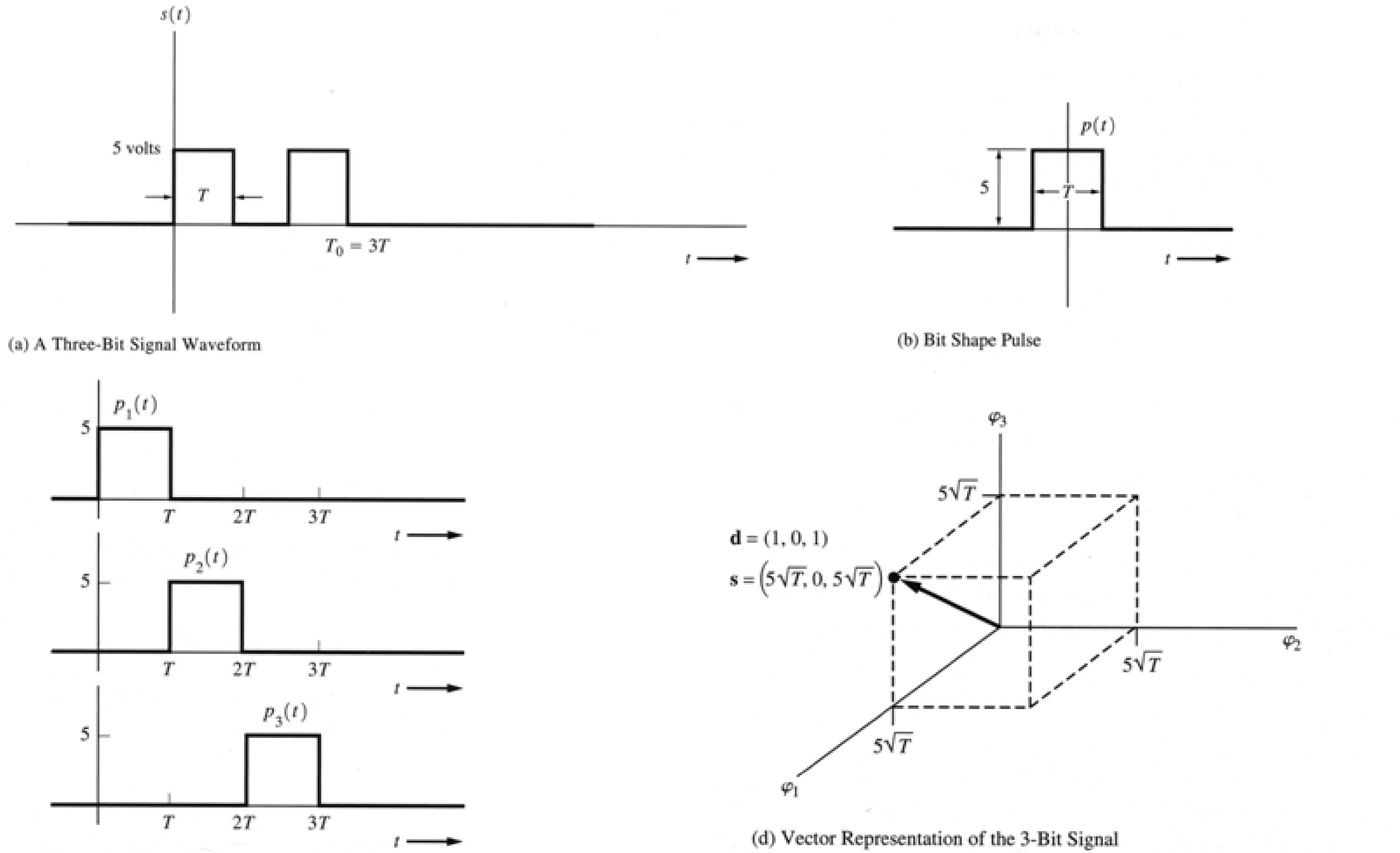


Figure 3–11 Representation for a 3-bit binary digital signal.

Review digital signaling (3)

The N functions $\varphi_k(t)$ (k = 1, .. N) span an N-dimensional space.
A message of N symbols can be represented by a vector in this N-dimensional space:

$$\underline{w} = \sum_{k=1}^{N} w_k \varphi_k$$

How do we detect the data after transmission over a channel?
By correlation with the complex conjugate of the orthogonal functions $\varphi_k(t)$:

$$w_k = \int_0^{T_0} \underline{w} \cdot \varphi_k^*(t)dt \quad \text{for } k = 1, 2, .., N$$

This is optimal detection for a channel with Additive White Gaussian Noise (AWGN): "matched-filter" detector.

Baseband transmission

Transmission of digital baseband signals: usually line transmission.

$$w(t) = \sum_{k=-\infty}^{\infty} w_k \varphi(t - kT_s)$$ with T_s the symbol time.

How do we transmit a "1" or a "0" over a line?

Usually in a serial manner, but different formats exist:

line codes

There are two main categories of *line codes*:

a) Non-Return-to-Zero (NRZ)

b) Return-to-Zero (RZ)

TU Delft

Line codes

Different line codes:
- punch tape code
- Unipolar NRZ (On-Off-Keying, OOK)
- Polar NRZ
- Unipolar RZ
- Bipolar RZ
  (or pseudo ternary
  or AMI (Alternate Mark Inversion)
- Manchester coding
  (or split phase coding)

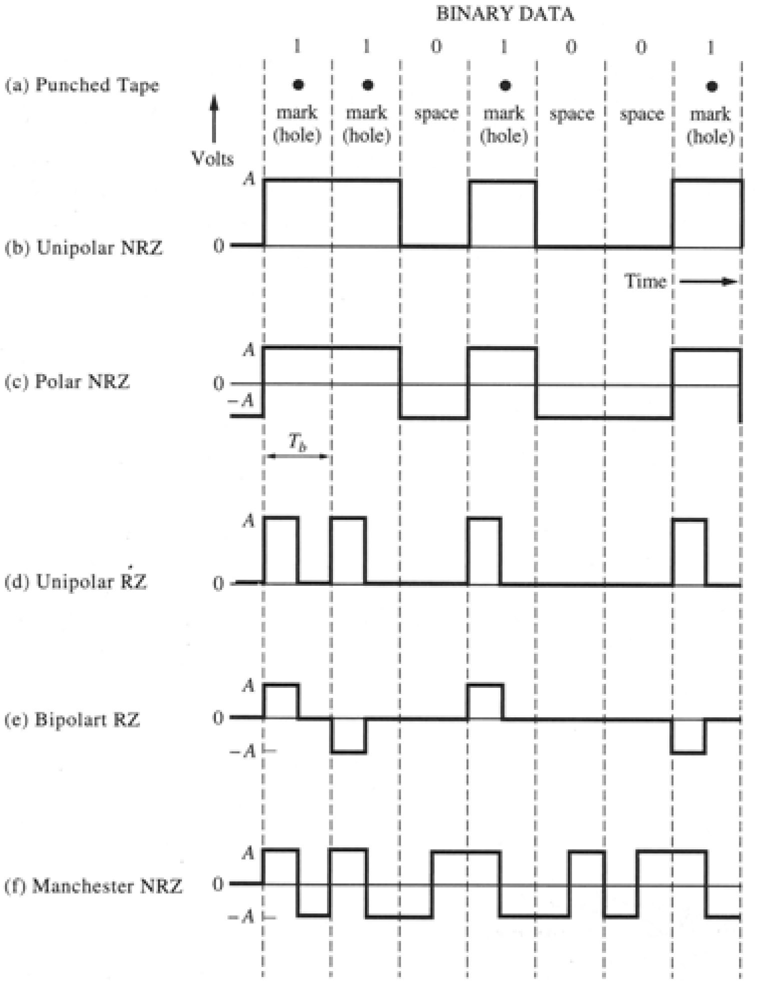


Figure 3–15 Binary signaling formats.

Features of line codes

Desirable features of *line codes:*

1. Spectrum adapted to the *available* baseband channel
2. Self-synchronizing (timing information available also for long sequences of identical symbols)
3. Low bit error probability
4. Small bandwidth
5. Build-in error detection -correction
6. Transparent and unambiguous

Power spectra line codes (1)

For a general line code:

$$s(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s) = f(t) * \sum_{n=-\infty}^{\infty} a_n \delta(t - nT_s)$$

In general, the *Power Spectral Density* is defined as:

$$P_s(f) = \lim_{T\to\infty}\left(\frac{|S_T(f)|^2}{T}\right) = \mathfrak{F}\{R_s(\tau)\}$$

with: $R_s(\tau) = \langle s(t)s(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s(t)s(t+\tau)dt$

Power Spectral Density (PSD) of a digital signal waveform (6.70):

$$P_s(f) = \frac{|F(f)|^2}{T_s}\left[\sum_{k=-\infty}^{+\infty} R(k)e^{-j\omega k T_s}\right]$$

TU Delft

Power spectra line codes (2)

For the *PSD* of a general line code, two factors are important:

* <u>Pulse shape</u>: $F(f)$ is the amplitude spectrum of waveform $f(t)$.

* <u>Data</u>: $R(k) = \overline{a_n a_{n+k}} = \sum_{i=1}^{I} (a_n a_{n+k})_i P_i$

  where P_i is the probability of the i^{th} outcome of the product $a_n a_{n+k}$, which has I possibilities.

We use here *stochastic techniques* because we observe a sequence of random bits or symbols with equal probability.

The full derivation of the *PSD* of a line code signal is given in Chapter 6.2.

Power spectrum Polar NRZ

The signal $s(t)=\sum_{n=-\infty}^{\infty} a_n f(t-nT_b)$ with $a_n \in \{-A,+A\}$ uses

square pulses $f(t)=\prod\left(\frac{t}{T_b}\right) \Rightarrow F(f)=T_b\,\mathrm{sinc}(fT_b)=T_b\frac{\sin(\pi fT_b)}{\pi fT_b}$

Since: $R(k=0)=\overline{a_n^2}=A^2$: $\quad I=1,\ a_n^2=A^2,\ P_i=1$

$R(k\neq 0)=\overline{a_n a_{n+k}}=0$: $\quad I=2,\ a_n a_{n+k}\in\{-A^2,A^2\},\ P_i=1/2$

Now it follows:

$$P_{P-NRZ}(f)=\frac{|F(f)|^2}{T_b}\left[\sum_{k=-\infty}^{+\infty} R(k)e^{-j\omega kT_b}\right]=\frac{A^2}{T_b}|F(f)|^2=A^2T_b\,\mathrm{sinc}^2(fT_b)$$

with bit rate $R_b=\frac{1}{T_b}$.

Power spectrum Polar NRZ (2)

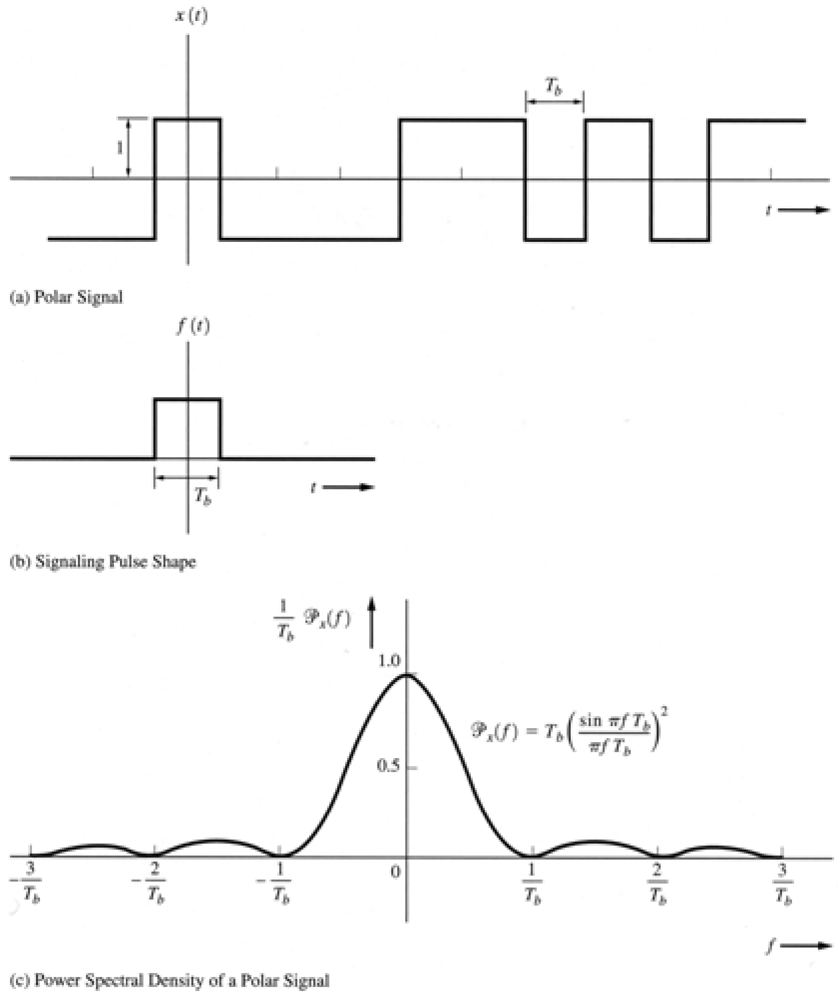


Figure 6–5 Random polar signal and its PSD.

Power spectrum Uni-polar NRZ (1)

In this case $a_n \in \{0, A\}$ and now we find for the autocorrelation of the data:

$$R(k) = \overline{a_n a_{n+k}} = \sum_{i=1}^{I} (a_n a_{n+k})_i P_i$$

$$k = 0 \;\rightarrow\; a_n^2 \quad = \begin{cases} A^2 & \text{with prob. } 1/2 \\ 0 & \text{with prob. } 1/2 \end{cases} \qquad \Rightarrow R(0) = \frac{A^2}{2}$$

$$k \neq 0 \;\rightarrow\; a_n a_{n+k} = \begin{cases} A \cdot A = A^2 & \text{with prob. } 1/4 \\ 0 \cdot A = 0 & \text{with prob. } 1/4 \\ \mathrm{A} \cdot 0 = 0 & \text{with prob. } 1/4 \\ 0 \cdot 0 = 0 & \text{with prob. } 1/4 \end{cases} \qquad \Rightarrow R(k \neq 0) = \frac{A^2}{4}$$

Power spectrum Unipolar NRZ (2)

Again we use square pulses. Using (3-36a), we find for the PSD:

$$P_{U-NRZ}(f) = \frac{|F(f)|^2}{T_b}\left[\sum_{k=-\infty}^{+\infty} R(k)e^{-j\omega k T_b}\right]$$

$$= T_b \frac{\sin^2(\pi f T_b)}{(\pi f T_b)^2}\left[\frac{A^2}{4} + \sum_{k=-\infty}^{\infty}\frac{A^2}{4}e^{-2\pi jkfT_b}\right] = \frac{A^2 T_b}{4}\operatorname{sinc}^2(fT_b)\left[1+\sum_{k=-\infty}^{\infty} e^{-2\pi jkfT_b}\right]$$

$$= \frac{A^2 T_b}{4}\operatorname{sinc}^2(fT_b)\left[1+\frac{1}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_b}\right)\right] = \frac{A^2 T_b}{4}\operatorname{sinc}^2(fT_b)\left[1+\frac{1}{T_b}\delta(f)\right]$$

$\dfrac{\sin \pi f T_b}{\pi f T_b} = \dfrac{\sin n\pi}{n\pi} = 0$ at $f = \dfrac{n}{T_b}$ for n $\neq 0$

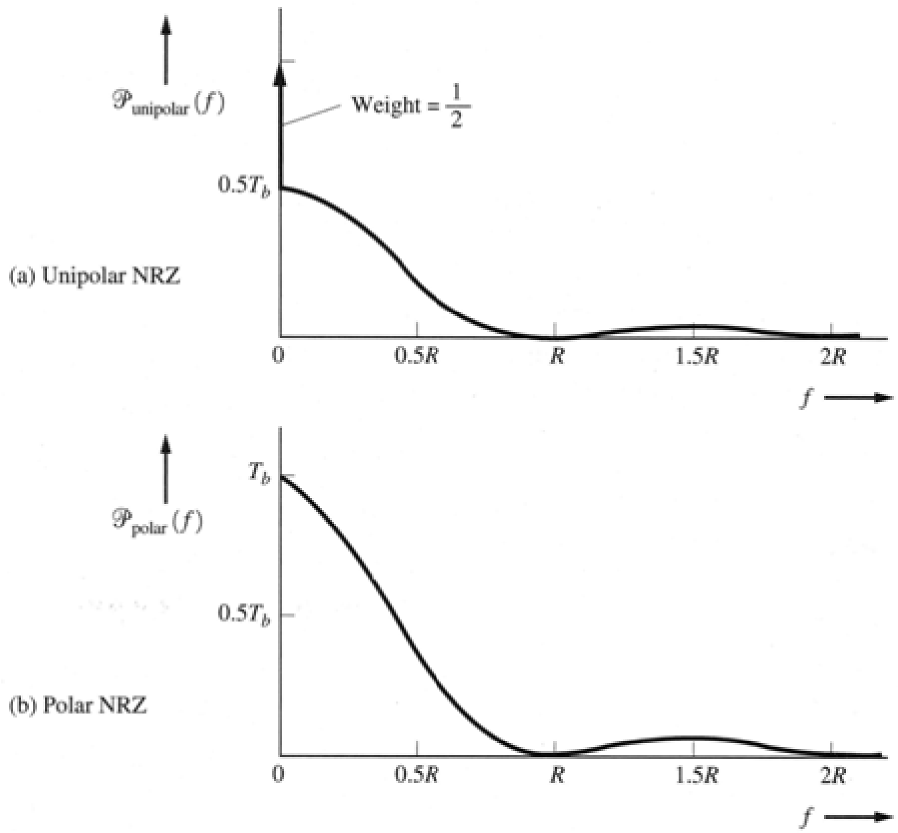
Compare Polar NRZ and Unipolar NRZ

$$P_{\substack{Unipolar\\NRZ}}(f) = \frac{A^2 T_b}{4}\operatorname{sinc}^2(fT_b)\left[1+\frac{1}{T_b}\delta(f)\right]$$

For a normalized power $P = 1 \rightarrow A = \sqrt{2}$
Due to a DC-level of $A/2$, power is wasted.

$$P_{\substack{Polar\\NRZ}}(f) = A^2 T_b \operatorname{sinc}^2(fT_b)$$

For a normalized power $P = 1 \rightarrow A = 1$



Power spectrum Unipolar RZ (1)

Square pulses: $f(t) = \prod\left(\frac{2t}{T_b}\right) \Rightarrow F(f) = \frac{T_b}{2}\mathrm{sinc}(\frac{fT_b}{2})$

Using (3-36a), we find for the PSD:

$$P_{Unipol.RZ}(f) = \frac{A^2 T_b}{16}\mathrm{sinc}^2(\frac{fT_b}{2})\left[1+\sum_{k=-\infty}^{+\infty} e^{j\omega k T_b}\right]$$

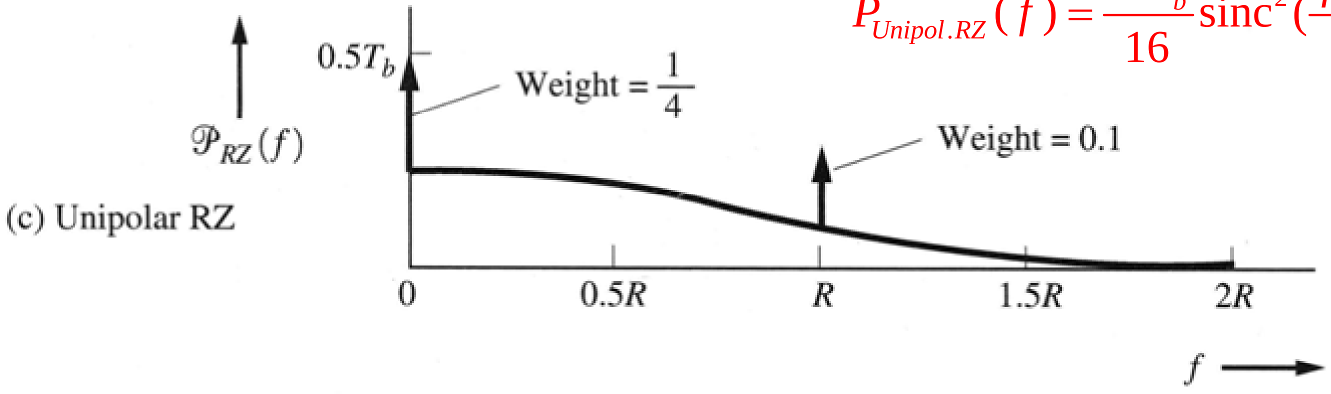
$$= \frac{A^2 T_b}{16}\mathrm{sinc}^2(\frac{fT_b}{2})\left[1+\frac{1}{T_b}\sum_{n=-\infty}^{+\infty}\delta(f-\frac{n}{T_b})\right]$$

For power $P = 1 \rightarrow A = 2$

$\frac{\sin\frac{\pi}{2}fT_b}{\frac{\pi}{2}fT_b} = \frac{\sin n\frac{\pi}{2}}{n\frac{\pi}{2}} = 0$ at $f = \frac{n}{T_b}$ for $n = 2m \neq 0$

Power spectrum Unipolar RZ (2)

$$P_{Unipol.RZ}(f) = \frac{A^2 T_b}{16} \text{sinc}^2\left(\frac{fT_b}{2}\right)\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T_b}\right)\right]$$



- an infinite train of δ-functions at odd multiples of R!
- spectrum twice as wide as for NRZ
- synchronization simple, because of the δ-function at the clock frequency $f = 1/T_b = R_b$

Bipolar RZ and Manchester NRZ

Power spectra:

$$P_{Bip.RZ}(f) = \frac{A^2 T_b}{4} \mathrm{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2 \pi f T_b$$

For power $P = 1 \rightarrow A = 2$

$$P_{Manchester}(f) = A^2 T_b \, \mathrm{sinc}^2\left(\frac{fT_b}{2}\right) \sin^2 \frac{\pi f T_b}{2}$$

For power $P = 1 \rightarrow A = 1$

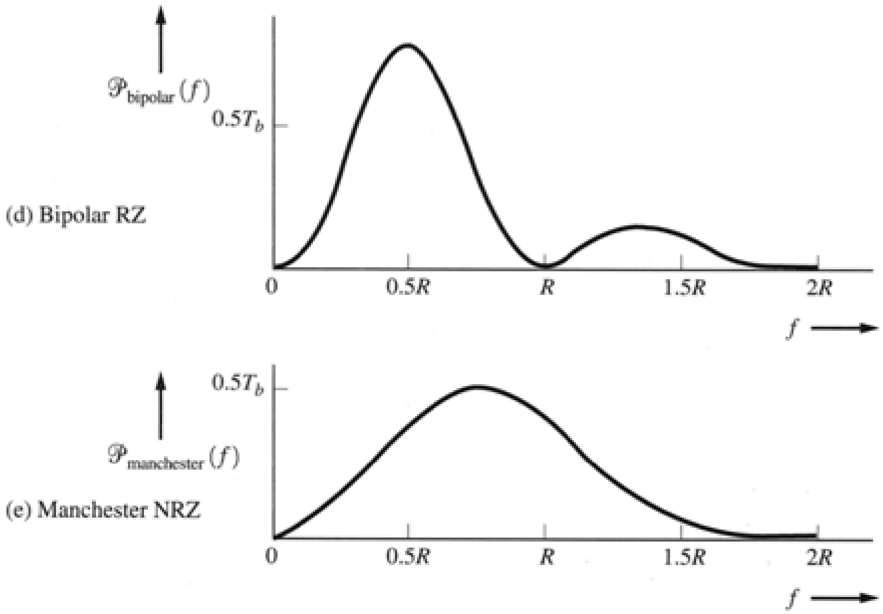


Figure 3–16 PSD for line codes (positive frequencies shown).

Synchronization Bipolar-RZ?

Line codes

Different line codes:
- punch tape code
- Unipolar NRZ (On-Off-Keying, OOK)
- Polar NRZ
- Unipolar RZ
- Bipolar RZ
  (or pseudo ternary
  or AMI (Alternate Mark Inversion)
- Manchester coding
  (or split phase coding)

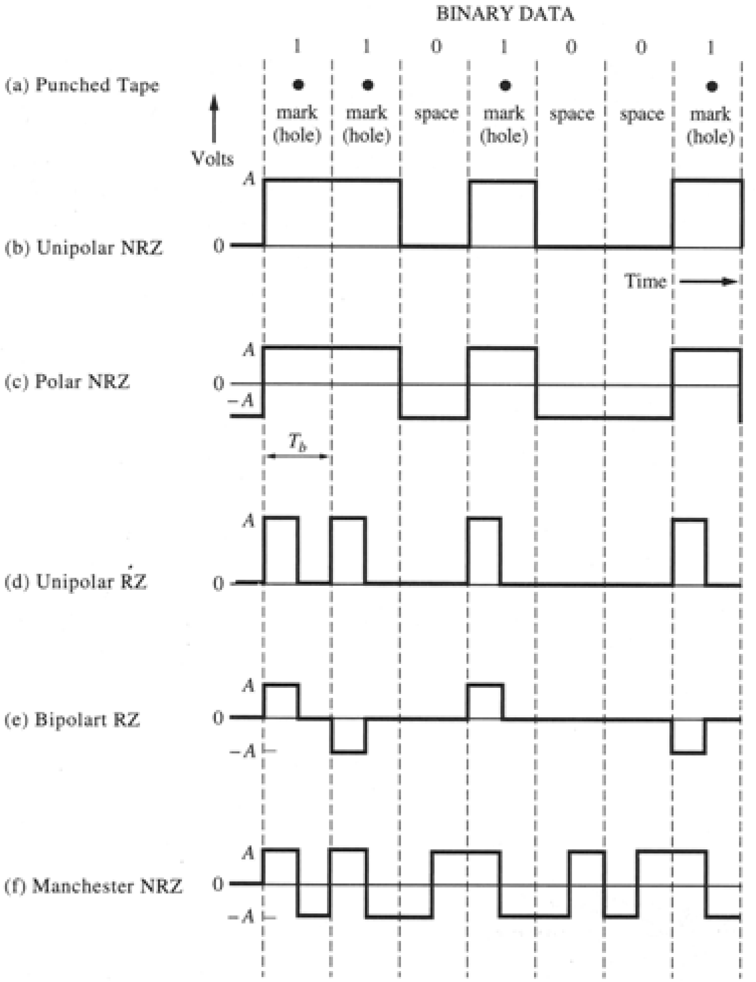


Figure 3–15 Binary signaling formats.

Bipolar RZ and Manchester NRZ

Features:

- no DC-component
- no/little energy around $f = 0$
- synchronization Bip. RZ based on rectified signal
- Bi-polar RZ has sync. problems for long sequences of zeros (HDBn)
- BW_Manchester > BW_Bip. RZ

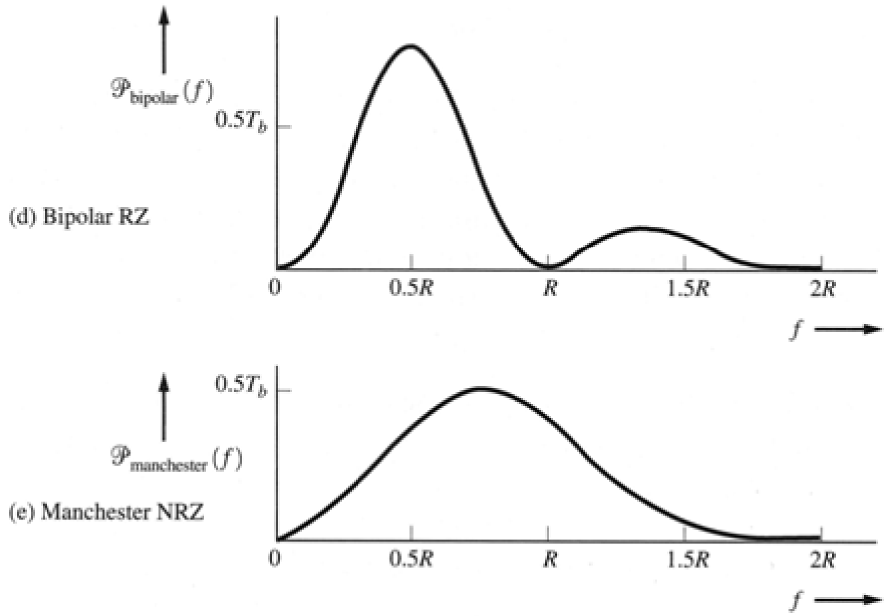
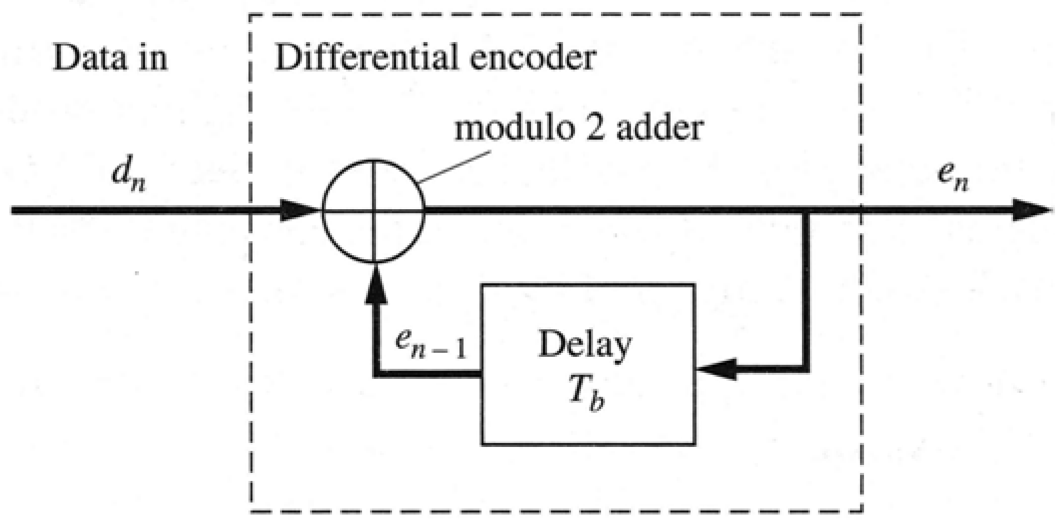


Figure 3–16 PSD for line codes (positive frequencies shown).

Differential coding

Polarity errors result in: "1" $\leftrightarrow$ "0". This can be prevented by *differential coding* : $e_n = d_n \oplus e_{n-1}$ (modulo 2 addition, XOR).



d_n	e_{n-1}	e_n
0	0	0
0	1	1
1	0	1
1	1	0

A "1" in the data signal changes the sign of the line signal.

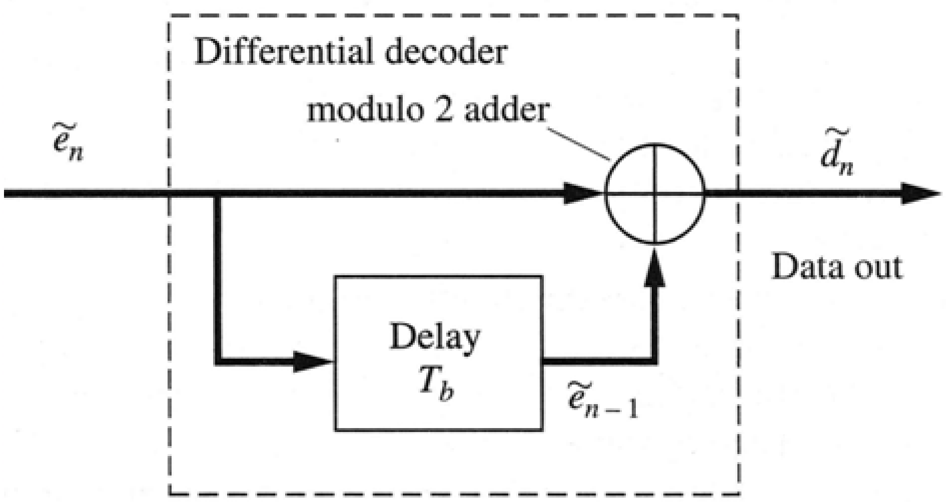
Differential decoding

Decoding:

$$\hat{d}_n = e_n \oplus e_{n-1} \quad \Rightarrow$$

A change of sign $\rightarrow$ "1";

No change of sign $\rightarrow$ "0".



The information is *not coded in the symbol values, but in the transitions*.

Differential coding - decoding

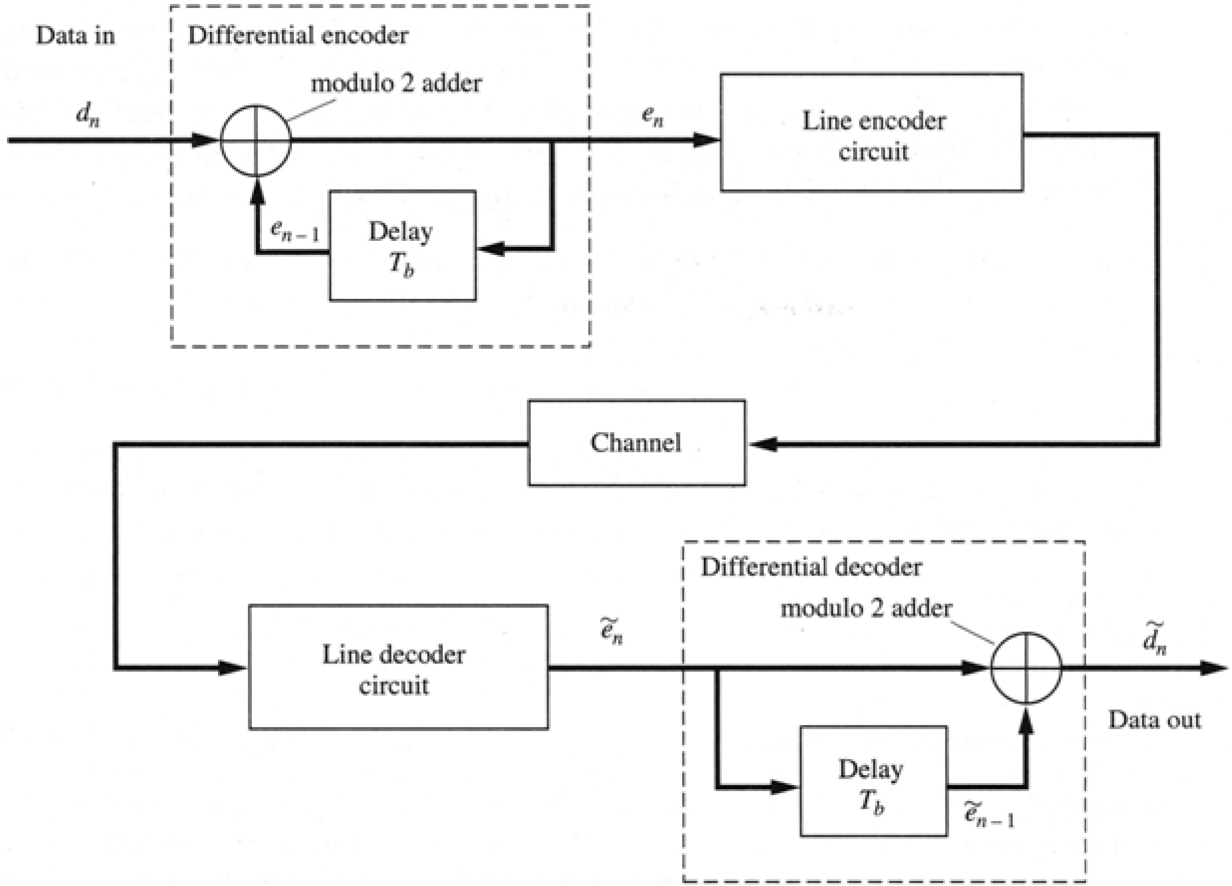


Figure 3–17 Differential coding system.

Example: differential coding (1)

Data sequence :

$$d = \quad 1\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1$$

Differential encoded sequence:

$$e = 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 1$$

or **Reference symbol**

$$e = 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 1\ 0$$

In both cases, with and without inversion the original sequence is obtained after decoding:

$$\hat{d} = \quad 1\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1$$

TUDelft

Example: differential coding (2)

Data sequence :

$$d = \quad 1\ 0\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1$$

What happens when ERRORS occur in the reception of the encoded sequence:

$$e = 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 1$$

Reference symbol

$$\hat{e} = 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 1$$

Sequence is obtained after decoding:

$$\hat{d} = \quad 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1$$

Every single or burst of errors results in two decoding errors.

Transmission quality: Eye pattern

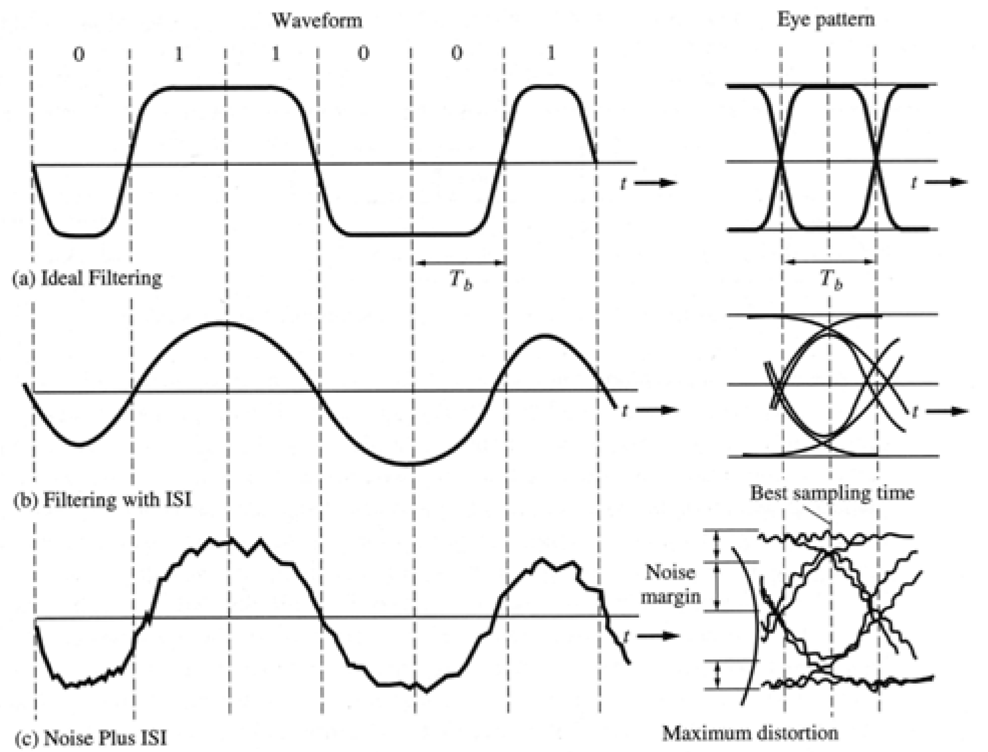


Figure 3–18 Distorted polar NRZ waveform and corresponding eye pattern.

Signal distortion and noise result in closure of the eye: in width and height.

The effect of filtering (rounding and dispersion of pulses) and noise (distortion) can be observed in the eye-pattern For a good detection quality, the eye-pattern should be open as far as possible!

PCM System

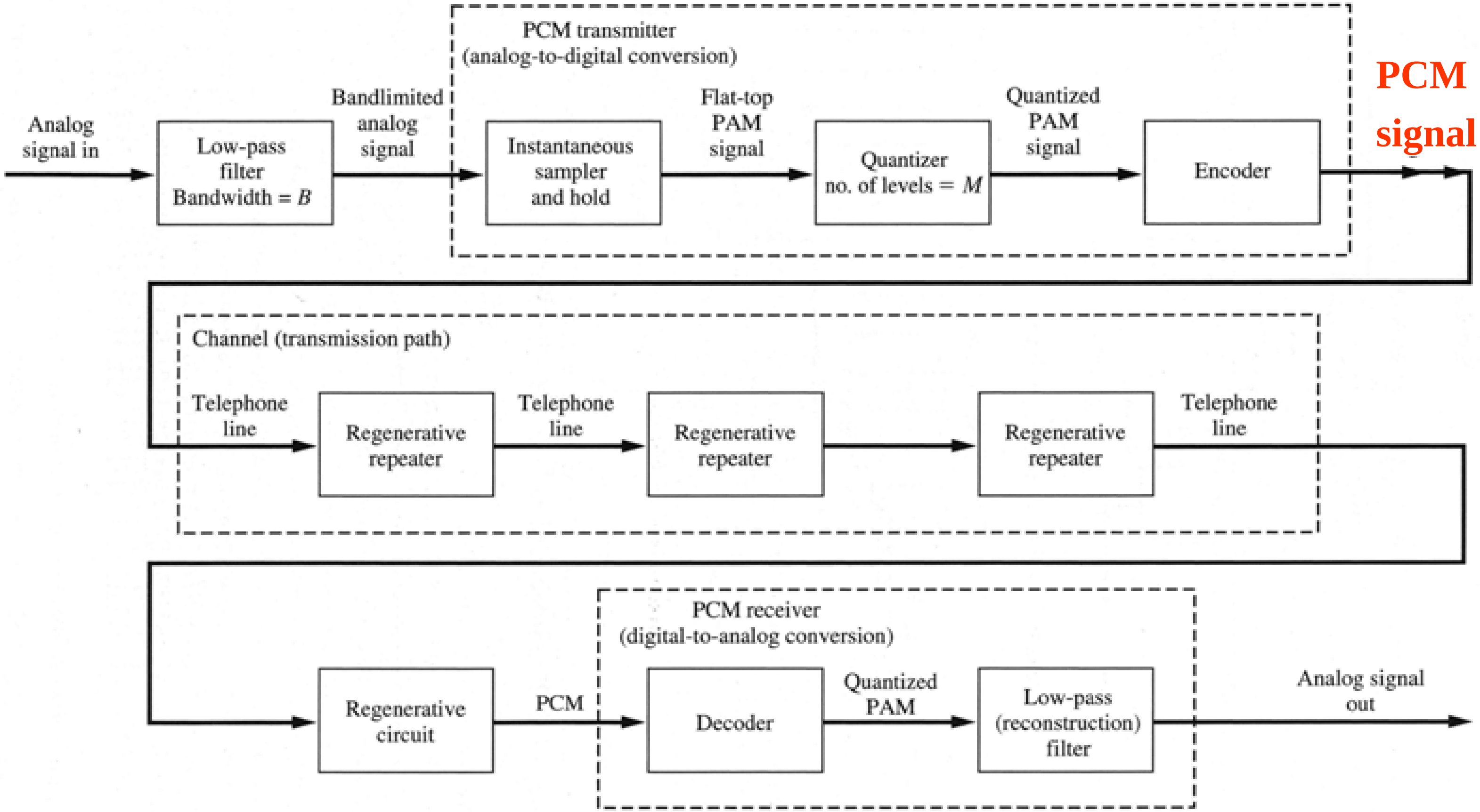


Figure 3–7 PCM trasmission system.

Regenerative repeater

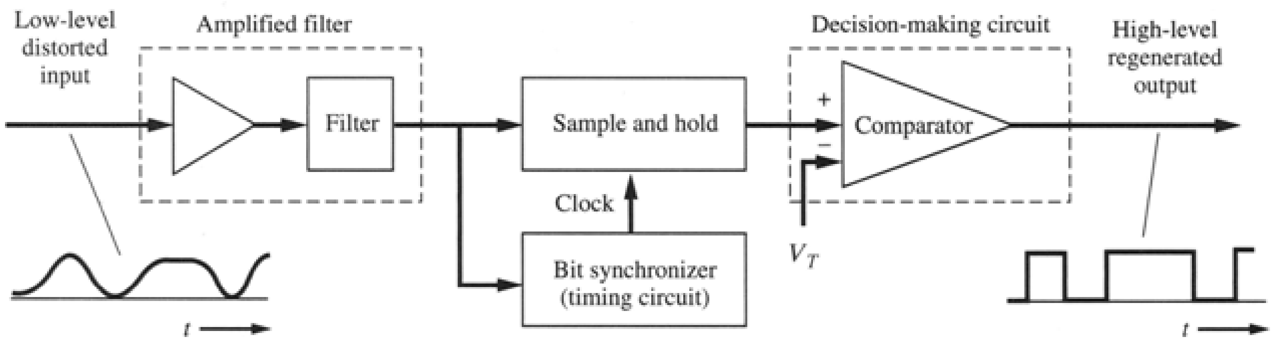


Figure 3–19 Regenerative repeater for unipolar NRZ signaling.

In a regenerative repeater, degenerated pulses due to filtering effects and noise are restored after detection at the cost of occasional errors.

With m repeaters the probability of i errors is: $P_i = \binom{m}{i} P_e^i (1-P_e)^{m-i}$

However, only an odd number of errors really result in a detection error in the end (for binary signals), so:

$$P_{me} = \sum_{i=1,3,5,..} P_i = \sum_{i=1,3,5,..} \binom{m}{i} P_e^i (1-P_e)^{m-i} \approx mP_e(1-P_e)^{m-1}$$

$$\text{thus for } P_e << 1 \Rightarrow P_{me} \approx mP_e$$

Multilevel signaling (1)

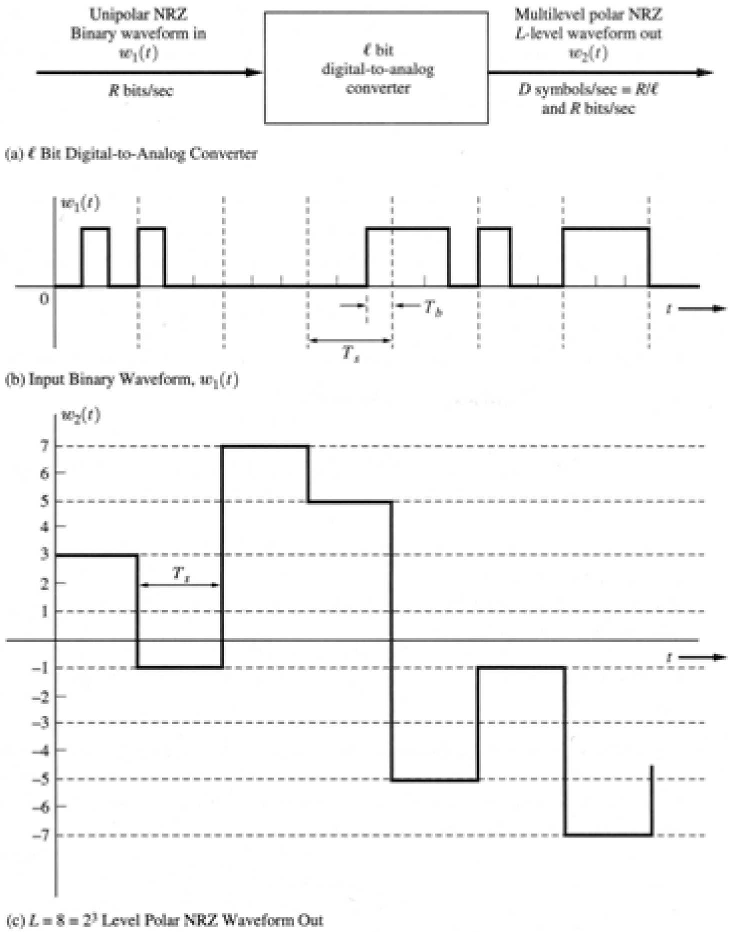


Figure 3–22 Binary-to-multilevel polar NRZ signal conversion.

Multilevel signaling needs less bandwidth than binary signaling because more bits are transmitted per symbol.

With l bits per symbol:

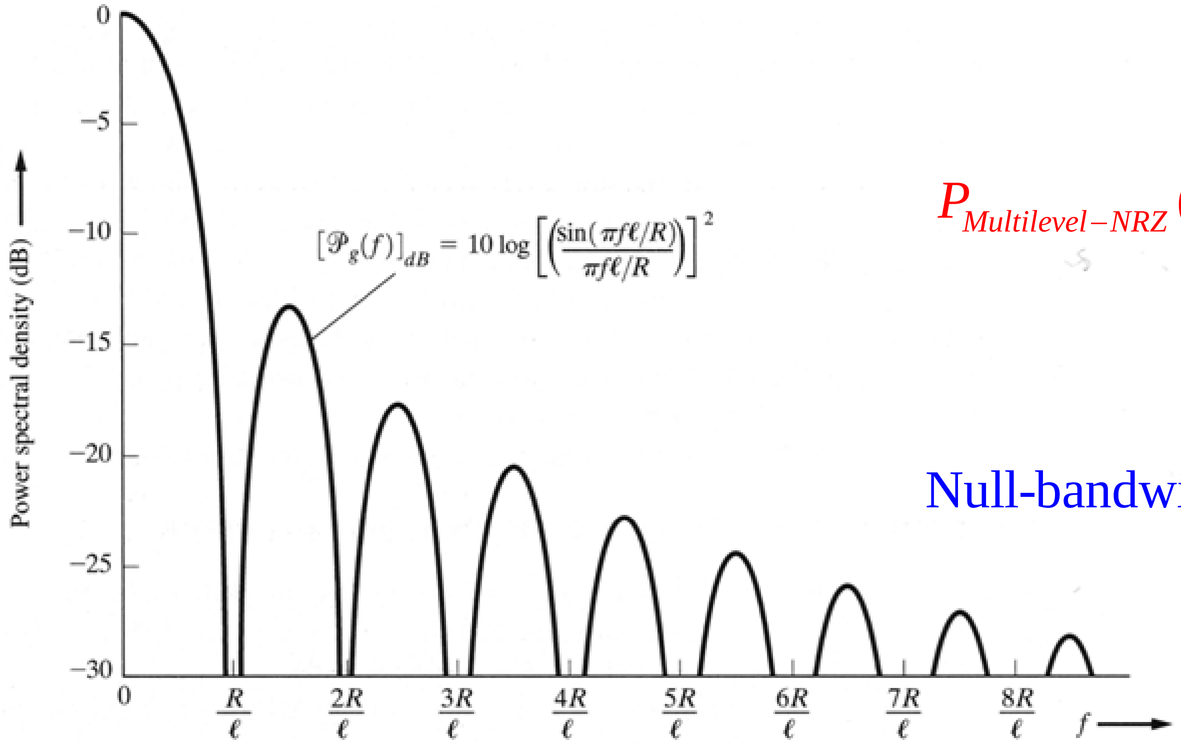
- transmission levels $\rightarrow$ $L = 2^l$
- symbol rate $\rightarrow$ $D = \dfrac{R_b}{l}$ [baud]

TABLE 3–5 THREE-BIT DAC CODE

Digital Word	Output Level, $(a_n)_i$
000	+7
001	+5
010	+3
011	+1
100	−1
101	−3
110	−5
111	−7

TU Delft

Multilevel signaling with rectangular pulses (2)



$$P_{Multilevel-NRZ}(f) = K \left(\frac{\sin l\pi fT_b}{l\pi fT_b} \right)^2$$
$$= K \operatorname{sinc}^2 lfT_b$$

Null-bandwidth: $B_{null} = \dfrac{R}{l}$

For L-level signaling the symbol duration increases to $T_s = lT_b$ with $l = {}^2\log L$.

Spectral efficiency (1)

Bandwidth is a scarce (expensive) resource!

Spectral efficiency: number of bits/s R_b that can be transmitted in a bandwidth B.

$$\eta \triangleq \frac{bit\ rate}{transmission\ bandwidth} = \frac{R_b}{B} \quad [\text{(bits/s)/Hz}]$$

With efficient L-level signaling, spectral efficiency is increased:

$$\eta = \frac{lR_s}{B} \quad [\text{(bits/s)/Hz}]$$

Upper bound on spectral efficiency (Shannon):

$$\eta_{\max} \triangleq \frac{C}{B} = {}^{2}\log(1+\text{SNR})$$

Spectral efficiency (2)

Design constraint: limit the required bandwidth! ⇒ Choose the signaling technique with the highest spectral efficiency. Of course you need to consider other boundary conditions as power, bit error probability and cost.

TABLE 3–6 SPECTRAL EFFICIENCIES OF LINE CODES

Code Type	First Null Bandwidth (Hz)	Spectral Efficiency $\eta = R/B$ [(bits/s)/Hz]
Unipolar NRZ	R	1
Polar NRZ	R	1
Unipolar RZ	$2R$	$\frac{1}{2}$
Bipolar RZ	R	1
Manchester NRZ	$2R$	$\frac{1}{2}$
Multilevel polar NRZ	R/ℓ	ℓ

Inter-Symbol Interference (1)

When pulses are transmitted in a smaller bandwidth than needed, they will be distorted: time dispersion and pulse overlap →

Inter-symbol Interference (ISI).

Question: How can we restrict the channel bandwidth without causing ISI?

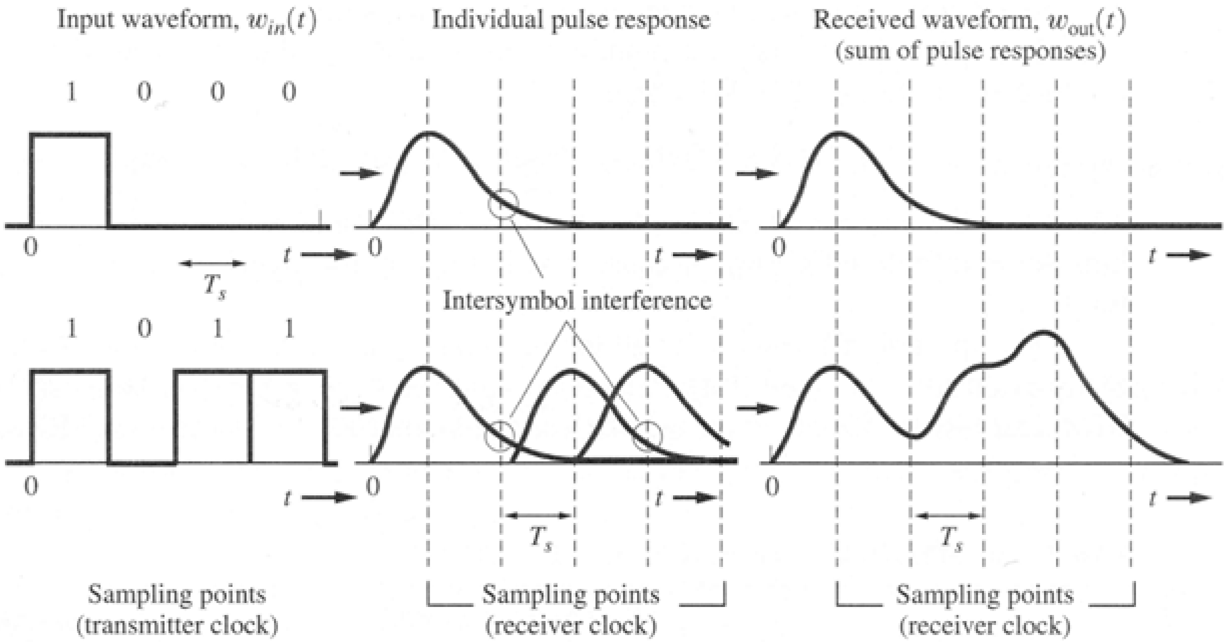


Figure 3–23 Examples of ISI on received pulses in a binary communication system.

TU Delft

Inter-Symbol Interference (2)

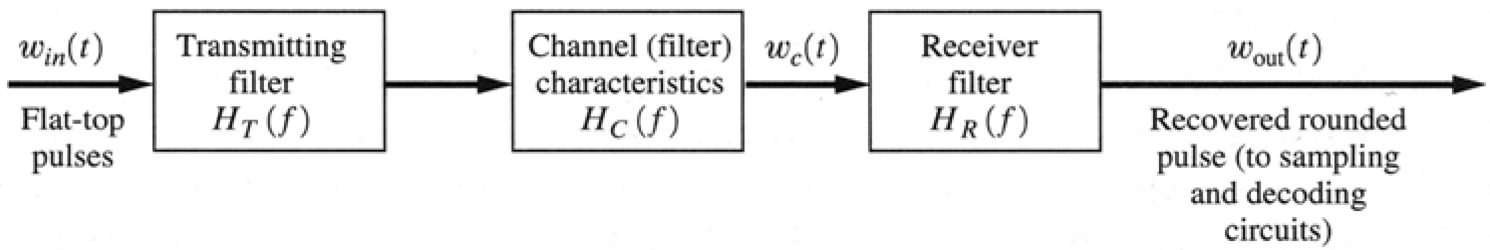


Figure 3–24 Baseband pulse-transmission system.

Let: $w_{in}(t) = \sum_{n=-\infty}^{+\infty} a_n h(t - nT_s)$ with $h(t) = \prod\left(\frac{t}{T_s}\right)$ and symbol rate $D = \frac{1}{T_s} = \frac{1}{lT_b}$.

The values a_n can be those of a multilevel signal.

Now we find for: $w_{in}(t) = \sum_{n=-\infty}^{+\infty} a_n h(t) * \delta(t - nT_s) = \left[\sum_{n=-\infty}^{+\infty} a_n \delta(t - nT_s)\right] * h(t)$

and $w_{out}(t) = \left[\sum_{n=-\infty}^{+\infty} a_n \delta(t - nT_s)\right] * h_e(t)$

with
$$h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t)$$
$$H_e(f) = H(f) \cdot H_T(f) \cdot H_C(f) \cdot H_R(f)$$

Inter-Symbol Interference (3)

For elimination of ISI, the equalizing filter is:

Spectrum of the required pulse shape.

$$H_R(f) = \frac{H_e(f)}{H(f)H_T(f)H_C(f)}$$

When $H_C(f)$ variable, then $H_R(f)$ should be adaptive (switched telephone line, radio channel).

This requires an adaptive receiver, i.e by means of:

- feedback
- learning filter based on a training-sequence (known word).

Design problem: Determine the optimum $H_T(f)$ and $H_R(f)$ for given $H_C(f)$ and the desired $H_e(f)$.

Elimination of ISI (1)

Nyquist (1928) found three methods to obtain ISI-free transmission.

Nyquist's 1e methode

Requirements for a pulse on the decision circuit's input:

$$h_e(\tau + kT_s) = \begin{cases} C \rightarrow k = 0 \\ 0 \rightarrow k \neq 0 \end{cases}$$

Where τ is a timing offset of the receiver clock. I.e. choose:

$$h_e(t) = \mathrm{sinc}(f_s t) = \frac{\sin \pi f_s t}{\pi f_s t} \quad \text{for a symbol rate } f_s = \frac{1}{T_s}$$

with $H_e(f) = \frac{1}{f_s} \prod \left(\frac{f}{f_s} \right) \quad \Rightarrow \; B = f_s / 2$,

for which $H_T(f)$ and $H_R(f)$ need to be determined. No ISI will occur since the 1e Nyquist criterion has been fulfilled.

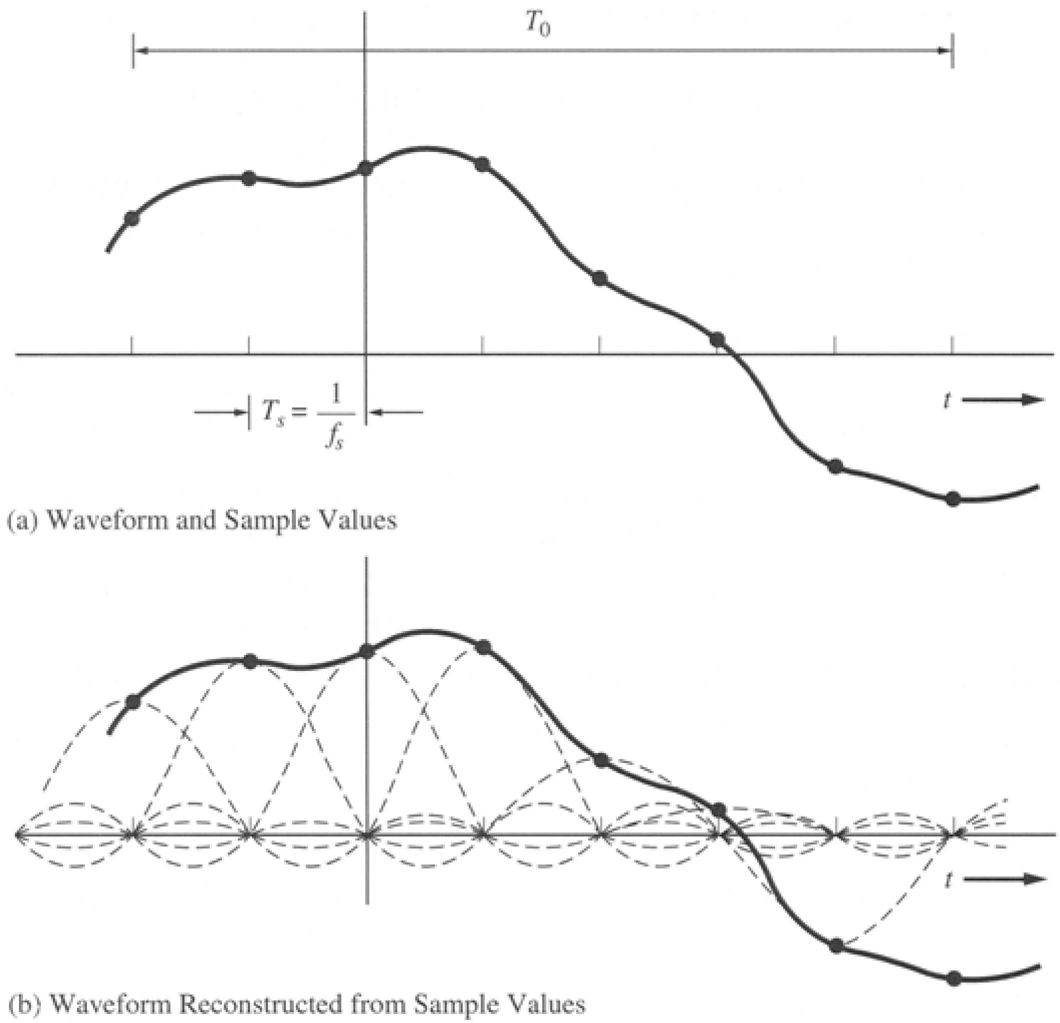


Figure 2–17 Sampling theorem.

Elimination of ISI (2)

The optimal filter $\Rightarrow$ sinc-pulses:

- minimum bandwidth: $B = f_s / 2$
- symbol rate: $D = 1/T_s = 2B$ [baud]

In practice not realizable since:

- $h_e(t)$ is non-causal: $h_e(t<0) \neq 0$ and is difficult to implement due to steep skirts in the frequency domain,
- synchronization is very critical; small timing errors result in a large increase of errors due to ISI.

In practice a slightly broader filter is used, i.e. the

Raised Cosine filter

Raised Cosine filter (1)

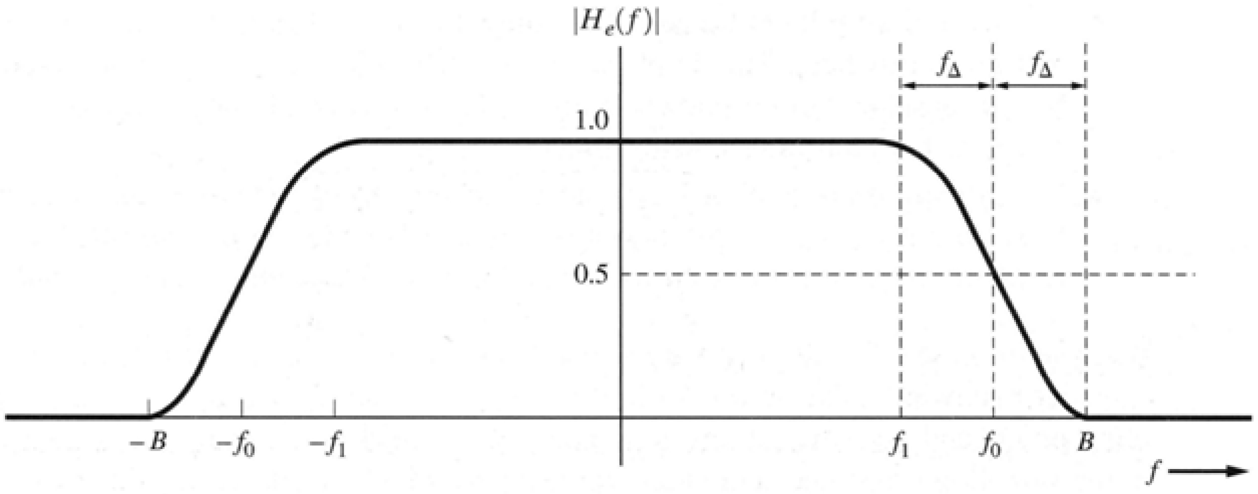


Figure 3–25 Raised cosine-rolloff Nyquist filter characteristics.

With:

$$f_\Delta \triangleq B - f_0$$

$$f_1 \triangleq f_0 - f_\Delta$$

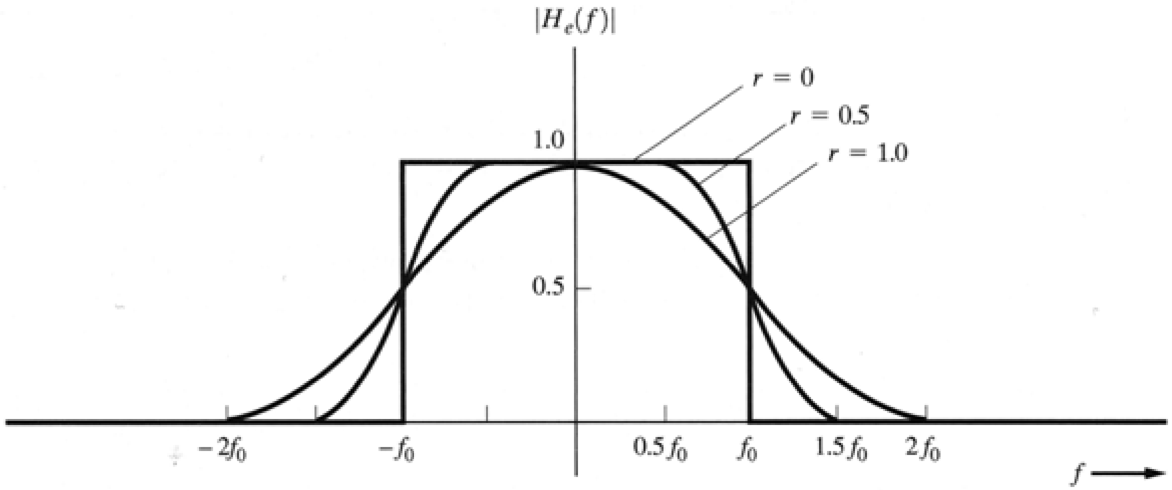
$f_0 =$ -6 dB frequency $= \dfrac{D}{2}$

Roll-off factor: $r = \dfrac{f_\Delta}{f_0} \quad (0 \le r \le 1)$

Absolute bandwidth $= B$

TU Delft

Raised Cosine filter (2)

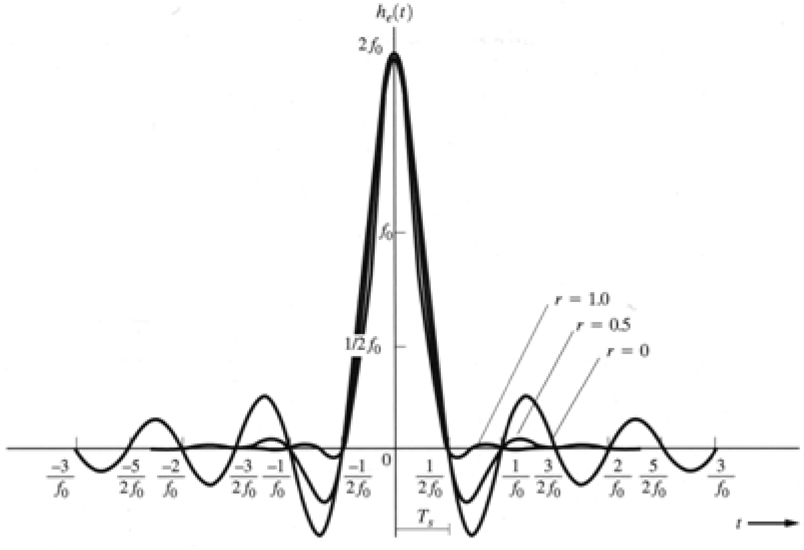


(a) Magnitude Frequency Response

$$H_e(f) = \begin{cases} 1 & |f| < f_1 \\ \frac{1}{2}\left(1 + \underbrace{\cos\left(\frac{\pi(|f| - f_1)}{2f_\Delta}\right)}\right) & f_1 < |f| < B \\ 0 & |f| > B \end{cases}$$

Cosine roll-off of the signal spectrum

Raised Cosine filter (3)



(b) Impulse Response

Figure 3–26 Frequency and time response for different rolloff factors.

$$h_e(t) = \mathfrak{F}^{-1}\{H_e(f)\}$$

$$= 2f_0 \operatorname{sinc}(2f_0 t)\frac{\cos 2\pi r f_0 t}{1-(4rf_0 t)^2}$$

A weighted sinc-pulse.

- no ISI according to Nyquist_1 for $t = nT_s$ (and $\tau = 0$)
- roll-off factor: $r = \frac{f_\Delta}{f_0} = \frac{B - f_0}{f_0} \Rightarrow r = \frac{B - D/2}{D/2}$
- maximum symbol rate without ISI: $D = 1/T_s = 2f_0 = \frac{2B}{1+r}$
- absolute bandwidth: $B = f_0 + f_\Delta = (1+r)D/2$

Class of ISI-free filters

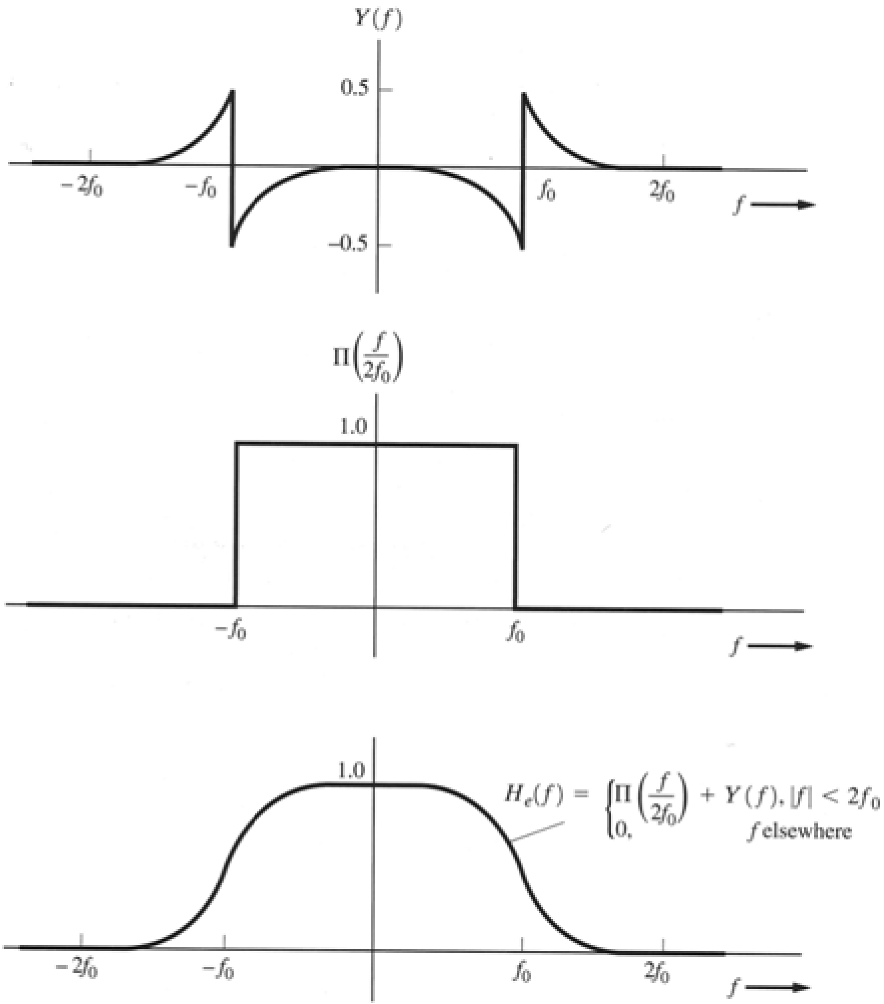


Figure 3–27 Nyquist filter characteristic.

$$H_e(f) = \begin{cases} \Pi\left(\dfrac{f}{2f_0}\right) + Y(f) & |f| \leq 2f_0 \\ 0 & |f| > 2f_0 \end{cases}$$

With:

1. $Y(f)$ real
2. $Y(f)$ even-symmetric around $f = 0$
3. $Y(f)$ uneven-symmetric $f = f_0$

No ISI for $R_s = 2f_0$

TUDelft

ISI-free filters and noise

With Nyquist_1: ISI = 0 but noise is not minimum!

Using:

$$H_T(f) = \frac{\sqrt{H_e(f)}[P_{n\_rx}(f)]^{1/4}}{\alpha\,|H(f)|\sqrt{H_C(f)}}$$

$P_{n\_rx}(f)$: is noise PSD at the receiver

$$H_R(f) = \frac{\alpha\sqrt{H_e(f)}}{\sqrt{H_C(f)}[P_{n\_rx}(f)]^{1/4}}$$

Now we obtain ISI = 0 and minimum noise at the receiver (*matched filter*):

- large transmit power at frequencies with a high noise level at RX,

- receive filter suppresses frequencies with a high noise level

and: $H_e(f) = H(f)H_T(f)H_C(f)H_R(f)$

TU Delft

Other methods of ISI-free filtering

Nyquist's 2e method:

- allow for controlled (known) ISI, which can be compensated in a later stage.

Nyquist's 3rd method:

- not based on $h_e(nT)$ but on $\int h_e(t)dt$, i.e. the area under the equivalent impulse.