Telecommunications Techniques (EE2T21)

Lecture 14 overview:

BER in coherent bandpass systems
* OOK
* BPSK
* FSK

BER for non-coherent detectors
* OOK
* FSK
* DPSK

BER for multilevel signals
* QPSK, MSK
* M-PSK

EE2T21 Telecommunication Techniques

Dr.ir. Gerard J.M. Janssen

May 6, 2016

Faculty of Electrical Engineering, Mathematics, and Computer Science

TU Delft

Colleges en Instructies Telecommunicatie B

Colleges:

Maandag 6-6 5e+6e uur, EWI-CZ Chip

Instructies:

Maandag 13-6 5e+6e uur, EWI-CZ Chip

Resume: optimal detection of binary signals

In general for additive white Gaussian noise AWGN and arbitrary type of filter:

$$P_e = Q\left(\sqrt{\frac{(s_{01}-s_{02})^2}{4\sigma_0^2}}\right) = Q\left(\left|\frac{s_{01}-s_{02}}{2\sigma_0}\right|\right)$$

s_{01} and s_{02} are the sampled values without noise.

Under the conditions:

- equally likely symbols: $P_1 = 1 - P_2 = 0.5$
- optimum decision threshold: $V_T = \dfrac{s_{01}+s_{02}}{2}$

For the matched filter: $P_e = Q\left(\sqrt{\dfrac{E_d}{2N_0}}\right)$

with $E_d \triangleq \int_0^T [s_{01}(t_0) - s_{02}(t_0)]^2\,dt$ is the "difference symbol" energy.

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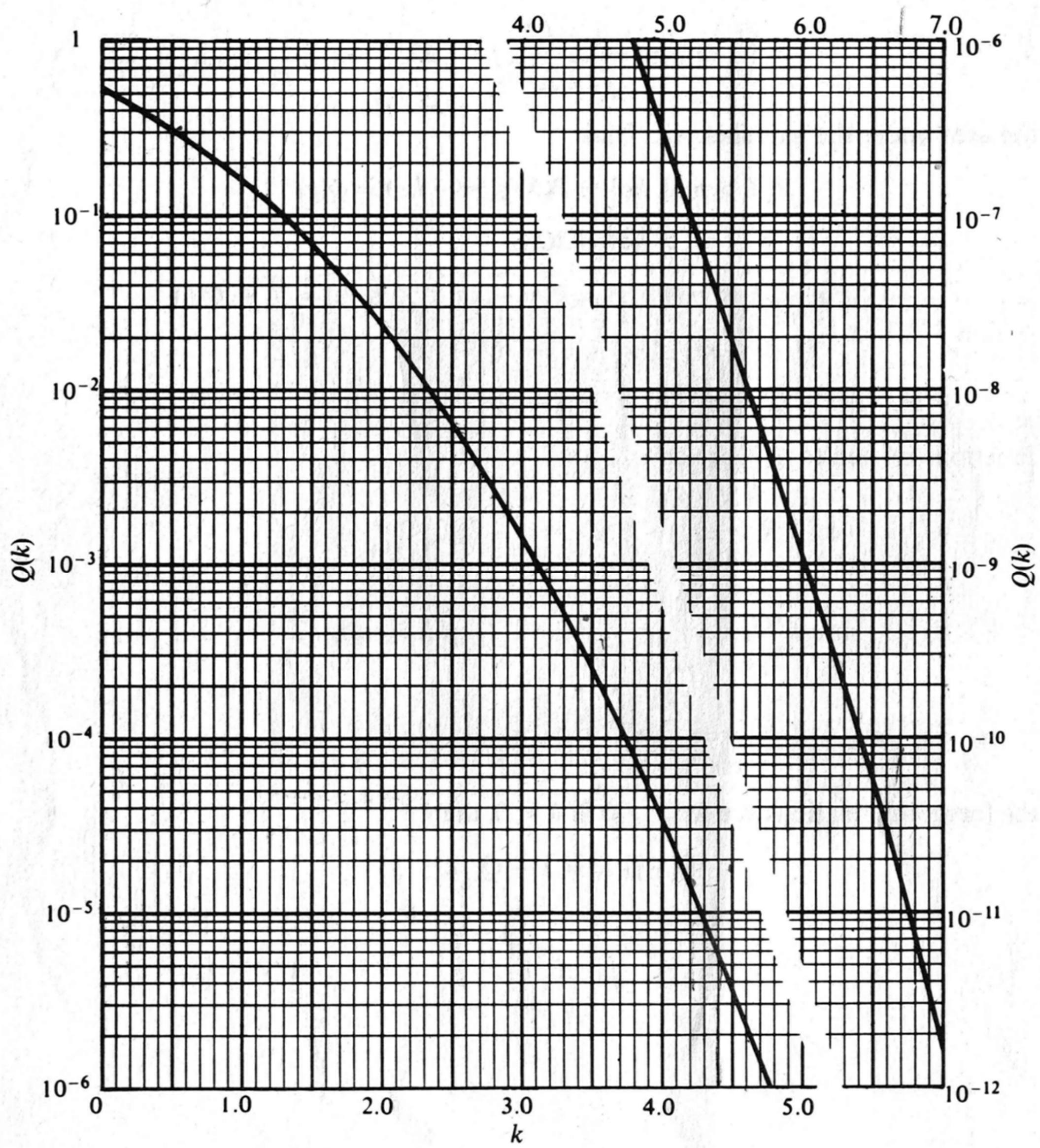
Q-function



For larger values of k the Q-function can be approximated by:

$$Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^\infty e^{-\frac{\lambda^2}{2}} d\lambda$$

$$\approx \frac{1}{k\sqrt{2\pi}} e^{-k^2/2}$$

which is quite accurate for $k > 3$.

Q-function: Couch pp. 700-701, 722-725, and cover page in the back.

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Detection of binary bandpass signals

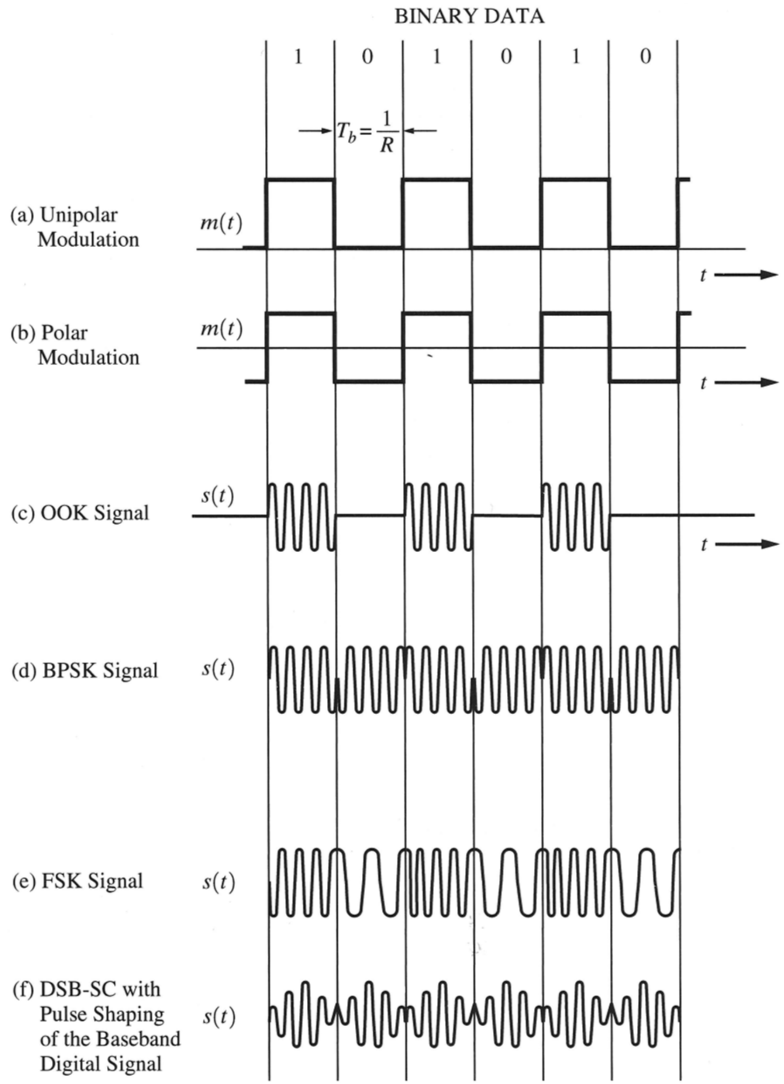

Figure 5–19 Bandpass digitally modulated signals.

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Coherent detection of binary bandpass signals

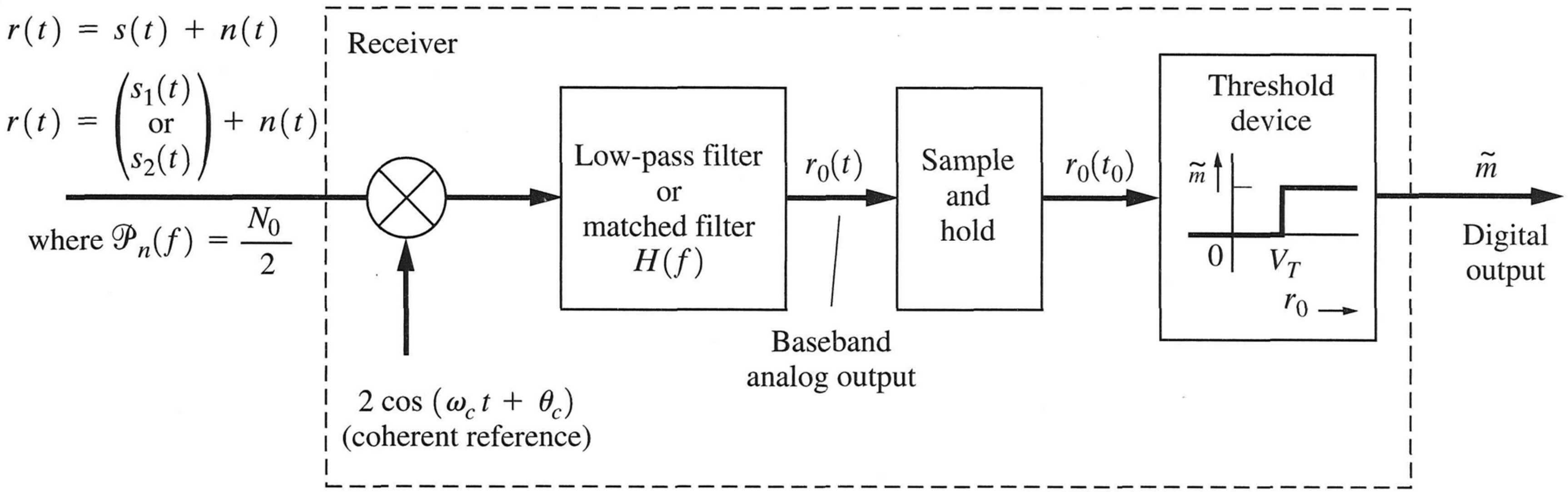

Figure 7–7 Coherent detection of OOK or BPSK signals.

The signal is down-converted to baseband using a product detector. For optimal detection, maximum power should be available in baseband:

$\Rightarrow$ reference carrier should be phase synchronous to the signal.

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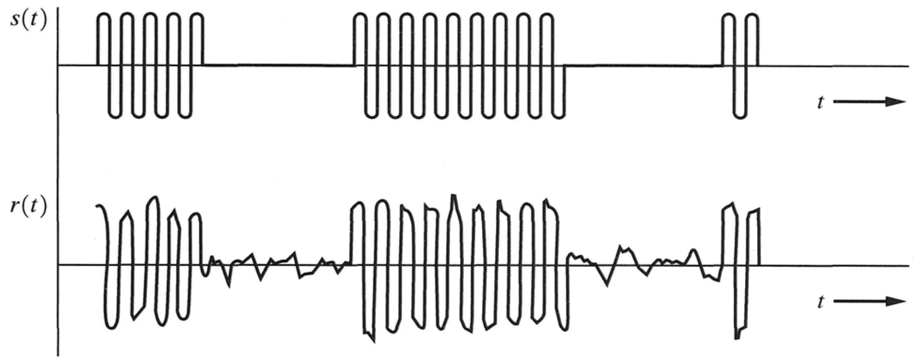
OOK: On-Off-Keying



(b) OOK Signaling

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_c t + \theta_c) & \text{"1"} \\ s_2(t) = 0 & \text{"0"} \end{cases}$$

$$0 \le t < T$$

Besides the signal, also bandpass noise is present which can be described as:

$$n(t) = x_n(t)\cos\omega_c t - y_n(t)\sin\omega_c t = R_n(\mathrm{t})\cos(\omega_c t + \theta_n) \qquad \theta_n \in [0, 2\pi]$$

$$P_n(f) = P_{x_n}(f) = P_{y_n}(f) = \frac{N_0}{2}$$

$$\overline{n^2(t)} = \overline{x_n^2(t)} = \overline{y_n^2(t)} = \sigma_n^2 \qquad \overline{n(t)} = \overline{x_n(t)} = \overline{y_n(t)} = 0$$

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OOK: On-Off-Keying, LPF (1)

1. $H(f)$ = LPF, $B_{eq} \geq 2/T$.

$$r_0(t) = s_0(t) + n_0(t)$$

$$s_0(t) = \begin{Bmatrix} LPF\{2A\cos^2(\omega_c t + \theta_c)\} \\ 0 \end{Bmatrix} = \begin{Bmatrix} LPF\{2A(\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_c t + 2\theta_c)\} \\ 0 \end{Bmatrix} \cong \begin{Bmatrix} A \\ 0 \end{Bmatrix}$$

$$\begin{aligned} n_0(t) &= LPF\{2n(t)\cos(\omega_c t + \theta_c)\} \\ &= LPF\{2x_n(t)\cos\omega_c t\cos(\omega_c t + \theta_c) \\ &\qquad - 2y_n(t)\sin\omega_c t\cos(\omega_c t + \theta_c)\} \\ &= LPF\{x_n(t)[\cos\theta_c + \cos(2\omega_c t + \theta_c)] \\ &\qquad - y_n(t)[\sin(-\theta_c) + \sin(2\omega_c t + \theta_c)]\} \\ &\cong x_n(t)\cos(\theta_c) + y_n(t)\sin(\theta_c) \end{aligned}$$

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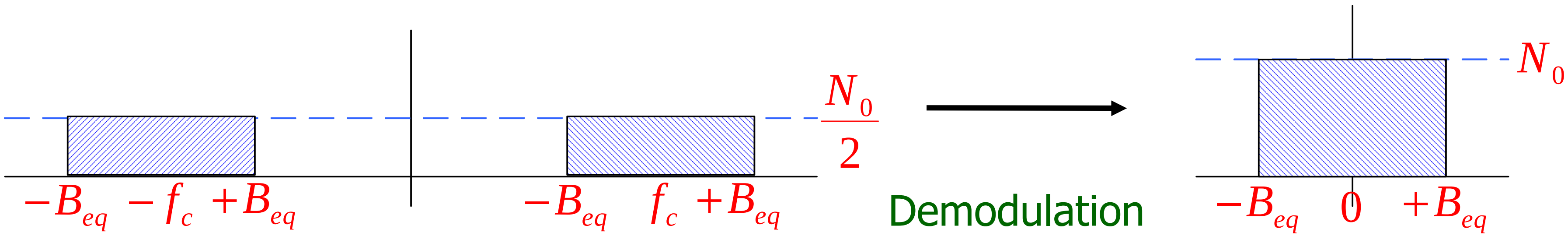
OOK: On-Off-Keying, LPF (2)

$$\overline{n_0^2} = \overline{x_n^2 \cos^2\theta_c + 2x_n y_n \cos\theta_c \sin\theta_c + y_n^2 \sin^2\theta_c}$$

$$= \overline{x_n^2}\cos^2\theta_c + \overline{y_n^2}\sin^2\theta_c$$

$$= \overline{x_n^2} = \overline{y_n^2} = \sigma_0^2$$



Now we find:

$$\overline{n_0^2} = \overline{x_n^2} = \sigma_0^2 = 2(2B_{eq})\frac{N_0}{2} = 2B_{eq}N_0$$

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OOK: On-Off-Keying, LPF (3)

With $s_{01} \cong A, \ s_{02} = 0$

and $P(s_{01}) = P(s_{02}) = \frac{1}{2} \quad \Rightarrow \quad V_T = \frac{s_{01} + s_{02}}{2} = \frac{A}{2}$

we find for the bit-error-probability when using a low-pass filter:

$$P_e = Q\left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}}\right) = Q\left(\sqrt{\frac{A^2}{8N_0 B_{eq}}}\right)$$

Half the value of unipolar baseband transmission! Why???

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OOK: On-Off-Keying, MF

2. $H(f)$ = matched filter

$$E_d \triangleq \int_0^T [s_1(t) - s_2(t)]^2 dt = \int_0^T A^2 \cos^2(\omega_c t + \theta_c) dt = \frac{A^2 T}{2}$$

$$E_b \triangleq \overline{s^2(t)} \cdot T = \frac{\frac{A^2 T}{2} + 0}{2} = \frac{A^2 T}{4} = \frac{E_d}{2}$$

and we find the bit-error-probability for the matched-filter:

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 T}{4N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

The same as for unipolar signaling!

where the optimum decision threshold is:

$$V_{T\_opt} = \frac{s_{01} + s_{02}}{2} = \frac{s_{01}}{2} = \frac{1}{2}\int_0^T 2A\cos^2(\omega_c t + \theta_c) dt = \frac{AT}{2}$$

For rectangular pulses, the MF-filter is an Integrate & Dump filter.

BER for binary modulations with matched filter detection

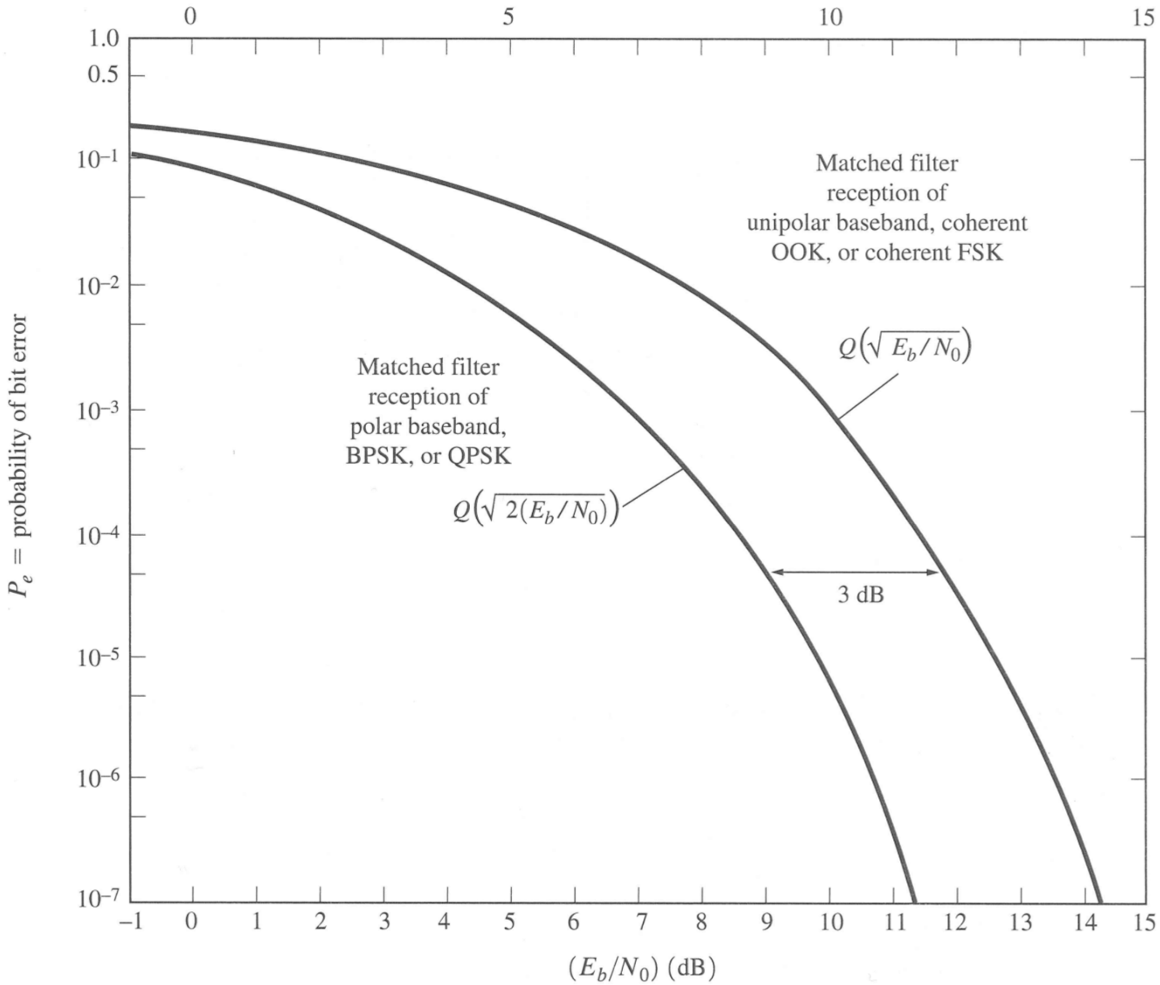

Figure 7–5 P_e for matched-filter reception of several binary signaling schemes.

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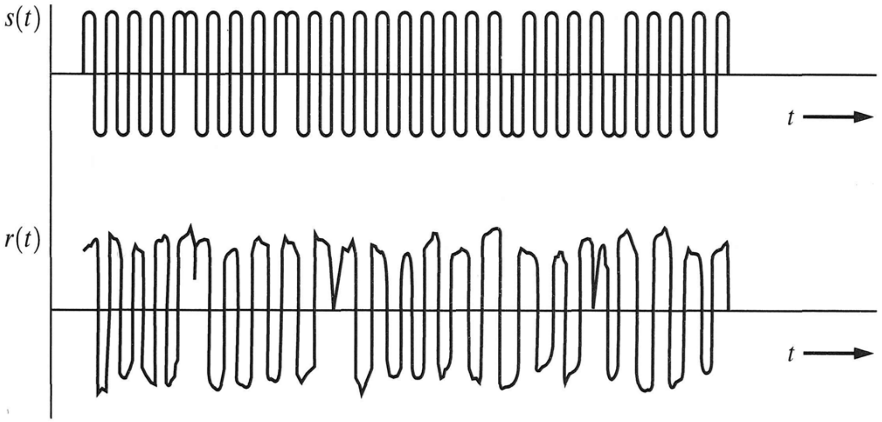
BPSK: Binary Phase Shift Keying (1)



(c) BPSK Signaling

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_c t + \theta_c) & \text{"1"} \\ s_2(t) = -A\cos(\omega_c t + \theta_c) & \text{"0"} \end{cases}$$

$$0 \le t < T$$

$s_1(t) = -s_2(t) \Rightarrow$ antipodal signals.

Similar to OOK, we find for $H(f)$ = LPF, $B_{eq} \ge 2/T$:

$$r_0(t) = s_0(t) + n_0(t) \cong \begin{Bmatrix} +A & 0 \le t \le T & \text{"1"} \\ -A & 0 \le t \le T & \text{"0"} \end{Bmatrix} + x_n(t) \text{ (assuming that } \theta_c = 0)$$

with: $\overline{x_n^2} = \sigma_0^2 = 2B_{eq}N_0$

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BPSK: Binary Phase Shift Keying (2)

Because $s_{01} = -s_{02} \cong A$

A fixed threshold is simple in case of time-variant (e.g. fading) channels.

and $P(s_{01}) = P(s_{02}) = \frac{1}{2} \quad \Rightarrow \quad V_T = \frac{s_{01} + s_{02}}{2} = 0$

Now, we find for the bit-error-probability when using a low-pass filter:

$$P_e = Q\left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}}\right) = Q\left(\sqrt{\frac{A^2}{2N_0 B_{eq}}}\right)$$

Comparison:

- based on PEP: OOK needs 6 dB more power than BPSK
- based on average power: BPSK needs 3 dB less power than OOK.

BPSK: Binary Phase Shift Keying (3)

2. $H(f)$ = matched filter

$$E_d \triangleq \int_0^T [s_1(t) - s_2(t)]^2 dt = \int_0^T 4A^2 \cos^2(\omega_c t + \theta_c) dt = 2A^2T$$

$$E_b \triangleq \overline{s^2(t)} \cdot T = \frac{A^2T}{2} = \frac{E_d}{4}$$

and we find the bit-error-probability for the matched-filter:

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Identical to polar baseband signaling!

BPSK needs only half the power of OOK.

This is a clear advantage of anti-podal signals over orthogonal signals: maximum distance between the states at minimum signal power!!

BER for binary modulations with matched filter detection

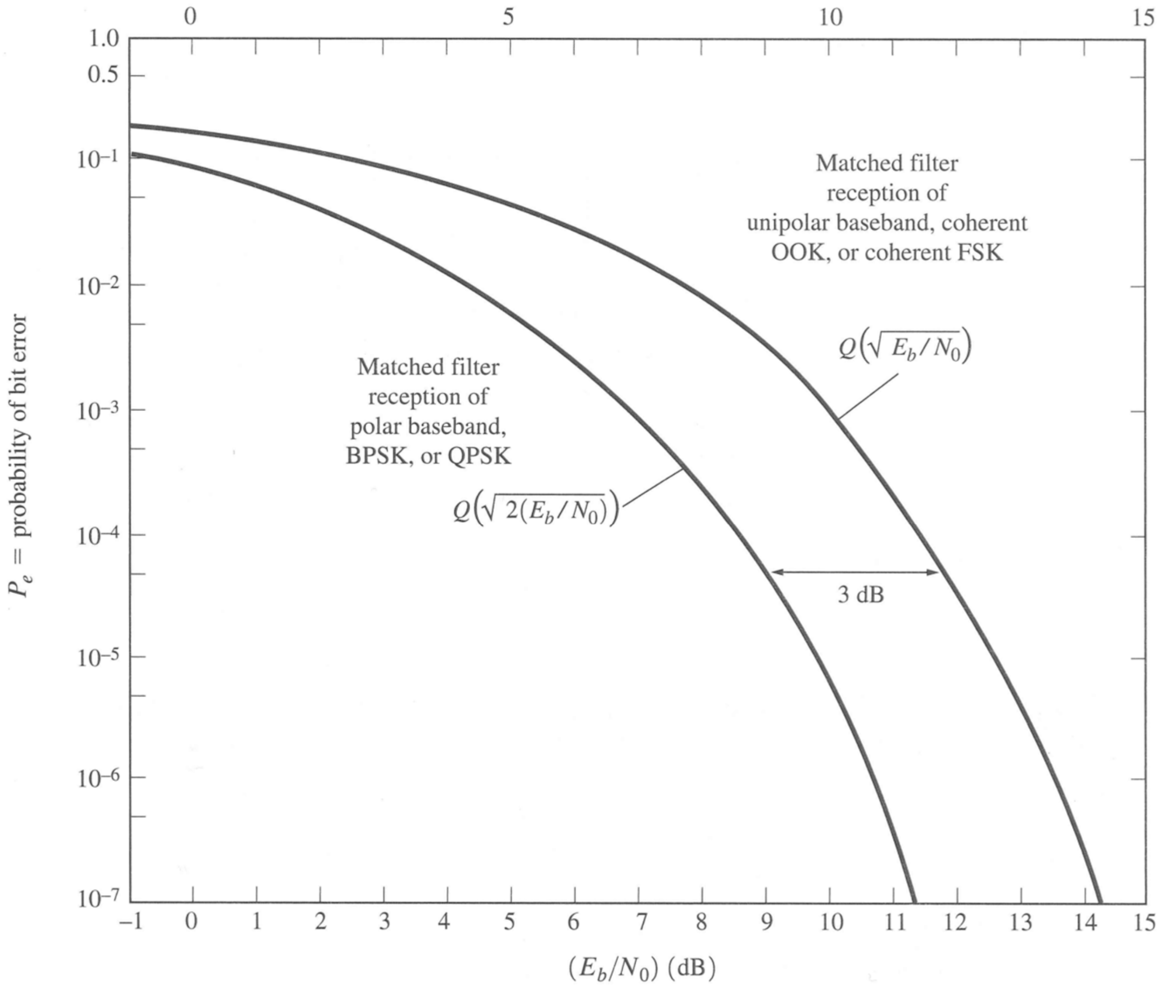

Figure 7–5 P_e for matched-filter reception of several binary signaling schemes.

FSK: Frequency Shift Keying (1)

For FSK, we have the signal states:

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_1 t + \theta_c) & 0 \le t \le T \quad "1" \\ s_2(t) = A\cos(\omega_2 t + \theta_c) & 0 \le t \le T \quad "0" \end{cases}$$

where the bandwidth of the FSK signal is: $B_T = 2(\Delta F + R)$
with $2\Delta F = f_1 - f_2 \;\; (f_1 > f_2)$.

Note that the effective transmission bandwidth around each of the subcarriers at f_1 and f_2 is $2R$ for rectangular symbols.

Receiver: two coherent product detectors and a difference stage.

FSK: Frequency Shift Keying (2)

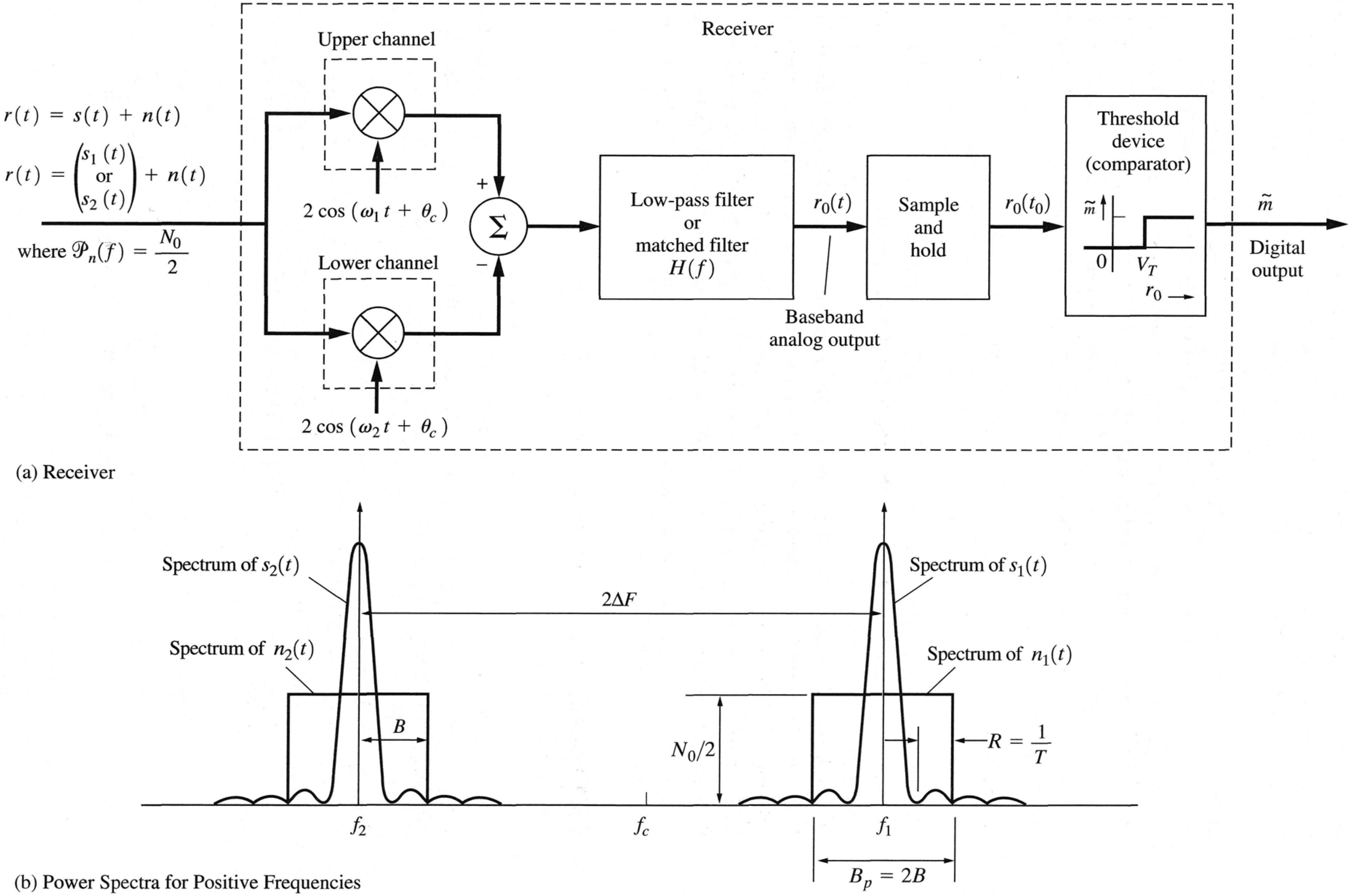

Figure 7–8 Coherent detection of an FSK signal.

FSK: Frequency Shift Keying (3)

1. $H(f)$ = LPF, $B_{eq} \geq 2/T \ll \Delta F \;\Rightarrow\; R \ll \Delta F,$

$$r(t) = r_1(t) + r_2(t)$$

$$r_1(t) = \begin{Bmatrix} s_1(t) & "1" \\ 0 & "0" \end{Bmatrix} + n_1(t) \qquad r_2(t) = \begin{Bmatrix} 0 & "1" \\ s_2(t) & "0" \end{Bmatrix} + n_2(t)$$

Two "OOK" signals!

$$n_1(t) = x_{n1}(t)\cos\omega_1 t - y_{n1}(t)\sin\omega_1 t = R_{n1}(t)\cos(\omega_1 t + \theta_{n1})$$

$$n_2(t) = x_{n2}(t)\cos\omega_2 t - y_{n2}(t)\sin\omega_2 t = R_{n2}(t)\cos(\omega_2 t + \theta_{n2})$$

$$\overline{n_1^2} = \overline{n_2^2} = \overline{x_{n1}^2} = \overline{x_{n2}^2} = \overline{y_{n1}^2} = \overline{y_{n2}^2} = \overline{\sigma_n^2} = 2B_{eq}N_0$$

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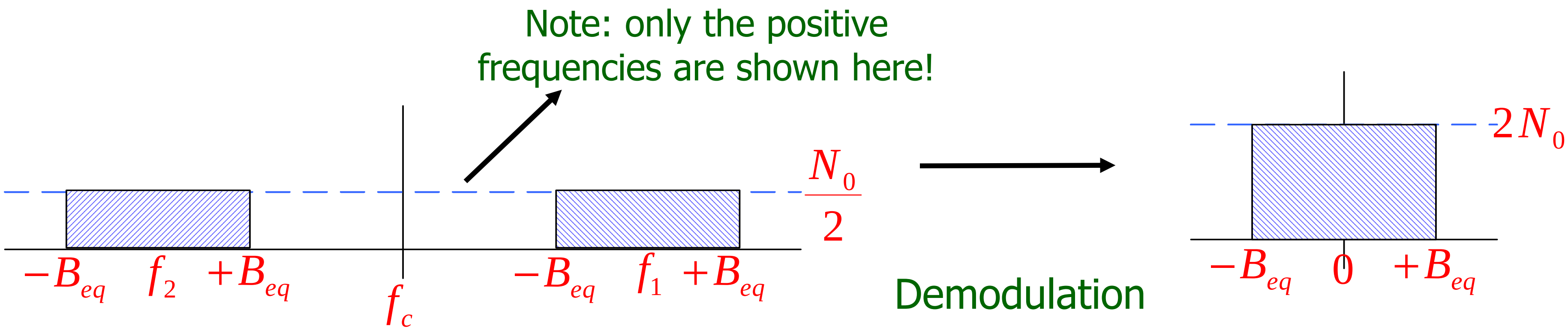
FSK: Frequency Shift Keying (4)



Now we find:

$$r_0(t) = r_{01}(t) - r_{02}(t) = \begin{Bmatrix} +A & 0 \le t \le T & "1" \\ -A & 0 \le t \le T & "0" \end{Bmatrix} + n_0(t)$$

$$\begin{aligned} \overline{n_0^2(t)} &= \overline{(n_{01}(t) - n_{02}(t))^2} = \overline{(x_{n1}(t) - x_{n2}(t))^2} \\ &= \overline{x_{n1}^2(t)} + \overline{x_{n2}^2(t)} = \sigma_0^2 = 2\sigma_n^2 = 4B_{eq}N_0 \end{aligned}$$

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FSK: Frequency Shift Keying (5)

Because $s_{01} = -s_{02} \cong A$

Again a fixed threshold: good for time-varying channels.

and $P(s_{01}) = P(s_{02}) = \frac{1}{2} \quad \Rightarrow \quad V_T = \frac{s_{01} + s_{02}}{2} = 0$

Now, we find for the bit-error-probability when using a lowpass filter:

$$P_e = Q\left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}}\right) = Q\left(\sqrt{\frac{4A^2}{16N_0B_{eq}}}\right) = Q\left(\sqrt{\frac{A^2}{4N_0B_{eq}}}\right)$$

PEP comparison:

- FSK needs 3 dB more PEP power than BPSK
- FSK needs 3 dB less PEP power than OOK.

FSK: Frequency Shift Keying (6)

2. $H(f)$ = matched filter

$$E_d \triangleq \int_0^T [s_1(t)-s_2(t)]^2\,dt = \int_0^T (A\cos(\omega_1 t+\theta_c) - A\cos(\omega_2 t+\theta_c))^2\,dt$$

$$= \int_0^T \begin{array}{l}[A^2\cos^2(\omega_1 t+\theta_c) - 2A^2\cos(\omega_1 t+\theta_c)\cos(\omega_2 t+\theta_c) \\ \quad + A^2\cos^2(\omega_2 t+\theta_c)]\,dt\end{array}$$

$$\cong \tfrac{1}{2}A^2T + \tfrac{1}{2}A^2T = A^2T$$

$$= 0 \text{ iff } \frac{\omega_1-\omega_2}{2\pi} = 2\Delta F = \frac{n}{2T} = \frac{nR}{2}$$

$$\text{or } \Delta F = \frac{nR}{4}$$

$$E_b \triangleq \overline{s^2(t)}\cdot T = \frac{A^2T}{2} = \frac{E_d}{2}$$

s_1 will hardly contribute to the output of detector 2 and v.v. if $\Delta F >> R$.

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FSK: Frequency Shift Keying (7)

Now we find the bit-error-probability for the matched-filter:

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

The same as for OOK!

The same quality as OOK $\Rightarrow$ 3 dB degradation compared to BPSK.

OOK and FSK: orthogonal signals

BPSK: antipodal signals

$V_T = 0$, no carrier component nor noise enhancement.

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BER for binary modulations with matched filter detection

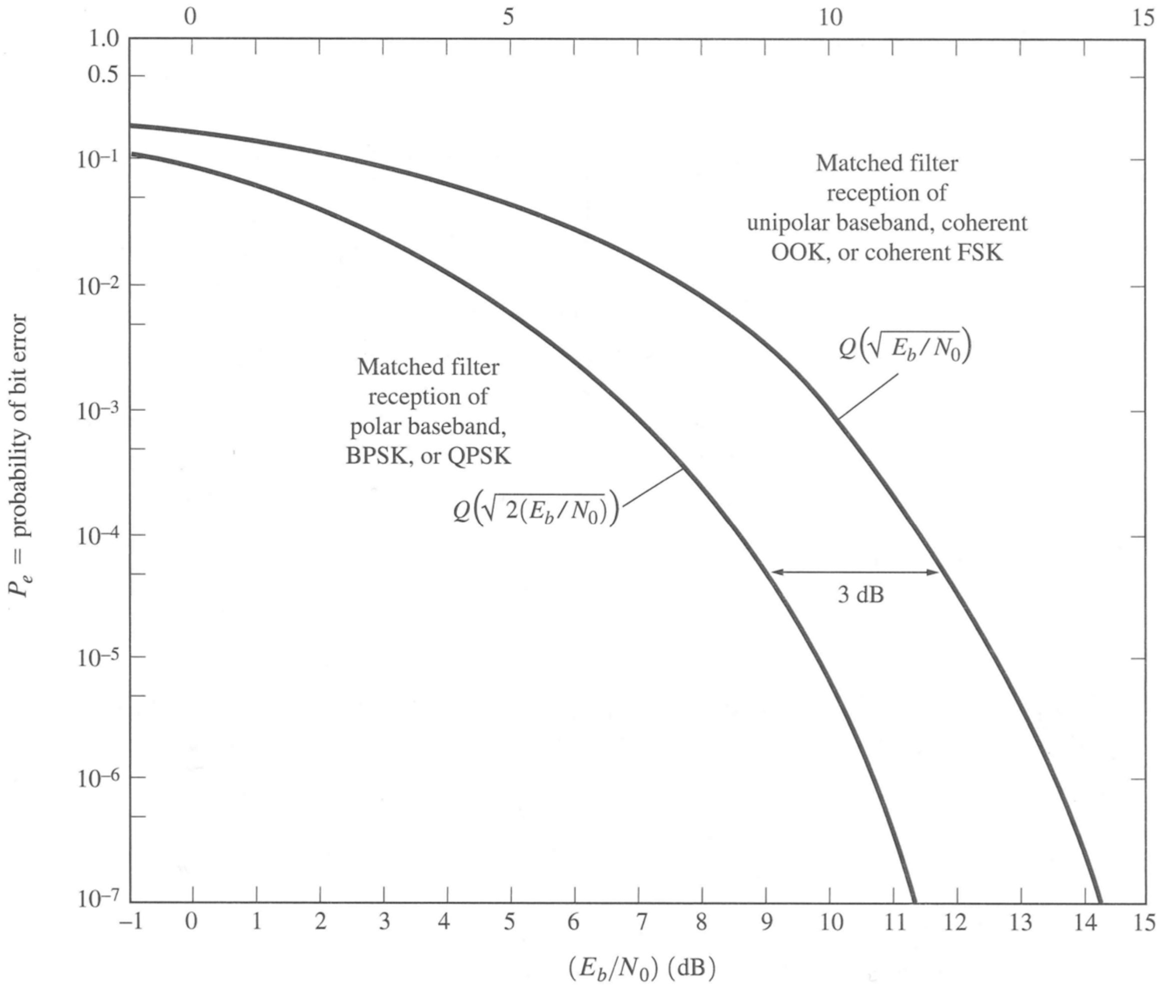

Figure 7–5 P_e for matched-filter reception of several binary signaling schemes.

Non-coherent detection: OOK, FSK and DPSK

1. Coherent detection: superior performance
   However, carrier recovery often very complex and expensive.

2. PSK signals, in general cannot be detected with a non-coherent detector.

3. The mathematical analysis of non-coherent detection (non-linear) is in general much more complex than for coherent detection (linear operations).

4. The performance of non-coherent detectors is sub-optimum (a loss of several dB's).

5. The implementation complexity of non-coherent detectors is low: cheaper.

Non-coherent OOK detection

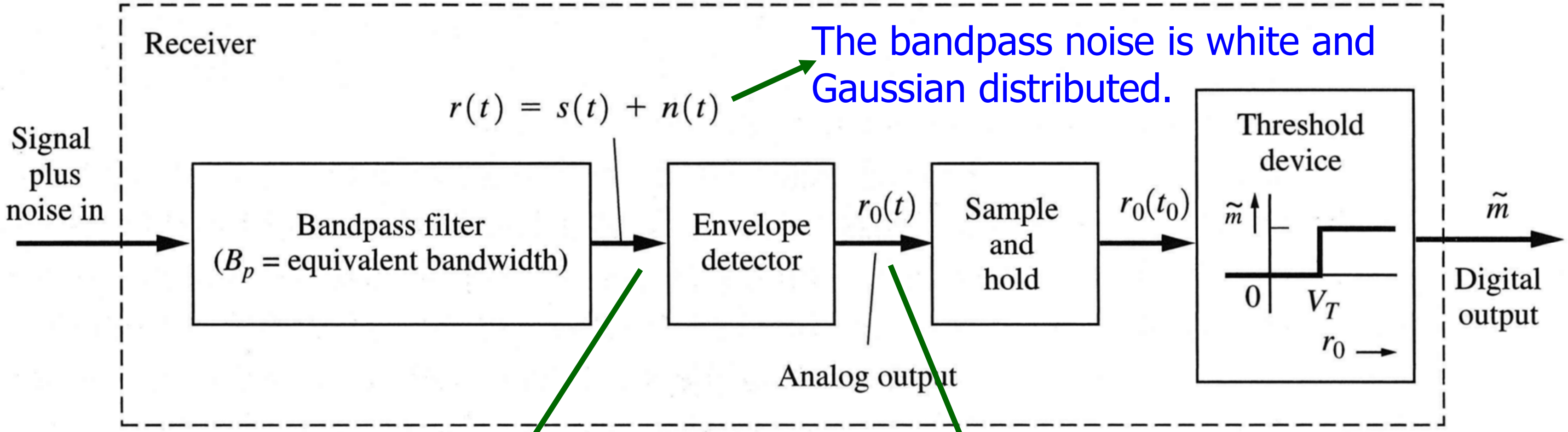

Figure 7–9 Noncoherent detection of OOK.

The bandpass noise is white and Gaussian distributed.

$$r_0(t) = \begin{cases} \sqrt{(A + x_n(t))^2 + y_n^2(t)} & \text{"1"} \\ \sqrt{x_n^2(t) + y_n^2(t)} & \text{"0"} \end{cases}$$

$$r(t) = \begin{cases} A\cos(\omega_c t + \theta_c) + n(t) & \text{"1"} \\ n(t) & \text{"0"} \end{cases}$$

$$= \begin{cases} [A + x_n(t)]\cos(\omega_c t + \theta_c) - y_n(t)\sin(\omega_c t + \theta_c) & \text{"1"} \\ x_n(t)\cos(\omega_c t + \theta_c) - y_n(t)\sin(\omega_c t + \theta_c) & \text{"0"} \end{cases}$$

Non-coherent FSK detection

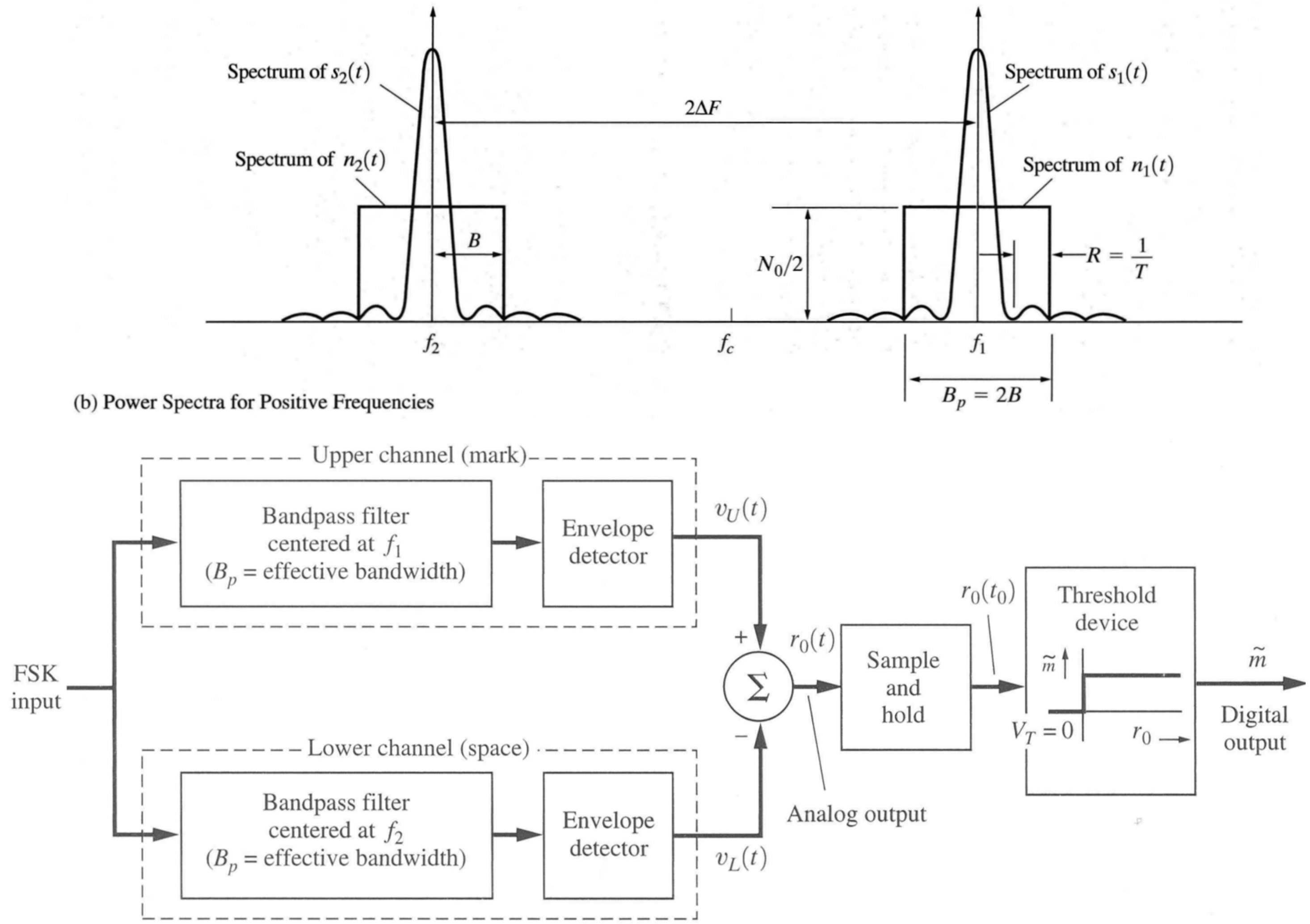

Figure 7–11 Noncoherent detection of FSK.

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Non-coherent detection of OOK and FSK

Under the assumptions:

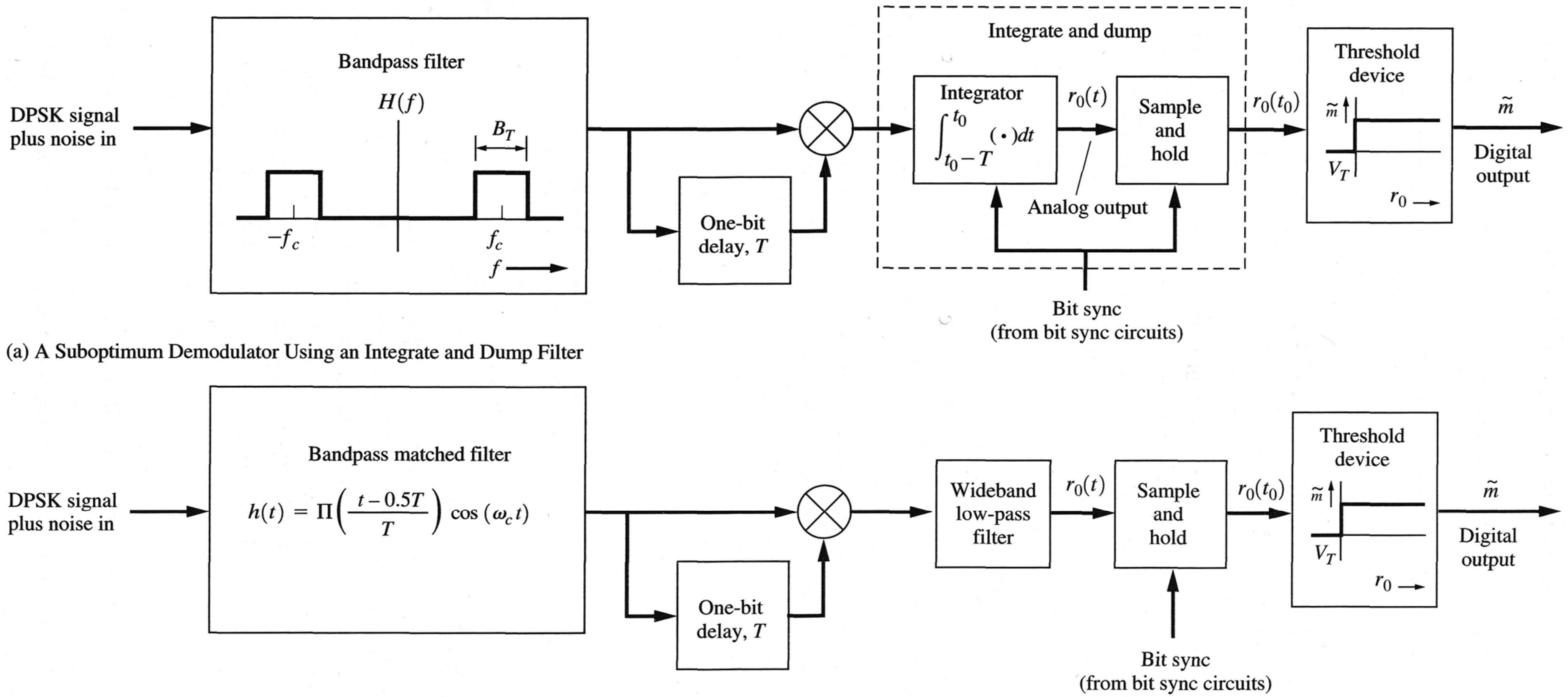
- high SNR at the input of the envelope detector,
- threshold at half the symbol amplitude for OOK and at zero for FSK,
- optimum filtering before envelope detection, $T_b B_p = 1$

the minimum BER for OOK and FSK is found as:

$$P_e = \tfrac{1}{2}\exp\left\{-\frac{1}{2}\frac{E_b}{N_0}\right\}$$

$\Rightarrow$ optimum performance of a non-coherent detector for OOK or FSK

Differential Phase Shift Keying (1)



(a) A Suboptimum Demodulator Using an Integrate and Dump Filter

(b) An Optimum Demodulator Using a Bandpass Matched Filter

Figure 7–12 Demodulation of DPSK.

Differential Phase Shift Keying (2)

In a DPSK detector:

- the previous symbol's phase is used as reference for the current symbol

- because of differential encoding: combined detection and decoding

$$P_e = \tfrac{1}{2}\exp\left\{-\frac{E_b}{N_0}\right\}$$

$\Rightarrow$ 3 dB better than non-coherent detection of OOK and FSK

Why is the performance slightly worse than for coherent BPSK?

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BER for non-coherent detection of OOK, FSK and DPSK

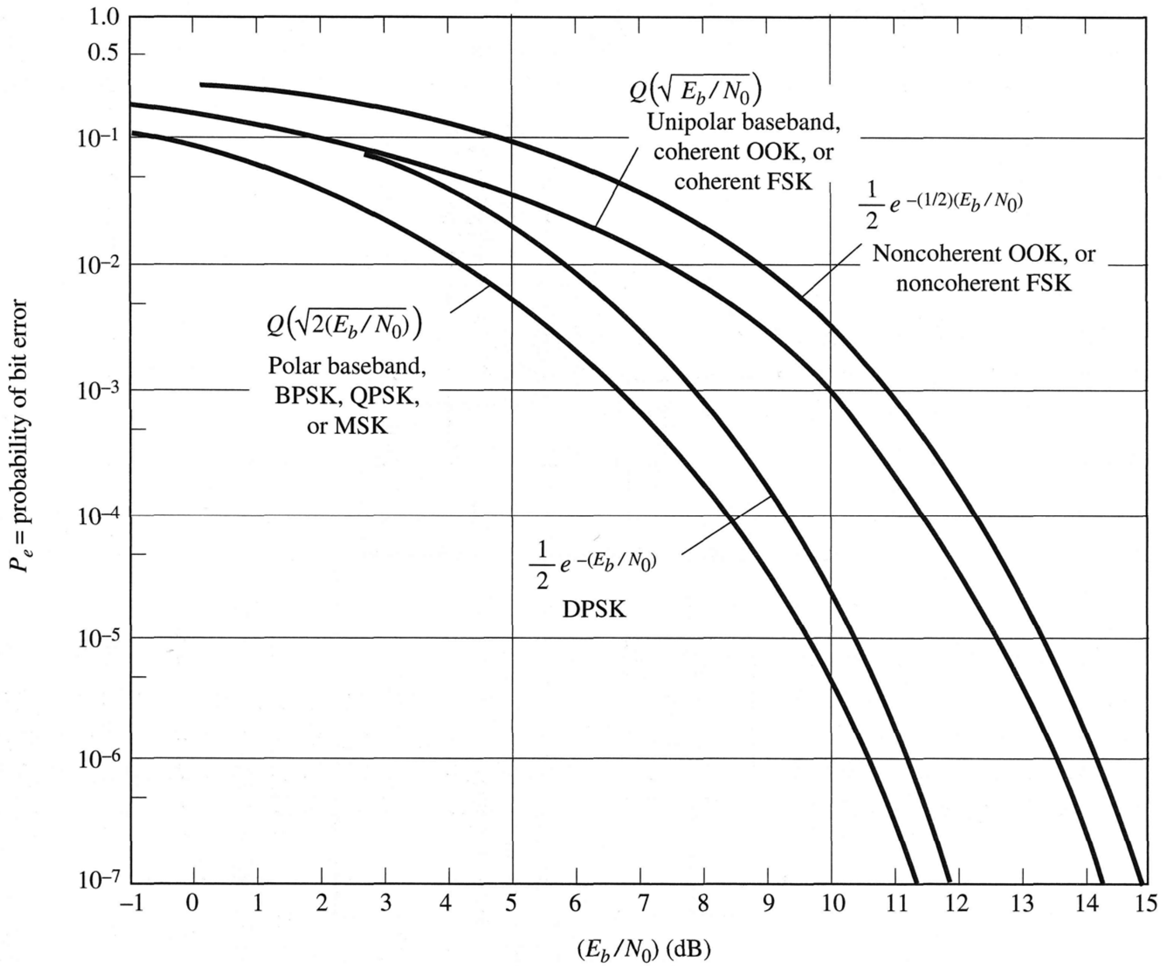

Figure 7–14 Comparison of the probability of bit error for several digital signaling schemes.

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Differential Phase Shift Keying (4)

The slight performance degradation ($\approx$ 1dB) is because:
the phase of the previous symbol is reference for the current symbol, but it is a noisy reference!

Note that for a good operation, the channel should be stable for the duration of at least two symbols.

How does this compare to carrier recovery required for coherent detection?

Detection of multilevel signals

In M-level modulated signals, one out of M possible signal states is transmitted. It is the task of the detector to decide correctly which state is received.

With a higher modulation level, we gain in spectral efficiency, but often at the cost of increased power (a higher E_b is needed), as well as increased detector complexity.

In a detector for M-level modulation signals, the in-phase and quadrature-phase (I&Q) signal components are often processed separately.

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I-Q-detector

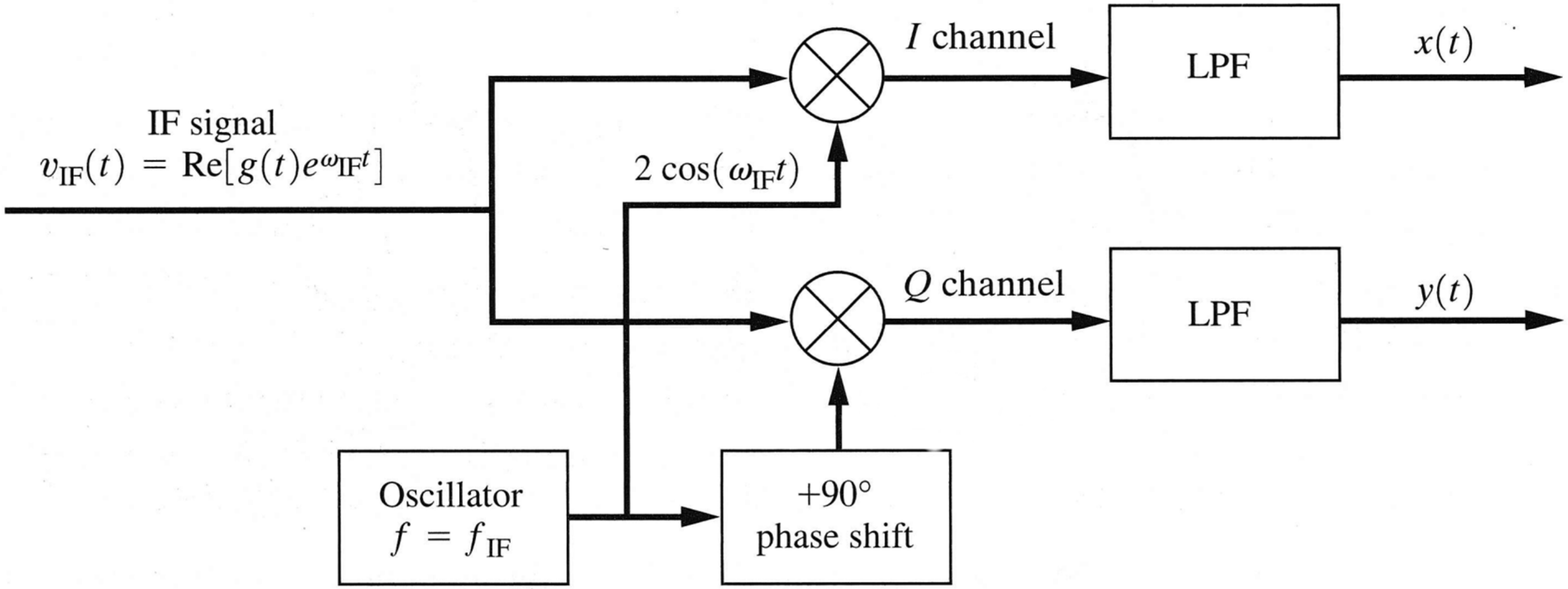


Figure 4–31 IQ (in-phase and quadrature-phase) detector.

QPSK modulation (1)

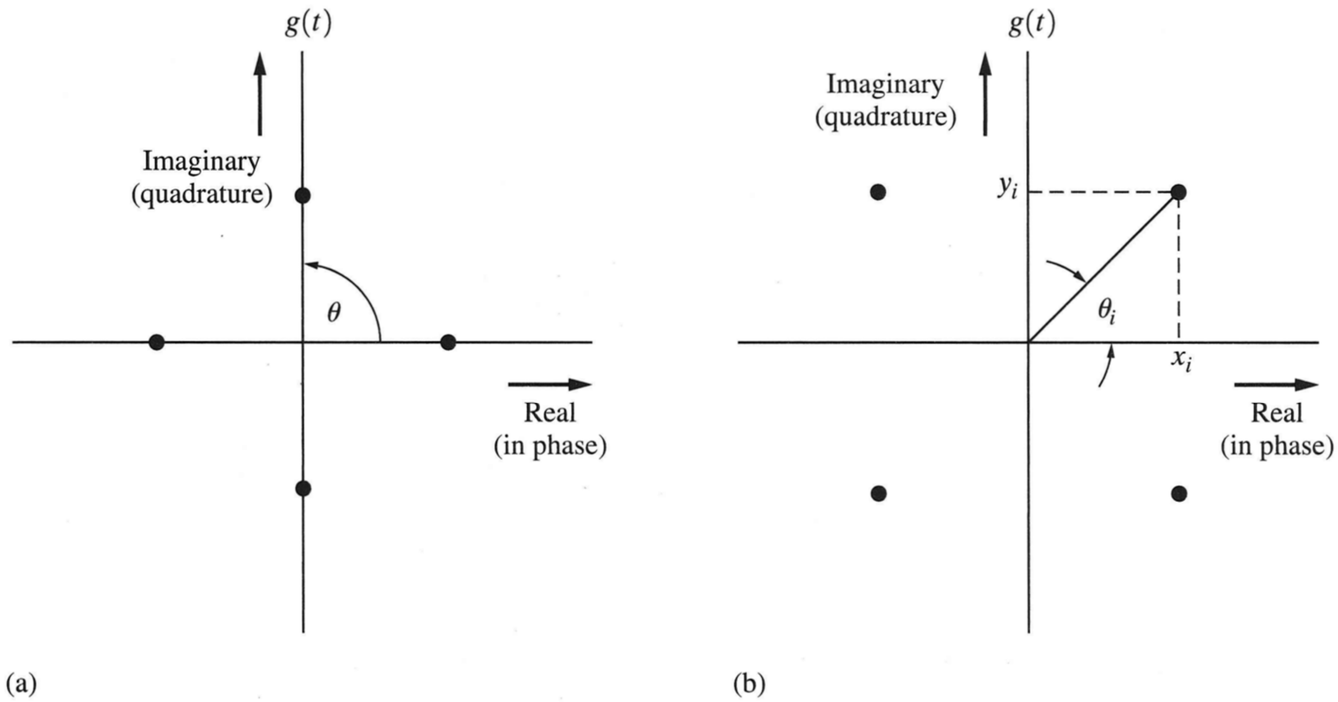


Figure 5–30 QPSK and $\pi/4$ QPSK signal constellations (permitted values of the complex envelope).

QPSK can be seen as 4-PSK or as the sum of two independent BPSK signals on quadrature carriers:

$$s(t) = A[d_1(t)\cos\omega_c t + d_2(t)\sin\omega_c t] \quad d_1, d_2 \in \{-1,1\}$$

QPSK modulation (2)

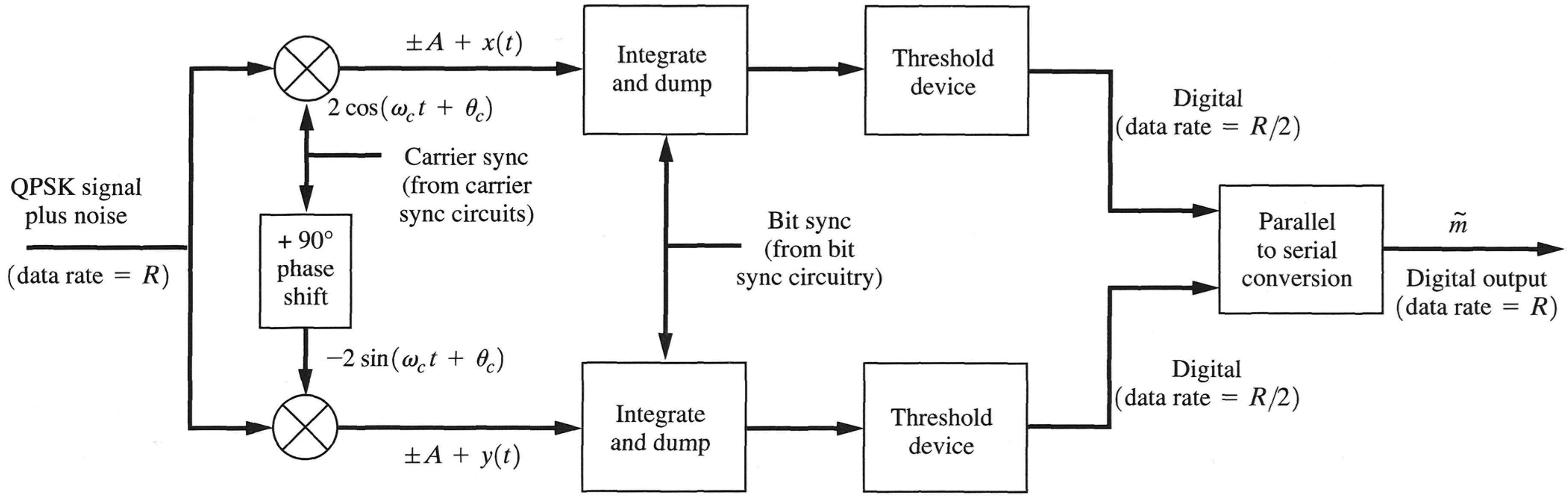


Figure 7–13 Matched-filter detection of QPSK.

QPSK ≡ 2 orthogonal BPSK signals ⇒ detection by means of 2 BPSK detectors in the I&Q branches.

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Note: in the equation below (7-68), the variable θ_n should replaced by θ_c.

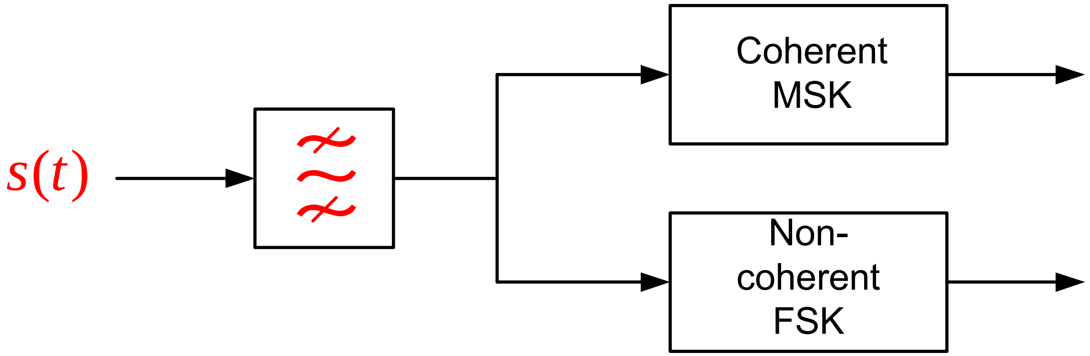
Bandwidth of the I&D-filters is matched to $T_s = 2/R_b = 2T_b$

MSK modulation

Minimum Shift Keying can be seen as:

1. OQPSK with half sine-wave pulses
2. FSK with a frequency deviation $\Delta F = \dfrac{R}{4}$

Thus MSK can be coherently detected like a QPSK signal using a filter matched to the half sine-wave pulse shape, or with a non-coherent FSK detector.



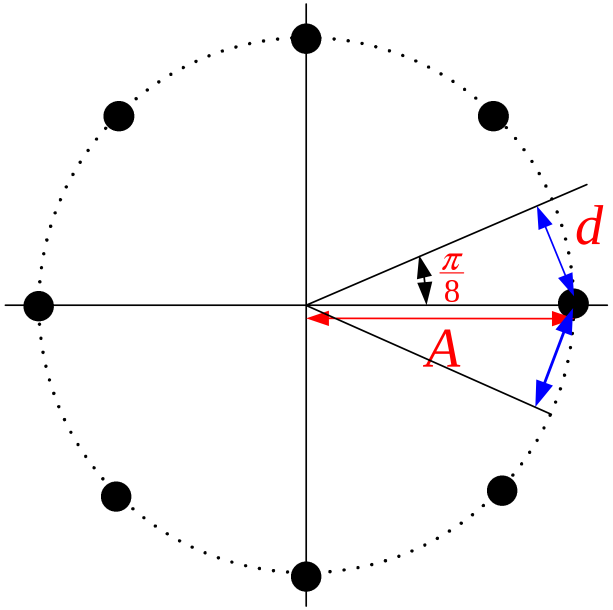
$$P_{e\_opt.} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_{e\_subopt.} = \tfrac{1}{2}\exp\left\{-\frac{1}{2}\frac{E_b}{N_0}\right\}$$

M-PSK modulation (1)

The exact calculation of the BER for M-PSK ($M > 4$) signals is complex. However, with some minor assumptions it can be done with the techniques discussed so far:

1. symbol errors only occur between neighboring symbols,
2. neighboring symbols differ in only one bit: Gray-coding.



Signal state diagram for 8-PSK

$$s(t) = A\cos(\omega_c t + \theta_i), \quad \theta_i = 2\pi(i-1)/M, \quad i = 1,2 \,..\, M$$

$$d = \text{distance to decision threshold} = A\sin\frac{\pi}{M}$$

$$l = \text{number of bits/symbol} = {}^2\log M$$

$$E_s = \text{symbol energy} = PT_s = \tfrac{1}{2}A^2T_s$$

$$= \tfrac{1}{2}A^2T_b\,{}^2\log M = E_b\,{}^2\log M$$

$$E_b = \frac{E_s}{{}^2\log M}$$

TU Delft

M-PSK modulation (2)

First we determine the SEP (symbol error probability):

$$P_{es} \le 2Q\left(\sqrt{\frac{(s_1 - s_2)^2}{4\sigma_0^2}}\right) \overset{MF}{=} 2Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

In the equation on page 545, the factor 2 is missing.

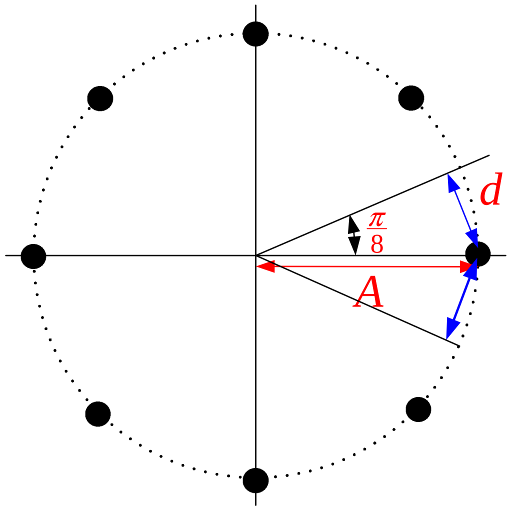
$$= 2Q\left(\sqrt{\frac{2E_s \sin^2 \frac{\pi}{M}}{N_0}}\right) = 2Q\left(\sqrt{\frac{2E_b \cdot {}^2\log M \cdot \sin^2 \frac{\pi}{M}}{N_0}}\right)$$

An error may occur against each of the neighbor symbols.

where we used:

$$E_d \triangleq \int_0^{T_s} [s_1(t) - s_2(t)]^2 dt = \tfrac{1}{2}(2A \sin \tfrac{\pi}{M})^2 T_s = 2A^2 T_s \sin^2 \tfrac{\pi}{M}$$

$$= 4E_s \sin^2 \tfrac{\pi}{M} = 4E_b \sin^2 \tfrac{\pi}{M} \cdot {}^2\log M$$



Now the BER is: $P_{eb} \le \dfrac{P_{es}}{{}^2\log M}$

Comparison of digital modulations

TABLE 7–1 COMPARISON OF DIGITAL SIGNALING METHODS

Type of Digital Signaling	Minimum Transmission Bandwidth Required[a] (Where R Is the Bit Rate)		Error Performance			
Baseband signaling						
Unipolar	$\frac{1}{2}R$	(5-105)			$Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$	(7–24b)
Polar	$\frac{1}{2}R$	(5-105)			$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–26b)
Bipolar	$\frac{1}{2}R$	(5-105)			$\frac{3}{2}Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$	(7–28b)
Bandpass signaling			*Coherent detection*		*Noncoherent detection*	
OOK	R	(5-106)	$Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$	(7–33)	$\frac{1}{2}e^{-(1/2)(E_b/N_0)}$	(7–58)
BPSK	R	(5-106)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–38)	Requires coherent detection	
FSK	$2\Delta F + R$ where $2\Delta F = f_2 - f_1$ is the frequency shift	(5-89)	$Q\left[\sqrt{\left(\frac{E_b}{N_0}\right)}\right]$	(7–47)	$\frac{1}{2}e^{-(1/2)(E_b/N_0)}$	(7–65)
DPSK	R	(5-106)	Not used in practice		$\frac{1}{2}e^{-(E_b/N_0)}$	(7–67)
QPSK	$\frac{1}{2}R$	(5-106)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–69)	Requires coherent detection	
MSK	$1.5R$ (null bandwidth)	(5-115)	$Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right]$	(7–69)	$\frac{1}{2}e^{-(1/2)(E_b/N_0)}$	(7–65)

[a] Typical bandwidth specifications by ITU are larger than these minima [Jordan, 1985].