

Telecommunicatie B (EE2T21)

Lecture 12 overview:

Modulation techniques for digital signals

- **Multi-level modulation schemes , e.g.:**
 - * **M-PSK**
 - * **Quadrature Phase Shift Keying (QPSK)**
 - * **Quadrature Amplitude Modulation (QAM)**
- **Novel robust or bandwidth efficient modulation schemes**
 - * **Orthogonal Frequency Division Multiplexing (OFDM)**
 - * **Direct Sequence-Spread Spectrum (DS-SS) modulation**

EE2T21 Telecommunication Techniques

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Colleges en Instructies Telecommunicatie B

Colleges:

Maandag 30-5, 6-6

5e+6e uur, EWI-CZ Chip

Dinsdag 10-5

7e+8e uur, EWI-CZ Pi

Instructies:

Dinsdag 17-5

5e+6e uur, EWI-CZ Boole

Dinsdag 31-5

7e+8e uur, EWI-CZ Pi

Maandag 13-6

5e+6e uur, EWI-CZ Chip

Multi-level modulation techniques

In M -level modulation: for every sequence of l source bits one out of $M = 2^l$ symbols is generated by the transmitter.

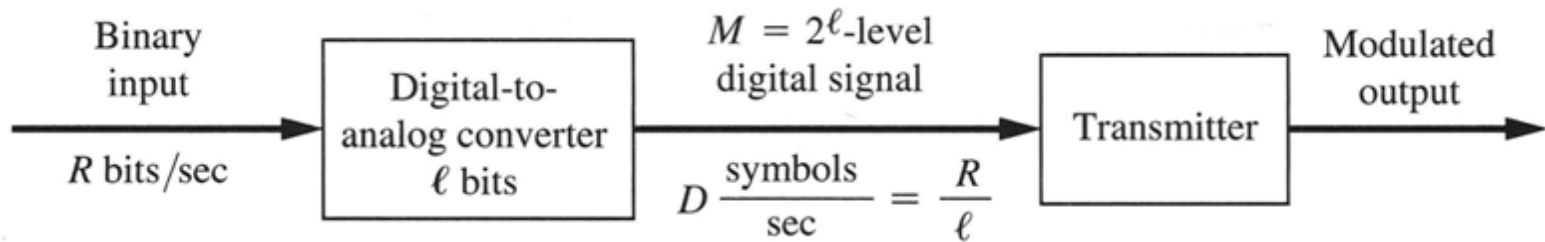


Figure 5–29 Multilevel digital transmission system.

The symbol rate $D = \frac{R_b \text{ [bit/s]}}{l \text{ [bit/symb]}} = \frac{R_b}{l} \text{ [symb/s] or [baud]}$

Per symbol, l bits are transmitted and the symbol time is $T_s = lT_b$.

M-PSK

The complex envelope for M -PSK is:

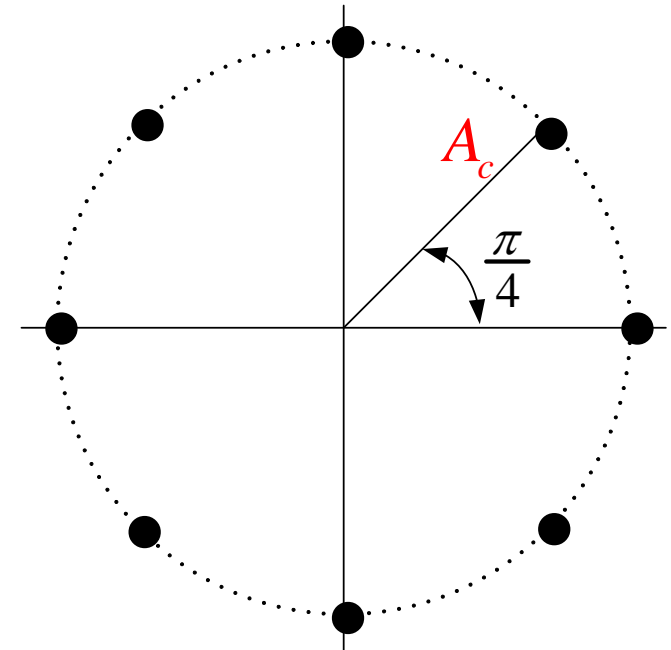
$$g(t) = A_c e^{j\theta(t)} = x(t) + jy(t)$$

For the i^{th} symbol, the phase θ_i is transmitted, with

$$\theta_i = \frac{2\pi(i-1)}{M} \quad i = 1, 2, \dots, M$$

and $x_i = A_c \cos \theta_i$, $y_i = A_c \sin \theta_i$

Usually, all symbols are equally likely.



Signal state diagram for 8-PSK

Quadrature Phase Shift Keying (QPSK)

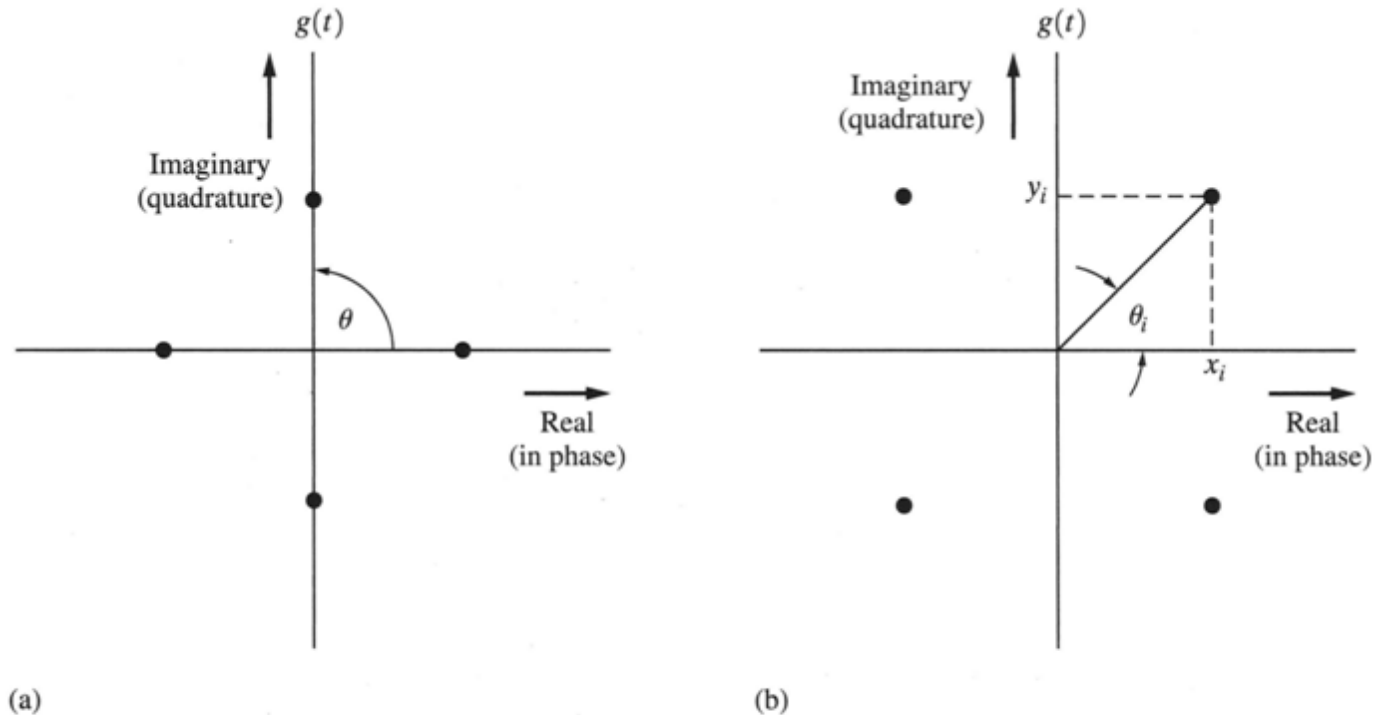
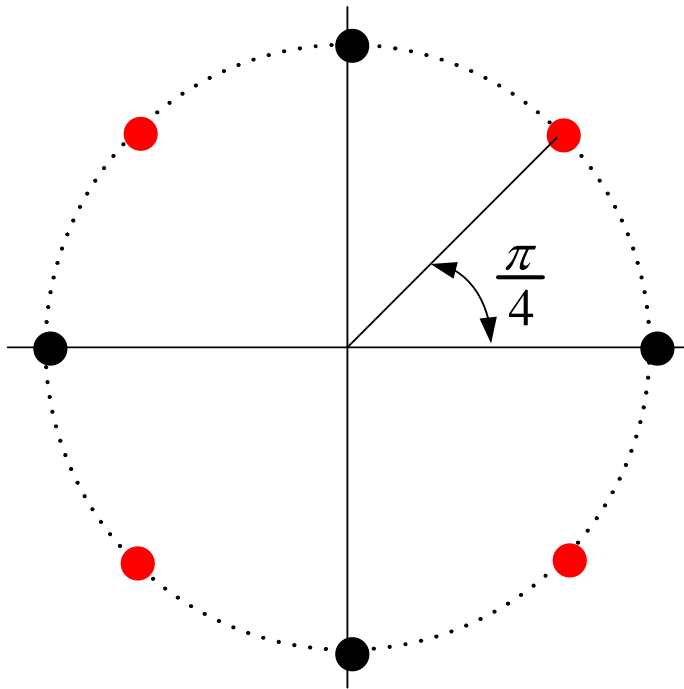


Figure 5-30 QPSK and $\pi/4$ QPSK signal constellations (permitted values of the complex envelope).

QPSK can be seen as 4-PSK or two independent BPSK signals on quadrature carriers:

$$s(t) = A_c [d_1(t) \cos \omega_c t + d_2(t) \sin \omega_c t] \quad d_1, d_2 \in \{-1, 1\}$$

$\frac{\pi}{4}$ -DQPSK



In $\frac{\pi}{4}$ -DQPSK (Differential QPSK) a form of differential coding is applied.

The $\frac{\pi}{4}$ -DQPSK states are those of two QPSK constellations shifted over $\frac{\pi}{4}$ rad.

The bits to be transmitted determine the phase shift.

The next state is one of the other QPSK states, i.e.:

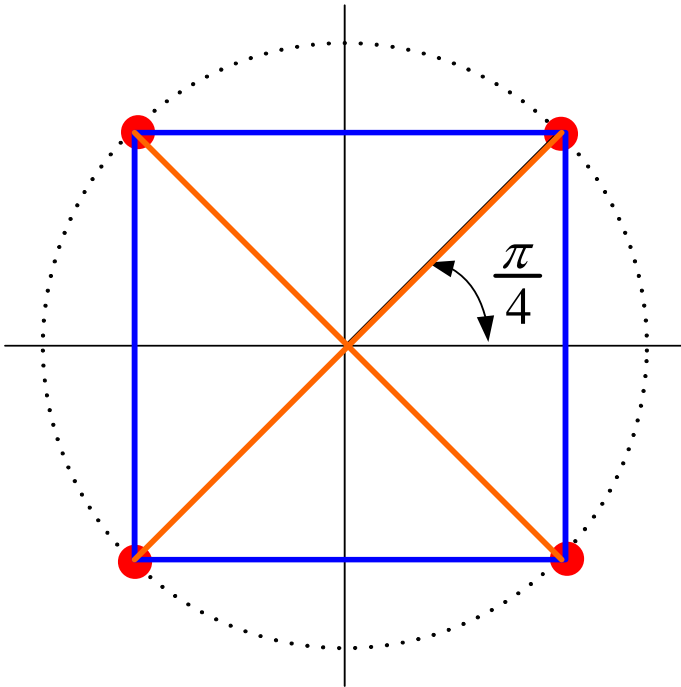
$$11 \rightarrow \Delta\theta = 45^\circ$$

$$10 \rightarrow \Delta\theta = -45^\circ$$

$$01 \rightarrow \Delta\theta = -135^\circ$$

$$00 \rightarrow \Delta\theta = 135^\circ$$

Offset QPSK (OQPSK)



In OQPSK:

- no 180° phase jumps,
- no zero-crossings
- less amplitude distortion

In OQPSK, the in-phase and quadrature phase symbols are time shifted over $1/(2D)$:

$$x'(t) = x(t)$$

$$y'(t) = y(t - \frac{1}{2D})$$

Half a symbol time delayed.

The signal spectrum is not affected by this time-delay between I- and Q- symbols.

Why do we do this?

Quadrature Amplitude Modulation (QAM)

In QAM we allow discrete phase values as well as discrete amplitude values. The general QAM structure is:

$$s(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t$$

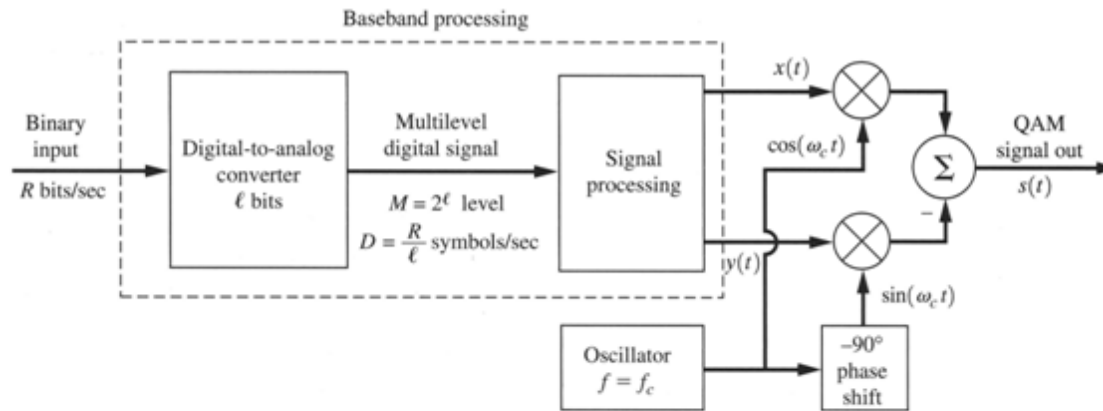
with: $g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}$

$x(t)$, $y(t)$ can take on a limited number of discrete values.

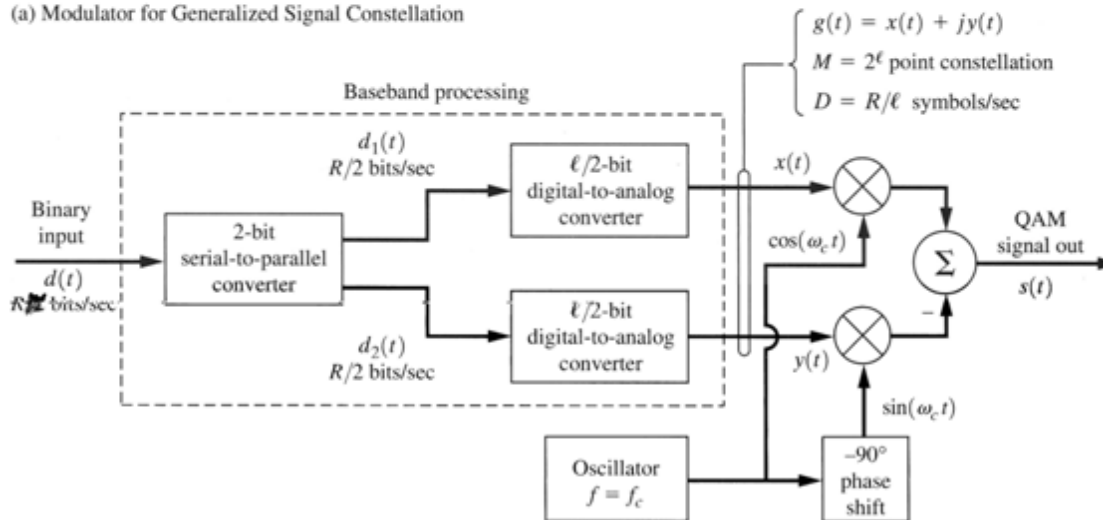
or

$R(t)$, $\theta(t)$ can take on a limited number of discrete amplitude- and phase values.

Quadrature Amplitude Modulation (QAM)



(a) Modulator for Generalized Signal Constellation



(b) Modulator for Rectangular Signal Constellation

Figure 5-31 Generation of QAM signals.

Quadrature Amplitude Modulation (QAM)

As seen from the I,Q-description, general QAM can be seen as two M-ASK signals modulating quadrature carriers:

$$x(t) = \sum_{n=-\infty}^{\infty} x_n h\left(t - \frac{n}{D}\right) = \sum_{n=-\infty}^{\infty} x_n h(t - nT_s)$$

$$y(t) = \sum_{n=-\infty}^{\infty} y_n h\left(t - \frac{n}{D}\right) = \sum_{n=-\infty}^{\infty} y_n h(t - nT_s)$$

$h(t)$ is the pulse shape: - rectangular
- raised cosine
- half sinewave

$$D = \frac{R_b}{l} = \frac{1}{T_s} \text{ is the symbol rate}$$

Quadrature Amplitude Modulation (QAM)

QAM consists of two orthogonal M-ASK signals modulating quadrature carriers:

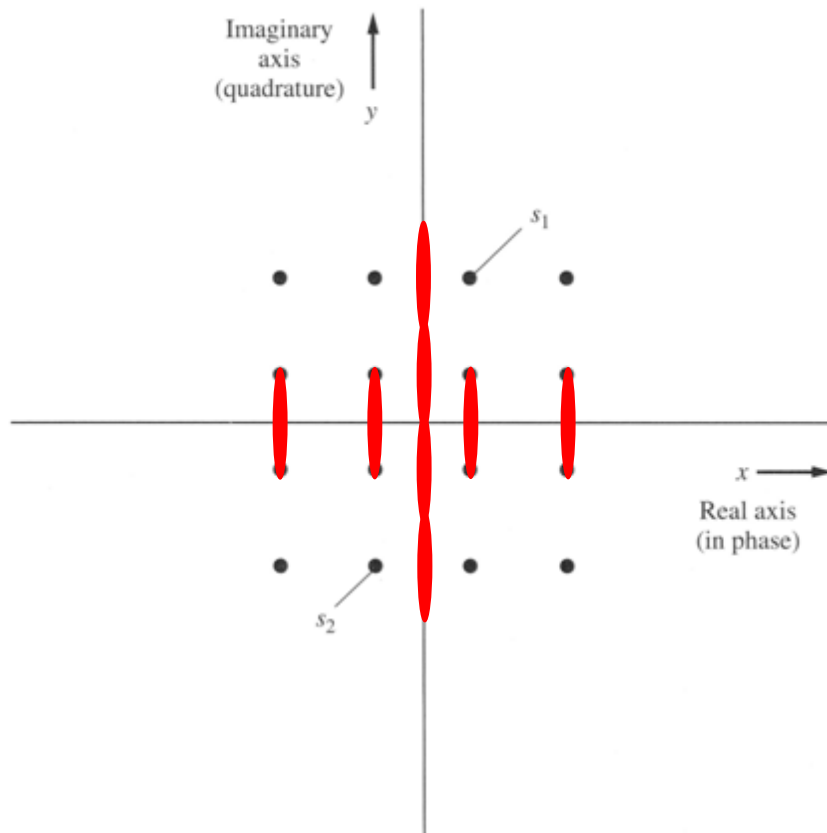


Figure 5-32 16-symbol QAM constellation (four levels per dimension).

$$x(t) = \sum_{n=-\infty}^{\infty} x_n h(t - nT_s)$$

$$y(t) = \sum_{n=-\infty}^{\infty} y_n h(t - nT_s)$$

For 16-QAM (black dots):

$$x_n, y_n \in \{-3, -1, +1, +3\}$$

(shown as red dots)

$$\text{Symbol rate } R_s = \frac{R_b}{4}$$

Power Spectral Density (PSD)

M-PSK and M-QAM with rectangular symbol shapes have the same spectral shapes as BPSK. However, the spectrum is a factor l (bits/symbol) narrower since $T_s = lT_b$.

$$P_g(f) \propto T_s \frac{\sin^2 \pi f T_s}{(\pi f T_s)^2} = lT_b \operatorname{sinc}^2 flT_b$$

Proof in Couch p. 397.

The *null-null* bandwidth is: $B_{T_{-0-0}} = \frac{2R_b}{l} = \frac{2}{T_s} = 2R_s (= 2D)$

Power Spectral Density (PSD)

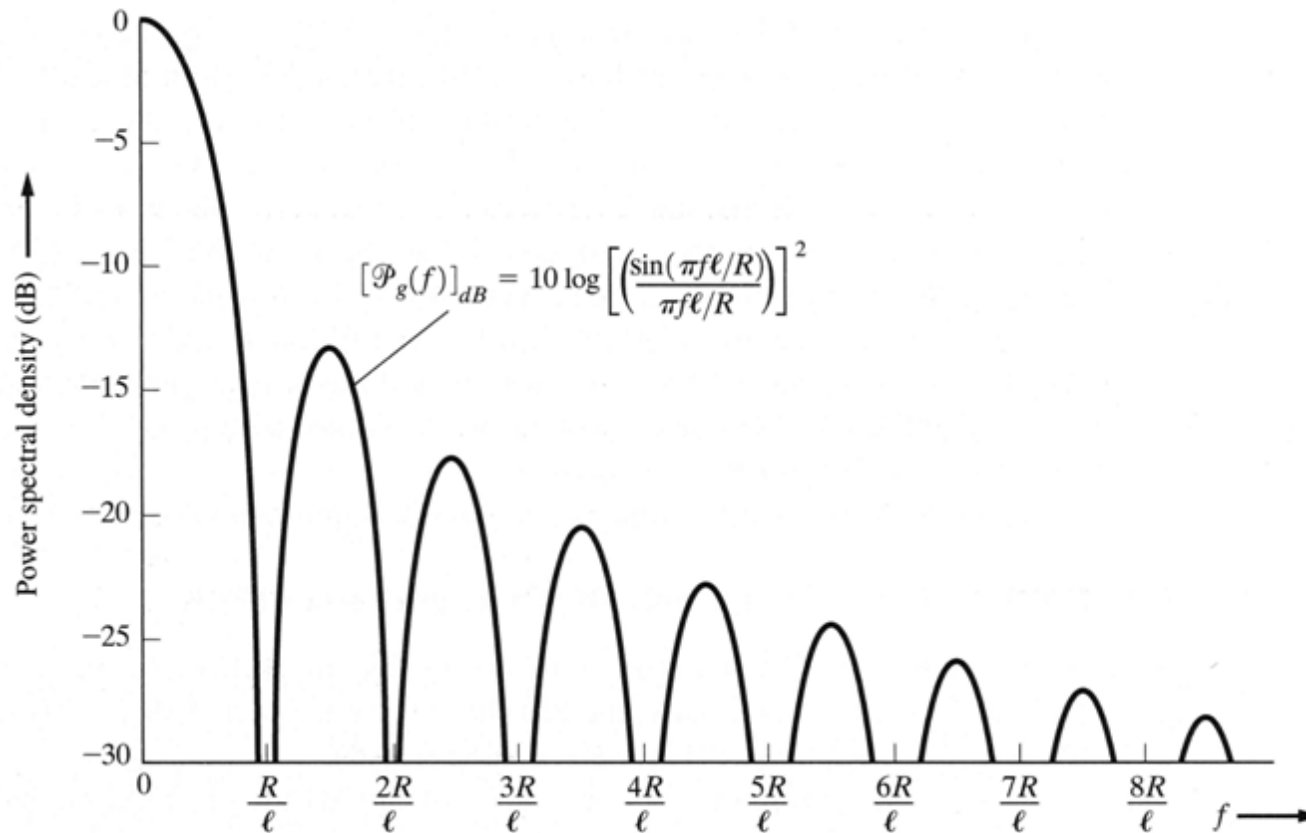


Figure 5–33 PSD for the complex envelope of MPSK and QAM with rectangular data pulses, where $M = 2^\ell$, R is the bit rate, and $R/\ell = D$ is the baud rate (positive frequencies shown). Use $\ell = 2$ for PSD of QPSK, OQPSK, and $\pi/4$ QPSK complex envelope.

Spectral efficiency: profit (1)

The spectral efficiency is defined as:

$$\eta \triangleq \frac{R_b \text{ [bit/s]}}{B_T \text{ [Hz]}} = \frac{R_b}{2D} = \frac{1/T_b}{2/lT_b} = \frac{l}{2} \left[\frac{\text{bit/s}}{\text{Hz}} \right] \quad \text{where } l = 2 \log M$$

For raised-cosine pulses we find with $B_T = (1+r)D = (1+r)R_b/l$:

$$\eta = \frac{l}{1+r} \left[\frac{\text{bit/s}}{\text{Hz}} \right]$$

When we can increase η by a factor of two, twice the data rate can be transmitted in the same bandwidth: twice as many users/turnover \Rightarrow doubling of the profit!

Spectral efficiency: cost

According to Shannon:

$$R_b \leq C = B \log^2 \left(1 + \frac{S}{N} \right) \Rightarrow \eta = \frac{R_b}{B} \leq \frac{C}{B} = \log^2 \left(1 + \frac{S}{N} \right)$$

So, for a larger η a larger S/N is required.

This is not so strange when we study Fig. 5.32 once again.

For larger l (\Rightarrow larger η), the signal states will be closer together when the signal power remains constant \Rightarrow larger probability of detection error due to noise.

Reliable detection is only possible if also S/N is increased!

Minimum Shift Keying (1)

What is the minimum frequency deviation ΔF which allows for orthogonal signaling when using FSK?

Let the FSK states be given by:

$$\begin{aligned} s_1(t) &= A_c \cos(\omega_1 t + \theta_1) \\ s_2(t) &= A_c \cos(\omega_2 t + \theta_2) \end{aligned} \quad kT_b \leq t \leq (k+1)T_b$$

For orthogonal signaling the following should hold:

$$\int_{kT_b}^{(k+1)T_b} s_1(t)s_2(t)dt = \int_{kT_b}^{(k+1)T_b} A_c^2 \cos(\omega_1 t + \theta_1) \cos(\omega_2 t + \theta_2) dt = 0$$

Minimum Shift Keying (2)

With application of:

$$2 \cos x \cos y = \cos(x - y) + \cos(x + y)$$

we get:

$$\begin{aligned} & \frac{\sin((\omega_1 - \omega_2)T_b + (\theta_1 - \theta_2)) - \sin(\theta_1 - \theta_2)}{\omega_1 - \omega_2} \\ & + \frac{\sin((\omega_1 + \omega_2)T_b + (\theta_1 + \theta_2)) - \sin(\theta_1 + \theta_2)}{\omega_1 + \omega_2} \\ & \approx \frac{\sin(2\pi h + (\theta_1 - \theta_2)) - \sin(\theta_1 - \theta_2)}{2\pi h / T_b} = 0 \end{aligned}$$

where we used $(\omega_1 - \omega_2)T_b = 2\pi \cdot 2\Delta F \cdot T_b \approx 2\pi h$.

Minimum Shift Keying (3)

Two situations:

1.) Continuous Phase FSK (CPFSK): $\theta_1 = \theta_2$

Orthogonal signaling when

$$\sin(2\pi h + (\theta_1 - \theta_2)) - \sin(\theta_1 - \theta_2) = 0$$

Thus the minimum h which holds: $2\pi h = \pi \Rightarrow h = 0.5$,
which corresponds to the minimum shift:

$$\omega_1 - \omega_2 = 2\pi \cdot 2\Delta F = \frac{2\pi h}{T_b} = \frac{\pi}{T_b} \Rightarrow \Delta F = \frac{1}{4T_b} = \frac{R_b}{4}$$

FSK with $h = 0.5$ is also indicated as FFSK (Fast FSK) or Minimum Shift Keying (MSK).

Minimum Shift Keying (4)

2.) Discontinuous phase FSK: $\theta_1 \neq \theta_2$

Orthogonal signaling when

$$\sin(2\pi h + (\theta_1 - \theta_2)) - \sin(\theta_1 - \theta_2) = 0$$

Now we find: $2\pi h = 2\pi \Rightarrow h = 1$

$$\text{and } \Delta F = \frac{1}{2T_b} = \frac{R_b}{2},$$

which is twice as large as for CPFSK.

Minimum Shift Keying (5)

MSK is a form of FSK with $h = 0.5$.

The complex envelope for Minimum Shift Keying is given by:

$$g(t) = A_c e^{j\theta(t)}$$

with $\theta(t) = 2\pi\Delta F \int_0^t m(\lambda) d\lambda = \frac{\pi t}{2T_b} m(t) \quad 0 \leq t \leq T_b$

A $\frac{\pi}{2}$ phase shift per T_b .

where

$$\Delta F = \frac{R_b}{4} = \frac{1}{4T_b}, \quad D_f = 2\pi\Delta F = \frac{\pi}{2T_b} = \frac{\pi R_b}{2}, \quad \text{and } m(t) \in \{-1, +1\},$$

Minimum Shift Keying (6)

Now $g(t) = A_c \exp\left(jm(t)\frac{\pi t}{2T_b}\right) = x(t) + jy(t) \quad 0 \leq t \leq T_b$

With $x(t) = A_c \cos\left(\pm \frac{\pi t}{2T_b}\right) \quad kT_b \leq t \leq (k+2)T_b$

$$y(t) = A_c \sin\left(\pm \frac{\pi t}{2T_b}\right) \quad (k+1)T_b \leq t \leq (k+3)T_b$$

Since $s(t) = x(t)\cos\omega_c t - y(t)\sin\omega_c t$ we can conclude that MSK is equivalent to OQPSK with half-sine symbol shapes with period $2T_b$. The odd bits determine $y(t)$ and the even bits $x(t)$.

Minimum Shift Keying (7)

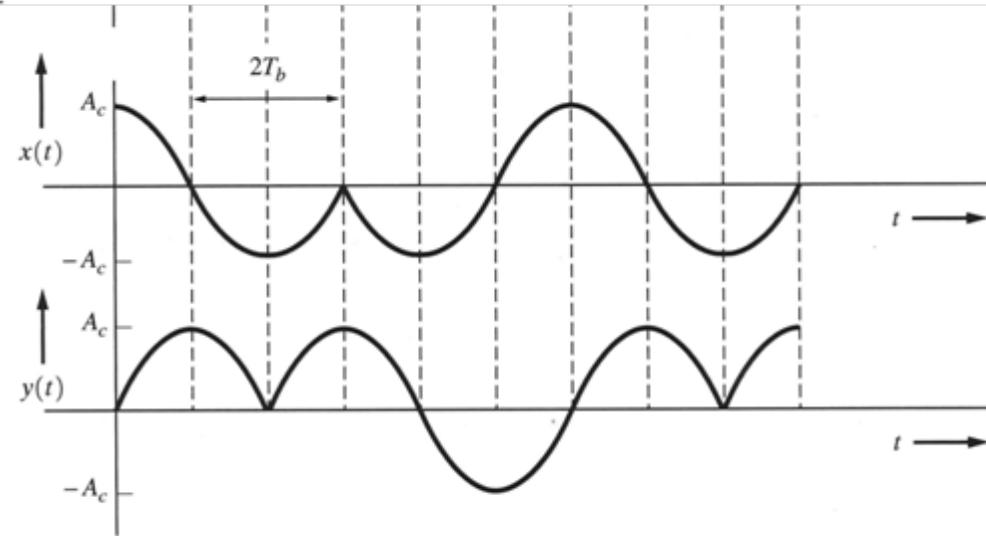
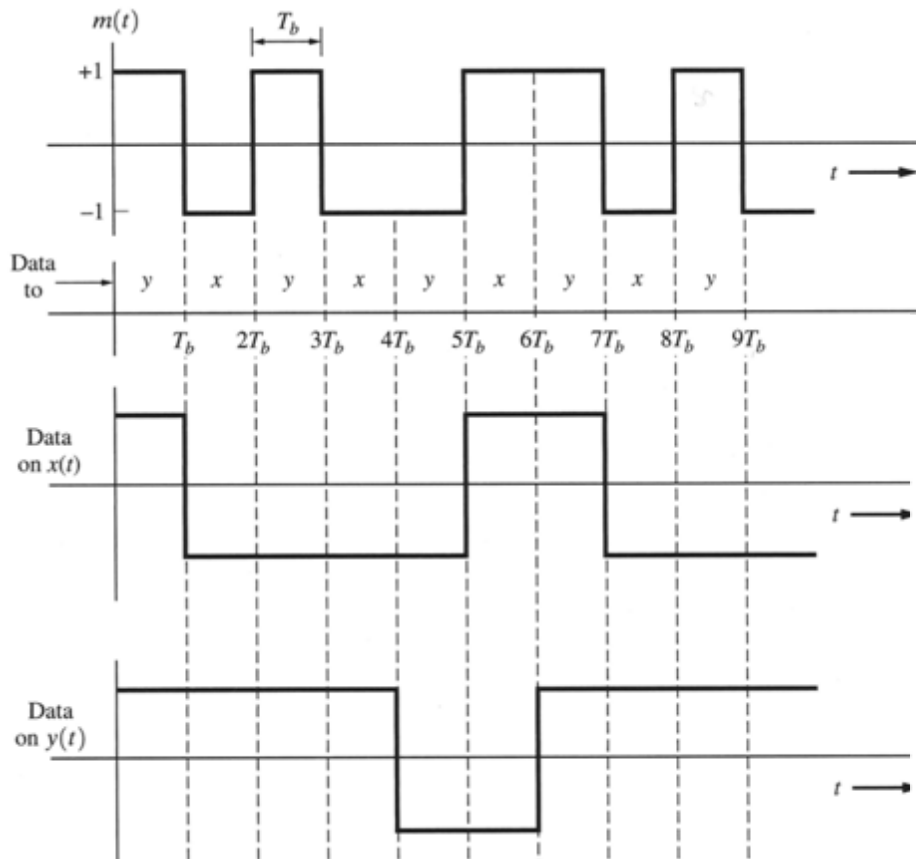


Figure 5-34 MSK quadrature component waveforms (Type II MSK).

Power Spectral Density of MSK

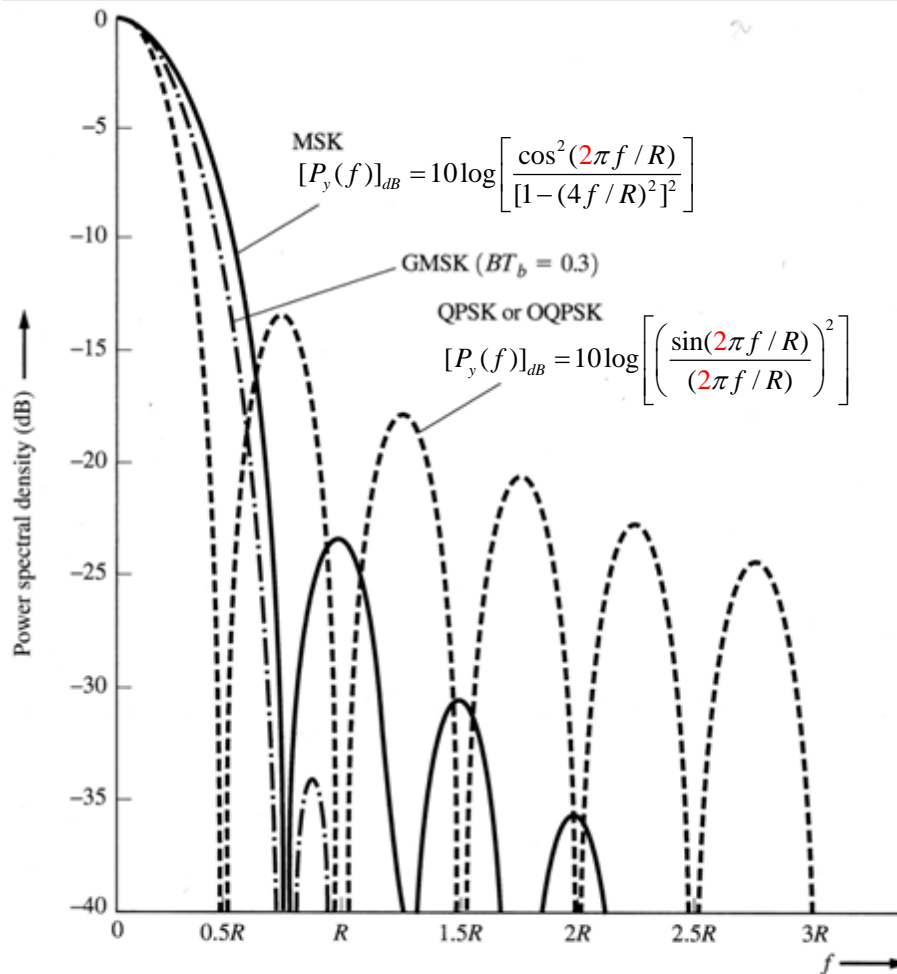
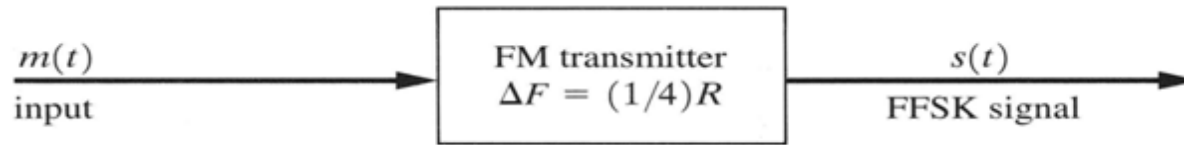


Figure 5-35 PSD for complex envelope of MSK, GMSK, QPSK, and OQPSK, where R is the bit rate (positive frequencies shown).

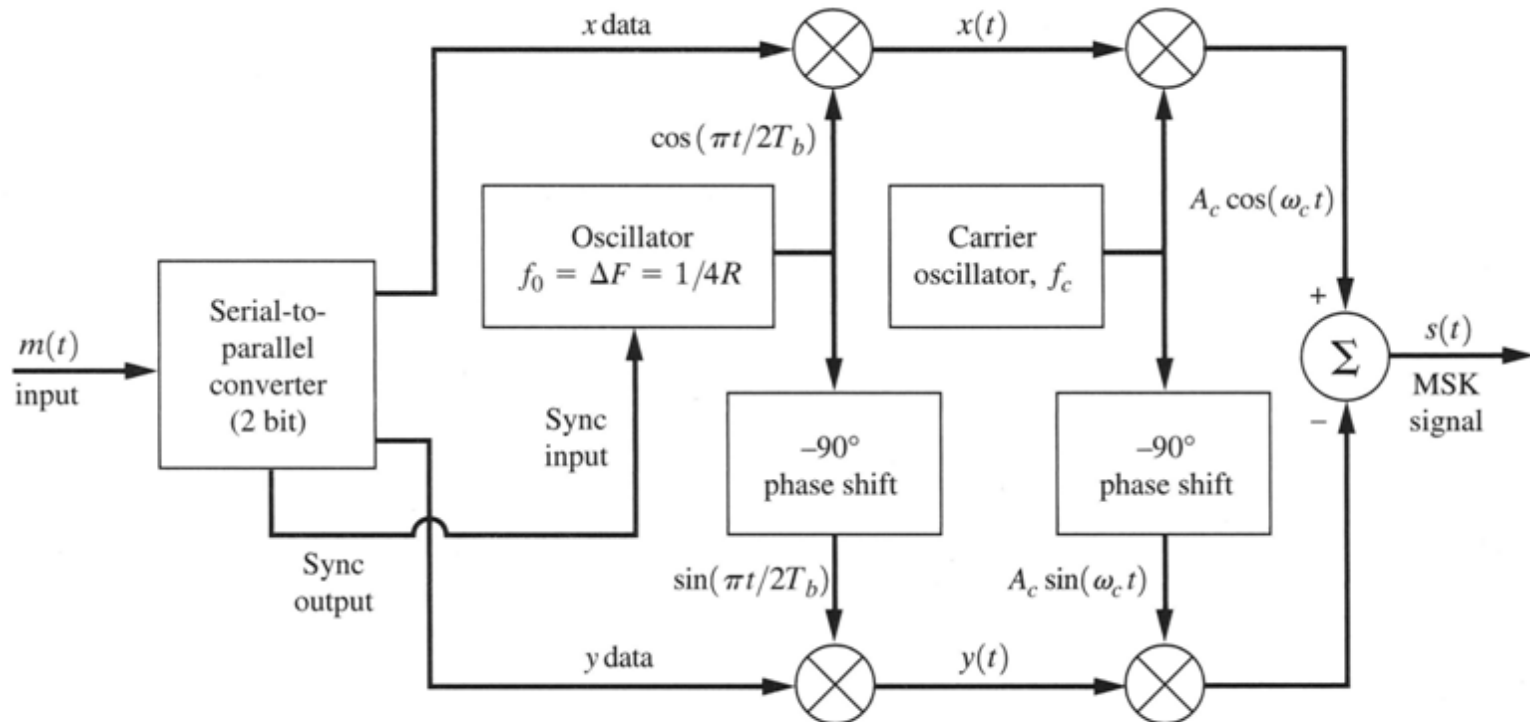
The spectral main lobe of MSK is 1.5x wider than for QPSK due to the half-sine pulse shape, however, the side lobes decrease much faster, because the pulse shape contains less high frequency components (no abrupt changes): better adjacent channel properties.

In GMSK (Gaussian Minimum Shift Keying), Gaussian shaped pulses are used with even more gradual slopes and therefore less high frequency components and better adjacent channel properties.

Generation of MSK



(a) Generation of Fast Frequency-Shift Keying (FFSK)



(b) Parallel Generation of Type I MSK (This will generate FFSK if a differential encoder is inserted at the input.)

Spectral efficiency of digital signals

Table 5–7 SPECTRAL EFFICIENCY OF DIGITAL SIGNALS

Type of Signal	Spectral Efficiency, $\eta = \frac{R}{B_T} \left(\frac{\text{bits/s}}{\text{Hz}} \right)$	
	Null-to-Null Bandwidth	30-dB Bandwidth
OOK and BPSK	0.500	0.052
QPSK, OQPSK, and $\pi/4$ QPSK	1.00	0.104
MSK	0.667	0.438
16 QAM	2.00	0.208
64 QAM	3.00	0.313

Orthogonal Frequency Division Multiplexing (OFDM)

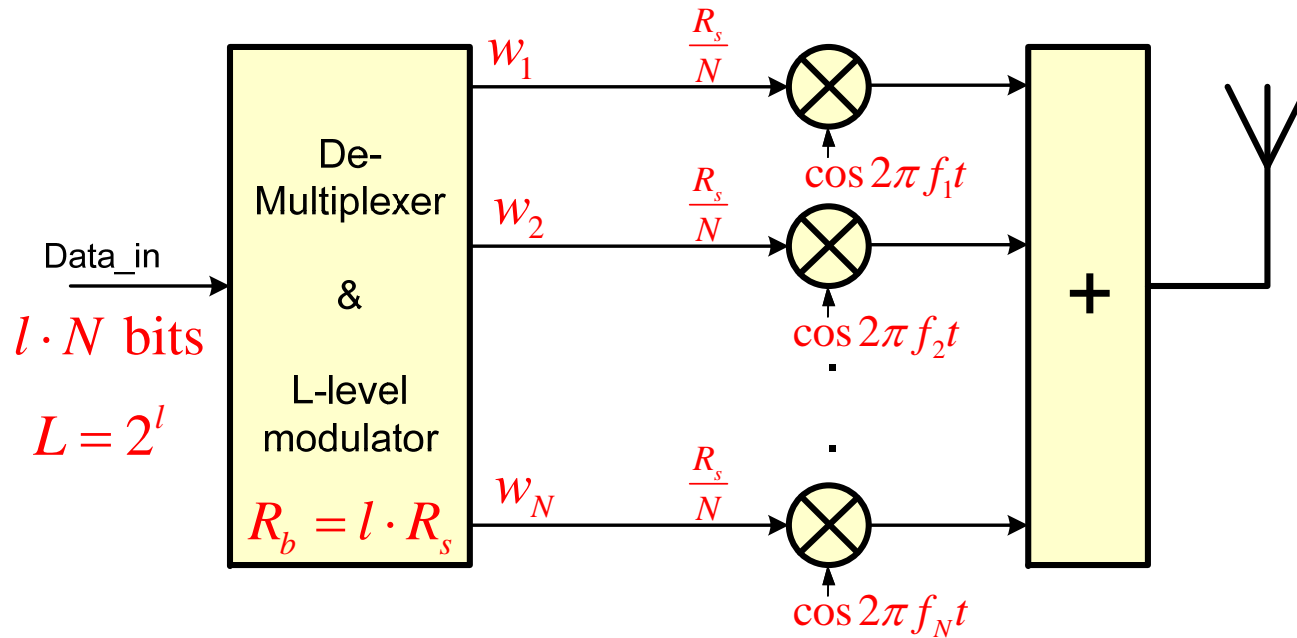
In Orthogonal Frequency Division Multiplexing (OFDM), a high bit rate signal with symbol rate R_s is split up in N parallel data sub-streams of a lower symbol rate $R_{ss} = R_s / N$.

Each of the N low rate sub-streams modulates one of N parallel independent sub-carriers.

⇒ the symbol time is N times increased: $T_{ss} = NT_s$

⇒ very bandwidth efficient

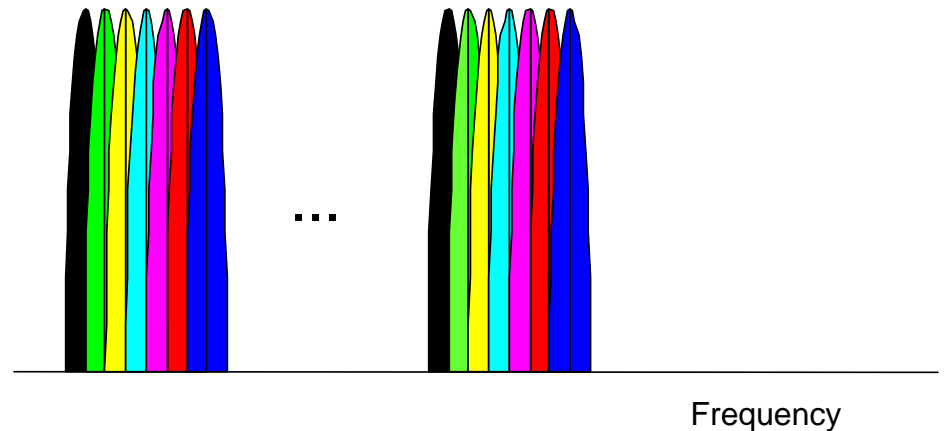
Orthogonal Frequency Division Multiplexing (OFDM)



OFDM transmission

Subcarrier symbol rate $R_{ss} = \frac{R_s}{N}$

Subcarrier symbol time $T_{ss} = NT_s = \frac{N}{R_s}$



Generation of OFDM (1)

OFDM belongs to the family of multi-carrier (MC) modulation.

An OFDM signal consists of a number of N carriers indicated as subcarriers $\{1, 2, \dots, N\}$.

The signal has the following characteristics:

1. Subcarriers are orthogonal
2. Arbitrary modulation of the subcarriers: BPSK, QPSK, QAM, etc.
3. High spectral efficiency
4. Relatively simple generation and detection

Generation of OFDM (2)

For orthogonality between two modulated subcarriers: $s_1(t)$, $s_2(t)$

$$\int_0^{T_{ss}} s_1(t) \cdot s_2(t) dt = 0 \Leftrightarrow \int_0^{T_{ss}} w_1 \cos(2\pi f_1 t + \theta_1) \cdot w_2 \cos(2\pi f_2 t + \theta_2) dt$$

Due to modulation $\theta_1 \neq \theta_2$

$$\cong \frac{w_1 w_2}{2} \left[\frac{\sin(2\pi(f_1 - f_2)T_{ss} + (\theta_1 - \theta_2)) - \sin(\theta_1 - \theta_2)}{2\pi(f_1 - f_2)} \right]$$

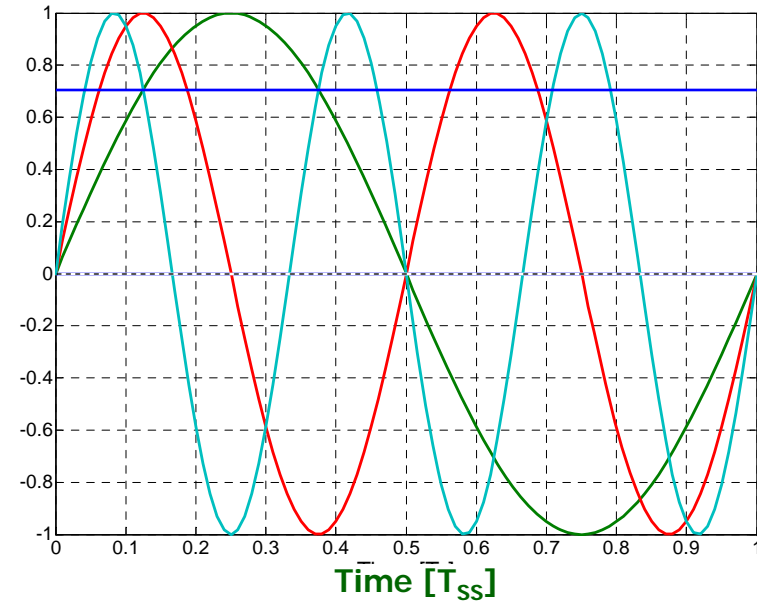
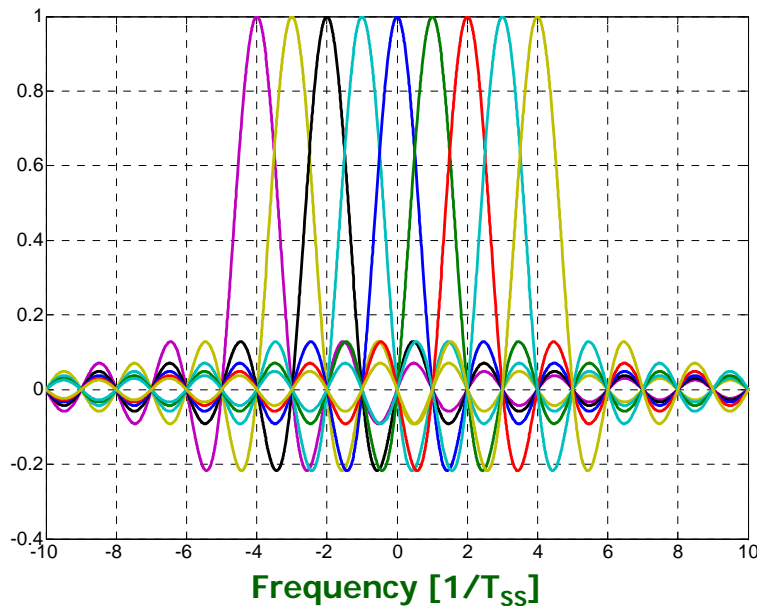
With $w_1 e^{j\theta_1}$, $w_2 e^{j\theta_2}$ complex amplitudes of the modulated signal states.

Orthogonality is obtained for: $f_1 - f_2 = \Delta f = \frac{n}{T_{ss}} = n \cdot R_{ss}$ with $n \in \mathbb{Z}$, $n \neq 0$

For minimum separations between subcarrier center frequencies: $\Delta f = R_{ss}$

Generation of OFDM (3)

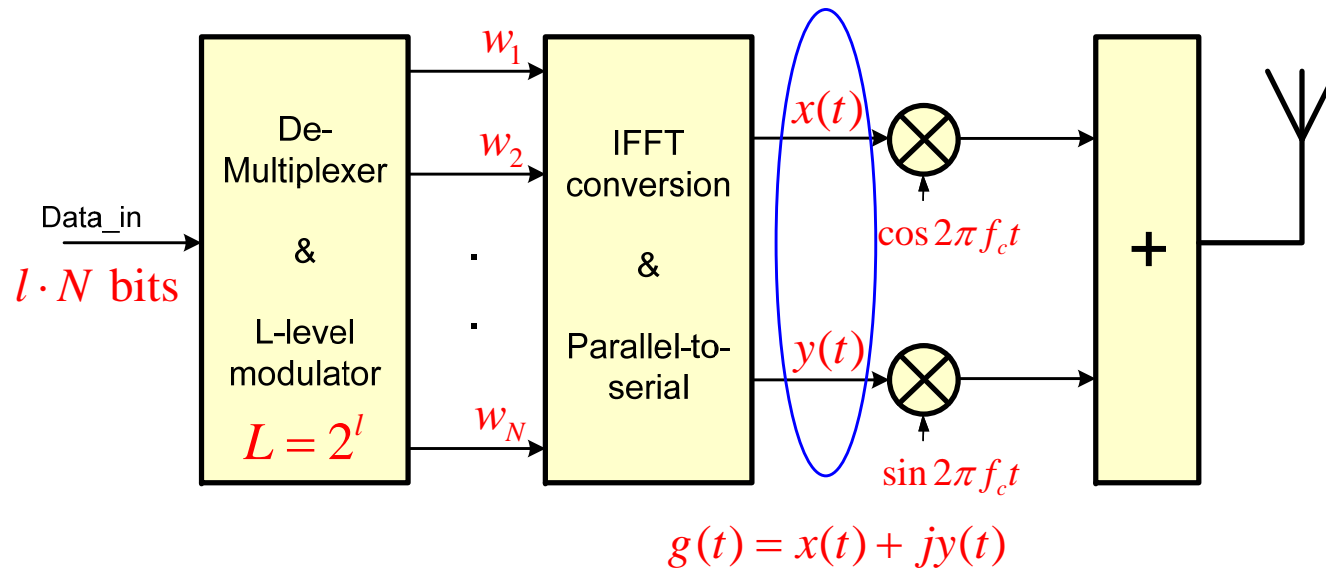
Orthogonal sub-carriers (in baseband) in frequency and time domain.



The sub-carrier frequencies are integer multiples of the sub-carrier symbol rate:

$$f_k = k \cdot \Delta f \quad \text{with} \quad \Delta f = \frac{1}{T_{ss}} = R_{ss}$$

Generation of OFDM (4)



A simple way to generate the OFDM baseband signal is by using the Inverse Fast Fourier Transform (IFFT):

$$g_i = \frac{1}{N} \sum_{k=1}^N w_k e^{j\{\theta_k + 2\pi(k-1)(i-1)/N\}} \quad \begin{array}{l} \text{for } i = 1, \dots, N \text{ samples} \\ \text{for } k = 1, \dots, N \text{ frequencies} \end{array} \Rightarrow \begin{array}{l} N \text{ complex samples} \\ \text{per OFDM symbol of} \\ \text{duration } T_{ss} \end{array}$$

$\{g_i\}$ is the i^{th} sample of a symbol of the complex envelope $g(t)$ of an OFDM signal. By I&Q-mixing, the signal is brought to the desired center frequency.

Generation of OFDM (5)

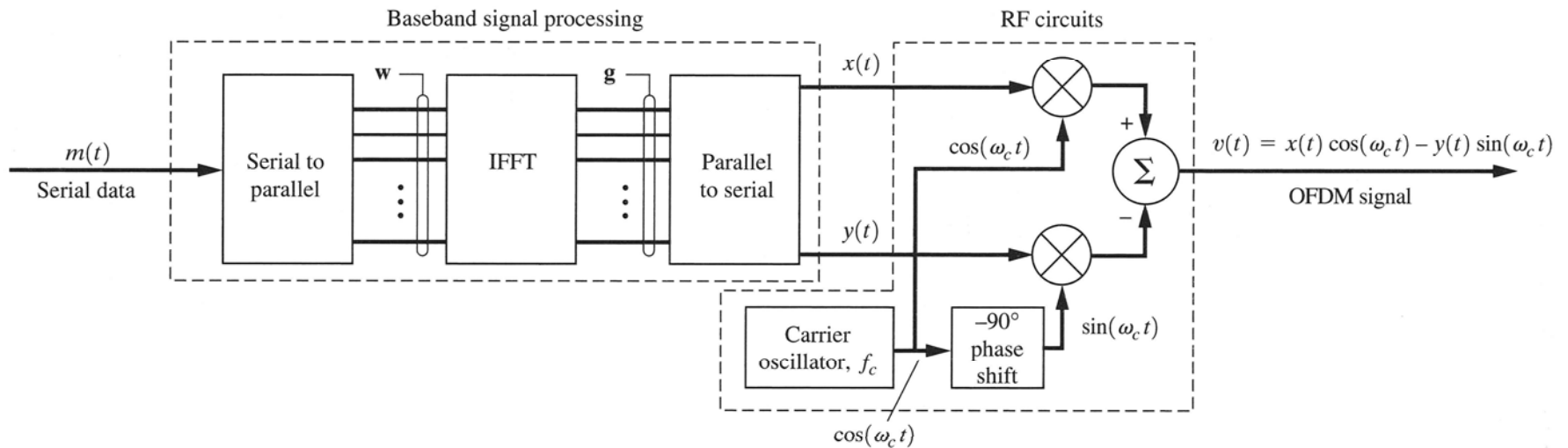


Figure 5-37 OFDM transmitter.

Generation of OFDM (2)

Some important parameters of OFDM:

1. The symbol rate is equal for each subcarrier: R_{ss}
2. The corresponding symbol period $T_{ss} = \frac{1}{R_{ss}}$ is therefore also fixed per subcarrier.
3. An OFDM symbol consists of the sum of all subcarrier symbols transmitted during a symbol time T_{ss} .
4. An OFDM symbol time is equal to the subcarrier symbol time T_{ss} .
5. For equal L -level modulation of the subcarriers, the symbol rate is given by:

$$R_{ss} = \frac{R_b}{l \cdot N} = \frac{R_s}{N} \quad l = 2^{\log L}$$

Spectral efficiency

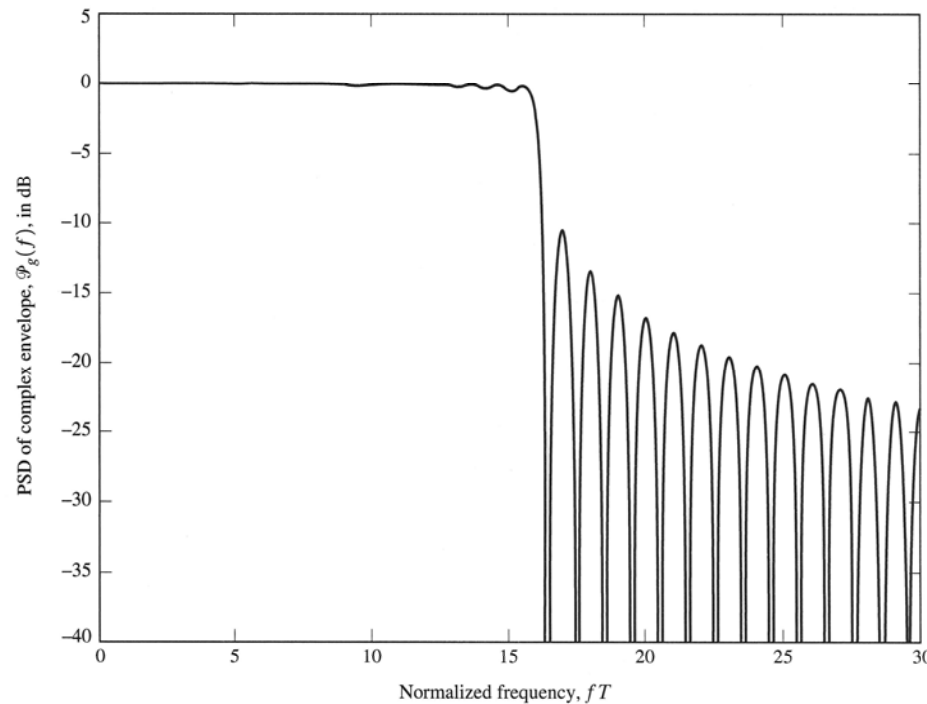


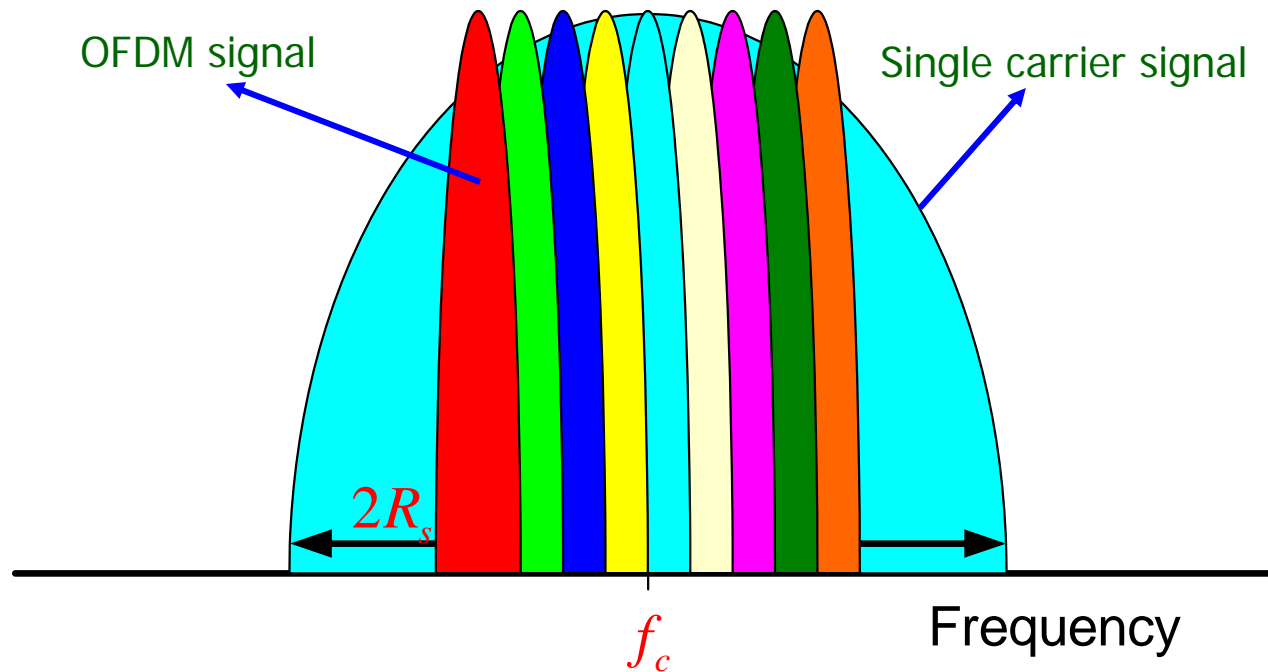
Figure 5-38 PSD for the complex envelope of OFDM with $N = 32$.

The PSD of the equivalent baseband signal is:

$$P_g(f) = A_c^2 \overline{|w_n|^2} T_{ss} \sum_{n=0}^{N-1} \left| \frac{\sin[\pi(f - nf_\Delta)T_{ss}]}{\pi(f - nf_\Delta)T_{ss}} \right|^2$$

where $\overline{|w_n|^2}$ is the mean power in w_n and it is assumed that $\overline{w_n} = 0$.

Spectral efficiency



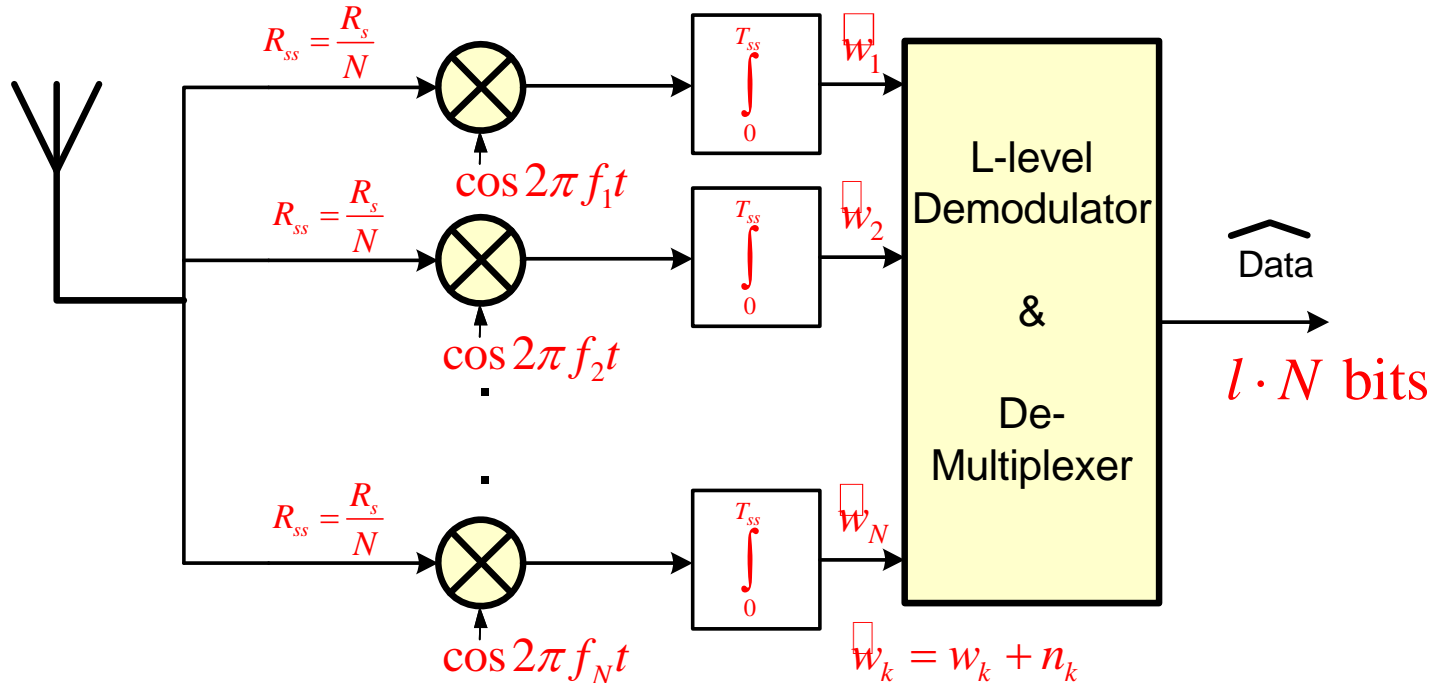
A single carrier system with a symbol rate R_s requires a transmission bandwidth $B_{T,0-0} = 2R_s$

An OFDM system with N subcarriers, each with a symbol rate $R_{ss} = R_s / N$

$$\Rightarrow B_T = (N + 1)R_{ss} = (N + 1)\frac{R_s}{N} \approx R_s = D_s \text{ for } N \gg 1$$

Due to overlapping subcarrier spectra: \approx doubling of the spectral efficiency

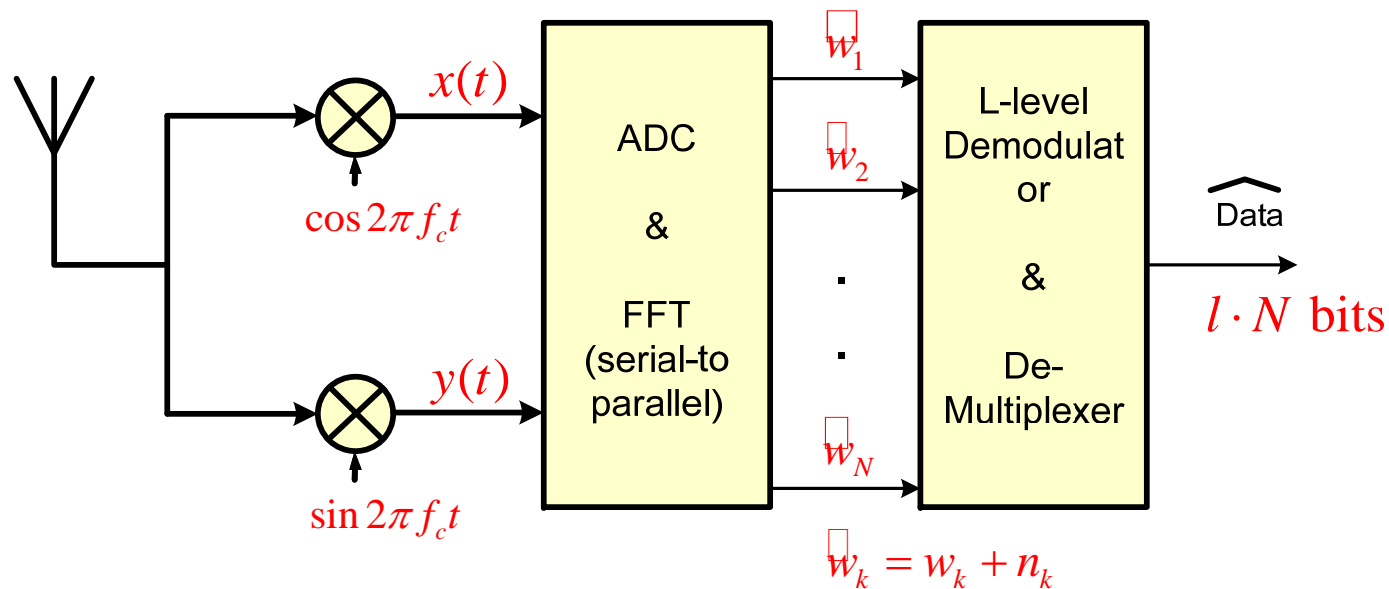
OFDM receiver (1)



The OFDM receiver performs basically the reverse operations of the transmitter. A direct analog implementation becomes very complex.

This has delayed practical application of OFDM till the end of the 1990s when fast digital signal processing components (especially hardware implemented FFT/IFFT) became available.

OFDM receiver (2)



The FFT operates on N complex samples taken uniformly over T_{ss} .
The output of the FFT are N complex amplitudes (symbol states) belonging to each of the subcarriers.

Accurate synchronization in time and frequency is crucial. Errors result in ISI and inter-carrier interference (ICI), respectively.

Adaptive loading

In slowly changing channels (like in static systems), the variable fading of the subcarriers can be exploited to maximize the data rate.

1. good subcarriers are loaded with a higher order modulation (more bits per symbol), but maintain the same symbol rate,
2. bad carrier use a low order modulation or are switched off.

The subcarrier qualities have to be known at the transmitter and the assigned modulation levels have to be known at the receiver!

Direct Sequence Spread Spectrum

Direct Sequence Spread Spectrum (DS-SS) modulation was originally developed for military communication systems between 1950 - 1980 to ensure robust and covered communications.

In DS-SS modulation, the information signal is spread over a much wider bandwidth than required based on the symbol rate.

A narrowband signal is made wideband !

How?

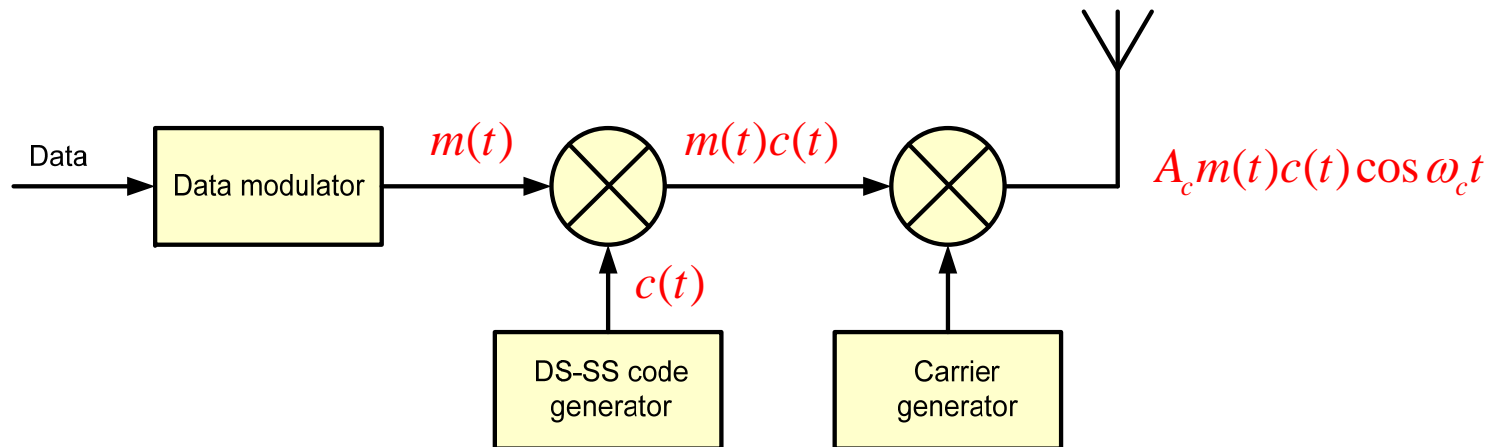
Signal spreading over a much wider bandwidth is achieved by multiplying the signal at the transmitter by a wideband code signal.

Why?

- to create robustness against: multipath fading, time dispersion, and interference
- to allow multiple access: multiple users can share the same channel;
- coveredness: the low PSD makes it difficult to detect and eavesdrop.

DS-SS Transmitter (1)

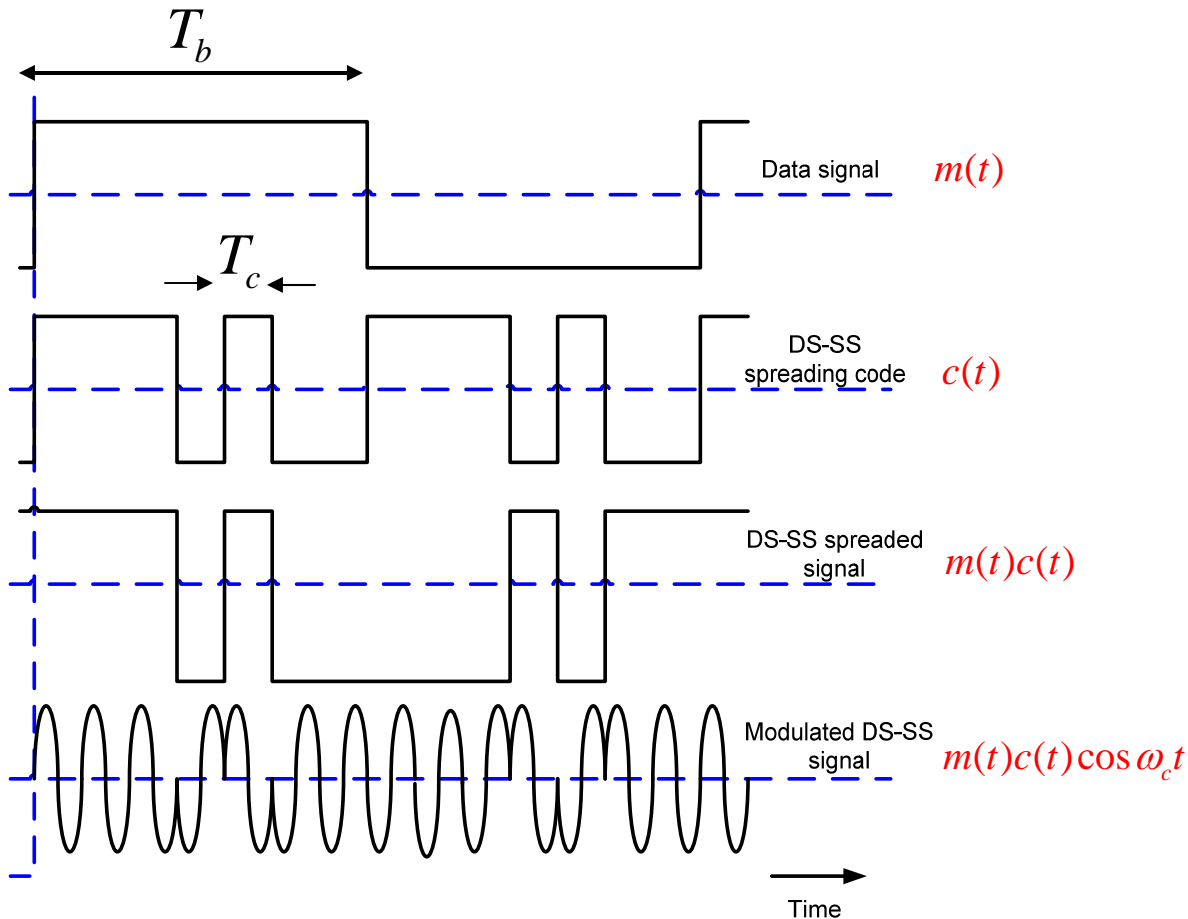
In DS-SS, spreading of the signal over a much wider bandwidth is achieved by multiplying the signal at the transmitter by a code sequence $c(t) \in \{-1, +1\}$ consisting of code chips.



Transmitted signal: $s(t) = A_c m(t)c(t) \cos \omega_c t$ where $m(t)$ represents the information signal, e.g. for BPSK $m(t) = \pm 1$.

DS-SS Transmitter (2)

Signals in a DS-SS transmitter for a BPSK modulated data signal.



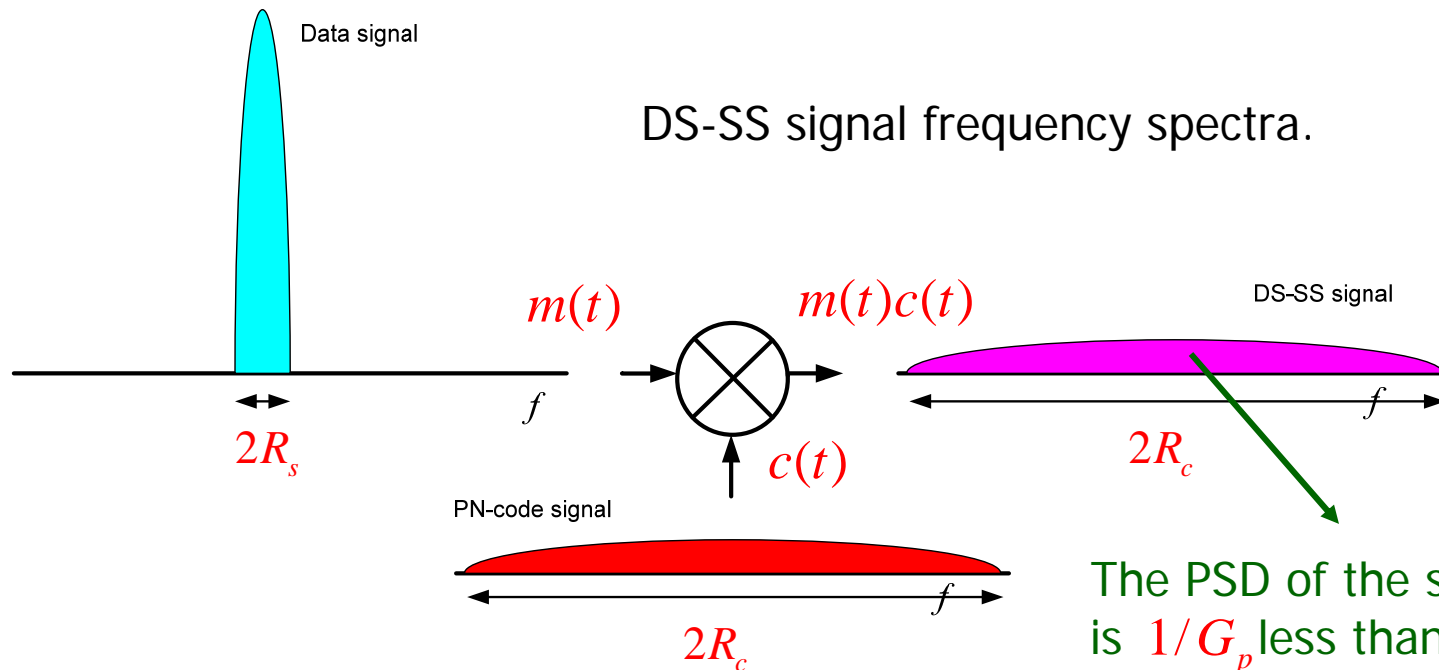
The data modulation can be arbitrary, e.g. BPSK, DPSK, QPSK, MSK, QAM, etc.

The code causes 'random' phase jumps of $\{0, \pi\}$ rad.

Short code: repeats every symbol time

Long code: extends over many symbol times

DS-SS Transmitter (3)

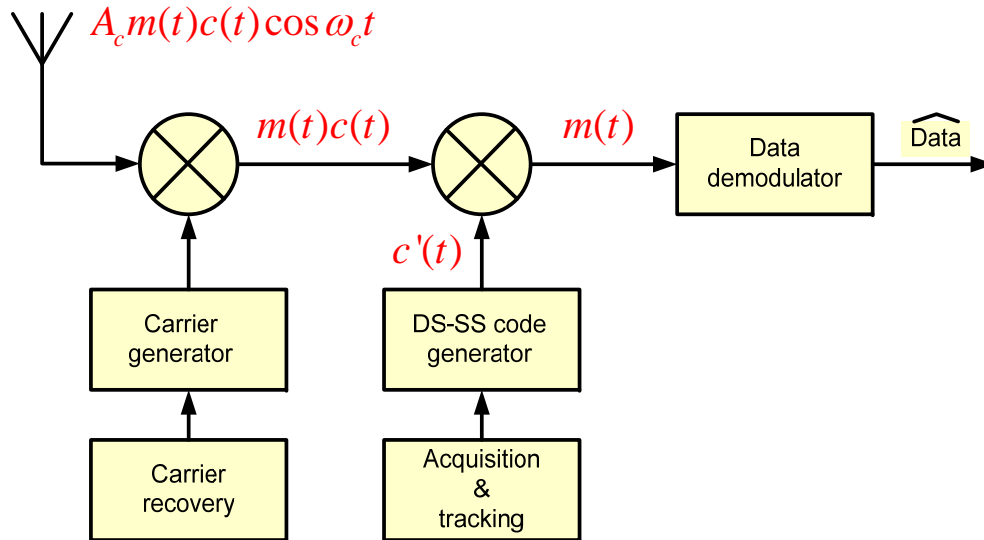


The PSD of the spread signal is $1/G_p$ less than the PSD of the un-spread signal.

The chip rate $R_c = \frac{1}{T_c} \gg$ data symbol rate $R_s = \frac{1}{T_s}$.

The ratio $G_p \triangleq \frac{BW_{\text{DS-SS}}}{BW_{\text{Data signal}}} = \frac{2R_c}{2R_s} = \frac{T_s}{T_c}$ is called the Processing Gain or spreading factor.

DS-SS Reception (1)

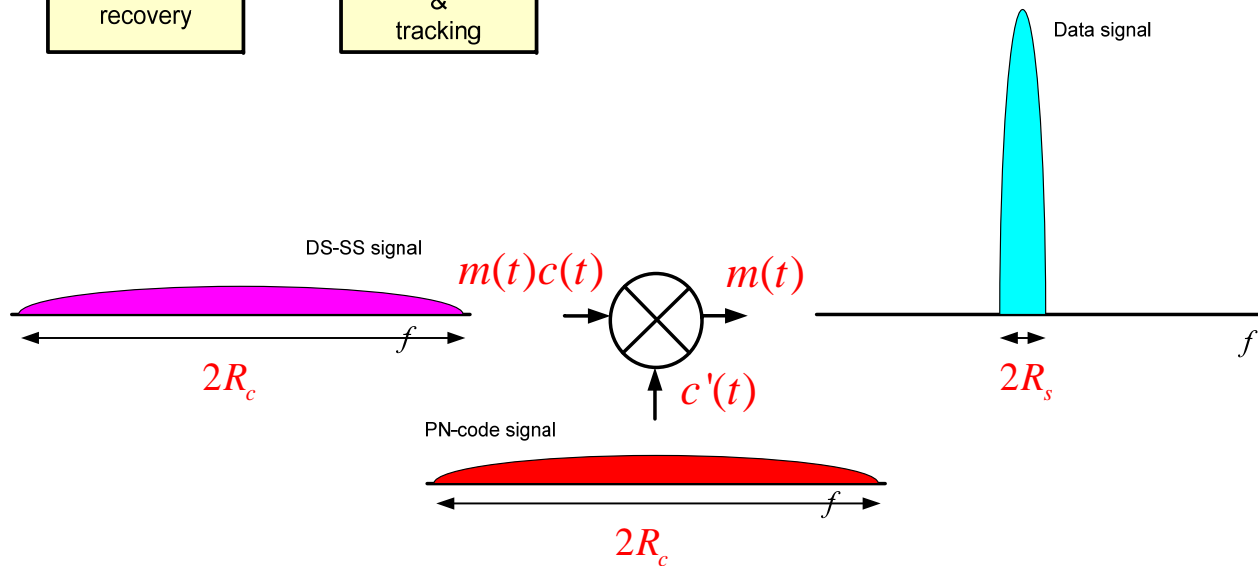


At the DS-SS receiver the reverse operations are performed.

We can recover the signal at the receiver only iff:

$$c(t)c'(t)m(t) = m(t) \Rightarrow$$

- the code is known,
- perfect synchronization.

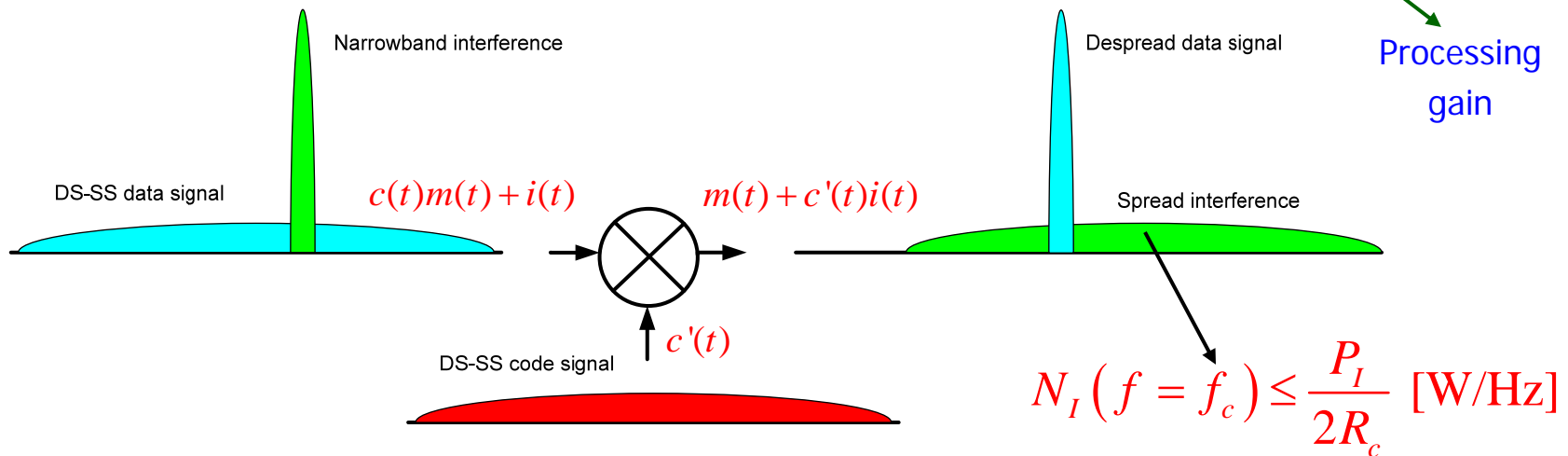


DS-SS Reception (2)

DS-SS is very robust against interference. When de-spreading the desired signal, the interference is spread.

Only a fraction $1/G_p$ of the spread interference overlaps with the de-spread data signal spectrum:

Effective interference power:
$$I_{eff} = P_I \frac{BW_{Data}}{BW_{Spread-Interference}} = P_I \frac{2R_s}{2R_c} = \frac{P_I}{G_p}$$



DS-SS Reception (3)

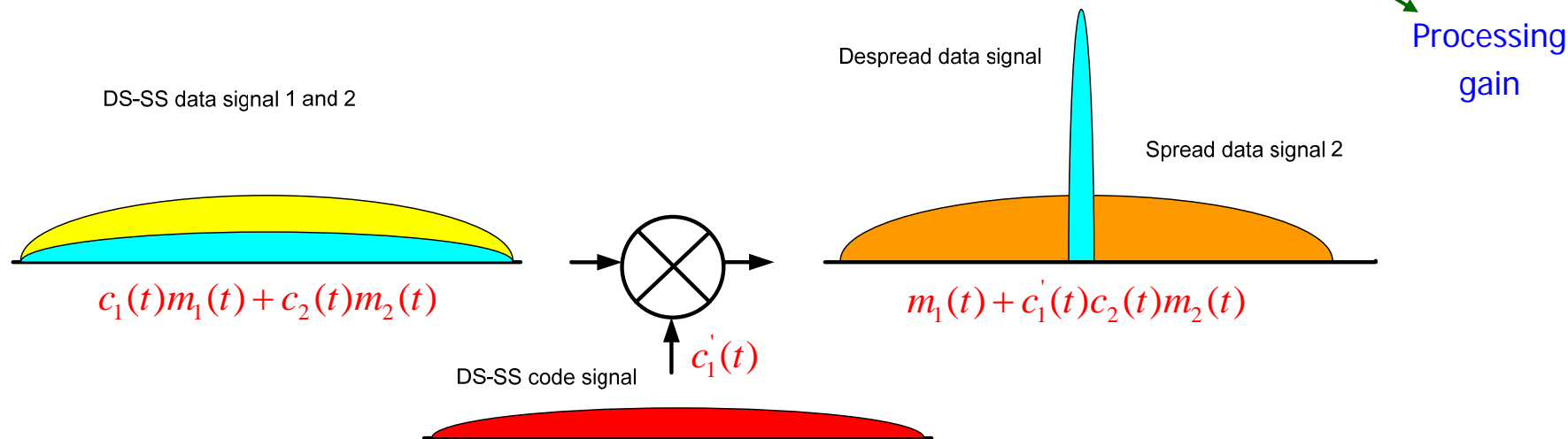
DS-SS is also robust against other DS-SS interfering signals. When the SS-code or code phase is different, the signal remains spread.

Also in this situation, only a fraction $1/G_p$ of the interfering spectrum overlaps with the desired de-spread data signal spectrum:

Effective interference power: $I_{eff} = P_I \frac{BW_{Data}}{BW_{DS-SS}} = P_I \frac{2R_s}{2R_c} = \frac{P_I}{G_p}$

Annotations for the equation:

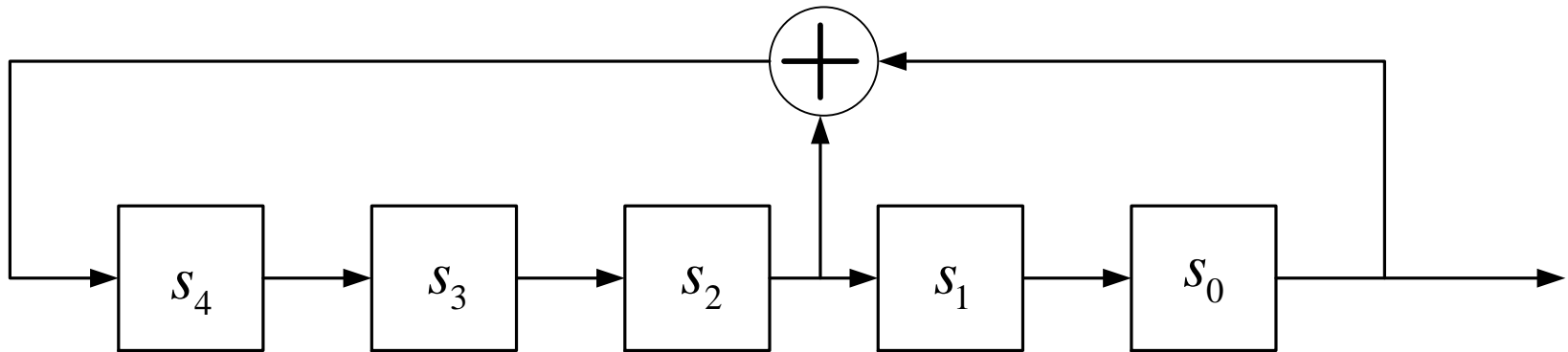
- $c_1(t), c_2(t)$ synchronized (points to the $c_1(t)$ and $c_2(t)$ terms in the numerator)
- Processing gain (points to the G_p term in the denominator)



Pseudo-Noise code generators (1)

A DS-SS code is a "pseudo random" sequence of $\{-1, 1\}$. There are several ways to generate such codes depending on the required properties. One way is to use **Maximum Length sequences** which are also called **Pseudo-Noise (PN)** codes because of their noise like spectral properties.

Maximum Length (ML) sequences can be digitally generated in a simple way using a shift register with feedback.



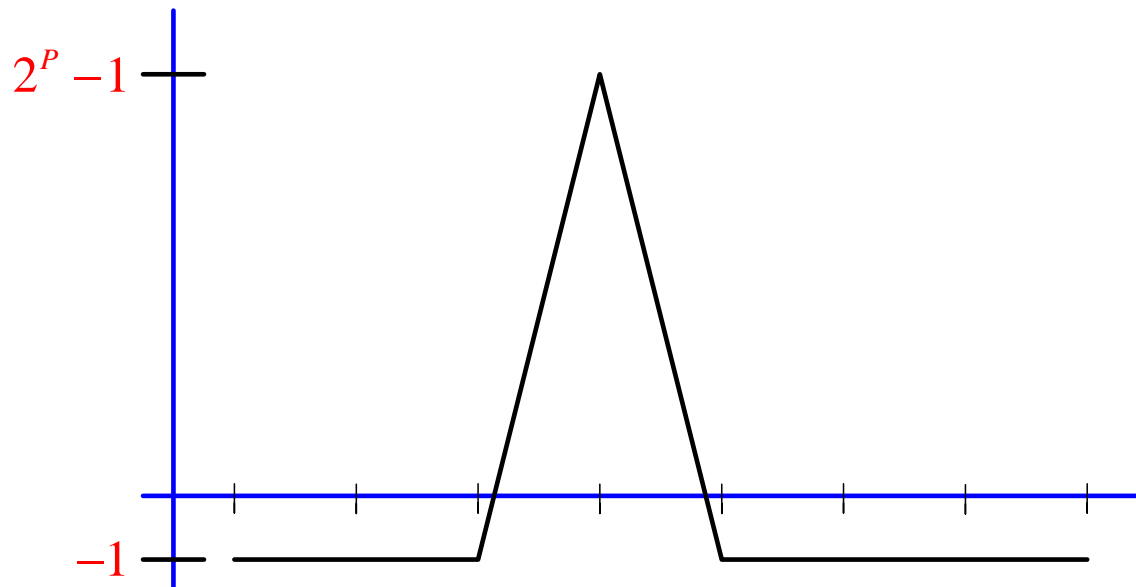
A ML-code generator using a 5-stage shift register with code length **31**.

Pseudo-Noise code generators (2)

A shift register length of P stages results in an ML-code of $2^P - 1$ chips.

The autocorrelation of an ML-code is given by: $R_{cc}(\tau) = \int_0^{T_{code}} c(t)c(t-\tau)dt$

Good auto-correlation properties are important for reliable synchronization and detection. The ML-code has very good auto-correlation features.



Auto-correlation of a 7 chip
($P = 3$) ML-code:

- width autocorrelation peak is $2T_c$
- outside correlation peak very low correlation value -1
- maximum at $2^P - 1$

Figure 10.10 shows the power spectrum $\mathcal{P}_c(f)$ versus frequency f . The plot features a central peak at $f=0$ and side lobes. The weight of the central peak is labeled as $\text{Weight} = 1/N^2$. The weight of the side lobes is labeled as $\text{Weight} = \left(\frac{N+1}{N^2}\right) \left(\frac{\sin(\pi n/N)}{\pi n/N}\right)^2$. The frequency axis is marked with $-3/T_c, -2/T_c, -1/T_c, 0, 1/T_c, 2/T_c, 3/T_c$. The central frequency is labeled $f_0 = 1/(NT_c)$.

Figure 5-40 Autocorrelation and PSD for an m -sequence PN waveform.

Spectral efficiency of DS-SS

For DS-SS, with M -level modulation, the spectral efficiency is given by:

$$\eta \approx \frac{R_b}{B_T} = \frac{lR_s}{B_T} = \frac{lR_s}{2R_c} = \frac{l}{2G_p} \left[\frac{\text{bit/s}}{\text{Hz}} \right] \quad \text{where } l = 2 \log M$$

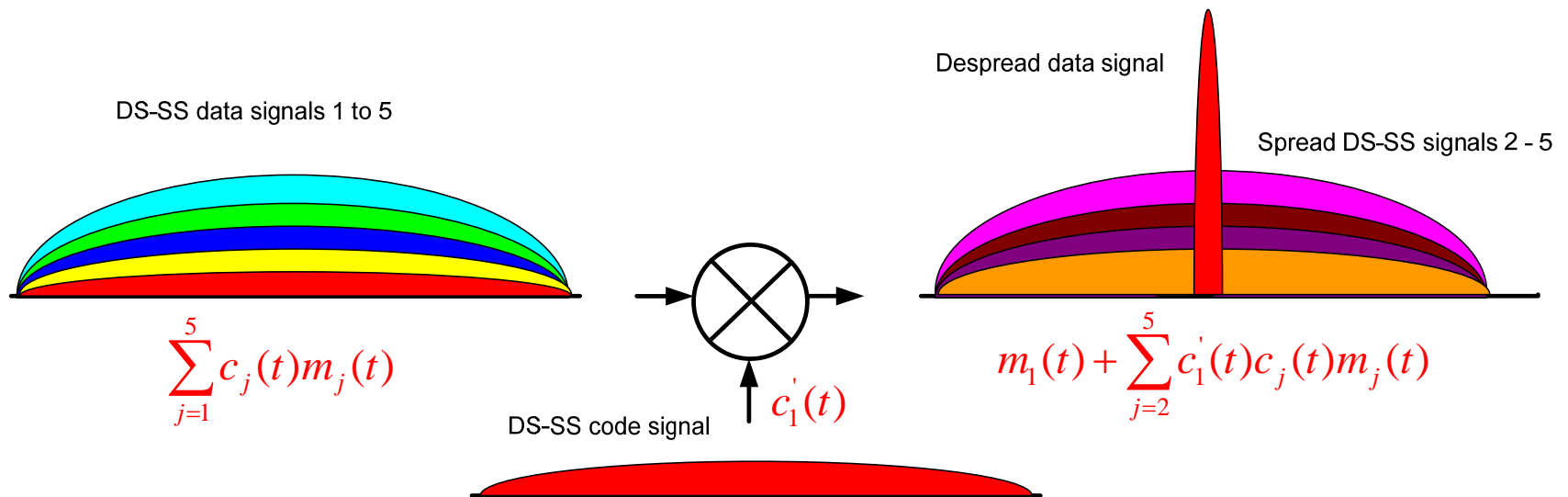
The price we pay for increased robustness against interference and multipath fading (large G_p) is a very low spectral efficiency.

This can be improved by letting multiple users share the same bandwidth.

Code Division Multiple Access (CDMA)

In Code Division Multiple Access (CDMA) the spectral resources (bandwidth) are shared among multiple users using different DS-SS codes.

Each user is identified by its own code. The cross-correlation between the codes has to be low (ideally zero \Rightarrow orthogonal codes).

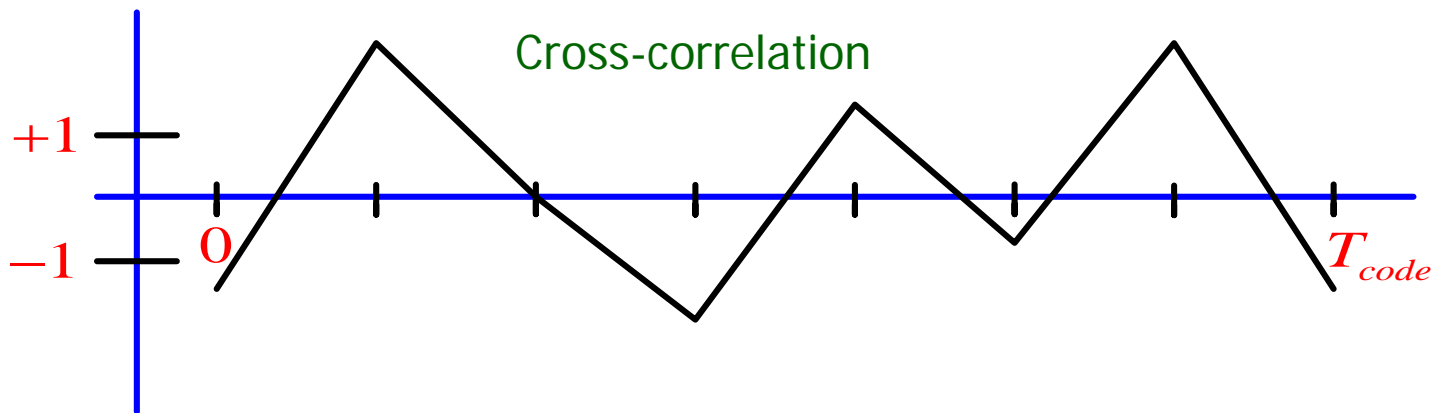


Pseudo-Noise code generators (3)

The cross-correlation of two codes, $c_1(t)$ and $c_2(t)$, of the same length, is given by:

$$R_{c_1 c_2}(\tau) = \int_0^{T_{code}} c_1(t) c_2(t - \tau) dt$$

The cross-correlation properties of a set of codes is very important for the separation of users using different codes, like in a Code Division Multiple Access (CDMA) system, e.g. UMTS.



Codes with poor cross-correlation properties result in bad separation or a high level of interference for certain delays between the codes.

Direct Sequence Spread Spectrum – Review

In DS-SS modulation the bandwidth of the signal is increased by multiplying the data with a code known to the receiver.

