Telecommunicatie B (EE2T21)

Lecture 9 overview:

Review of bandpass signal description

Linear modulation techniques for analog signals

*** Amplitude modulation (AM)**

*** Double sideband modulation (DSB)**

*** Single sideband modulation (SSB)**

EE2T21 Telecommunicatie B

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Colleges en Instructies Telecommunicatie B

Colleges:

Maandag	25-4, 2-5, 9-5	5e+6e uur, EWI-CZ Chip
	30-5, 6-6	
Dinsdag	10-5	7e+8e uur, EWI-CZ Pi

Instructies:

Dinsdag	17-5	5e+6e uur, EWI-CZ Boole
Dinsdag	31-5	7e+8e uur, EWI-CZ Pi
Maandag	13-6	5e+6e uur, EWI-CZ Chip

Modulation (1)

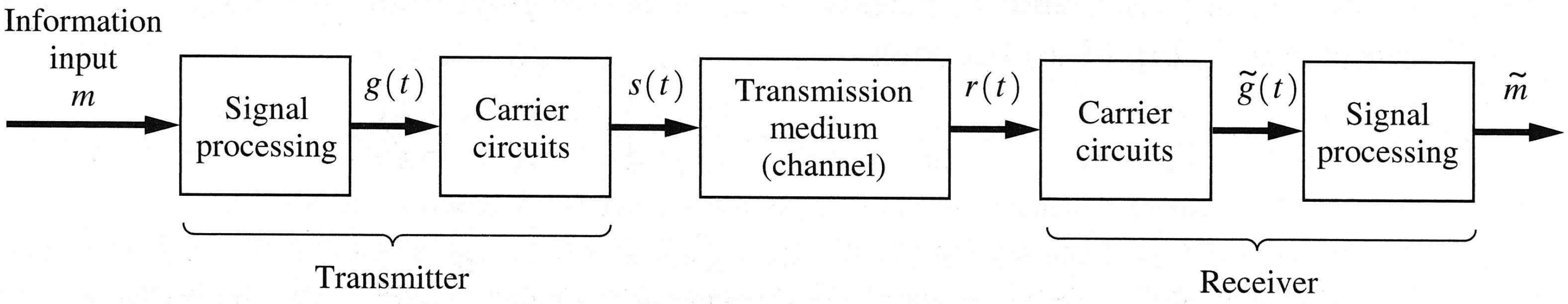


Figure 4–1 Communication system.

What is modulation?

Modulation is the method to adapt one or more carrier signal parameters according to an information signal, in order to make it suitable for transmission over the available channel.

Bandpass Signals

Every physical signal is real!

Mathematical description of bandpass signals:

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{R(t)e^{j[\omega_c t+\theta(t)]}\}$$
$$= \mathrm{Re}\{R(t)\left(\cos[\omega_c t+\theta(t)] + j\sin[\omega_c t+\theta(t)]\right)\}$$
$$= R(t)\cos[\omega_c t+\theta(t)] = x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

Complex envelope or complex equivalent baseband signal:

$$g(t) = f(m(t)) = x(t) + jy(t) \quad = \quad R(t)e^{j\theta(t)}$$

Modulation function

In-phase and Quadrature-phase component

Amplitude: AM

Phase: PM, FM

Bandpass signal characteristics (1)

Amplitude spectrum:

$$\begin{aligned} s(t) &= R(t)\cos[\omega_c t + \theta(t)] = x(t)\cos\omega_c t - y(t)\sin\omega_c t \\ &= \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \tfrac{1}{2}[g(t)e^{j\omega_c t} + g^*(t)e^{-j\omega_c t}] \end{aligned}$$

$$\begin{aligned} S(f) &= \mathfrak{F}\{s(t)\} = \mathfrak{F}\{\tfrac{1}{2}g(t)e^{j\omega_c t} + \tfrac{1}{2}g^*(t)e^{-j\omega_c t}\} \\ &= \tfrac{1}{2}G(f - f_c) + \tfrac{1}{2}G^*(-f - f_c)] \end{aligned}$$

where we used: $G(f) = \mathfrak{F}\{g(t)\}$

$$G^*(-f) = \mathfrak{F}\{g^*(t)\}$$

Bandpass signal characteristics (2)

Power Spectral Density (PSD):

$$P_s(f) = \tfrac{1}{4} P_g(f - f_c) + \tfrac{1}{4} P_g(-f - f_c) = |S(f)|^2 \quad \text{[W/Hz]}$$

where we used: $P_g(f) = PSD\{g(t)\} = |G(f)|^2$ Proof on p. 237

Normalized average signal power (normalized to a 1Ω load):

$$P_s = \langle s^2(t) \rangle = \tfrac{1}{2} \langle |g(t)|^2 \rangle = \tfrac{1}{2} \langle R^2(t) \rangle$$

due to sinewave carrier

Peak Envelope Power (PEP): the average power when $|g(t)|$ is maximum

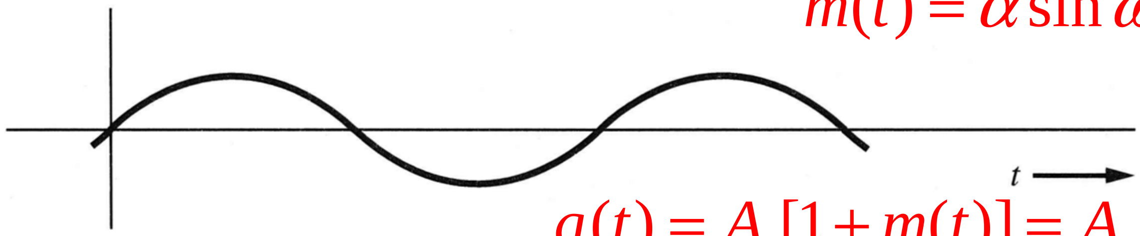
Normalized PEP: $P_{s\_PEP} = \tfrac{1}{2}[\max\{|g(t)|\}]^2$

Modulation techniques (1)

TABLE 4–1 COMPLEX ENVELOPE FUNCTIONS FOR VARIOUS TYPES OF MODULATION[a]

Type of Modulation	Mapping Functions $g(m)$	Corresponding Quadrature Modulation $x(t)$	$y(t)$
AM	$A_c[1 + m(t)]$	$A_c[1 + m(t)]$	0
DSB-SC	$A_c m(t)$	$A_c m(t)$	0
PM	$A_c e^{jD_p m(t)}$	$A_c \cos[D_p m(t)]$	$A_c \sin[D_p m(t)]$
FM	$A_c e^{jD_f \int_{-\infty}^{t} m(\sigma)\, d\sigma}$	$A_c \cos\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$	$A_c \sin\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$
SSB-AM-SC[b]	$A_c[m(t) \pm j\hat{m}(t)]$	$A_c m(t)$	$\pm A_c \hat{m}(t)$
SSB-PM[b]	$A_c e^{jD_p[m(t) \pm j\hat{m}(t)]}$	$A_c e^{\mp D_p \hat{m}(t)} \cos[D_p m(t)]$	$A_c e^{\mp D_p \hat{m}(t)} \sin[D_p m(t)]$
SSB-FM[b]	$A_c e^{jD_f \int_{-\infty}^{t}[m(\sigma) \pm j\hat{m}(\sigma)]d\sigma}$	$A_c e^{\mp D_f \int_{-\infty}^{t} \hat{m}(\sigma)d\sigma} \cos\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$	$A_c e^{\mp D_f \int_{-\infty}^{t} \hat{m}(\sigma)d\sigma} \sin\left[D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma\right]$
SSB-EV[b]	$A_c e^{\{\ln[1+m(t)] \pm j\hat{\ln}\vert 1+m(t)\vert\}}$	$A_c[1 + m(t)] \cos\{\hat{\ln}[1 + m(t)]\}$	$\pm A_c[1 + m(t)] \sin\{\hat{\ln}[1 + m(t)]\}$
SSB-SQ[b]	$A_c e^{(1/2)\{\ln[1+m(t)] \pm j\hat{\ln}\vert 1+(t)\vert\}}$	$A_c\sqrt{1 + m(t)} \cos\{\frac{1}{2} \hat{\ln}[1 + m(t)]\}$	$\pm A_c\sqrt{1 + m(t)} \sin\{\frac{1}{2} \hat{\ln}[1 + m(t)]\}$
QM	$A_c[m_1(t) + jm_2(t)]$	$A_c m_1(t)$	$A_c m_2(t)$

Amplitude Modulation (1)

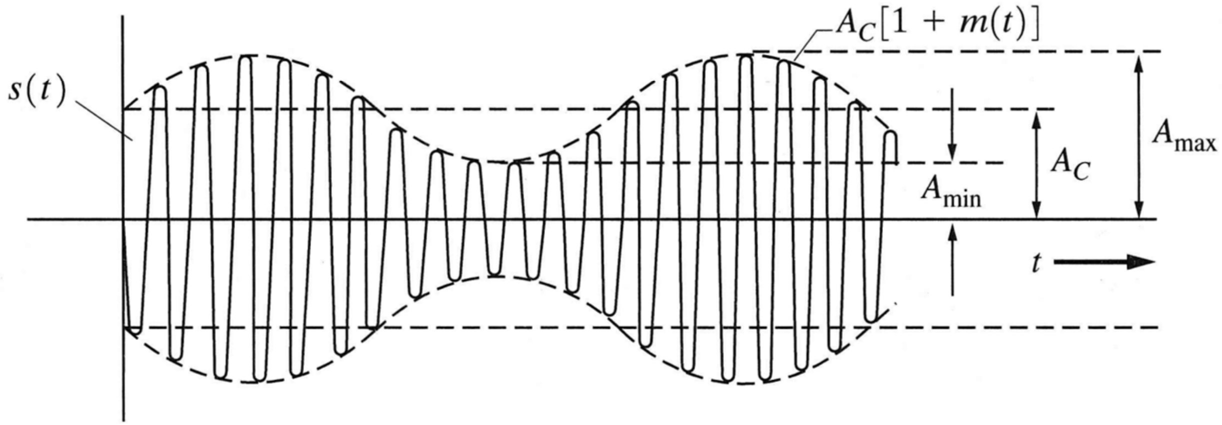


$$m(t) = \alpha \sin \omega_m t$$

$$g(t) = A_c[1 + m(t)] = A_c[1 + \alpha \sin \omega_m t]$$

(a) Sinusoidal Modulating Wave

$$R(t) = |g(t)| = |A_c[1 + m(t)]|$$



(b) Resulting AM Signal

$$s(t) = A_c[1 + \alpha \sin \omega_m t] \cos \omega_c t$$

Figure 5–1 AM signal waveform.

Amplitude Modulation (2)

AM-signal: $s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \underbrace{A_c[1+m(t)]}\cos\omega_c t$

Tab. 4.1
Fig. 5.1

$$g(t) = A_c[1+m(t)] = x(t)$$

Modulation depth [%] $\triangleq \dfrac{A_{\max} - A_{\min}}{2A_c} \cdot 100\%$

$$= \frac{A_c[1+m_{\max}] - A_c[1+m_{\min}]}{2A_c} \cdot 100\% \overset{|m(t)|\leq 1}{=} \frac{m_{\max} - m_{\min}}{2} \cdot 100\%$$

In an AM-transmitter, the modulation depth usually can be adapted.
This requires a small change in $g(t) = f\{m(t)\}$:

$$g(t) = A_c[1+\mu \mathrm{m(t)}] \qquad \mu = \text{modulation index} \geq 0$$

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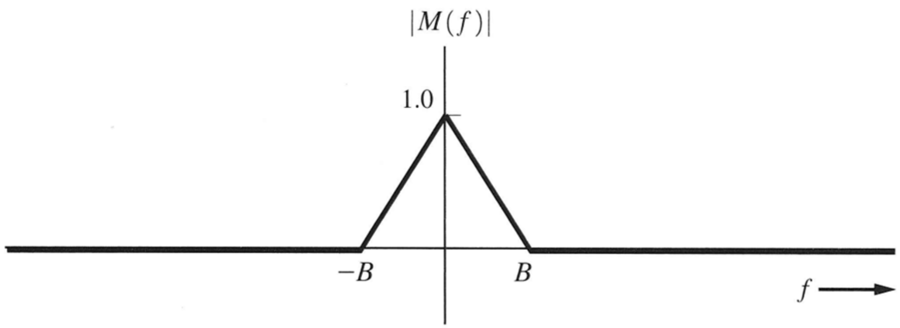
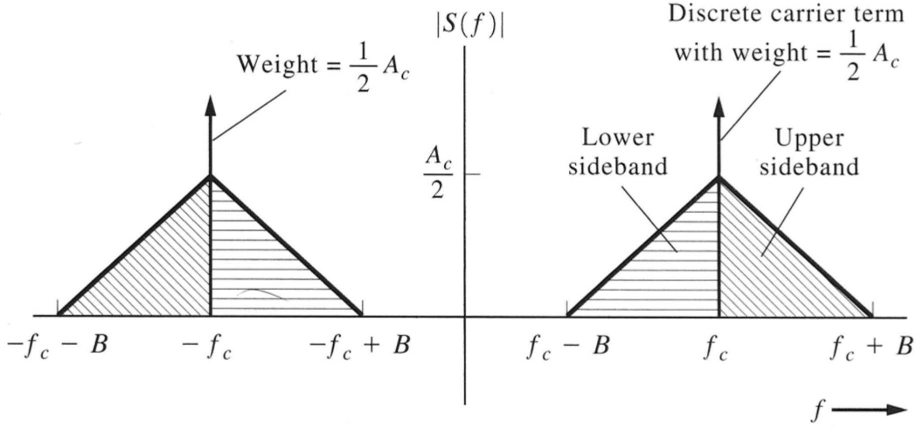
Amplitude Modulation (3)



(a) Magnitude Spectrum of Modulation



(b) Magnitude Spectrum of AM Signal

Figure 4–2 Spectrum of AM signal.

$$g(t) = A_c[1 + m(t)]$$

Transmissionbandwith: $B_T = 2B$

Spectrum $g(t)$: $G(f) = \mathfrak{F}\{g(t)\} = A_c\delta(f) + A_c M(f) \rightarrow M(f) = \mathfrak{F}\{m(t)\}$

Spectrum $s(t)$: $S(f) = \frac{1}{2}A_c[\delta(f - f_c) + M(f - f_c) + \delta(f + f_c) + M(f + f_c)]$

where we used that: $\mathfrak{F}\{m(t)\} = M^*(-f) = M(f)$ since $m(t)$ is real.

Amplitude Modulation (4)

Normalized average signal power:

$$P_s = \langle s^2(t) \rangle = \tfrac{1}{2} \langle |g(t)|^2 \rangle = \tfrac{1}{2} \langle R^2(t) \rangle = \tfrac{1}{2} A_c^2 \langle [1+\mu m(t)]^2 \rangle$$

$$= \tfrac{1}{2} A_c^2 \langle [1 + 2\mu m(t) + \mu^2 m^2(t)] \rangle \overset{\langle m(t) \rangle = 0}{=} \tfrac{1}{2} A_c^2 \langle [1 + \mu^2 m^2(t)] \rangle$$

$$= \underbrace{\tfrac{1}{2} A_c^2}_{P_{carrier}} + \underbrace{\tfrac{1}{2} A_c^2 \mu^2 \overline{m^2(t)}}_{P_{sidebands}}$$

Peak Envelope Power (PEP):

$$P_{PEP} = \tfrac{1}{2} [\max\{|g(t)|\}]^2 = \tfrac{1}{2} A_c^2 [1 + \max\{\mu m(t)\}]^2$$

The Peak Envelope Power is the average transmitted power at maximum information signal level.

Amplitude Modulation (5)

Which part of the signal power does effectively contribute to the transmission of information?

Modulation efficiency: $E \triangleq \dfrac{P_{side\ bands}}{P_{carrier} + P_{side\ bands}} \cdot 100\%$

$$= \frac{\frac{1}{2} A_c^2 \mu^2 < m^2(t) >}{\frac{1}{2} A_c^2 [1 + \mu^2 < m^2(t) >]} \cdot 100\% = \frac{\mu^2 < m^2(t) >}{1 + \mu^2 < m^2(t) >} \cdot 100\%$$

1. Let us take μ = 1 and $m(t) = \sin \omega_m t \;\Rightarrow\; < m^2(t) >= \frac{1}{2}$

$\Rightarrow E$ = 33.3% and modulation depth = 100%
$\Rightarrow$ 66.7% of the power is transmitted as carrier power,

2. Let us take μ = 1 and $m(t) = \pm 1$ (square wave) $\Rightarrow\; < m^2(t) >= 1$

$\Rightarrow E$ = 50%, the best we can achieve.

In AM, at least 50% of the power is transmitted as carrier power.

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Example: Amplitude Modulation

At the transmitter output $A_c = 500\text{V}$ and the antenna impedance is $Z_{ant} = 50\Omega$. Further signal details: $\overline{m^2} = \frac{1}{2}$, $m_{\max} = 1$, $\mu = \frac{1}{2}$.

Determine the average transmit power, modulation efficiency and PEP.

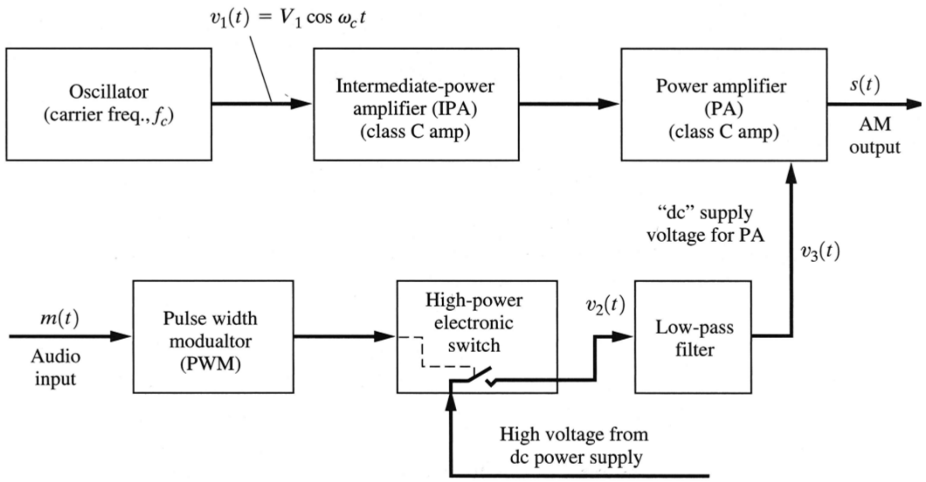
$$P_{av} = \frac{A_c^2}{2Z_{ant}}(1+\mu^2\overline{m^2}) = \frac{500^2}{2\cdot 50}(1+(\tfrac{1}{2})^2\cdot\tfrac{1}{2})$$

$$= 2500\cdot 1.125 = 2812.5\text{W} \equiv 34.5\text{dBW}$$

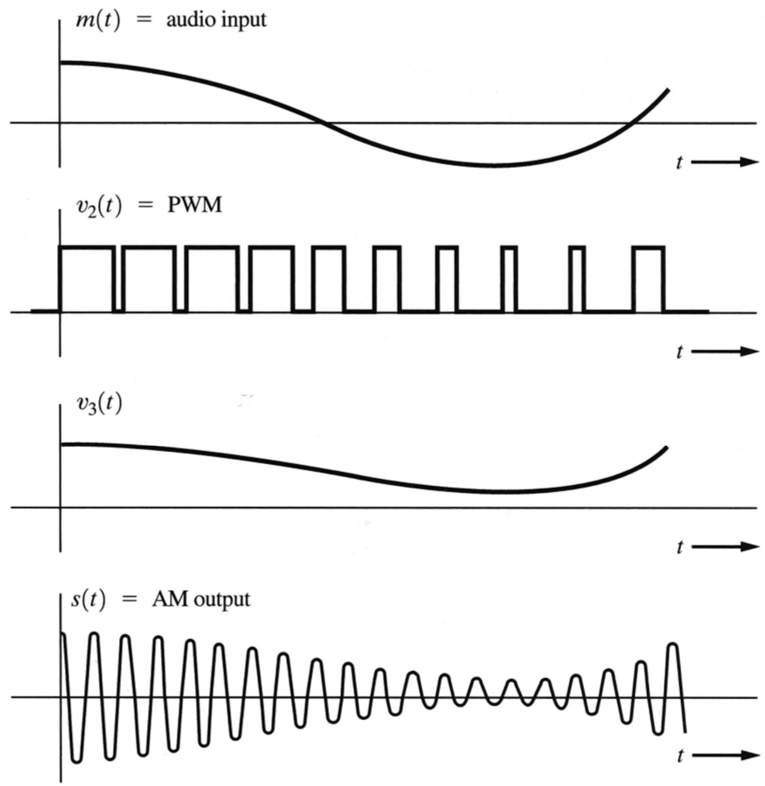
$$E = \frac{\mu^2 < m^2(t) >}{1+\mu^2 < m^2(t) >}\cdot 100\% = \frac{0.125}{1+0.125}\cdot 100\% = 11.1\%$$

$$P_{PEP} = \frac{A_c^2}{2Z_{ant}}(1+\mu m_{\max})^2 = \frac{500^2}{2\cdot 50}(1+\tfrac{1}{2})^2 = 2500\cdot(1.5)^2 = 5625\text{W} \equiv 37.5\text{dBW}$$

AM-transmitter using PWM



(a) Block Diagram



(b) Waveforms

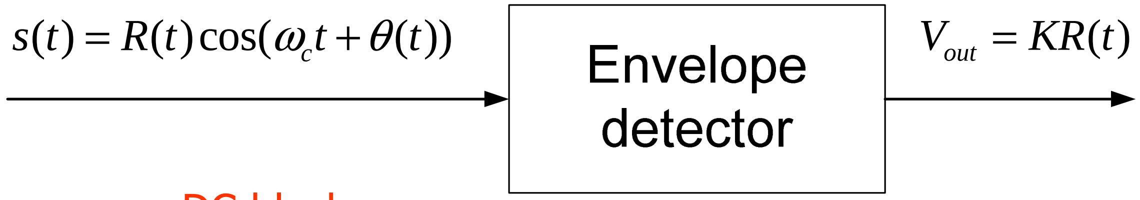
Figure 5–2 Generation of high-power AM by the use of PWM.

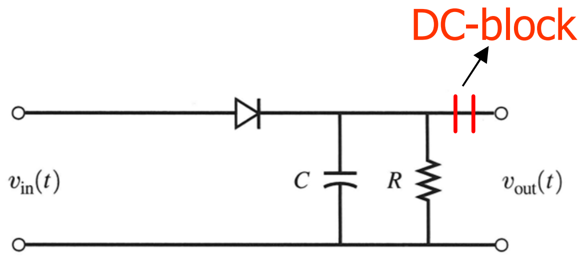
AM-transmission standards

Table 5–1 AM BROADCAST STATION TECHNICAL STANDARDS

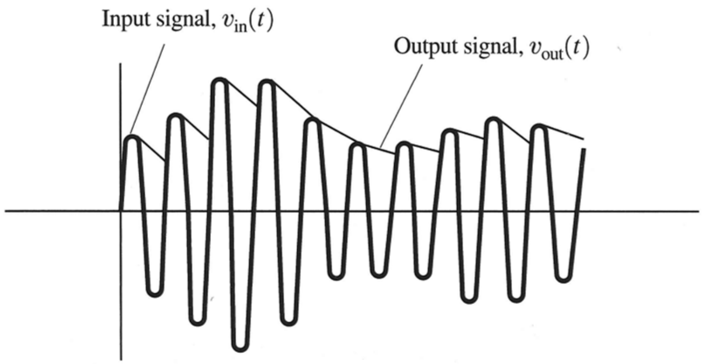
Item	FCC Technical Standard
Assigned frequency, f_c	In 10-kHz increments from 540 to 1,700 kHz
Channel bandwidth	10 kHz
Carrier frequency stability	±20 Hz of the assigned frequency
Clear-channel frequencies (One Class A, 50-kW station) (Nondirectional)	640, 650, 660, 670, 700, 720, 750, 760, 770, 780, 820, 830, 840, 870, 880, 890, 1,020, 1,030, 1,040, 1,070, 1,100, 1,120, 1,160, 1,180, 1,200, and 1,210 kHz
Clear-channel frequencies (Multiple 50-kW stations) (Directional night)	680, 710, 810, 850, 1,000, 1,060, 1,080, 1,090, 1,110, 1,130, 1,140, 1,170, 1,190, 1,500, 1,510, 1,520, and 1,530 kHz
Clear-channel frequencies (For Bahama, Cuba, Canada, or Mexico)	540, 690, 730, 740, 800, 860, 900, 940, 990, 1,010, 1,050, 1,220, 1,540, 1,550, 1,560, 1,570, and 1,580 kHz
Local channel frequencies (1-kW stations)	1,230, 1,240, 1,340, 1,400, 1,450, and 1,490 kHz
Maximum power licensed	50 kW

AM-detection: envelope detector





(a) A Diode Envelope Detector



(b) Waveforms Associated with the Diode Envelope Detector

Figure 4–13 Envelope detector.

$$V_{out} = KR(t) = KA_c[1 + \mu m(t)] = KA_c + KA_c\mu m(t)$$

$$\frac{1}{f_c} << \tau = RC << \frac{1}{B}$$

Note: the rectifying envelope detector is a non-linear component: $R(t) = \sqrt{x^2(t) + y^2(t)}$!

But … , it is cheap!!!

Product detector (1)

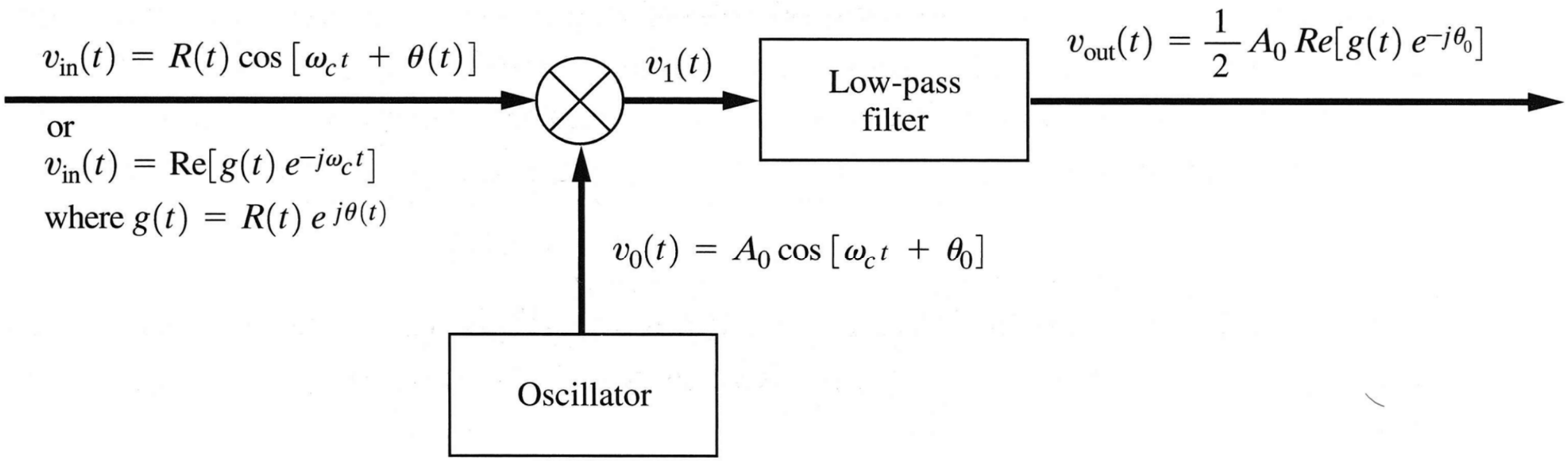


Figure 4–14 Product detector.

$$v_1(t) = v_{in}(t) \cdot v_0(t) = R(t)\cos(\omega_c t + \theta(t)) \cdot A_0 \cos(\omega_c t + \theta_0)$$

$$= \tfrac{1}{2} A_0 R(t)\cos(\theta(t) - \theta_0) + \tfrac{1}{2} A_0 R(t)\cos(2\omega_c t + \theta(t) + \theta_0)$$

removed by LPF

Product detector (2)

The product detector is sensitive to $R(t)$ as well as $\theta(t)$:

AM PM

For $\theta(t) \neq 0$ and $R(t)$ = 1 (PM) we find for $\theta_0 = \frac{\pi}{2}$:

$$v_{out}(t) = \tfrac{1}{2} A_0 A_c \sin\theta(t) \quad \text{(sine phase detector)}$$

and with $|\theta(t)| << 1$:

$$v_{out}(t) \approx \tfrac{1}{2} A_0 A_c \theta(t) \quad \text{(linear phase detector)}$$

For $\theta(t) = 0$ (AM) we find for $\theta_0 = 0$:

$$v_{out}(t) \approx \tfrac{1}{2} A_0 R(t) \quad \text{(linear amplitude detector)}$$

Note: we have to maintain phase synchronization!

Product detector (3)

In-phase and quadrature phase components can be detected by choosing the right values for θ_0 .

$$v_{out}(t) = \begin{cases} \frac{1}{2}A_0 R(t)\cos\theta(t) = \frac{1}{2}A_0 x(t) & \text{for } \theta_0 = 0 \\ \frac{1}{2}A_0 R(t)\sin\theta(t) = \frac{1}{2}A_0 y(t) & \text{for } \theta_0 = \frac{\pi}{2} \end{cases}$$

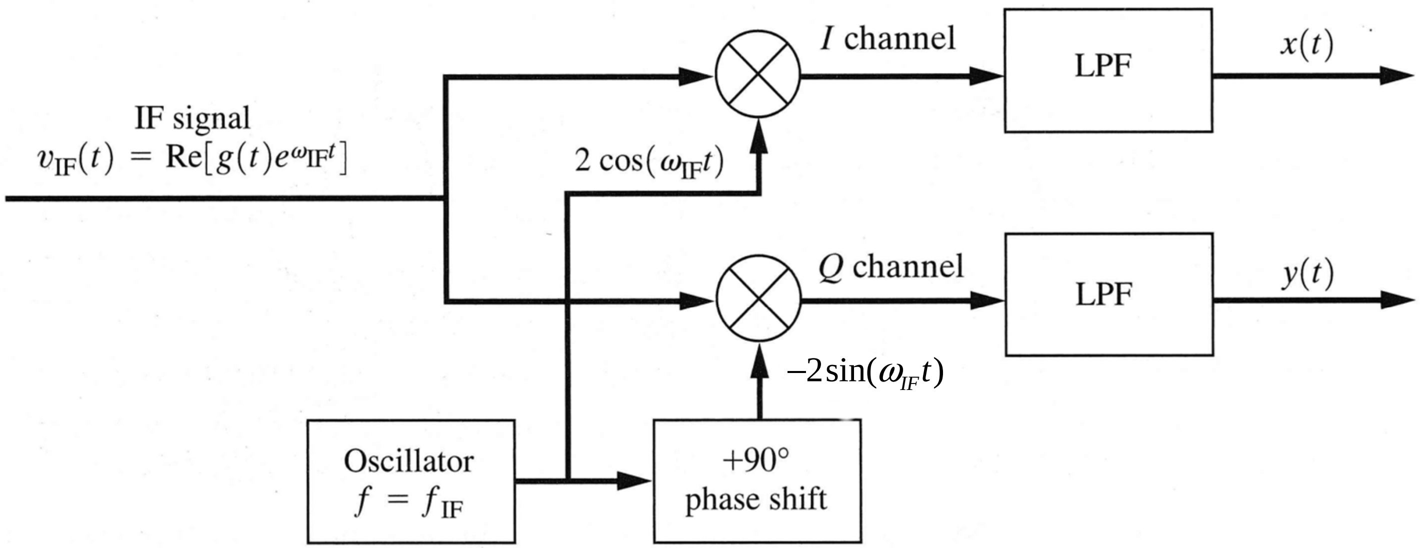


Figure 4–31 IQ (in-phase and quadrature-phase) detector.

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Over-modulation in AM

In case of over-modulation, the modulation depth is > 100%
or $\mu m(t) < -1$.

Is this allowed?

In case of over-modulation, what happens with the output signal of:
* an envelope detector?
* a product detector?

Double Sideband - Suppressed Carrier (DSB-SC) (1)

DSB-signal: $g(t) = A_c m(t) = x(t) \quad \Rightarrow$

Tab. 4.1

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{A_c m(t)e^{j\omega_c t}\}$$
$$= A_c m(t)\cos\omega_c t$$

SC = suppressed-carrier $\Rightarrow$ no carrier is added and also $\langle m(t)\rangle = 0$, i.e. no DC-component in the information signal $m(t)$.

No power is "lost" in a carrier $\Rightarrow$ the modulation efficiency $E \triangleq 100\%$.
This is an advantage compared to AM.

DSB-SC (2)

Normalized average signal power: $P_s = \langle s^2(t) \rangle = \frac{1}{2} A_c^2 \langle m^2(t) \rangle$

Peak Envelope Power (PEP):

$$P_{PEP} = \tfrac{1}{2}[\max\{|g(t)|\}]^2 = \tfrac{1}{2} A_c^2 [\max\{m(t)\}]^2$$

Let $|\max\{m(t)\}| = 1$:

$$\left.\begin{aligned} P_{PEP\_AM} &= \tfrac{1}{2} A_c^2 [1 + \max\{m(t)\}]^2 = 2A_c^2 \\ P_{PEP\_DSB} &= \tfrac{1}{2} A_c^2 [\max\{m(t)\}]^2 = \tfrac{1}{2} A_c^2 \end{aligned}\right\}$$

$\Rightarrow P_{PEP\_AM}$ is 6 dB higher than P_{PEP_DSB}

modulation depth 100%

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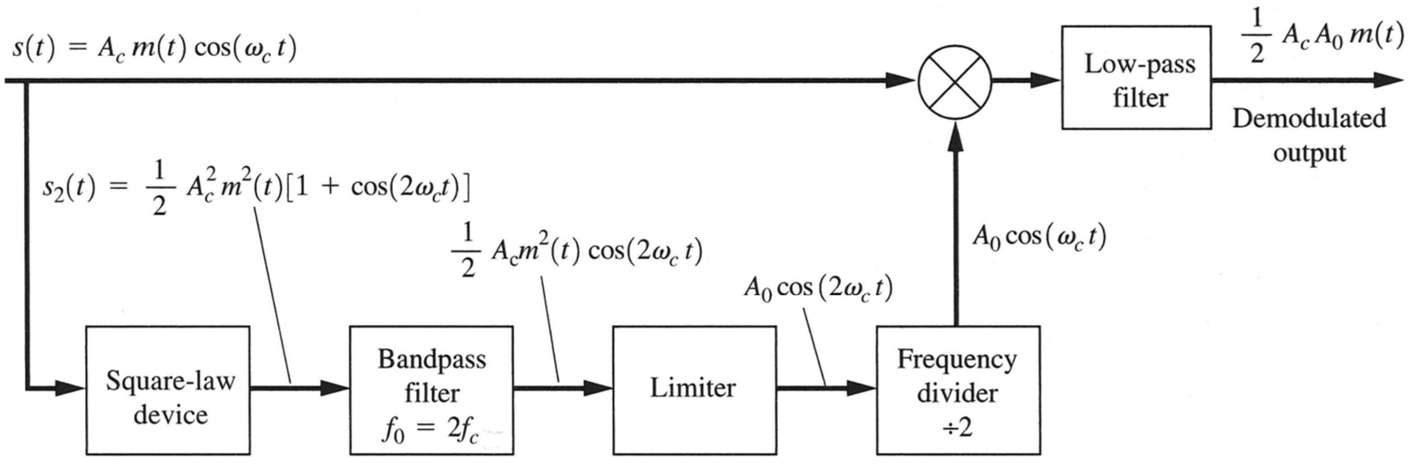
DSB-SC (3)

Since no carrier component is available, envelope detection is not possible. Product detection has to be used which requires carrier recovery with synchronized phase.

1. Squaring-loop:

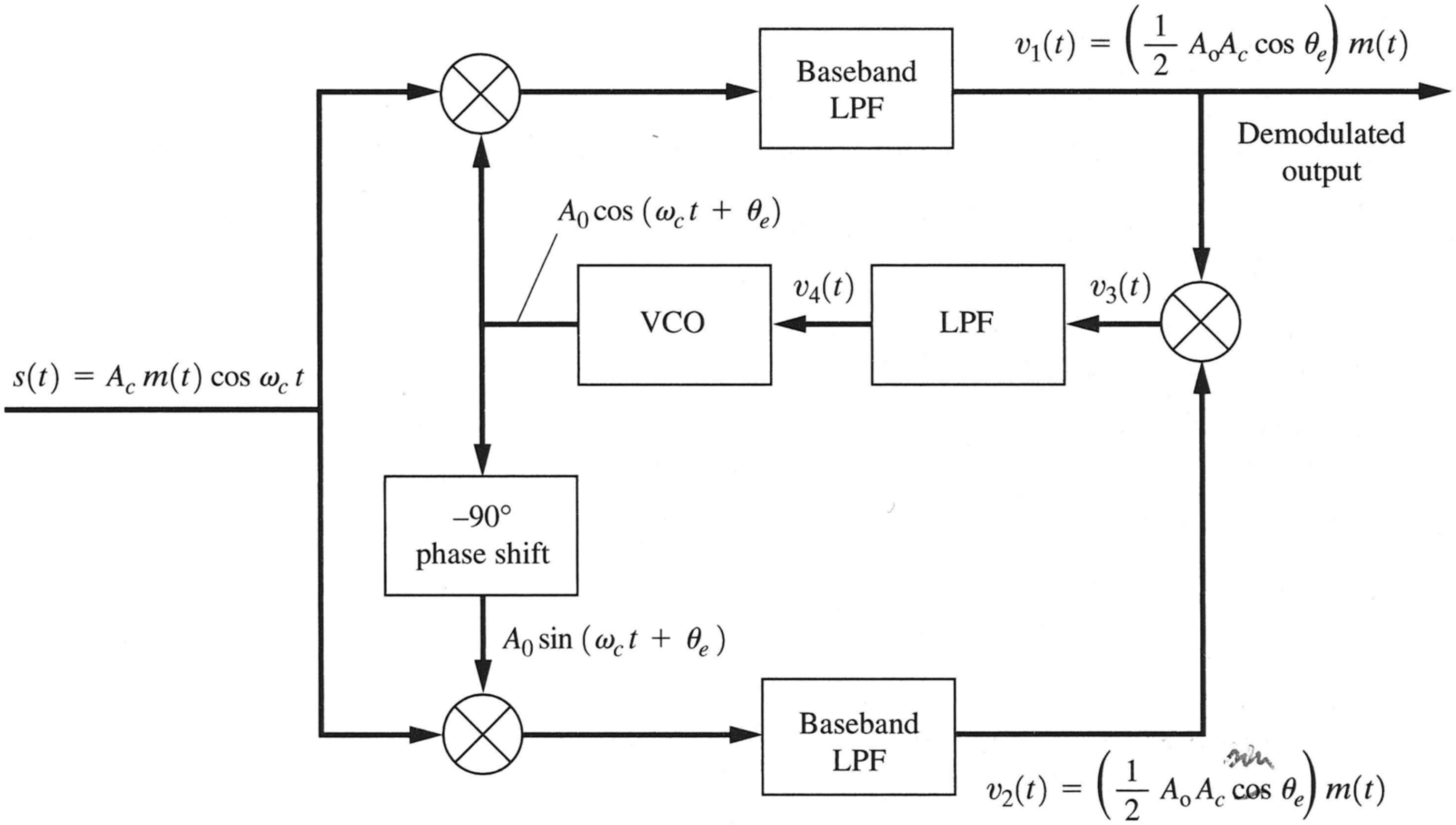


(b) Squaring Loop

Figure 5–3 Carrier recovery loops for DSB-SC signals.

DSB-SC (4)

2. Costas-loop



(a) Costas Phase-Locked Loop

Asymmetric linear modulation techniques

1. Single sideband (SSB) modulation (enkelzijband (EZB) modulatie)
   - USSB : Upper Single Sideband
   - LSSB : Lower Single Sideband

2. Vestigial Sideband (VSB) modulation (restzijband modulatie)
   - UVSB : Upper Vestigial Sideband
   - LVSB : Lower Vestigial Sideband

3. Independent Sideband (ISB) Modulation (e.g. stereo AM)

Single Sideband (SSB) modulation (1)

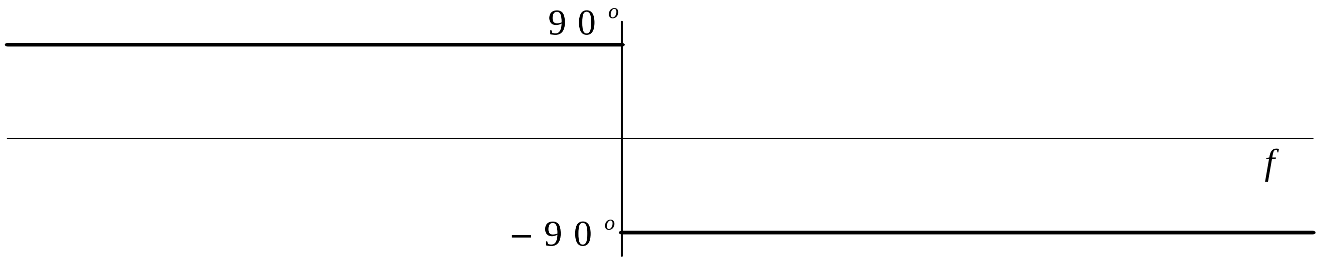
Complex envelope SSB-signal:

$$g(t) = A_c[m(t) \mp j\hat{m}(t)] \quad \begin{matrix} -\text{: USSB} \\ +\text{: LSSB} \end{matrix}$$

Hilbert transform: $\hat{m}(t) = \mathfrak{H}\{m(t)\} = m(t) * h(t)$ with $h(t) = \dfrac{1}{\pi t}$

with:

$$H(f) = \mathfrak{F}\{h(t)\} = \begin{cases} -j & f > 0 \\ j & f < 0 \end{cases} \Rightarrow \; -90^0 \text{ phase shifter}$$



Phase response Hilbert transform

Single Sideband (SSB) modulation (2)

Realization of the Hilbert transform is difficult over large frequency bandwidths and around $f = 0$.

For harmonic signals:

$$\mathfrak{H}\{\cos\omega_m t\} = \mathfrak{H}\{\tfrac{1}{2}e^{j\omega_m t} + \tfrac{1}{2}e^{-j\omega_m t}\} = \tfrac{1}{2}\{-je^{j\omega_m t} + je^{-j\omega_m t}\}$$

$$= \tfrac{1}{2j}\{e^{j\omega_m t} - e^{-j\omega_m t}\} = \sin\omega_m t$$

$$\mathfrak{H}\{\sin\omega_m t\} = -\cos\omega_m t$$

Single Sideband (SSB) modulation (3)

SSB-signal: Tab. 4.2

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = A_c\{m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t\} \quad \begin{matrix} -: USSB \\ +: LSSB \end{matrix}$$

with: $g(t) = A_c[m(t) \mp j\hat{m}(t)] \quad \begin{matrix} -: \mathrm{USSB} \\ +: \mathrm{LSSB} \end{matrix}$

$$= R(t)e^{j\theta(t)}$$

$$R(t) = A_c\sqrt{m^2(t) + \hat{m}^2(t)}$$

$$\theta(t) = \arctan\frac{\hat{m}(t)}{m(t)}$$

SSB involves a combination of AM and PM

Modulation techniques (2)

TABLE 4–1 COMPLEX ENVELOPE FUNCTIONS FOR VARIOUS TYPES OF MODULATION (*cont.*)

Type of Modulation	Corresponding Amplitude and Phase Modulation: $R(t)$	$\theta(t)$	Linearity	Remarks
AM	$A_c\|1+m(t)\|$	$\begin{cases} 0, & m(t) > -1 \\ 180°, & m(t) < -1 \end{cases}$	L[c]	$m(t) > -1$ required for envelope detection
DSB-SC	$A_c\|m(t)\|$	$\begin{cases} 0, & m(t) > 0 \\ 180°, & m(t) < 0 \end{cases}$	L	Coherent detection required
PM	A_c	$D_p m(t)$	NL	D_p is the phase deviation constant (rad/volt)
FM	A_c	$D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma$	NL	D_f is the frequency deviation constant (rad/volt-sec)
SSB-AM-SC[b]	$A_c\sqrt{[m(t)]^2 + [\hat{m}(t)]^2}$	$\tan^{-1}[\pm\hat{m}(t)/m(t)]$	L	Coherent detection required
SSB-PM[b]	$A_c e^{\pm D_p \hat{m}(t)}$	$D_p m(t)$	NL	
SSB-FM[b]	$A_c e^{\pm D_f \int_{-\infty}^{t} \hat{m}(\sigma) d\sigma}$	$D_f \int_{-\infty}^{t} m(\sigma)\, d\sigma$	NL	
SSB-EV[b]	$A_c\|1+m(t)\|$	$\pm \hat{\ln}[1+m(t)]$	NL	$m(t) > -1$ is required so that $\ln(\cdot)$ will have a real value
SSB-SQ[b]	$A_c\sqrt{1+m(t)}$	$\pm\frac{1}{2}\hat{\ln}[1+m(t)]$	NL	$m(t) > -1$ is required so that $\ln(\cdot)$ will have a real value
QM	$A_c\sqrt{m_1^2(t) + m_2^2(t)}$	$\tan^{-1}[m_2(t)/m_1(t)]$	L	Used in NTSC color television; requires coherent detection

[a] $A_c > 0$ is a constant that sets the power level of the signal as evaluated by the use of Eq. (4–17); L, linear; NL, nonlinear; $[\hat{\cdot}]$ is the Hilbert transform (i.e., the $-90°$ phase-shifted version) of $[\cdot]$. (See Sec. 5–5 and Sec. A–7, Appendix A.)

[b] Use upper signs for upper sideband signals and lower signs for lower sideband signals.

[c] In the strict sense, AM signals are not linear, because the carrier term does not satisfy the linearity (superposition) condition.

Single Sideband (SSB) modulation (4)

Proof: $$S(f)=A_c\int_{-\infty}^{\infty}\left\{m(t)\cdot\tfrac{1}{2}[e^{j\omega_c t}+e^{-j\omega_c t}]\mp\hat{m}(t)\cdot\tfrac{1}{2j}[e^{j\omega_c t}-e^{-j\omega_c t}]\right\}e^{-j\omega t}dt$$

$$=\frac{A_c}{2}\int_{-\infty}^{\infty}[m(t)\pm j\hat{m}(t)]e^{-j(\omega-\omega_c)t}dt+\frac{A_c}{2}\int_{-\infty}^{\infty}[m(t)\mp j\hat{m}(t)]e^{-j(\omega+\omega_c)t}dt$$

$$=\frac{A_c}{2}\left[M(f-f_c)\pm j\hat{M}(f-f_c)\right]+\frac{A_c}{2}\left[M(f+f_c)\mp j\hat{M}(f+f_c)\right]$$

Using $\hat{M}(f)=M(f)H(f)=M(f)\cdot\begin{cases}-j & \text{for } f>0\\ j & \text{for } f<0\end{cases}$ we find for USSB:

$$S(f)=\frac{A_c}{2}\begin{bmatrix}2M(f-f_c) & f>f_c\\ 0 & f<f_c\end{bmatrix}+\frac{A_c}{2}\begin{bmatrix}0 & f>-f_c\\ 2M(f+f_c) & f<-f_c\end{bmatrix}$$

$$=A_c\begin{bmatrix}M(f-f_c) & f>f_c\\ 0 & f<f_c\end{bmatrix}+A_c\begin{bmatrix}0 & f>-f_c\\ M(f+f_c) & f<-f_c\end{bmatrix}$$

Single Sideband (SSB) modulation (5)

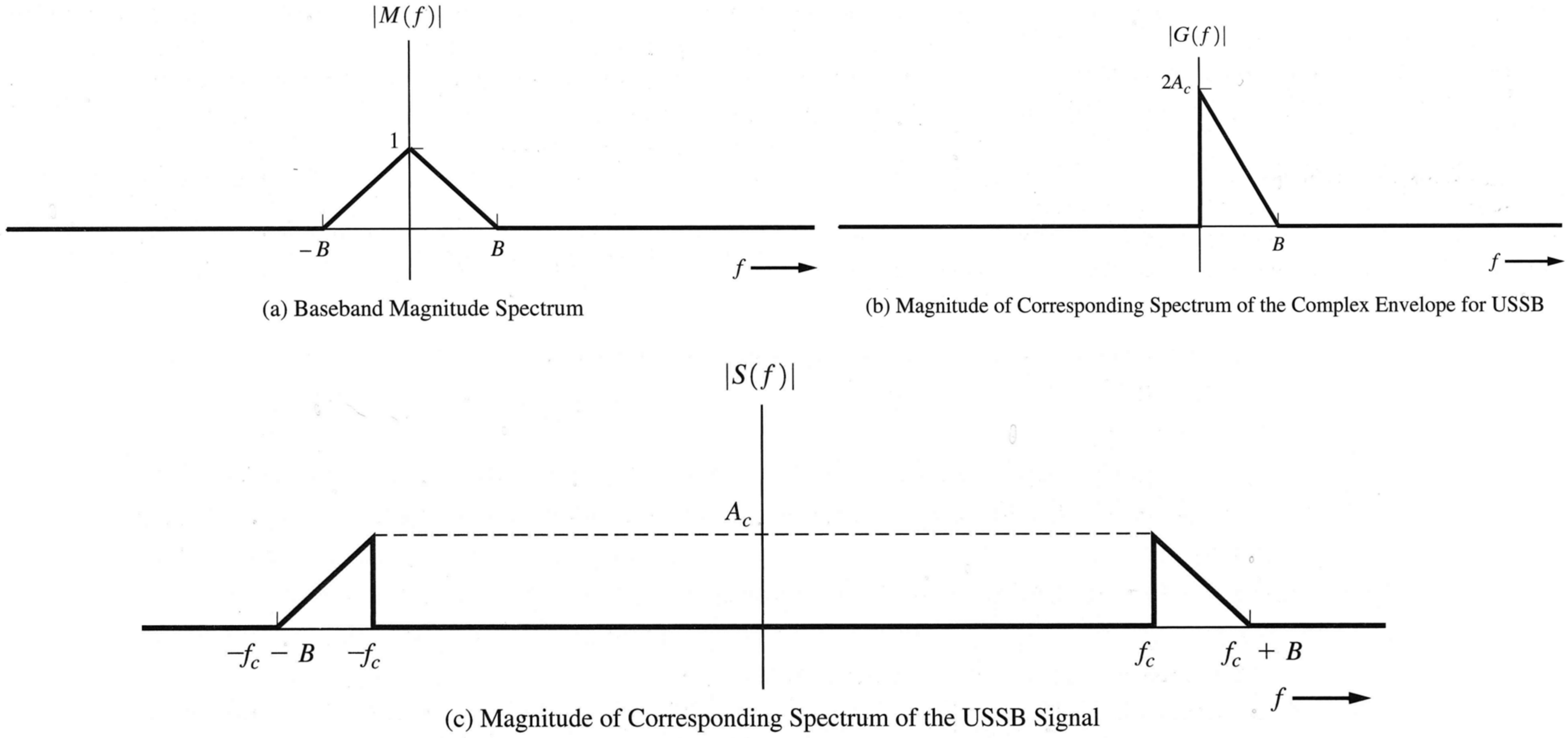


Figure 5–4 Spectrum for a USSB signal.

Transmissionbandwith: $B_T = B$

Example: USSB with a sine wave

The information signal $m(t) = A_m \cos \omega_m t$ is USSB modulated.

The corresponding Hilbert transform is $\hat{m}(t) = A_m \sin \omega_m t$.

Now:

$$
\begin{aligned}
s(t) &= A_c \{ m(t) \cos \omega_c t \overset{\text{USSB}}{-} \hat{m}(t) \sin \omega_c t \} \\
&= A_c A_m \{ \cos \omega_m t \cos \omega_c t - \sin \omega_m t \sin \omega_c t \} \\
&= A_c A_m \cos(\omega_c + \omega_m) t
\end{aligned}
$$

Single Sideband (SSB) modulation (6)

Normalized average signal power:

$$P_s = \langle s^2(t) \rangle = \tfrac{1}{2} \langle |g(t)|^2 \rangle = \tfrac{1}{2} \langle R^2(t) \rangle$$
$$= \tfrac{1}{2} A_c^2 \langle m^2(t) + \hat{m}^2(t) \rangle = A_c^2 \langle m^2(t) \rangle$$

where we have used: $\langle m^2(t) \rangle = \langle \hat{m}^2(t) \rangle$
and $m(t)$ and $\hat{m}(t)$ are orthogonal.

Peak Envelope Power (PEP):

$$P_{PEP} = \tfrac{1}{2} [\max\{|g(t)|\}]^2 = \tfrac{1}{2} A_c^2 \max\{m^2(t) + \hat{m}^2(t)\}$$

Single Sideband (SSB) modulation (6)

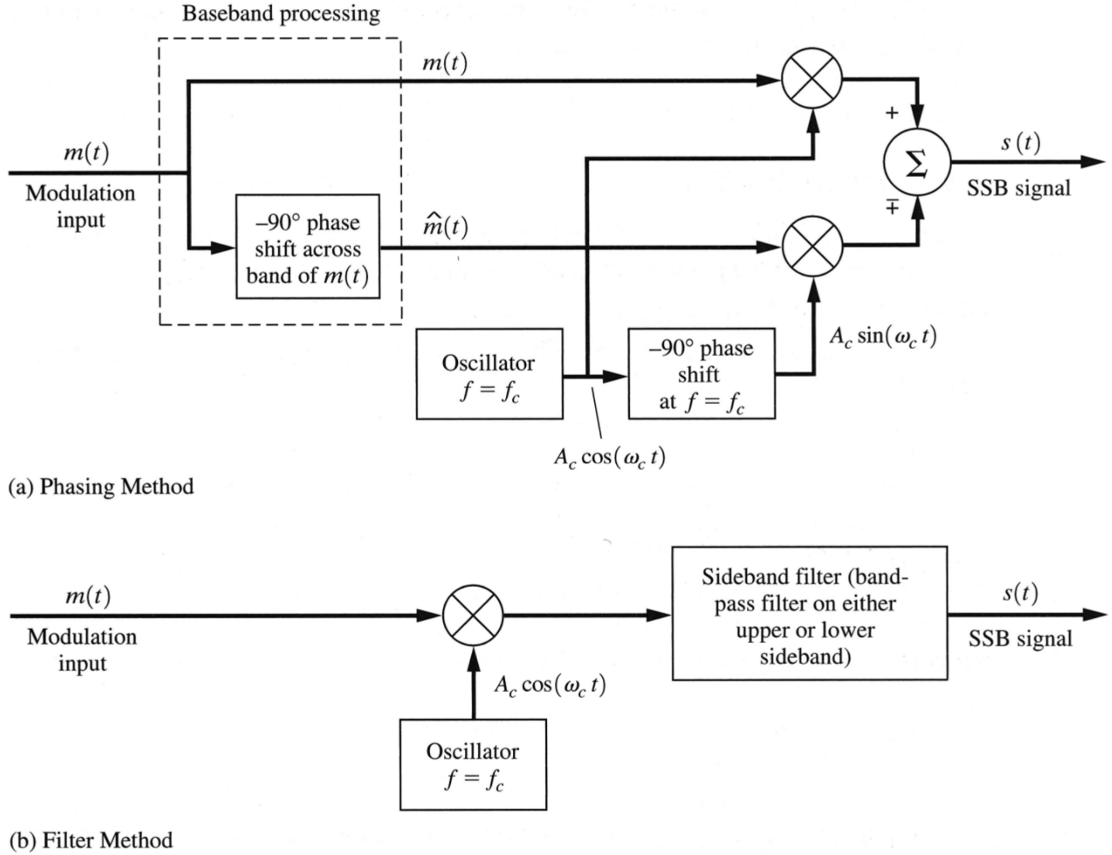


Figure 5–5 Generation of SSB.

Generation of SSB: (1) IQ-method, (2) Filter method