Telecommunicatie A (EE2T11)

Lecture 4 overview:

System noise calculations and link budget

- Brief review equivalent noise temperature and noise figure
- Noise figure and equivalent noise temperature of cascaded communication components
- Link budget calculation: SNR at the receive system input

Non-ideal components for analog signal processing in telecommunication systems

- filters, amplifiers, limiters, mixers, frequency multipliers

EE2T11 Telecommunicatie A

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Colleges en Werkcolleges Telecommunicatie A

Colleges:

Maandag	7-3, 14-3	5e en 6e uur, EWI-Pi
Dinsdag	15-3, 29-3	7e en 8e uur, EWI-Pi

Werkcolleges:

Donderdag	10-3	5e en 6e uur, EWI-Boole
Maandag	21-3, 4-4	5e en 6e uur, EWI-Pi

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Review: Available noise power

For $R_L = R$, a matched load, the PSD of the noise (transferred to the load) is equal to:

$$P_a(f) = \frac{V_L^2(f)}{R} = \frac{V_v^2(f)}{4R} = \frac{kT}{2} \equiv \frac{N_0}{2} \quad \text{[W/Hz]}$$

double sided

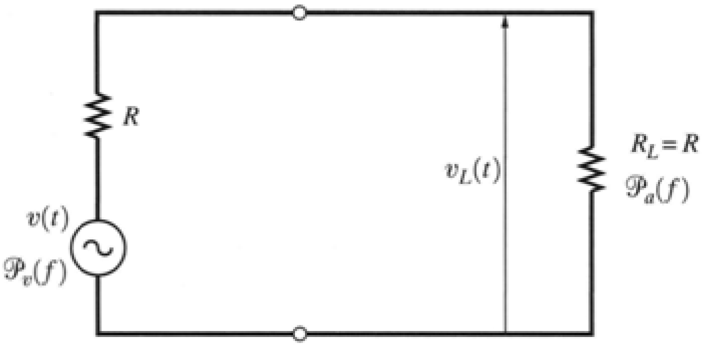

Figure 8–18 Thermal source with a matched load.

with $V_v(f) = \sqrt{2kTR}\ [\mathrm{V}/\sqrt{Hz}]$.

This is the RMS-voltage of the source per $\sqrt{Hz}$ of bandwidth.

The available noise power is given by:

$$P_a = \int_{-B_n}^{B_n} P_a(f)df = kTB_n \ \text{[W]}$$

- proportional with B_n and T
- independent of f and R

Review: Equivalent noise temperature (1)

An arbitrary white noise source can be characterized by an equivalent noise temperature. We represent here the source as a noise generating resistor at temperature:

$$\text{Equivalent noise temperature } T_n = \frac{P_a}{kB_n} = \frac{2P_a(f)}{k}$$

When the noise is not caused by thermal effects, T_n is not related to the physical temperature of the component.

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Review: equivalent noise temperature (1)

The equivalent noise temperature T_e of a two port:

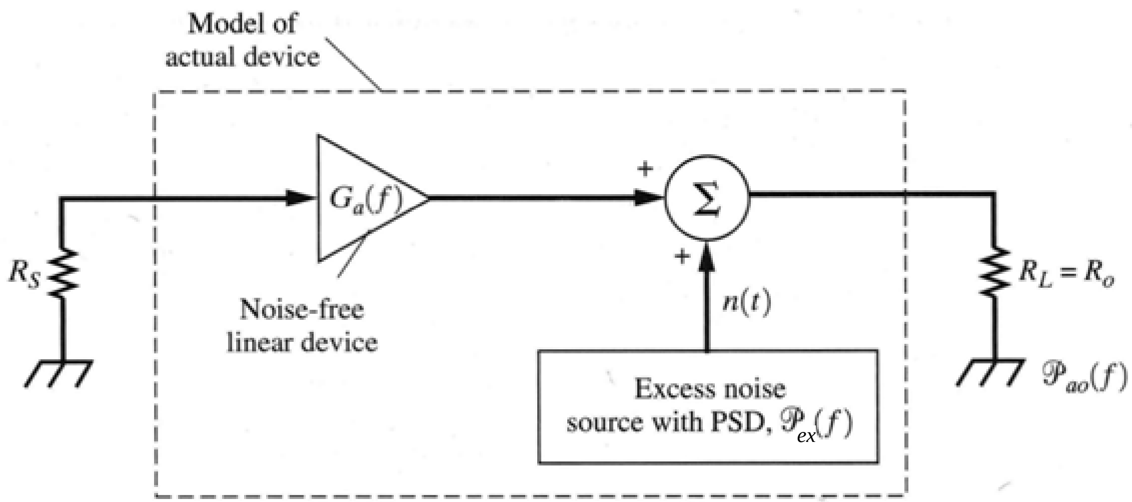

Figure 8–19 Noise model for an actual device.

$$P_{ao} \square k(T_i + T_e) \int_{f_0-\frac{B_n}{2}}^{f_0+\frac{B_n}{2}} G_a(f)df = k(T_i + T_e)G_a B_n$$

available output noise power

input noise temperature

if G_a constant over passband

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Review: equivalent noise temperature (2)

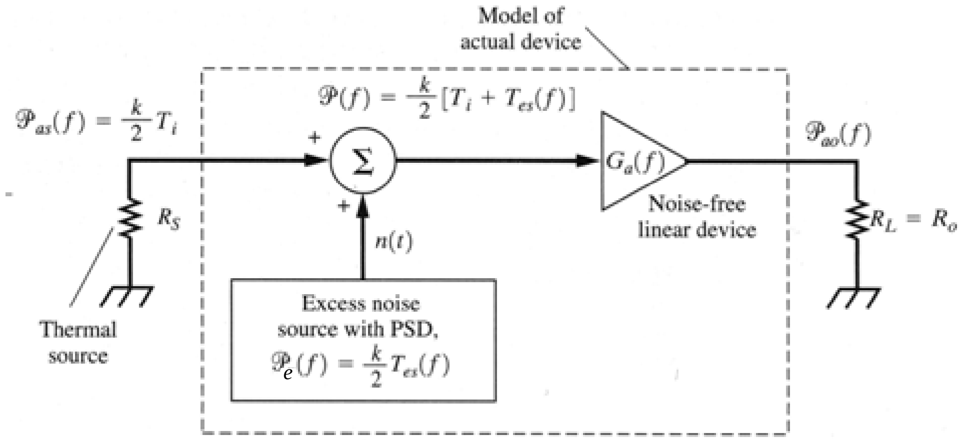


Figure 8–20 Another noise model for an actual device.

The equivalent noise temperature T_e is defined as:

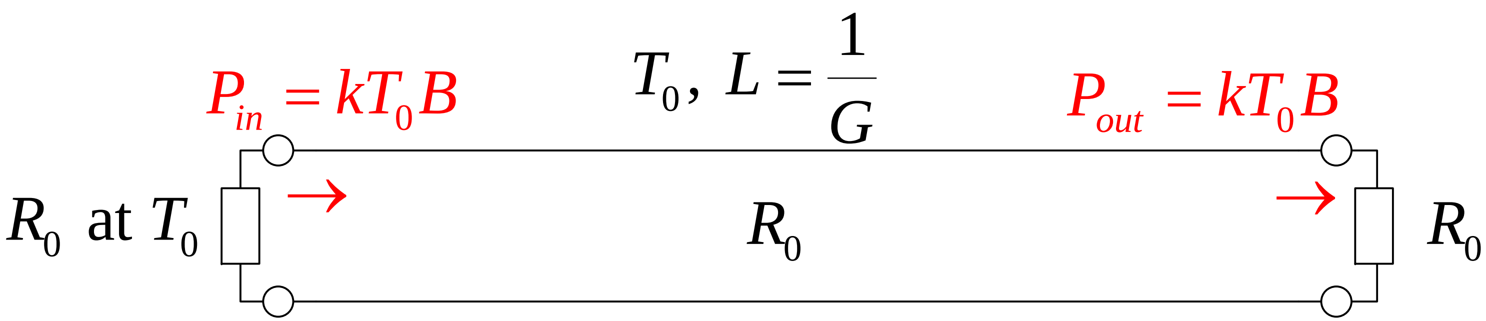
$$T_e = \frac{P_{ao} - kT_{n\_in}G_aB_n}{kG_aB_n} = (F-1)T_0$$

where $F = 1 + \frac{T_e}{T_0}$ is the Noise Figure and T_0 = 290 K is

the standard room temperature.

Review: Noise Figure and equivalent noise temperature of a transmission line

For a transmission line with attenuation $L = 1/G$, loaded at the input and output with its characteristic impedance R_0 , and which is at room temperature T_0:



$$T_e = \frac{P_{\text{cable noise}}}{kGB} = \frac{P_{\text{out}} - \frac{P_{in}}{L}}{\frac{kB}{L}} = \frac{kT_0B(1-\frac{1}{L})}{\frac{kB}{L}} = (L-1)T_0$$

and $F = 1 + \frac{T_e}{T_0} = L$ $\Rightarrow$ the Noise Figure of a cable at T_0 is equal to its loss.

Equivalent temperature of cascaded devices (1)

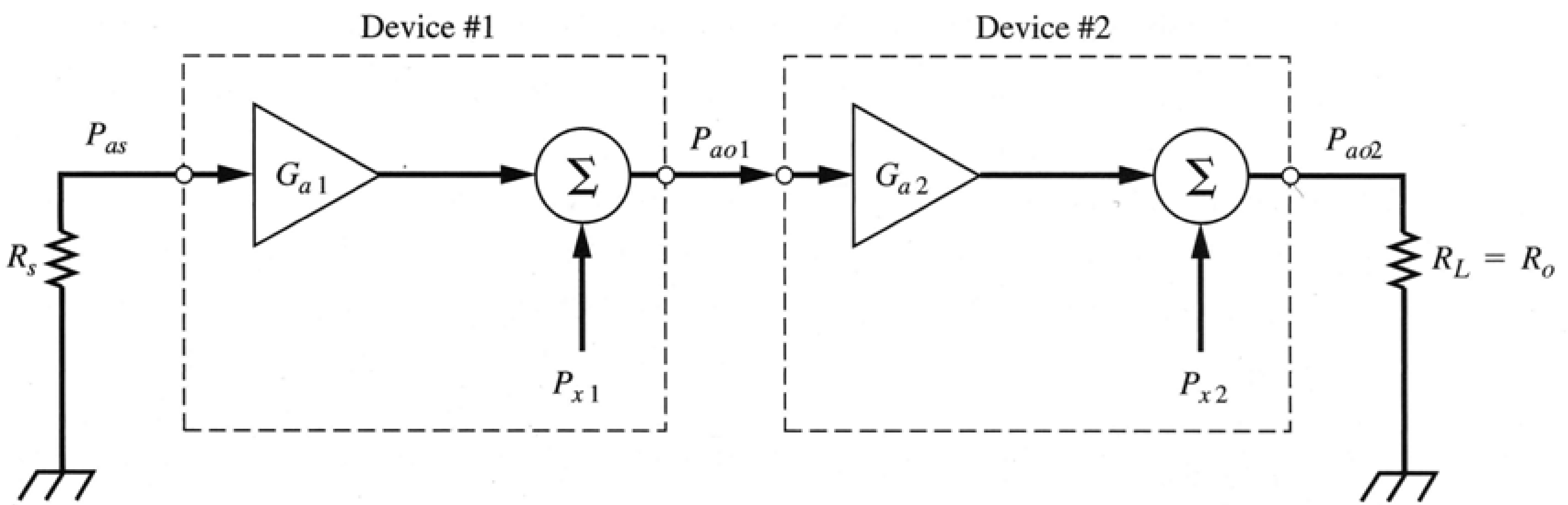
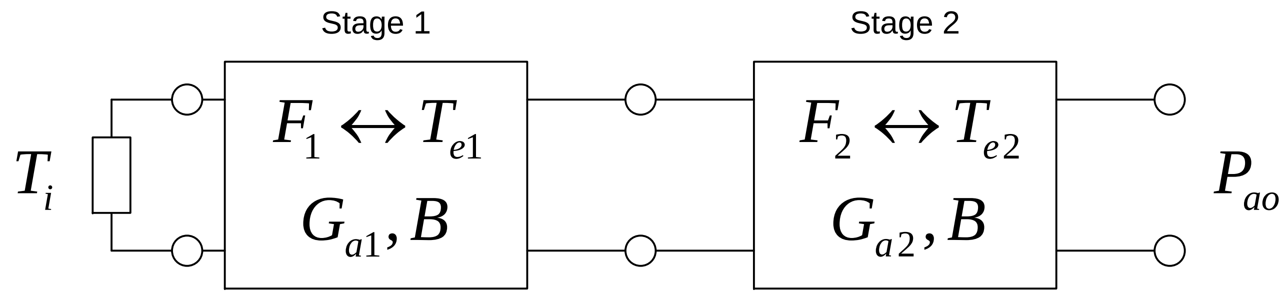


Figure 8–23 Noise model for two cascaded devices.

What is the equivalent noise temperature of a cascading of two-ports?

Effective temperature of cascaded devices (2)



$$\underbrace{\qquad}_{T_e,\ F\,?} \qquad\qquad G_a = G_{a1} G_{a2}$$

The available output noise power consists of the following contributions:

$$\left.\begin{array}{ll} \text{Source:} & kT_i B\, G_{a1} G_{a2} \\ 1^{st}\,\text{stage:} & kT_{e1} B\, G_{a1} G_{a2} \\ 2^{nd}\,\text{stage:} & kT_{e2} B\, G_{a2} \end{array}\right\} \Rightarrow \begin{array}{l} P_{oa} = k(G_{a1}G_{a2}T_i + G_{a1}G_{a2}T_{e1} + G_{a2}T_{e2})B \\ \quad = k(T_i + T_{e1} + \dfrac{T_{e2}}{G_{a1}})G_a B \approx k(T_i + T_e)G_a B \end{array}$$

and we find: $T_e = T_{e1} + \dfrac{T_{e2}}{G_{a1}}$

Equivalent temperature of cascaded devices (3)

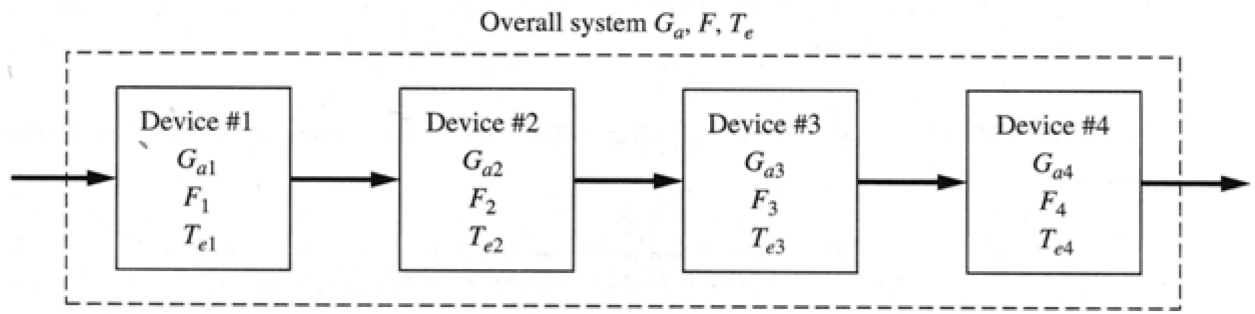


Figure 8–22 Cascade of four devices.

By induction we find Friis' formula:

$$T_e = T_{e1} + \frac{T_{e2}}{G_{a1}} + \frac{T_{e3}}{G_{a1}G_{a2}} + \frac{T_{e4}}{G_{a1}G_{a2}G_{a3}} + \dots$$

Using $T_e = (F\text{-}1)T_0$, we find a similar expression for the "system" noise figure:

$$(F-1)T_0 = (F_1-1)T_0 + \frac{(F_2-1)T_0}{G_{a1}} + \frac{(F_3-1)T_0}{G_{a1}G_{a2}} + \dots$$

$$\Rightarrow F = F_1 + \frac{(F_2-1)}{G_{a1}} + \frac{(F_3-1)}{G_{a1}G_{a2}} + \dots$$

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Equivalent temperature of cascaded devices (4)

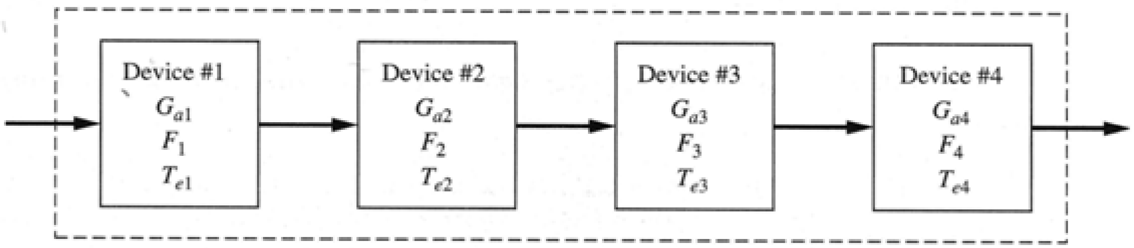


Figure 8–22 Cascade of four devices.

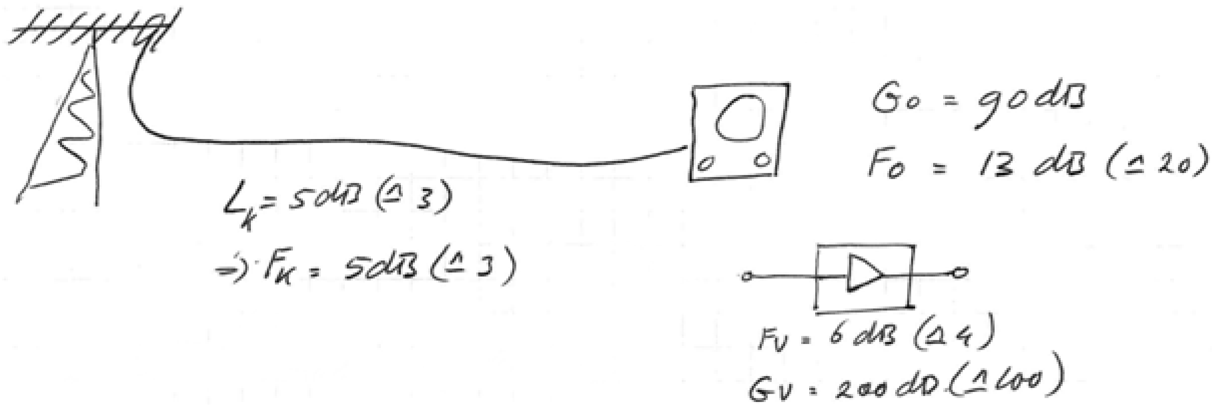
$$T_e = T_{e1} + \frac{T_{e2}}{G_{a1}} + \frac{T_{e3}}{G_{a1}G_{a2}} + \frac{T_{e4}}{G_{a1}G_{a2}G_{a3}} + \dots$$

When the gain of the 1st stage is high, the quality of the whole system will be mainly determined by its noise contribution!

However, when the 1st stage is lossy ($G_{a1} < 1$), then the other stages are also important!

What is the effective noise temperature at other locations in the chain?

Example: TV-station (1)



1. T_e and F without amplifier?

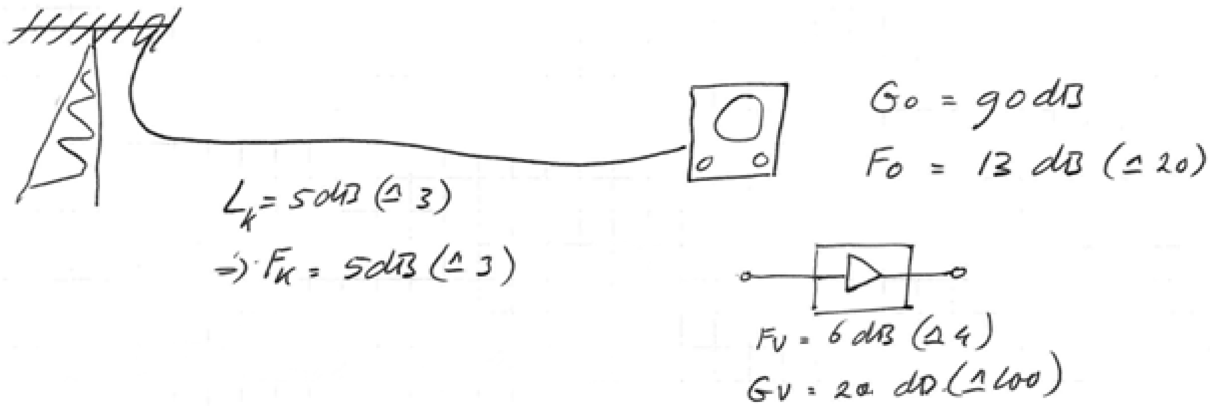
$$T_e = (F_1 - 1)T_0 + \frac{(F_2 - 1)T_0}{G_{a1}} = (L_k - 1)T_0 + (F_o - 1)T_0 L_k$$

$$= (3-1)T_0 + (20-1)\cdot T_0 \cdot 3 = 59T_0 = 17110 \text{ K} \equiv 42.3 \text{ dBK}$$

$$F = F_1 + \frac{(F_2 - 1)}{G_{a1}} = L_k + (F_o - 1)L_k$$

$$= L_k F_o = 1 + \frac{T_e}{T_0} = 60 \equiv 18 \text{ dB}$$

Example: TV-station (2)



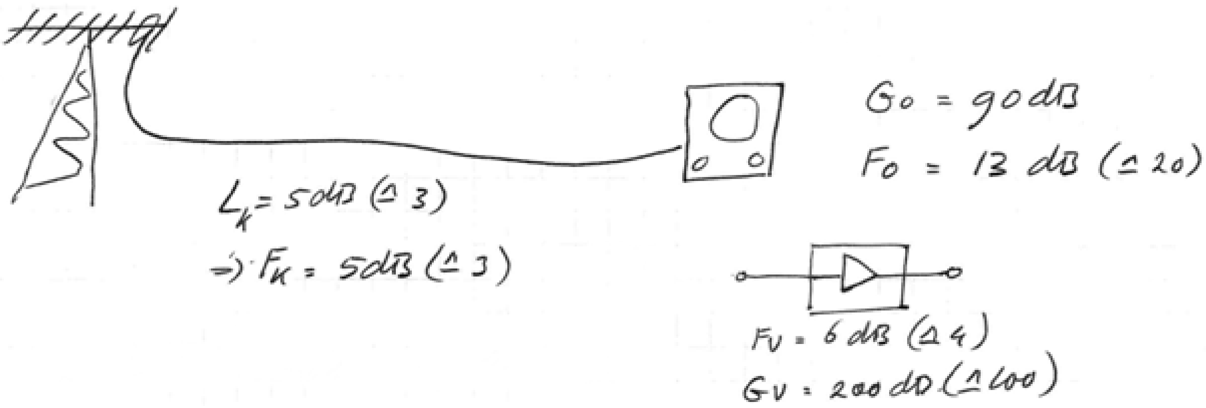
2. T_e and F with amplifier at the TV-input?

$$T_e = (F_1 - 1)T_0 + \frac{(F_2 - 1)T_0}{G_{a1}} + \frac{(F_3 - 1)T_0}{G_{a1}G_{a2}} = (L_k - 1)T_0 + (F_v - 1)T_0 L_k + \frac{(F_o - 1)T_0 L_k}{G_v}$$

$$= (3-1)T_0 + (4-1)\cdot T_0 \cdot 3 + \frac{(20-1)\cdot T_0 \cdot 3}{100} = 11.6T_0 = 3364\ \mathrm{K} \equiv 35.3\ \mathrm{dBK}$$

$$F = 1 + \frac{T_e}{T_0} = 12.6 \equiv 11.0\ \mathrm{dB}$$

Example: TV-station (3)



2. T_e and F with amplifier right after the antenna ?

$$F = F_v + \frac{(L_k - 1)}{G_v} + \frac{(F_o - 1)}{G_v / L_k}$$

$$= 4 + \frac{(3-1)}{100} + \frac{(20-1)\cdot 3}{100} = 4.59 \cong 6.6 \text{ dB}$$

$$T_e = (F - 1)T_0 = 1041 \text{ K}$$

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Link budget (1)

Which trade-off's can be made in achieving a required C/N?

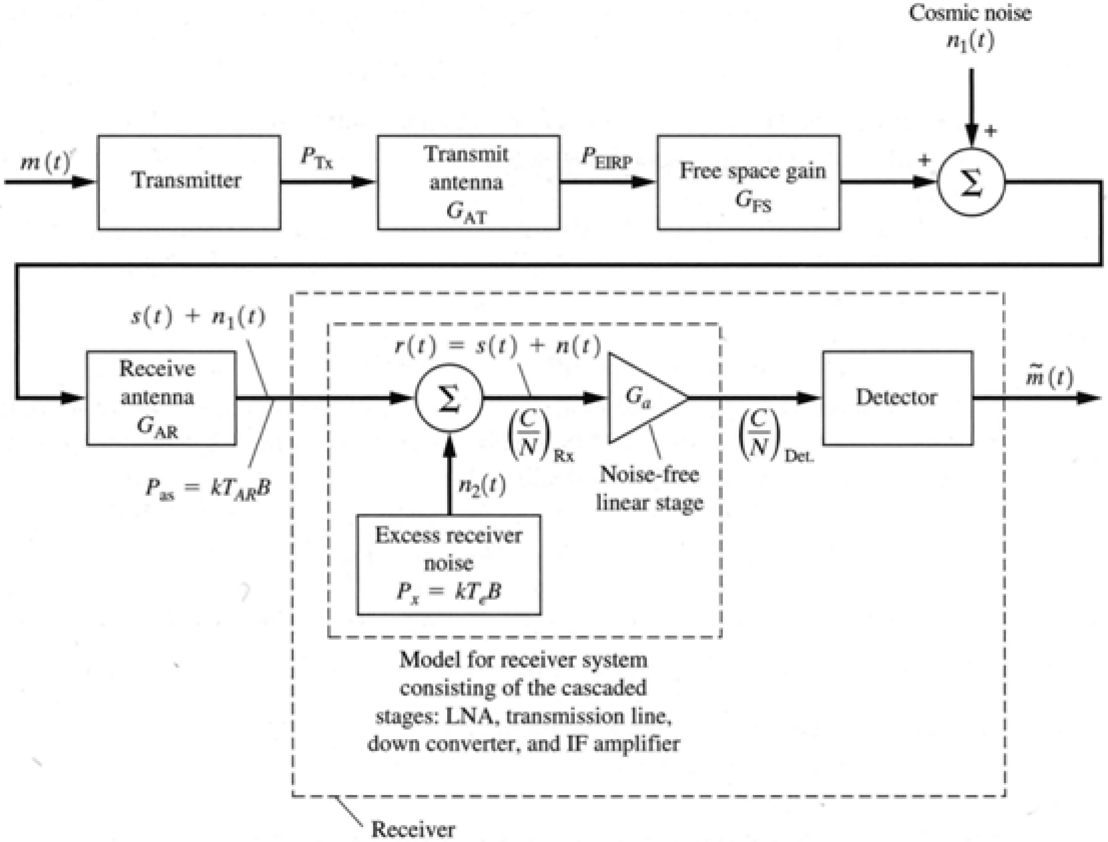
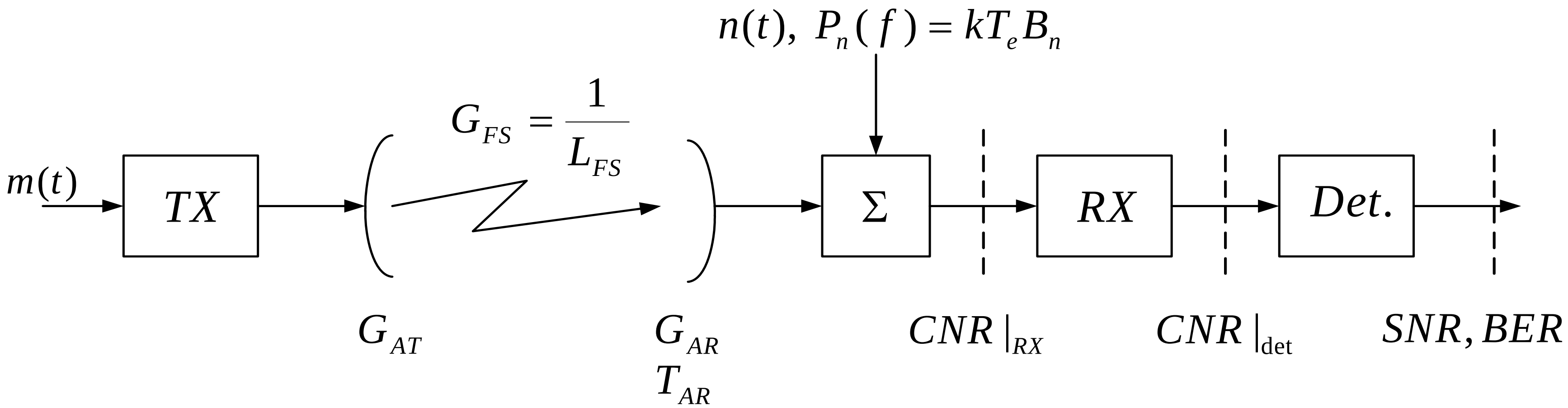


Figure 8–24 Communication system model for link budget evaluation.

Figure 8.24 assumes linearity of the system.

Link budget (2)

Link budget: budget of CNR.

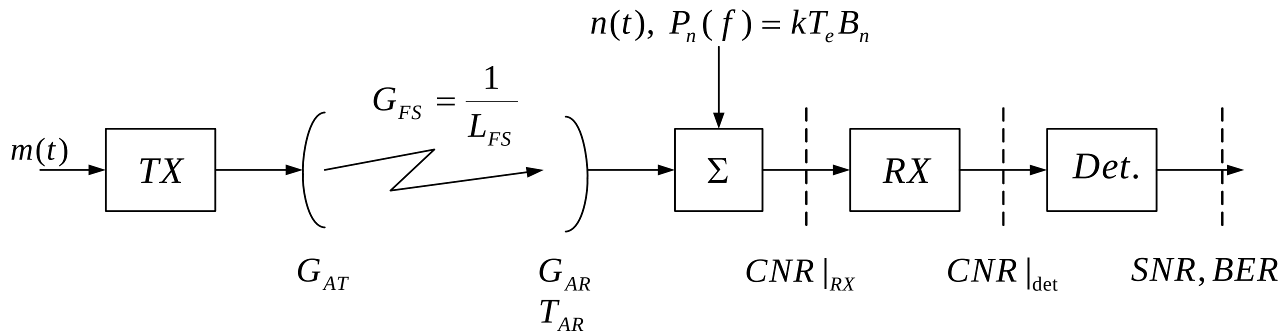


For an ideal receiver:

$$\left.\frac{C}{N}\right|_{Det} = \left.\frac{C}{N}\right|_{RX} = \frac{P_{EIRP} \cdot G_{FS} \cdot G_{AR}}{k \cdot T_{sys} \cdot B_n}$$

$$= \frac{P_{TX} \cdot G_{AT} \cdot G_{AR}}{L_{FS} \cdot k(T_{AR} + T_e)B_n}$$

Link budget (3)



Or in dB notation:

$$\left.\frac{C}{N}\right|_{dB} = (P_{EIRP})_{\mathrm{dBW}} - (L_{FS})_{\mathrm{dB}} + \left(\frac{G_{AR}}{T_{sys}}\right)_{\mathrm{dB/K}} - (k)_{\mathrm{dBW/Hz/K}} - (B_n)_{\mathrm{dBHz}}$$

Figure of merit of the receiver station, with $T_{sys}=T_a+T_e$

$$k_{\mathrm{dB}} = -228.6\ [\mathrm{dBW/Hz/K}]$$

$$L_{FS\_\mathrm{dB}} = 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2\ [\mathrm{dB}] \Rightarrow\ \text{free-space loss}$$

$$B_{n\_\mathrm{dB}} = 10\log_{10} B_n\ \ [\mathrm{dBHz}]$$

Telecommunication devices

In telecommunication systems several types of devices are used to perform analog signal processing functions:

- filters
- amplifiers
- limiters
- mixers (up- and down-converters)
- frequency multipliers

In general, these devices are non-ideal and several exploit non-linearity to achieve their goal.

Filters (1)

A filter is a linear device which modifies the frequency spectrum of its input signal:

- amplitude/phase/delay of the signal's frequency components can be changed
- but no new frequency components are added

A filter uses energy storage elements to obtain frequency discrimination.

Filters are classified based on:

- type of construction
- type of transfer function

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Filters (2)

The quality of a filter is determined by the quality of the storage elements (especially due to parasitic resistors).

One definition of quality is:

$$Q = \frac{2\pi(\text{maximum energy stored during one cycle})}{\text{energy dissipated per cycle}}$$

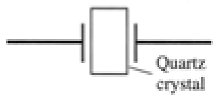
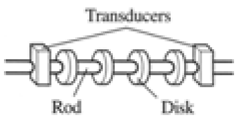
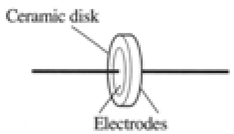
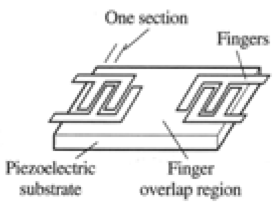
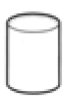
Perfect L 's and C 's have infinite Q.

Another definition of the quality of a bandpass filter:

$$Q = \frac{\text{center frequency}}{\text{-3 dB bandwidth}} = \frac{f_0}{B_{3dB}}$$

Filter constructions

TABLE 4–2 FILTER CONSTRUCTION TECHNIQUES

Type of Construction	Description of Elements or Filter	Center Frequency Range	Unloaded Q (Typical)	Filter Application[a]
LC (passive)		dc–300 MHz	100	Audio, video, IF, and RF
Active and Switched Capacitor		dc–500 kHz	200[b]	Audio
Crystal		1 kHz–100 MHz	100,000	IF
Mechanical		50–500 kHz	1,000	IF
Ceramic		10 kHz–10.7 MHz	1,000	IF
Surface acoustic waves (SAW)		10–800 MHz	[c]	IF and RF
Transmission line		UHF and microwave	1,000	RF
Cavity		Microwave	10,000	RF

[a] IF, intermediate frequency; RF, radio frequency. (See Sec. 4–16).
[b] Bandpass Q's.
[c] Depends on design: $N = f_0/B$, where N is the number of sections, f_0 is the center frequency, and B is the bandwidth. Loaded Q's of 18,000 have been.

Construction determines:

- frequency band,
- quality of the filter,
- allowable size,
- cost

Filters characteristics (1)

Another way to characterize a filter is by its transfer function.
A general transfer function of an n'th order filter:

$$H(f) = \frac{b_0 + b_1(j\omega) + b_2(j\omega)^2 + \ldots + b_k(j\omega)^k}{a_0 + a_1(j\omega) + a_2(j\omega)^2 + \ldots + a_n(j\omega)^n}$$

Different types of filters:

- low pass filter,
- high pass filter,
- band pass filter,
- band stop filter,
- notch filter.

Filters characteristics (2)

TABLE 4–3 SOME FILTER CHARACTERISTICS

Type	Optimization Criterion	Transfer Characteristic for the Low-Pass Filter[a]
Butterworth	Maximally flat: as many derivatives of $\lvert H(f)\rvert$ as possible go to zero as $f \to 0$	$\lvert H(f)\rvert = \dfrac{1}{\sqrt{1+(f/f_b)^{2n}}}$
Chebyshev	For a given peak-to-peak ripple in the passband of the $\lvert H(f)\rvert$ characteristic, the $\lvert H(f)\rvert$ attenuates the fastest for any filter of nth order	$\lvert H(f)\rvert = \dfrac{1}{\sqrt{1+\varepsilon^2 C_n^2(f/f_b)}}$ ε = a design constant; $C_n(f)$ is the nth-order Chebyshev polynomial defined by the recursion relation $C_n(x) = 2xC_{n-1}(x) - C_{n-2}(x)$, where $C_0(x) = 1$ and $C_1(x) = x$
Bessel	Attempts to maintain linear phase in the passband	$H(f) = \dfrac{K_n}{B_n(f/f_b)}$ K_n is a constant chosen to make $H(0) = 1$, and the Bessel recursion relation is $B_n(x) = (2n-1)\,B_{n-1}(x) - x^2 B_{n-2}(x)$, where $B_0(x) = 1$ and $B_1(x) = 1 + jx$

[a] f_b is the cutoff frequency of the filter.

Amplifiers

Ideal amplifier:

- linear,
- infinite bandwidth: no memory

Practical amplifier:

- approximately linear for small signals
- non-linear for large signals
- finite bandwidth: with memory

Distortionless amplifier:

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)} = Ke^{-j\omega\tau} \qquad \Rightarrow \qquad v_{out}(t) = Kv_{in}(t-\tau)$$

Non-linear amplifier characteristics (1)

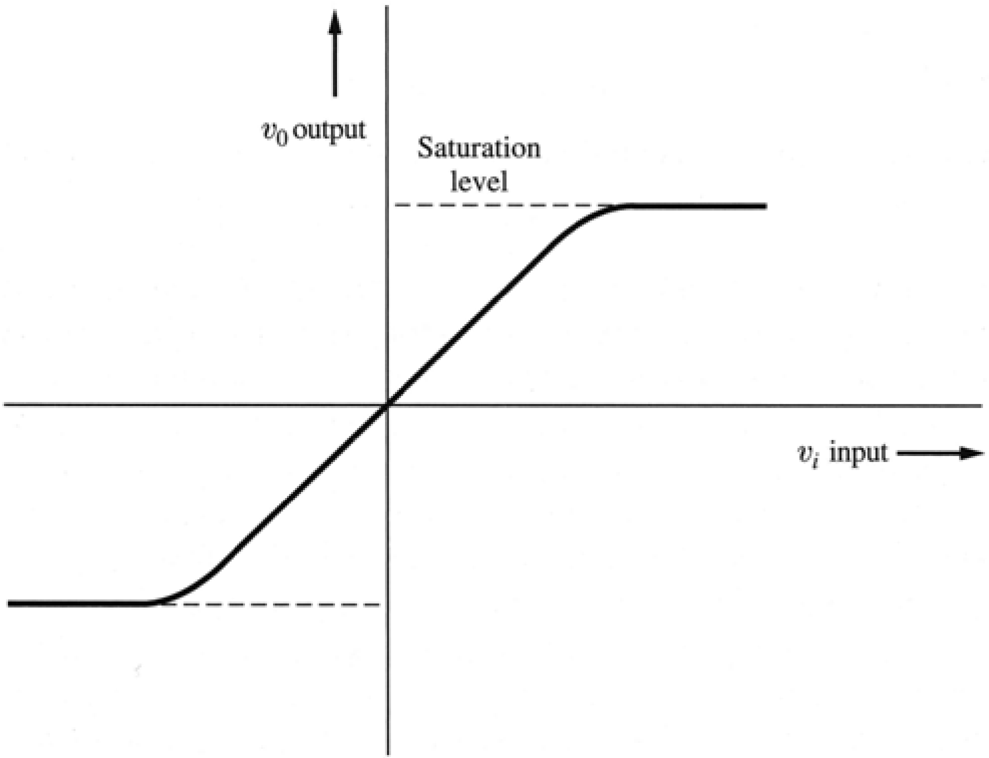


Figure 4–5 Nonlinear amplifier output-to-input characteristic.

For large input signals the output signal may contain 2nd, 3rd and higher powers of the input signal.

Causes: non-linear component behavior and output signal limitation due to limited supply voltage.

$$v_{out} = K_0 + K_1 v_{in} + K_2 v_{in}^2 + \ \ldots = \sum_{n=0}^{\infty} K_n v_{in}^n \quad \text{with} \quad K_n = \frac{1}{n!}\left(\frac{d^n v_{out}}{dv_{in}^n}\right)\Bigg|_{v_{in}=0}$$

Non-linear amplifier characteristics (2)

$$v_{out} = K_0 + K_1 v_{in} + K_2 v_{in}^2 + \ \dots \ = \sum_{n=0}^{\infty} K_n v_{in}^n$$

For a single-frequency input signal $v_{in}(t) = A_0 \sin \omega_0 t$ the output signal will contain higher order harmonics resulting from the 2nd, 3rd and higher order non-linearities:

$$K_2 v_{in}^2(t) = K_2 A_0^2 \sin^2 \omega_0 t = \frac{K_2 A_0^2}{2}\left(1 - \cos 2\omega_0 t\right)$$

$$K_3 v_{in}^3(t) = K_3 A_0^3 \sin^3 \omega_0 t = \frac{K_3 A_0^3}{4}\left(3 \sin \omega_0 t - \sin 3\omega_0 t\right)$$, etc.

In general: $v_{out}(t) = V_0 + V_1 \sin(\omega_0 t + \varphi_1) + V_2 \sin(2\omega_0 t + \varphi_2) + \ \dots$

$$= \sum_{n=0}^{\infty} V_n \sin(n\omega_0 t + \varphi_n)$$

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Non-linear amplifier characteristics (3)

An often used measure of non-linearity of an amplifier is the Total Harmonic Distortion (THD):

$$\mathrm{THD} = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} \cdot 100\%$$

Non-linear amplifier characteristics (4)

When the input signal contains 2 or more frequency components, amplifier non-linearity results in:

Inter-Modulation Distortion (IMD) $\Rightarrow$ the frequency components modulate each other.

For a two-frequency input signal $v_{in}(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$ the output signal will contain sum and differences (of multiples) of the input frequencies:

$$K_2 v_{in}^2(t) = K_2 (A_1 \sin \omega_1 t + A_2 \sin \omega_2 t)^2$$

$$= K_2 (A_1^2 \sin^2 \omega_1 t + 2 A_1 A_2 \sin \omega_1 t \sin \omega_2 t + A_2^2 \sin^2 \omega_2 t)$$

$DC,\ 2\omega_1$ $\qquad$ $\omega_1 - \omega_2,\ \omega_1 + \omega_2$ $\qquad$ $DC,\ 2\omega_2$

Non-linear amplifier characteristics (5)

$$K_3 v_{in}^3(t) = K_3(A_1 \sin\omega_1 t + A_2 \sin\omega_2 t)^3$$

$$= K_3(A_1^3 \sin^3\omega_1 t + 3A_1^2 A_2 \sin^2\omega_1 t \sin\omega_2 t$$

$$+ 3A_1 A_2^2 \sin\omega_1 t \sin^2\omega_2 t + A_2^3 \sin^3\omega_2 t)$$

$\omega_1,\ 3\omega_1$ $\quad$ $2\omega_2 - \omega_1$, $2\omega_2 + \omega_1$, ω_1 $\quad$ $2\omega_1 - \omega_2$, $2\omega_1 + \omega_2$, ω_2 $\quad$ $\omega_2,\ 3\omega_2$

Non-linear amplification results in:

- sum and difference frequencies of multiples of the input frequency components,
- components are amplitude modulated by each other,
- components are phase/frequency modulated by each other.

Non-linear amplifier characteristics (6)

RF component's non-linearity is often characterized by the ratio of the desired component and the 3rd order IMD component:

$$R_{IMD} = \frac{K_1 A}{\frac{3K_3A^3}{4}} = \frac{4}{3}\left(\frac{K_1}{K_3 A^2}\right)$$

With input signal: $v_{in}(t) = A\sin\omega_1 t + A\sin\omega_2 t$, the power of the intermodulation signal components $2\omega_1 \pm \omega_2$ and $2\omega_2 \pm \omega_1$ increase with A^6, whereas the power of the desired components increases with A^2. The input signal power A^2 at the virtual intersection point where the output power of both components becomes equal is called the 3rd order intercept point: $R_{IMD} = 1$.

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Non-linear amplifier output characteristics (7)

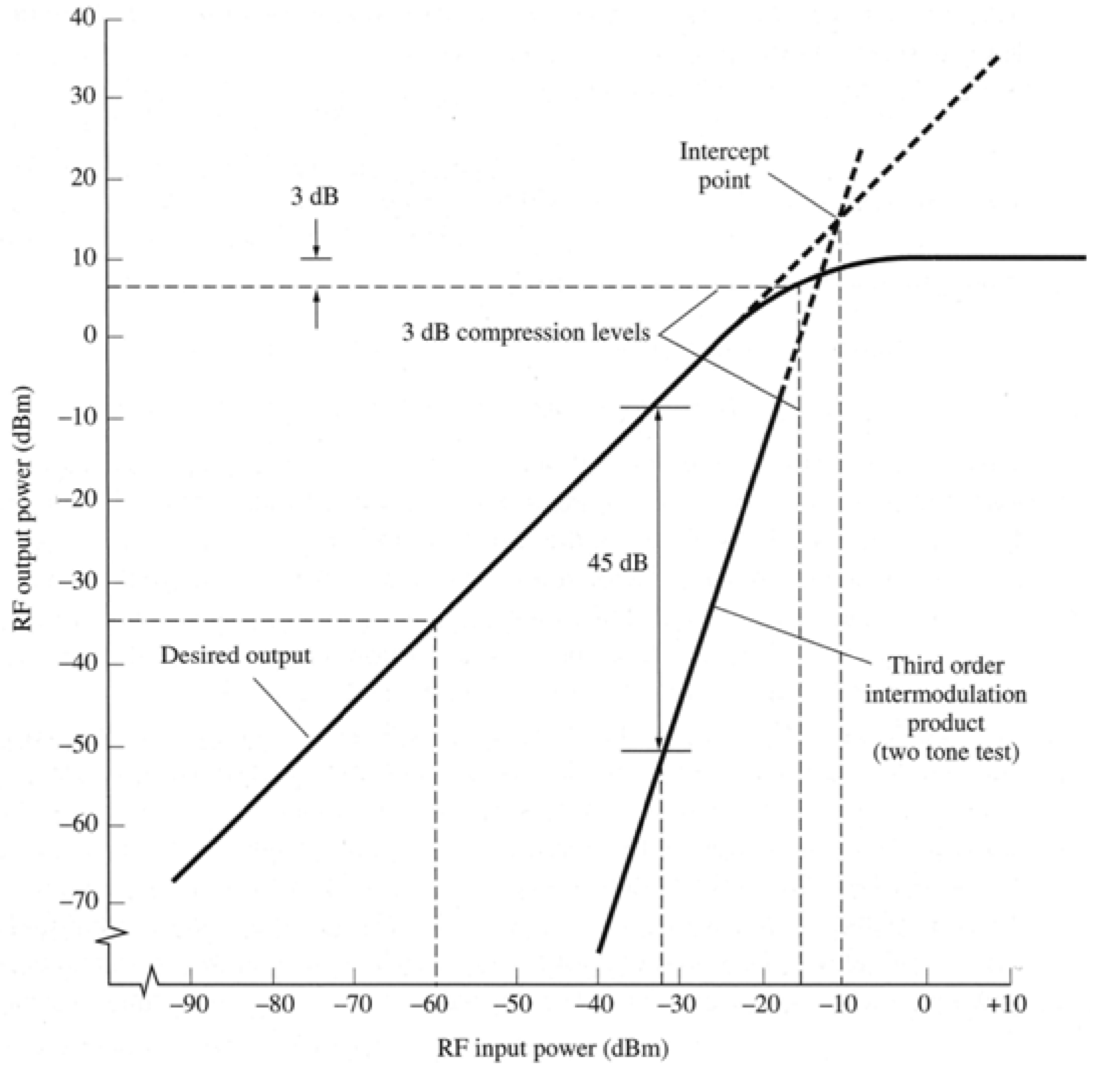


Figure 4–6 Amplifier output characteristics.

Classes of power amplifiers

Amplifiers are distinguished in various classes related to their basic setup. Each have their own power efficiency characteristics.

Class A: transistor biased in linear region, for linear operation, due to continuous current at least 50% of power is turned into heat.

Class B: push-pull setup, separate transistors for positive and negative signal parts, distortion around the cross-over point.

Class C: limiting amplifier, amplitude variations are removed, only useful when the information is in the zero-crossing instants: phase and frequency modulation, power efficient, also produces odd order harmonics of the signal (so filtering is required).

Class D: pulse-width modulation (currently popular in audio power amplifiers), very power efficient.

Ideal limiter

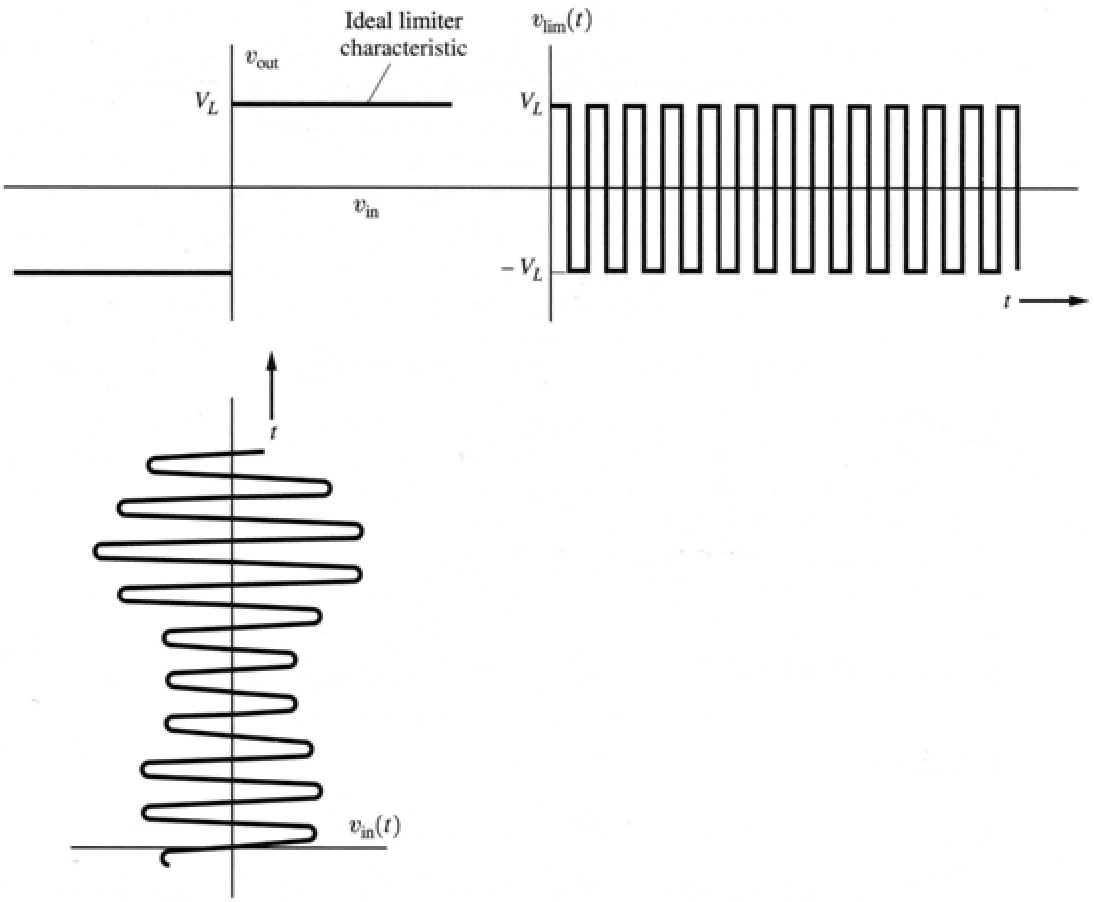


Figure 4–7 Ideal limiter characteristic with illustrative input and unfiltered output waveforms.

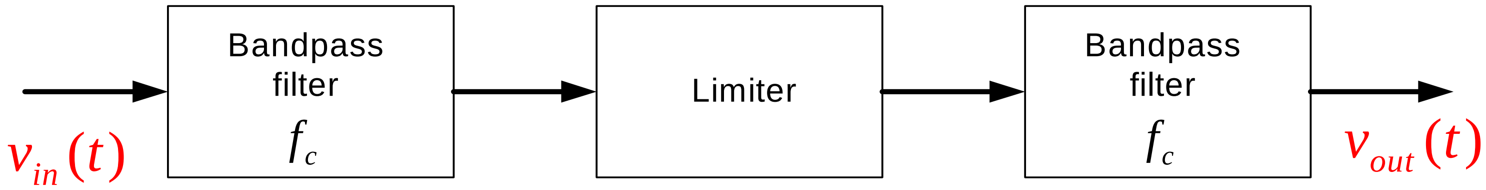
Ideal limiter: a comparator with zero reference:

- amplitude variations are removed
- phase information is preserved

but

- the output signal contains odd order harmonics of the input signal

Bandpass limiter



In RF-processing often bandpass limiters are applied.

- the first BPF removes all out-of-band noise which will otherwise disturb the limiting process,
- the second BPF will remove the odd harmonics of the input signal introduced by the limiter:

$$v_{in}(t) = R(t)\cos[\omega_c t + \theta(t)]$$

$$v_{out}(t) = KV_L \cos[\omega_c t + \theta(t)]$$

Ideal mixer

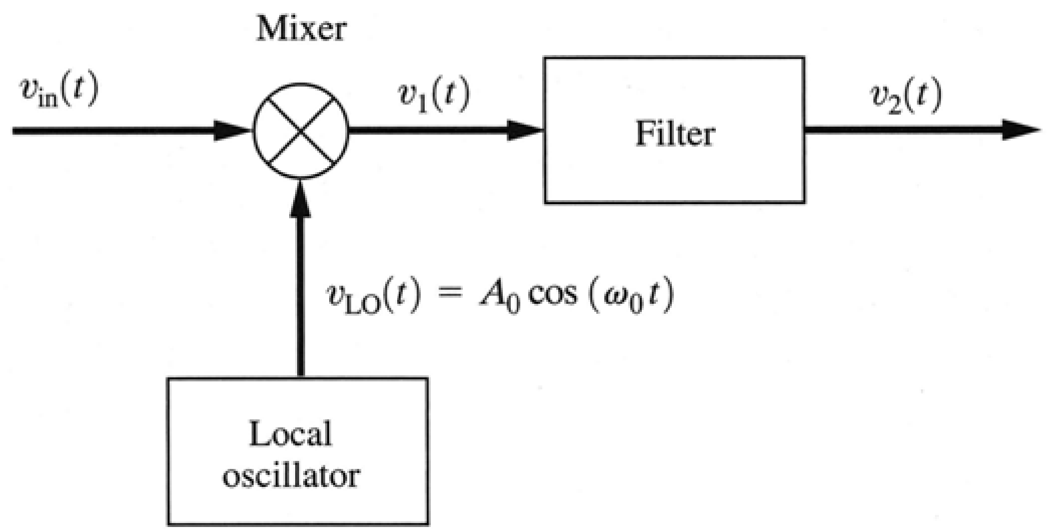


Figure 4–8 Mixer followed by a filter for either up or down conversion.

Ideal mixers:

- operate as a mathematical multiplier of two analog input signals,
- are used to:
  1. obtain a frequency transformation of a signal,
  2. or to modulate a signal.

Mixer as frequency converter (up or down)

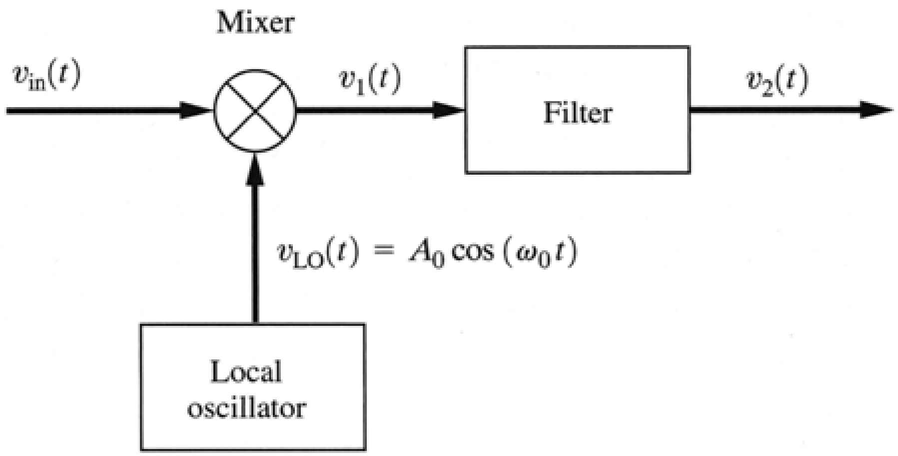


Figure 4–8 Mixer followed by a filter for either up or down conversion.

Let:

$$v_{in}(t) = R(t)\cos[\omega_c t + \theta(t)]$$

$$v_{LO}(t) = A_0 \cos \omega_0 t$$

$$\omega = 2\pi f$$

$$v_1(t) = v_{in}(t) v_{LO}(t) = A_0 R(t) \cos\omega_0 t \cos[\omega_c t + \theta(t)]$$

$$= \frac{A_0 R(t)}{2}\{\cos[(\omega_0 - \omega_c)t - \theta(t)] + \cos[(\omega_0 + \omega_c)t + \theta(t)]\}$$

$$v_2(t) = \begin{cases} LPF\{v_1(t)\} = \dfrac{A_0 R(t)}{2}\cos[(\omega_0 - \omega_c)t - \theta(t)] & \Rightarrow \text{down-conversion} \\ BPF\{v_1(t)\} = \dfrac{A_0 R(t)}{2}\cos[(\omega_0 + \omega_c)t + \theta(t)] & \Rightarrow \text{up-conversion} \end{cases}$$

TU Delft

Frequency conversion using a non-linearity

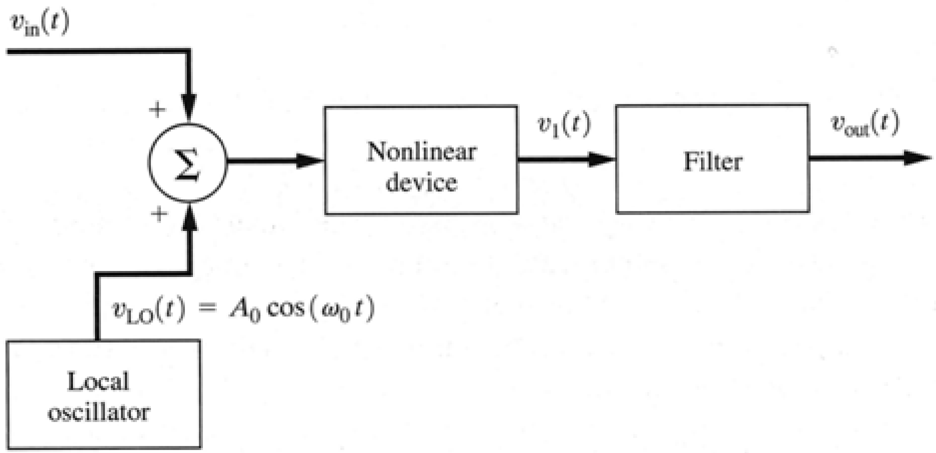


Figure 4–9 Nonlinear device used as a mixer.

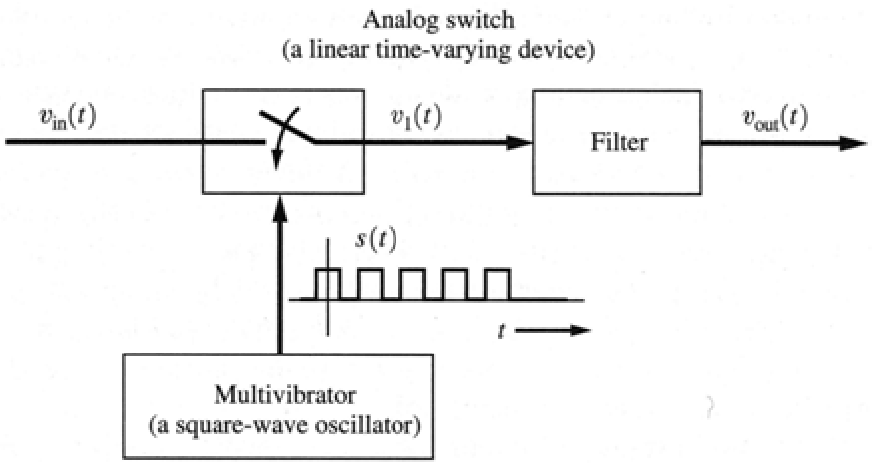


Figure 4–10 Linear time-varying device used as a mixer.

In practice a mixer does not operate as an ideal multiplier. The following devices are used:

- a continuously variable transconductance device, e.g. a dual gate FET,
- a non-linear device
- a linear device with a time-varying discrete gain (e.g. a switch)

Frequency conversion using a non-linearity

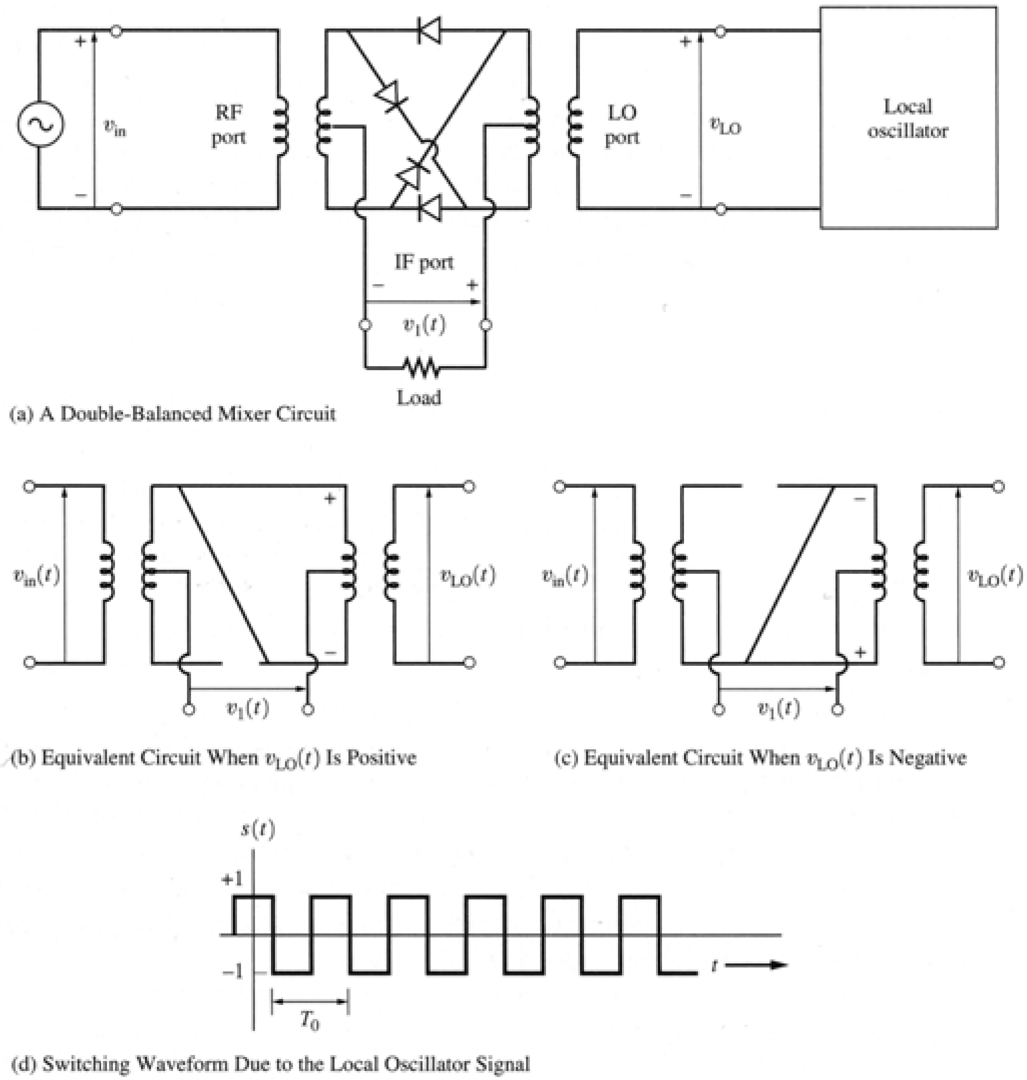
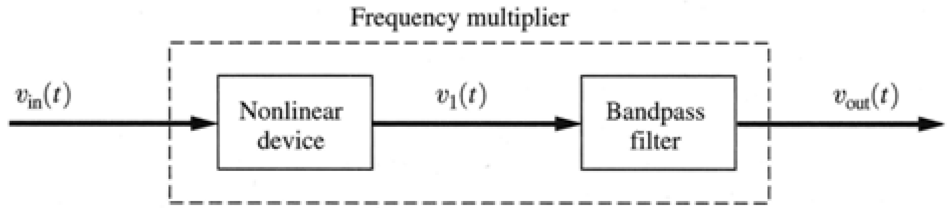
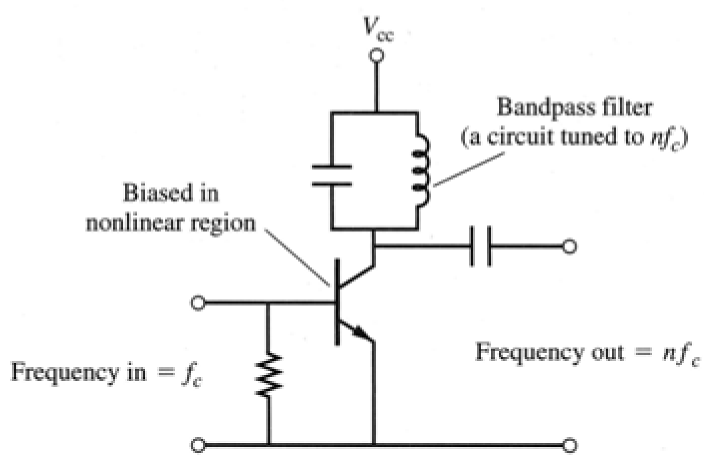


Figure 4–11 Analysis of a double-balanced mixer circuit.

Frequency multiplier using a non-linearity



(a) Block Diagram of a Frequency Multiplier



(b) Circuit Diagram of a Frequency Multiplier

Figure 4–12 Frequency multiplier.

Let the LC-filter be tuned to $3f_c$.

The component generated at this frequency will be amplified due to the high impedance of the LC-filter at this frequency.

Other generated components including the original component at f_c, are attenuated.

Let: $v_{in}(t) = A_c \cos \omega_c t$

$$Kv_{in}^3(t) = KA_0^3 \sin^3 \omega_c t = \frac{KA_0^2}{4}\left(3\sin \omega_c t - \sin 3\omega_c t\right)$$