Telecommunicatie B (EE2T12)

Lecture 11 overview:

Signal-to-Noise Ratio after detection for analog signals:

* Detection of FM, PM signals
* Maximum achievable output SNR

Modulation techniques for digital signals - binary schemes:

* Amplitude Shift Keying (ASK)
* Binary Phase Shift Keying (BPSK)
* Frequency Shift Keying (FSK)

EE2T12 Telecommunicatie B

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April 15, 2016

Colleges en Instructies Telecommunicatie B

Colleges:

Maandag	9-5, 30-5, 6-6	5e+6e uur, EWI-CZ Chip
Dinsdag	10-5	7e+8e uur, EWI-CZ Pi

Instructies:

Dinsdag	17-5	5e+6e uur, EWI-CZ Boole
Dinsdag	31-5	7e+8e uur, EWI-CZ Pi
Maandag	13-6	5e+6e uur, EWI-CZ Chip

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Phase- and Frequency modulation

The transmitted signal for angle modulation:

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{A_c e^{j\theta(t)} e^{j\omega_c t}\} = A_c \cos[\omega_c t + \theta(t)]$$

Phase Modulation (PM):

$$\theta(t) = D_p m(t)$$

$\Rightarrow$ so $\theta(t)$ is proportional with the information signal $m(t)$.

$\Rightarrow$ D_p = phase deviation constant [rad/V]

Frequency Modulation (FM):

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda)\,d\lambda$$

$\Rightarrow$ so $\theta(t)$ is proportional with the integral of information signal $m(t)$.

$\Rightarrow$ D_f = frequency deviation constant [rad/V.s]

SNR after detection: angle modulation

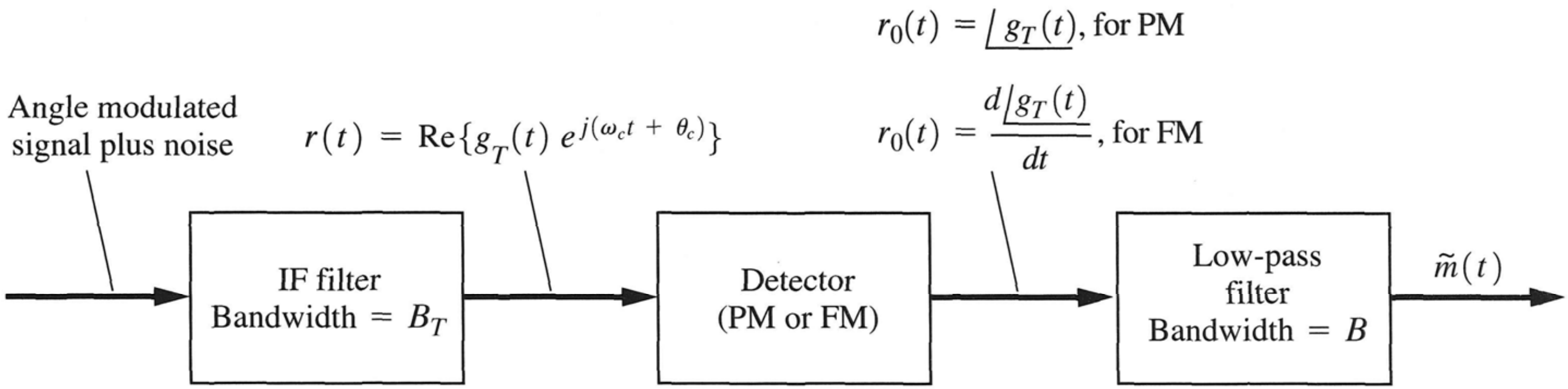

Figure 7–21 Receiver for angle-modulated signals.

SNR: PM modulation (1)

PM signal: $g_s(t) = A_c e^{j\theta_s(t)}$ with $\theta_s(t) = D_p m(t)$, $\beta_p = \Delta\theta = D_p \max\{|m(t)|\}$

Phase deviation constant

Modulation index

Received signal + noise:

$$g_T(t) = [g_s(t) + g_n(t)] = |g_T(t)| e^{j\theta_T(t)}$$
$$= A_c e^{j\theta_s(t)} + R_n(t) e^{j\theta_n(t)}$$

Phase detector with gain K: $r_0(t) = K \arg\{g_T(t)\} = K\theta_T(t)$

For $SNR_{in} >> 1 \quad \Rightarrow \quad A_c >> R_n(t)$:

$$r_0(t) = K\theta_T(t)$$
$$\approx K\left\{\theta_s(t) + \frac{R_n(t)\sin[\theta_n(t) - \theta_s(t)]}{A_c}\right\}$$

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SNR: PM modulation (2)

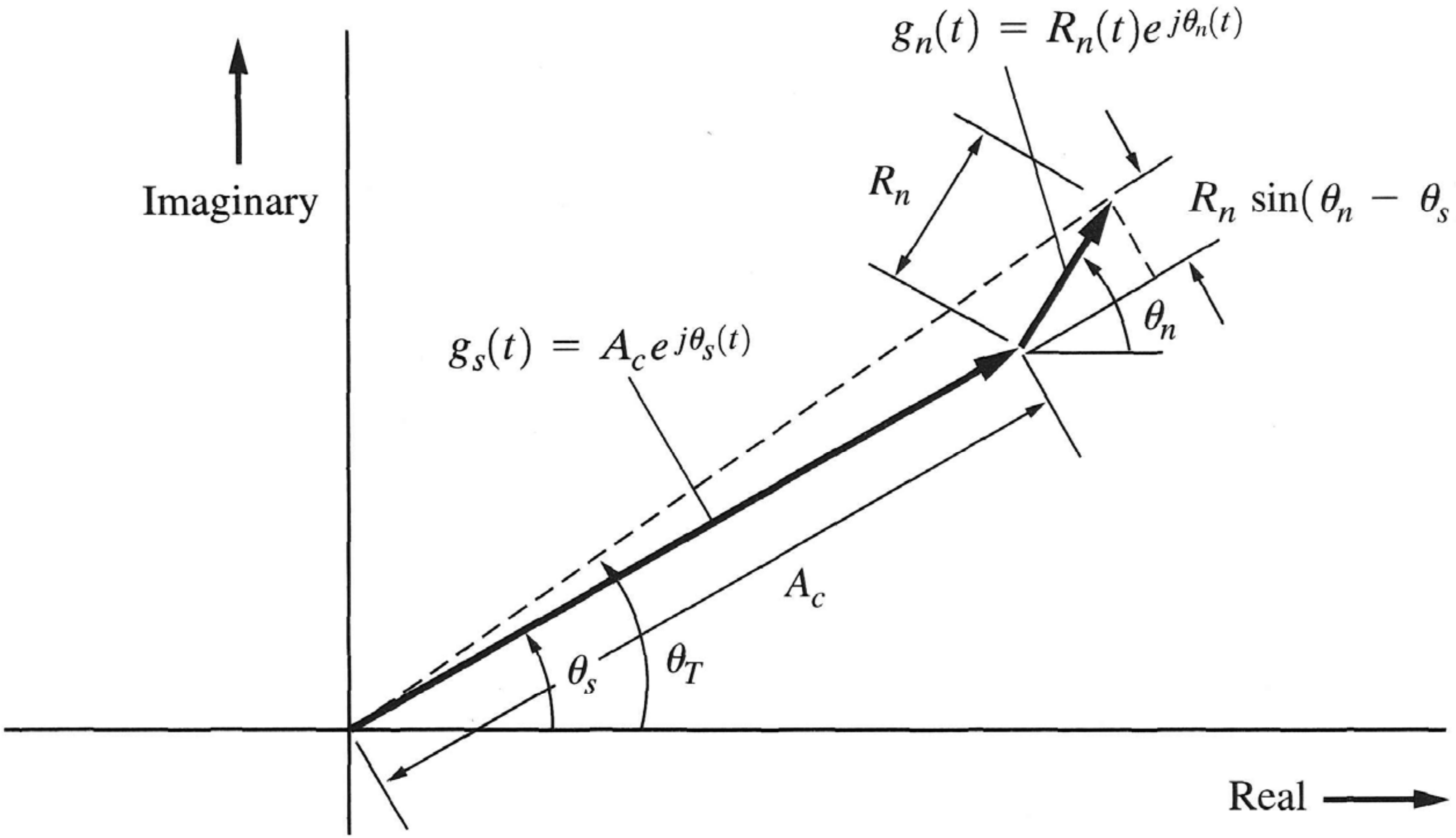

Figure 7–22 Vector diagram for angle modulation, $(S/N)_{\text{in}} \gg 1$.

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SNR: PM modulation (3)

Without modulation: $\theta_s(t) = 0$ and

$$r_0(t) \approx K\frac{R_n(t)}{A_c}\sin\theta_n(t) = K\frac{y_n(t)}{A_c} \approx K\theta_n' \rightarrow 0 \text{ for } A_c \rightarrow \infty$$

This is the "quieting–effect" or noise suppression effect when there is a strong carrier ($SNR_{in} >> 1$) at the input of the PM/FM detector (also without modulation: silence!).

At a certain time t, $\theta_s(t)$ is deterministic (but unknown), but $\theta_n(t) - \theta_s(t)$ is random and uniformly distributed at any moment. $\Rightarrow$ In the stochastic noise model, we don't need to take $\theta_s(t)$ into account.

For large SNR, the signal and noise terms are independent.

SNR: PM modulation (3)

For $SNR_{in} >> 1$, the detector output signal can be approximated by:

$$r_0(t) \approx s_0(t) + n_0(t)$$

For $SNR_{in} \not\gg 1$, the phase of the noise becomes dominant.

with $s_0(t) = K\theta_s(t) = KD_p m(t)$ and $n_0(t) = \frac{K}{A_c} y_n(t)$

$y_n(t)$ is the noise in quadrature (perpendicular) to the signal vector $g_s(t)$.

The noise PSD of the (two-sided) baseband spectrum is:

$$P_{n_0}(f) = \frac{K^2}{A_c^2} P_{y_n}(f) = \begin{cases} \frac{K^2}{A_c^2} N_0 & |f| \leq \frac{B_T}{2} \\ 0 & |f| > \frac{B_T}{2} \end{cases}$$

$\Rightarrow$ a white noise spectrum

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SNR: PM modulation (4)

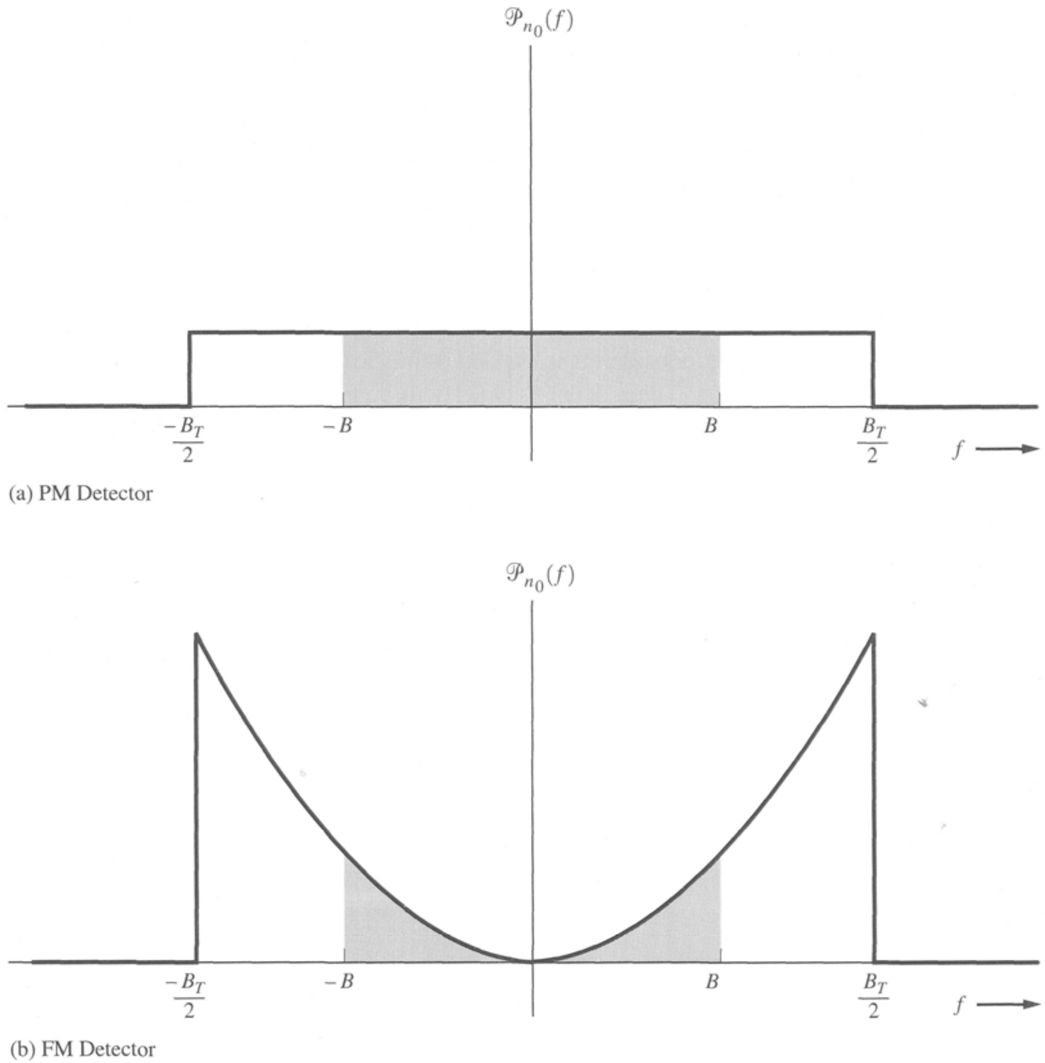

Figure 7–23 PSD for noise out of detectors for receivers of angle-modulated signals.

SNR: PM modulation (5)

In baseband bandwidth $B_{bb} \geq B$ (B = signal bandwidth in Couch)

$$\tilde{m}(t) = s_0(t) + n_0(t)$$

with signal power: $\overline{s_0^2(t)} = K^2 D_p^2 \overline{m^2(t)}$

and noise power: $\overline{n_0^2(t)} = \int_{-B_{bb}}^{B_{bb}} P_{n_0}(f)df = \dfrac{2K^2 N_0 B_{bb}}{A_c^2}$

Now the SNR at the detector output is given by:

$$SNR_{out} = \frac{\overline{s_0^2(t)}}{\overline{n_0^2(t)}} = \frac{A_c^2 D_p^2 \overline{m^2}}{2 N_0 B_{bb}}$$

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SNR: PM modulation (6)

Starting with

$$SNR_{out} = \frac{\overline{s_0^2(t)}}{\overline{n_0^2(t)}} = \frac{A_c^2 D_p^2 \overline{m^2}}{2N_0 B_{bb}}$$

and using: $D_p = \dfrac{\beta_p}{V_p}$

with D_p the phase deviation constant, β_p the phase modulation index and V_p the peak-value of $|m(t)|$, we find for $B_{bb} = B$:

$$SNR_{out} = \frac{A_c^2 \beta_p^2 (\overline{m^2} / V_p^2)}{2N_0 B}$$

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SNR: FM modulation (1)

For an FM-modulated signal: $g_s(t) = A_c e^{j\theta_s(t)}$

with $\theta_s(t) = D_f \int_{-\infty}^{t} m(\lambda) d\lambda$

Frequency deviation constant

The complex envelope of signal + noise:

$$g_T(t) = [g_s(t) + g_n(t)] = |g_T(t)| e^{j\theta_T(t)}$$

with $\theta_T(t) \approx \theta_s(t) + \dfrac{y_n(t)}{A_c}$ for $SNR_{in} >> 1$.

$y_n(t)$ is the noise in quadrature (perpendicular) to the signal vector $g_s(t)$.

FM-detector: output signal is proportional to the derivative of the phase of the total signal:

$$r_0(t) = \frac{K}{2\pi} \frac{d \arg\{g_T(t)\}}{dt} = \frac{K}{2\pi} \frac{d\theta_T(t)}{dt}$$

where K = FM-detector gain

SNR: FM modulation (2)

For $SNR_{in} >> 1$, the output signal can be approximated by:

$$r_0(t) \approx s_0(t) + n_0(t)$$

with $s_0(t) = \frac{K}{2\pi}\frac{d\theta_s(t)}{dt} = \frac{KD_f}{2\pi}m(t)$

and $n_0(t) = \frac{K}{2\pi A_c}\frac{dy_n(t)}{dt}$

The noise PSD of the (two-sided) baseband spectrum is:

$$P_{n_0}(f) = \left(\frac{K}{2\pi A_c}\right)^2 |2\pi jf|^2 P_{y_n}(f) = \begin{cases} \left(\frac{K}{A_c}\right)^2 N_0 f^2 & |f| \le \frac{B_T}{2} \\ 0 & |f| > \frac{B_T}{2} \end{cases}$$

$\Rightarrow$ a quadratic noise spectrum

SNR: FM modulation (3)

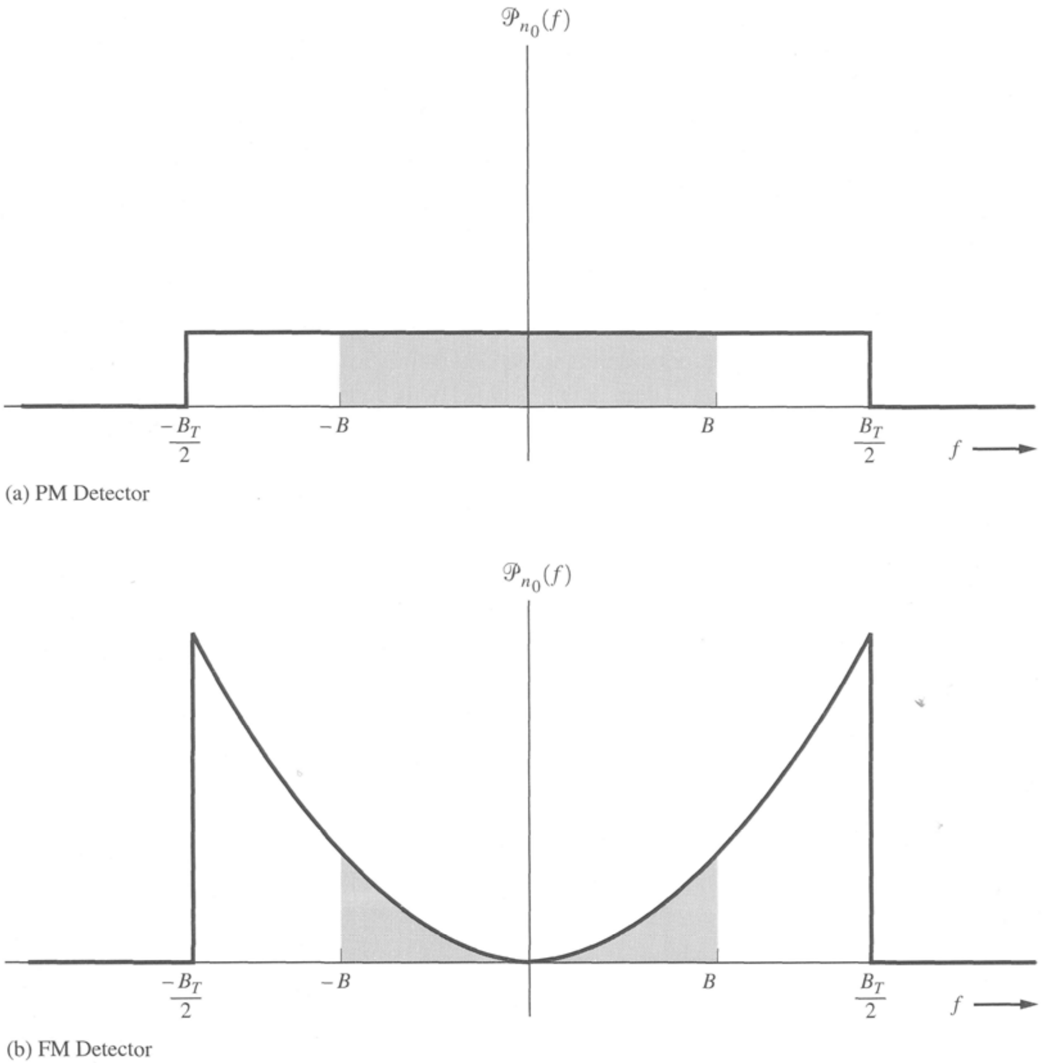

Figure 7–23 PSD for noise out of detectors for receivers of angle-modulated signals.

SNR: FM modulation (4)

In baseband bandwidth $B_{bb} \geq B$ (B = signal bandwidth in Couch)

$$\tilde{m}(t) = s_0(t) + n_0(t)$$

with signal power: $\overline{s_0^2(t)} = \left(\dfrac{KD_f}{2\pi}\right)^2 \overline{m^2(t)}$

and noise power: $\overline{n_0^2(t)} = \int_{-B_{bb}}^{B_{bb}} P_{n_0}(f)df = \dfrac{2}{3}\left(\dfrac{K}{A_c}\right)^2 N_0 B_{bb}^3$

Now the SNR at the detector output is given by:

$$SNR_{out} = \frac{\overline{s_0^2(t)}}{\overline{n_0^2(t)}} = \frac{3A_c^2[D_f/(2\pi B)]^2\,\overline{m^2}}{2N_0 B_{bb}} \cdot \frac{B^2}{B_{bb}^2}$$

SNR: FM modulation (5)

Starting with

$$SNR_{out} = \frac{\overline{s_0^2(t)}}{\overline{\tilde{n}_0^2(t)}} = \frac{3A_c^2[D_f/(2\pi B)]^2\,\overline{m^2}}{2N_0B_{bb}} \cdot \frac{B^2}{B_{bb}^2}$$

and using: $\dfrac{D_f V_p}{2\pi B} = \dfrac{\Delta F}{B} = \beta_f \quad \Rightarrow \quad \dfrac{D_f}{2\pi B} = \dfrac{\beta_f}{V_p}$

with D_f the frequency deviation constant, β_f the frequency modulation index and V_p the peak-value of $|m(t)|$, we find for $B_{bb} = B$:

$$SNR_{out} = \frac{3A_c^2\beta_f^2(\overline{m^2}/V_p^2)}{2N_0B}$$

Ideal case, when the receiver is matched to the signal.

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SNR: PM- and FM modulation (1)

With Carson's bandwidth $B_T = 2(\beta + 1)B$, SNR_{in} becomes:

$$SNR_{in} = \frac{A_c^2 / 2}{N_0 B_T} = \frac{A_c^2}{4N_0(\beta + 1)B}$$

and we find the following relation between SNR_{out} and SNR_{in}

PM: $$\frac{SNR_{out}}{SNR_{in}} = 2\beta_p^2(\beta_p + 1)\frac{\overline{m^2}}{V_p^2}$$

FM: $$\frac{SNR_{out}}{SNR_{in}} = 6\beta_f^2(\beta_f + 1)\frac{\overline{m^2}}{V_p^2}$$

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SNR: PM- and FM modulation (2)

Comparing SNR_{out} with the SNR in baseband:

$$SNR_{baseband} = \frac{P_s}{N_0 B} = \frac{A_c^2/2}{N_0 B}$$

PM: $$\frac{SNR_{out}}{SNR_{baseband}} = \beta_p^2 \frac{\overline{m^2}}{V_p^2}$$

FM: $$\frac{SNR_{out}}{SNR_{baseband}} = 3\beta_f^2 \frac{\overline{m^2}}{V_p^2}$$

Note: here the assumptions is made that the receiver bandwidth is matched to the signal bandwidth: $B = B_{bb}$.

By choosing a higher phase/frequency deviation, we can obtain a SNR_{out} higher than the SNR in baseband:

Trade-off between transmission power and bandwidth!!!

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SNR: PM- and FM modulation (3)

Gain PM: due to the maximum value of

$$\beta_p m(t)/V_p = D_p m(t) \le \pi \Leftrightarrow \beta_p = D_p V_p \le \pi$$

the maximum achievable gain compared to baseband for sine-wave modulation is limited to:

$$\beta_p^2 \frac{\overline{m^2}}{V_p^2} = \pi^2/2 \quad \equiv 6.9 \text{ dB}$$

Gain FM: for FM there is no such limitation on β_f. For sine-wave modulation with: $\overline{m^2(t)}/V_p^2 = 0.5$ this gain is:

$$3\beta_f^2 \frac{\overline{m^2}}{V_p^2} = \frac{3}{2}\beta_f^2$$

With increasing β_f the SNR_{in} will decrease due to increasing transmission bandwidth B_T.

Iff SNR_{in} is above the detection threshold!

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SNR: FM detection threshold

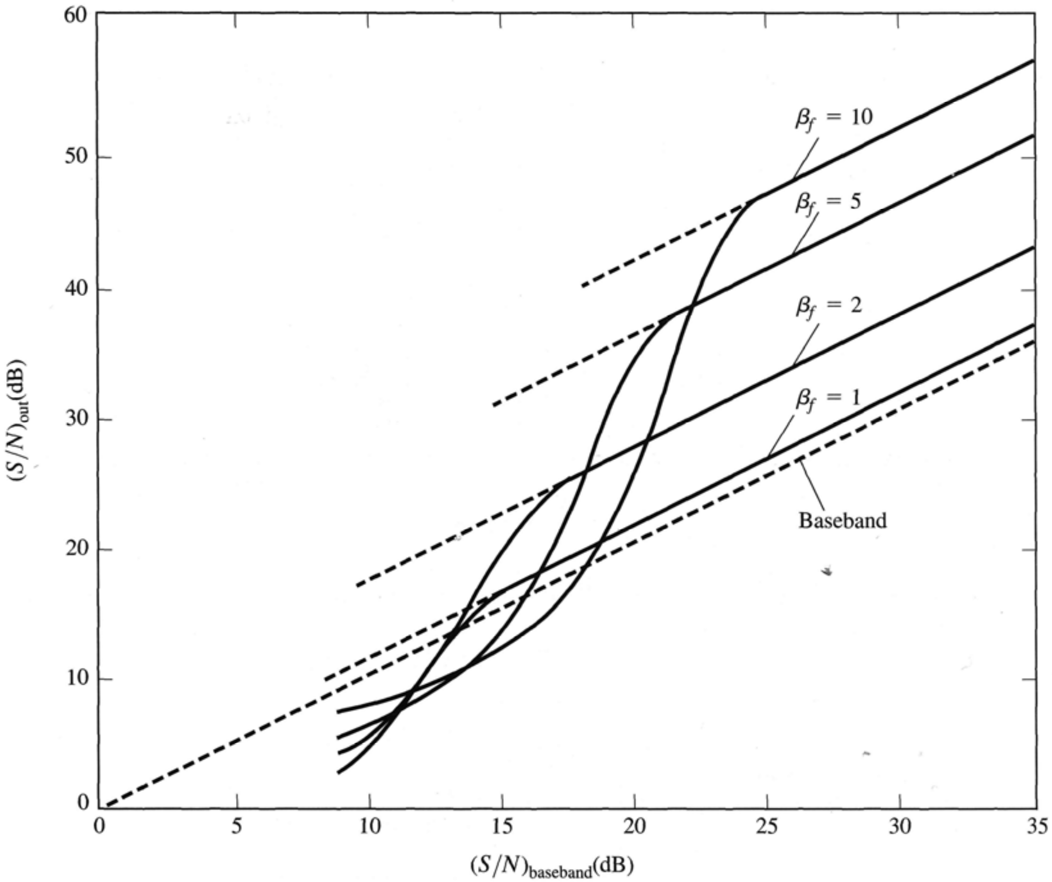

Figure 7–24 Noise performance of an FM discriminator for a sinusoidal modulated FM signal plus Gaussian noise (no deemphasis).

$$SNR_{baseband} = \frac{A_c^2/2}{N_0 B}$$

$$SNR_{in} = \frac{A_c^2/2}{N_0 B_T}$$

$$= \frac{A_c^2}{4N_0(\beta_f + 1)B}$$

Thus we find:

$$SNR_{baseband} = 2(\beta_f + 1) SNR_{in}$$

and

$$SNR_{in\_thr} = \frac{SNR_{baseband\_thr}}{2(\beta_f + 1)}$$

Exercise output SNR PM-modulation (1)

A PM signal with sine wave modulation $m(t) = \sin 2000\pi t$ is received with a power $S_R = A_c^2/2 = -40 \text{ dBm}$, $N_0 = -90 \text{ dBm/Hz}$, the baseband bandwidth of the receiver is $B_{bb} = 2 \text{ kHz}$ and the phase deviation constant is $D_p = \pi/2 \text{ rad/V}$.

Determine:

- SNR_{out}
- SNR_{out} for a baseband bandwidth matched to the signal bandwidth
- maximum achievable SNR_{out}

Exercise output SNR PM-modulation (2)

1. $$SNR_{out} = \frac{A_c^2}{2}\frac{D_p^2\overline{m^2}}{N_0 B_{bb}} = \frac{A_c^2}{2}\frac{\beta_p^2(\overline{m^2}/V_p^2)}{N_0 B_{bb}}$$

$$= -40 \text{ dBm} + 90 \text{ dBm/Hz} - 10\log_{10} B_{bb} \text{ dBHz} + 10\log_{10} \beta_p^2(\overline{m^2}/V_p^2)$$

$$= -40 + 90 - 10\log_{10} 2000 + 10\log_{10} \tfrac{\pi^2}{8} = 17.9 \text{ dB}$$

2. With optimal baseband bandwidth B_{bb} = B, we obtain a gain of 3 dB $\Rightarrow$ SNR_{out} = 20.9 dB

3. Maximum SNR is obtained with $\beta_p = V_p D_p = \pi$. Then:

$$SNR_{out} = -40 + 90 + -10\log_{10}1000 + 10\log_{10} \tfrac{\pi^2}{2} = 26.9 \text{ dB}$$

Exercise output SNR FM-modulation (1)

An FM signal with sine wave modulation $m(t) = \sin 2000\pi t$ is received with a power $S_R = A_c^2/2 = -40 \text{ dBm}$, $N_0 = -90 \text{ dBm/Hz}$, the baseband bandwidth of the receiver is $B_{bb} = 2 \text{ kHz}$ and the modulation index is $\beta_f = \pi/2.$

Determine:

- SNR_{out}
- SNR_{out} for a baseband bandwidth matched to the signal bandwidth
- maximum achievable SNR_{out}

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Exercise output SNR FM-modulation (2)

1. $$SNR_{out} = \frac{3A_c^2 (V_p D_f / 2\pi B)^2 \frac{\overline{m^2}}{V_p^2} \frac{B^2}{B_{bb}^2}}{2N_0 B_{bb}}$$

$$= \frac{3A_c^2 \beta_f^2 \frac{\overline{m^2}}{V_p^2} \frac{B^2}{B_{bb}^2}}{2N_0 B_{bb}}$$

$$= 10\log_{10} 3 \text{ dB} - 40 \text{ dBm} + 90 \text{ dBm/Hz} + 10\log_{10}\left(\tfrac{\pi}{2}\right)^2$$

$$- 3 \text{ dB} + 10\log_{10}\left(\tfrac{1}{2}\right)^2 \text{ dB} - 10\log_{10} B_{bb} \text{ dBHz}$$

$$= 4.77 - 40 + 90 + 3.92 - 3 - 6 - 33 = 16.7 \text{ dB}$$

2. With optimal baseband bandwidth $B_{bb} = B$, we obtain a gain of 3x3 dB = 9dB $\Rightarrow$ SNR_{out} = 25.7 dB

3. β_f can be increased to much larger values than π, however, $SNR_{in} > SNR_{thr}$. For $\beta_f = \pi$ $\Rightarrow$ *SNRout* = 31.7 dB

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Output SNR for ideal demodulation (1)

In an ideal system:

no loss of capacity by detection.

$$C_{in} = C_{out}$$

Shannon: $B_T \log_2\left(1+SNR_{in}\right) = B\log_2\left(1+SNR_{out}\right)$

$$\Rightarrow SNR_{out} = \left[1+SNR_{in}\right]^{B_T/B} - 1$$

With: $SNR_{in} = \dfrac{B}{B_T} SNR_{baseband}$

we find: $SNR_{out} = \left[1+\left(\dfrac{B}{B_T}\right)SNR_{baseband}\right]^{B_T/B} - 1$

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Ideal output SNR for analog modulations (2)

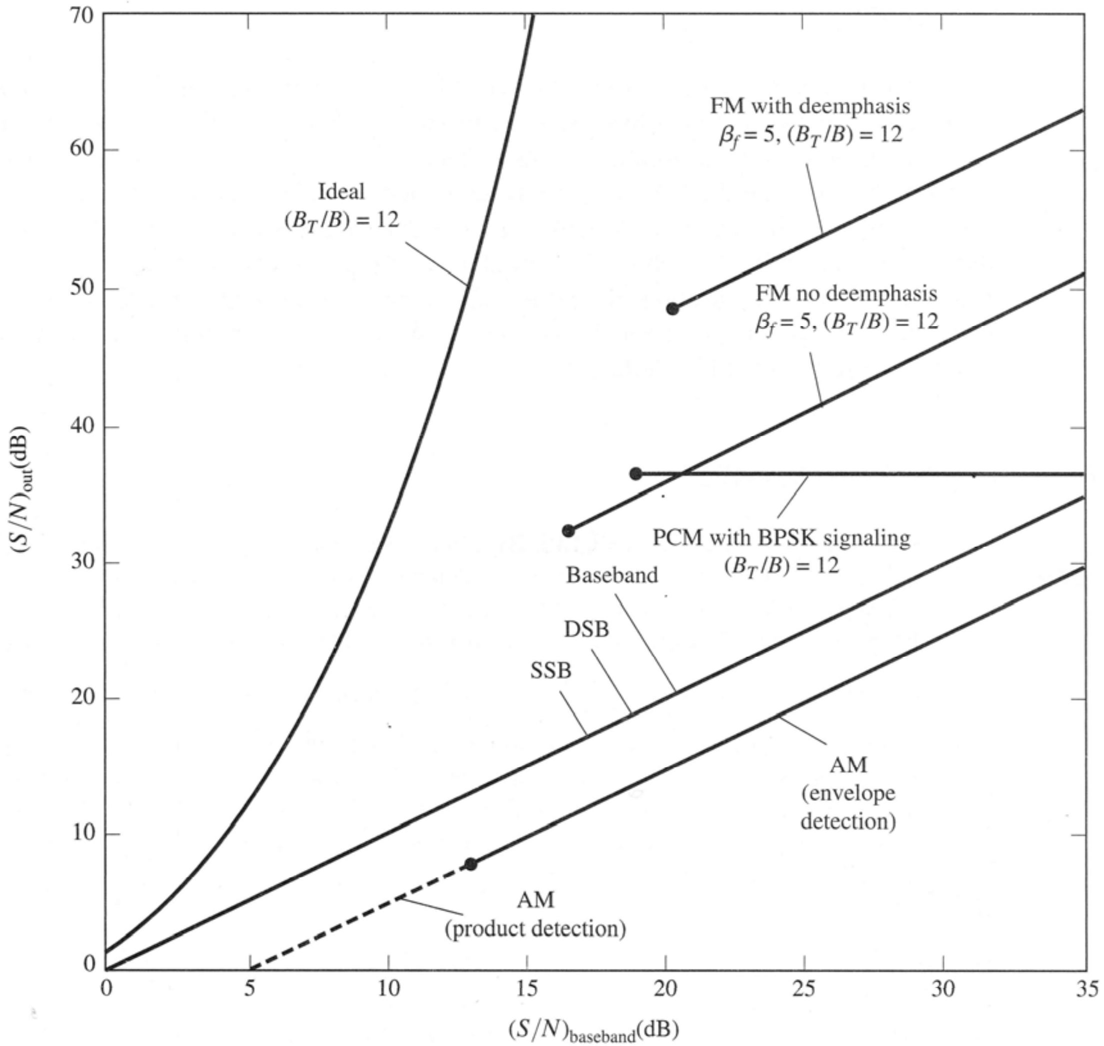


Figure 7–27 Comparison of the noise performance of analog systems.

Where do we find digital modulation?

Digital modulation is applied in:

- computer modems: analog as well as in ADSL, WiFi
- mobile communications: GSM, GSM-EDGE, UMTS, LTE, WiMAX
- digital audio and video broadcasting (DAB, DVB: e.g. digitenne)
- digital cable television: DVB-C
- etc, etc.

Binary modulation schemes

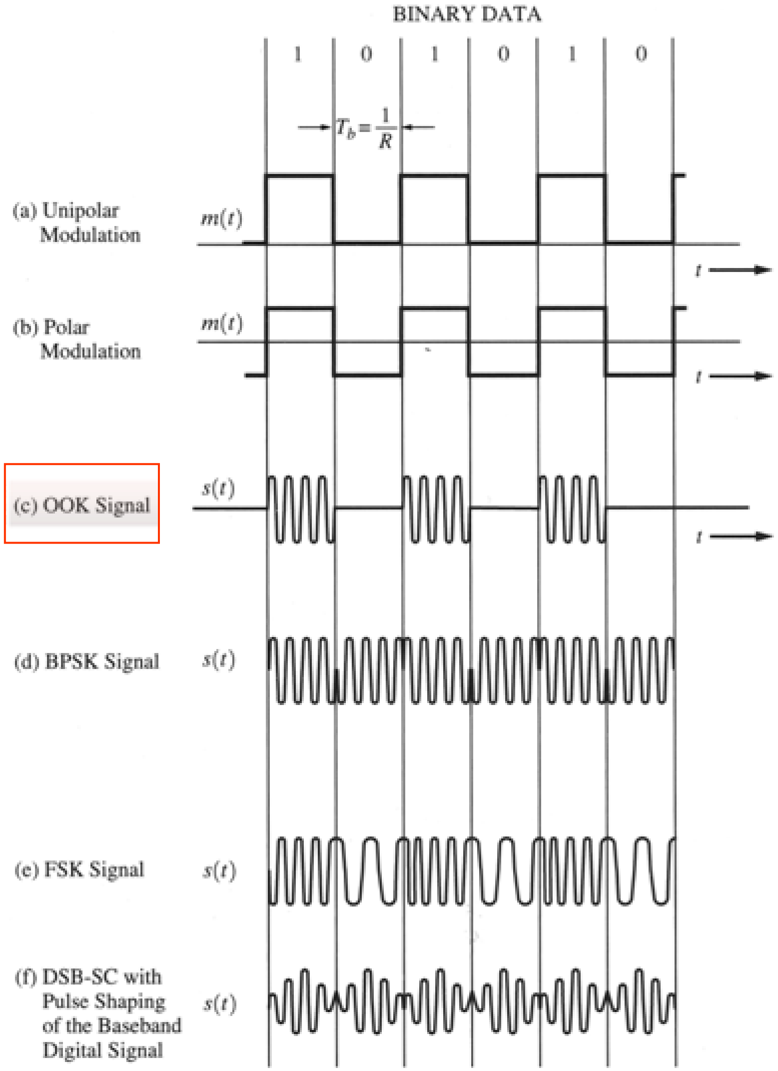


Figure 5–19 Bandpass digitally modulated signals.

On-Off Keying (1)

On-Off Keying (OOK) or Amplitude Shift Keying (ASK)

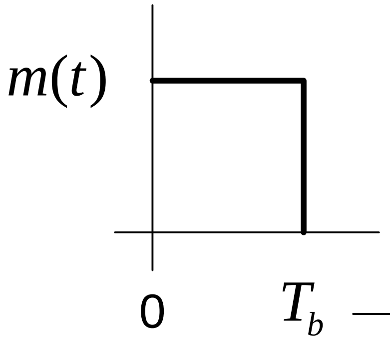
Carrier is switched on or off depending on the bit to be send

OOK-signal: $$s(t)=\begin{cases} A_c m(t)\cos\omega_c t & \text{with } m(t)\in\{0,1\} \\ \frac{1}{2}A_c[1+m'(t)]\cos\omega_c t & \text{with } m'(t)\in\{-1,1\} \end{cases}$$

Unipolar

Polar

Complex envelope: $$g(t)=\begin{cases} A_c m(t) & \text{with } m(t)\in\{0,1\} \\ \frac{1}{2}A_c[1+m'(t)] & \text{with } m'(t)\in\{-1,1\} \end{cases}$$



$\longrightarrow$ $R_b = 1/T_b$

For en average power $\overline{m^2(t)}=1$,

$m(t)\in\{0,\sqrt{2}\}$ with equal probability.

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On-Off Keying (2)

Power spectral density (PSD) of OOK

1. For rectangular pulses:

$$P_g(f) = \frac{A_c^2}{2}[\delta(f) + T_b \operatorname{sinc}^2 fT_b] \qquad \text{for } \overline{m^2(t)} = 1$$

$$P_s(f) = \frac{A_c^2}{8}[\delta(f+f_c) + T_b \operatorname{sinc}^2 (f+f_c)T_b]$$

$$+\frac{A_c^2}{8}[\delta(f-f_c) + T_b \operatorname{sinc}^2 (f-f_c)T_b]$$

What is the transmission bandwidth B_T?

The 0-0 bandwidth: $B_{T\_0-0} = 2R_b \quad \Rightarrow \quad B = R_b,\ B_T = 2B$

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Power spectral density OOK, BPSK

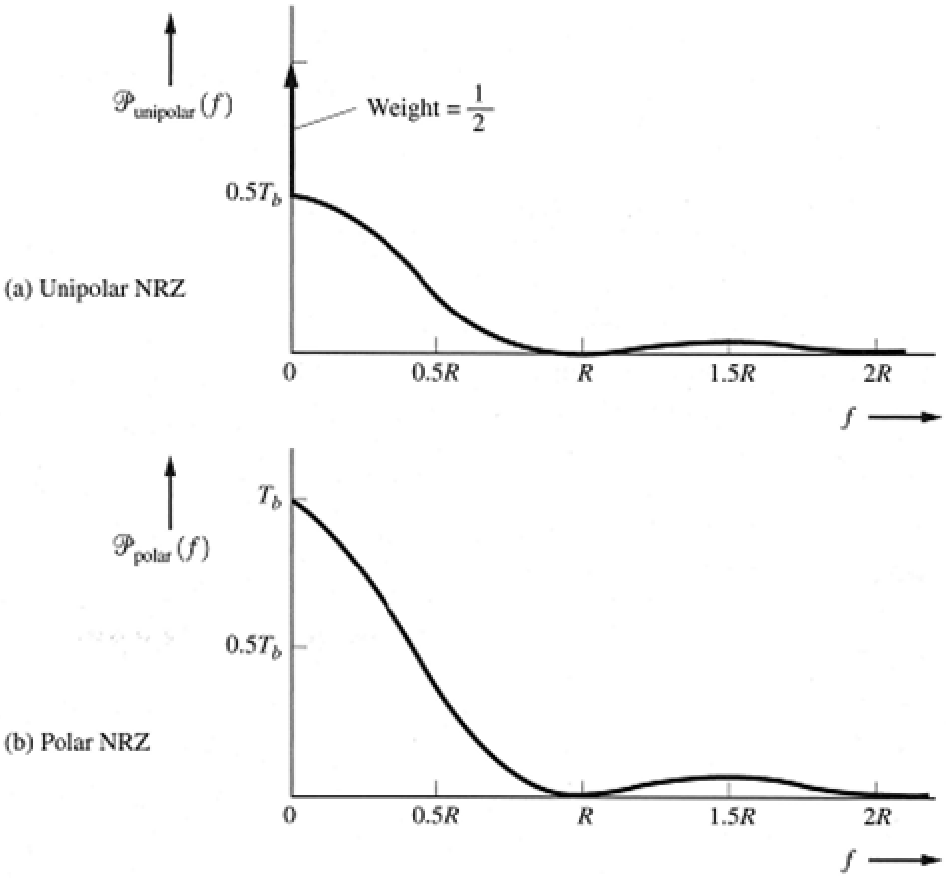

Figure 3–16 PSD for line codes (positive frequencies shown).

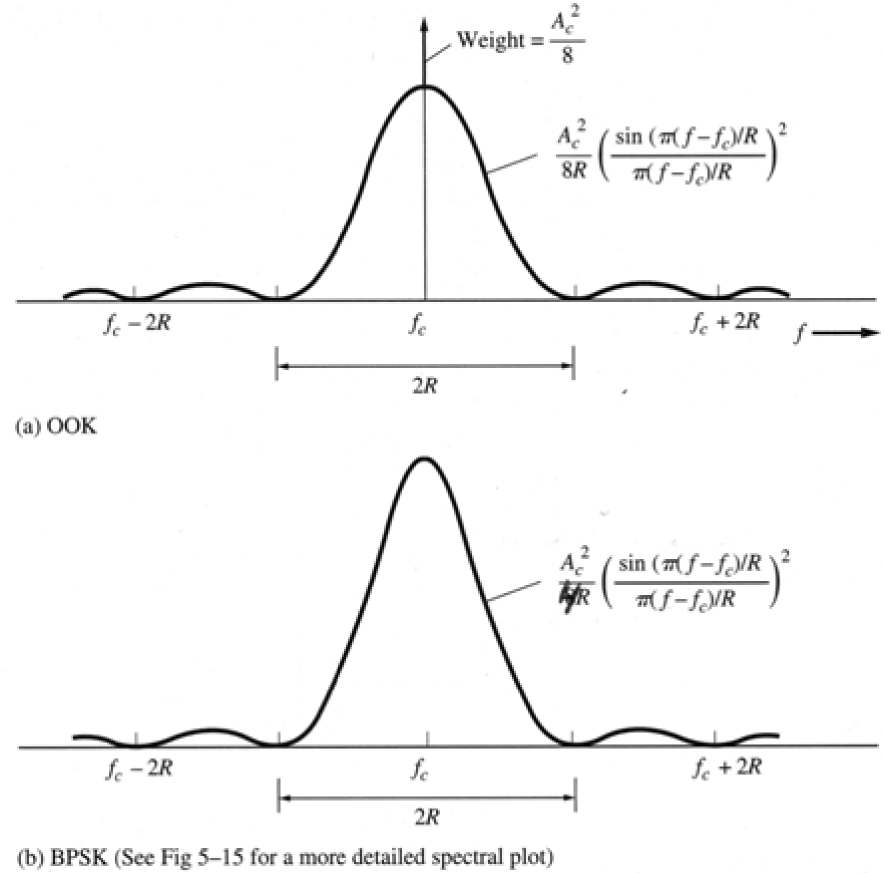

Figure 5–20 PSD of bandpass digital signals (positive frequencies shown).

Power spectral density RC-pulses

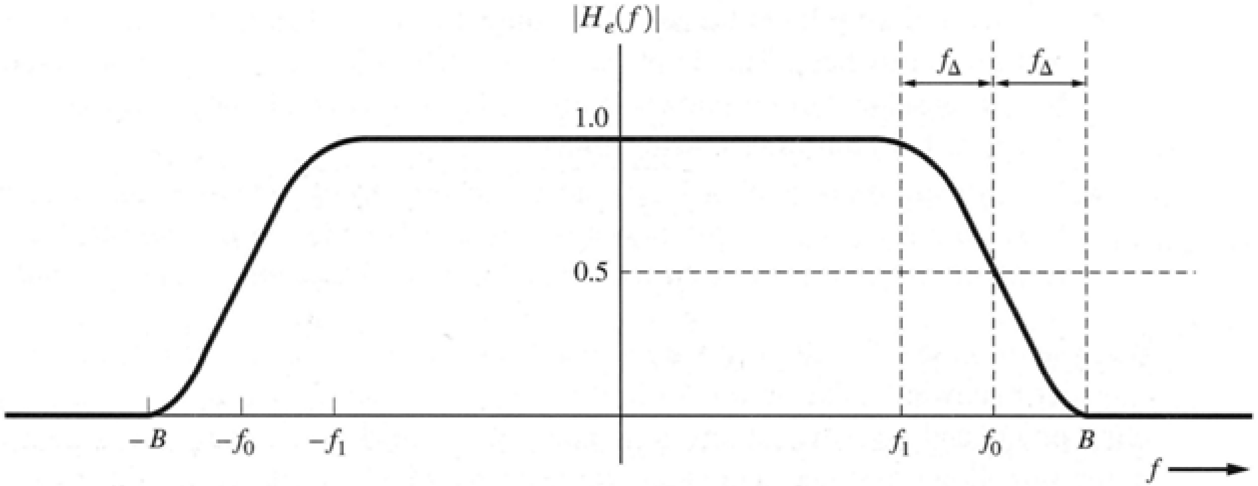

Figure 3–25 Raised cosine-rolloff Nyquist filter characteristics.

2. For raised-cosine pulses: $B = \frac{1}{2}(1+r)R_b = (1+r)\,f_0$

$$f_0 = R_b / 2$$

$$\Rightarrow \quad B_T = (1+r)R_b$$

$$r = \frac{f_\Delta}{f_0}$$: roll-off factor

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Detection of OOK signals

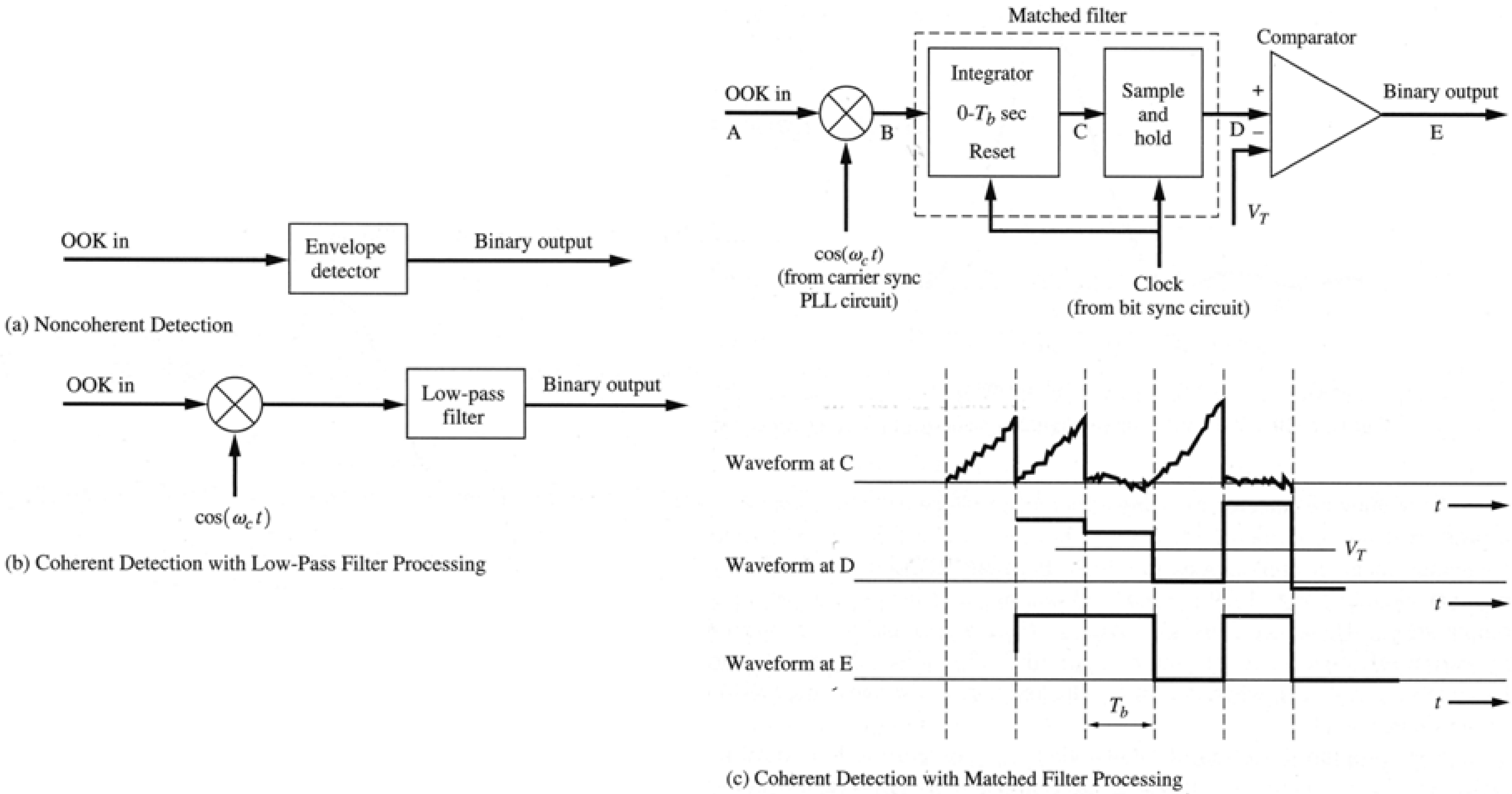

Figure 5–21 Detection of OOK.

Binary modulation schemes

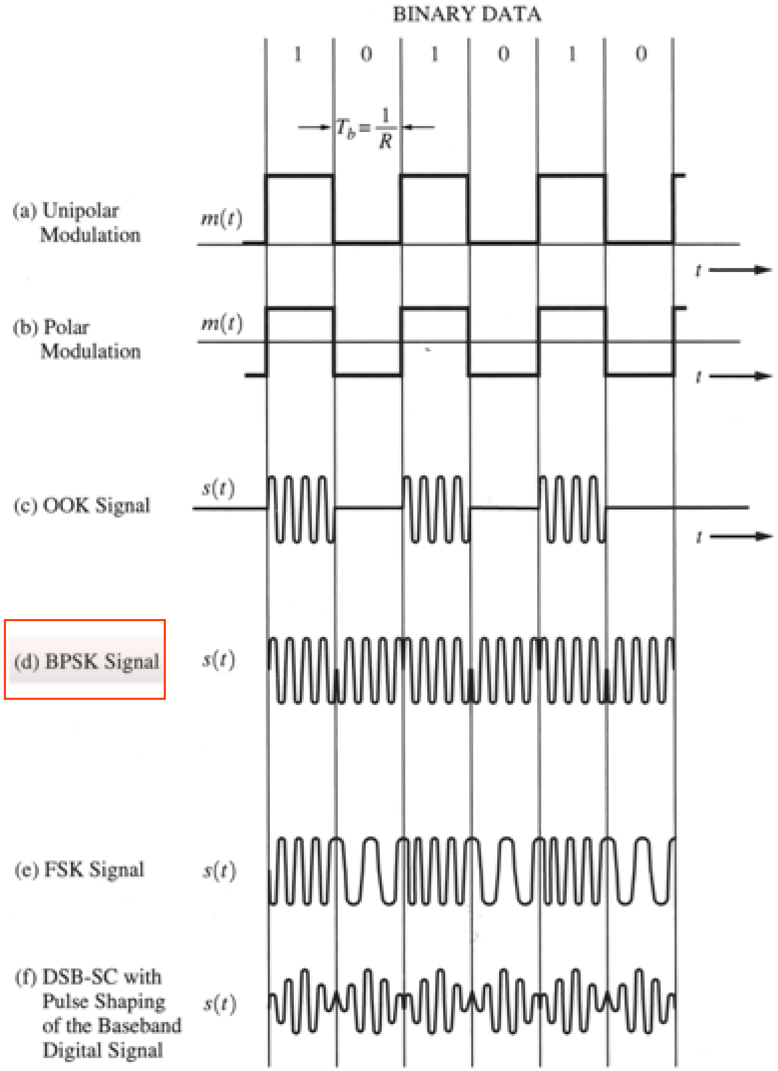

Figure 5–19 Bandpass digitally modulated signals.

Binary Phase Shift Keying (1)

In Binary Phase Shift Keying (BPSK) the carrier phase is switched between $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ depending of the bit to be sent.

Digital phase modulation

In general: $s(t)=A_c\cos[\omega_c t+D_p m(t)]$

Phase deviation constant

$m(t)\in\{-1,1\}$ is a polar baseband signal

$$=A_c\cos[D_p m(t)]\cos\omega_c t-A_c\sin[D_p m(t)]\sin\omega_c t$$

$$=A_c\cos D_p\cos\omega_c t-A_c m(t)\sin D_p\sin\omega_c t$$

Carrier component

Data component

Complex envelope: $g(t)=A_c e^{jD_p m(t)}$

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Binary Phase Shift Keying (2)

The digital modulation index is defined as: $h \triangleq \dfrac{2\Delta\theta}{\pi} = \dfrac{2D_p}{\pi}$

Where $2\Delta\theta$ is the maximum (peak-peak) phase deviation per symbol time $[\mathrm{rad}/T_s]$.

For maximum power in the data component (i.e. no power in the carrier component) :

$$\Delta\theta = D_p = \frac{\pi}{2} \quad \Rightarrow \quad h = 1$$

and $s(t) = -A_c m(t)\sin\omega_c t$

or by adding a phase shift of $\pi/2$: $s(t) = A_c m(t)\cos\omega_c t$

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Binary Phase Shift Keying (3)

By changing D_p we can choose for pure DSB-SC or DSB with a carrier component, which is however 90^o out of phase with the data component.

For BPSK, $D_p = \pi / 2$ and the complex envelope is:

$$g(t) = jA_c m(t) \quad (\text{or } \; g(t) = A_c m(t) \;)$$

and the power spectrum is

$$P_g(f) = A_c^2 T_b \,\mathrm{sinc}^2(fT_b)$$

$$P_s(f) = \frac{A_c^2 T_b}{4}\left[\mathrm{sinc}^2(f + f_c)T_b + \mathrm{sinc}^2(f - f_c)T_b\right]$$

For data detection a synchronous detector is required. When a carrier component is available, a Phase Locked Loop (PLL) can be used. Otherwise a Costas-loop or a squaring-loop is needed.

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Power spectral density OOK, BPSK

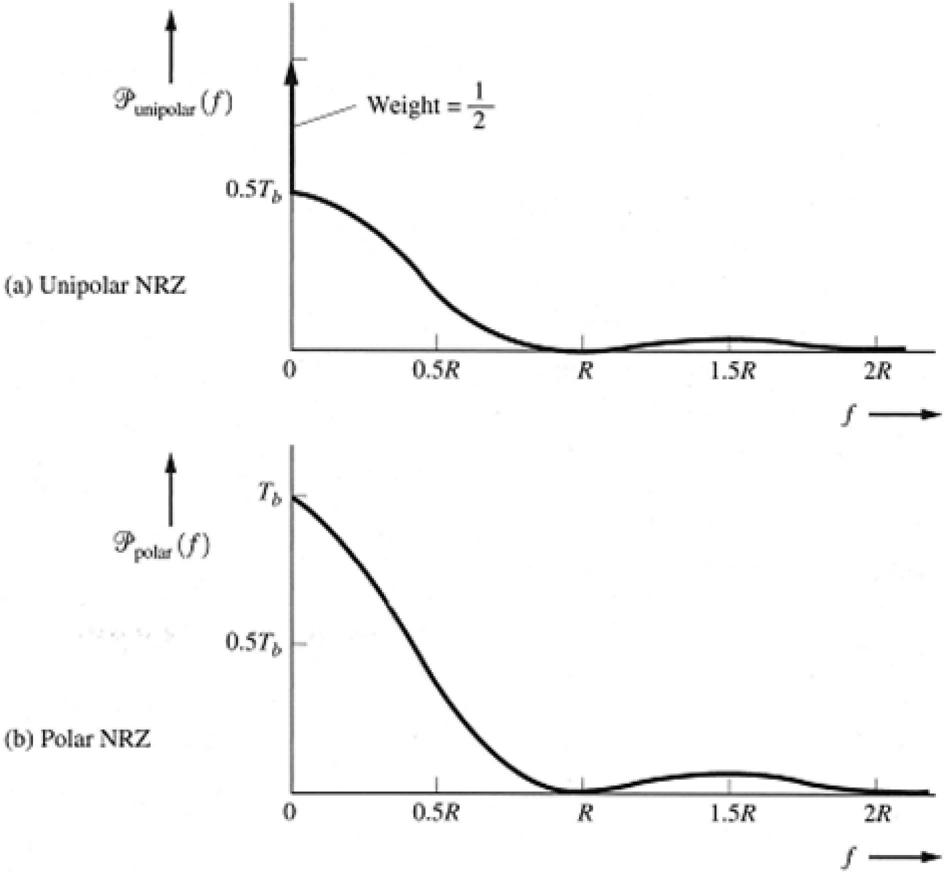


Figure 3–16 PSD for line codes (positive frequencies shown).

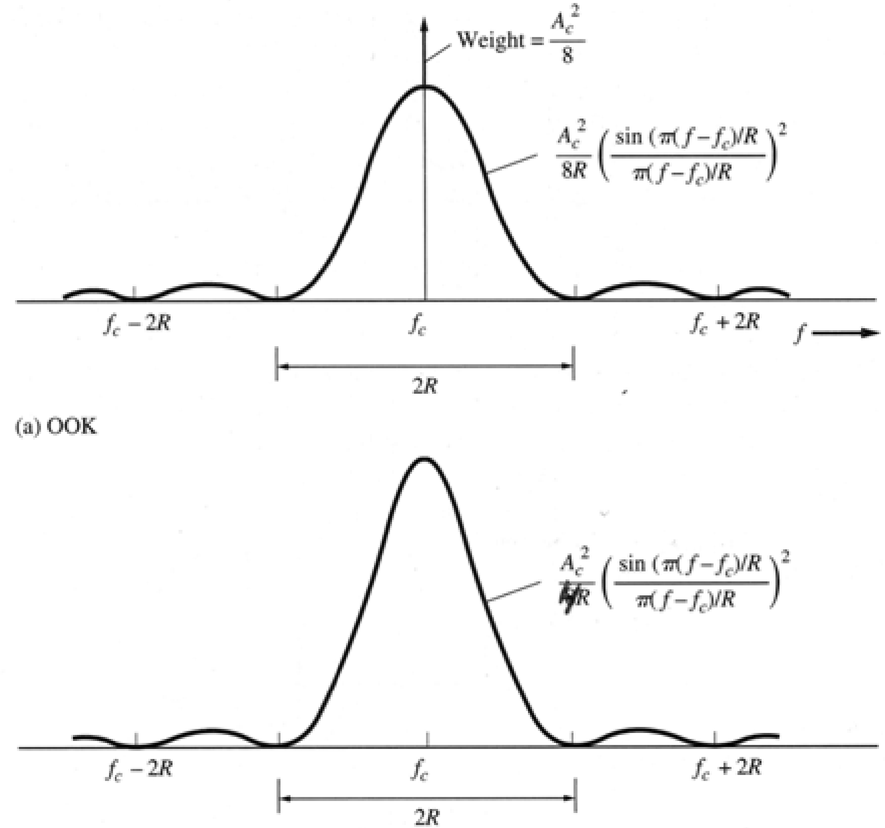
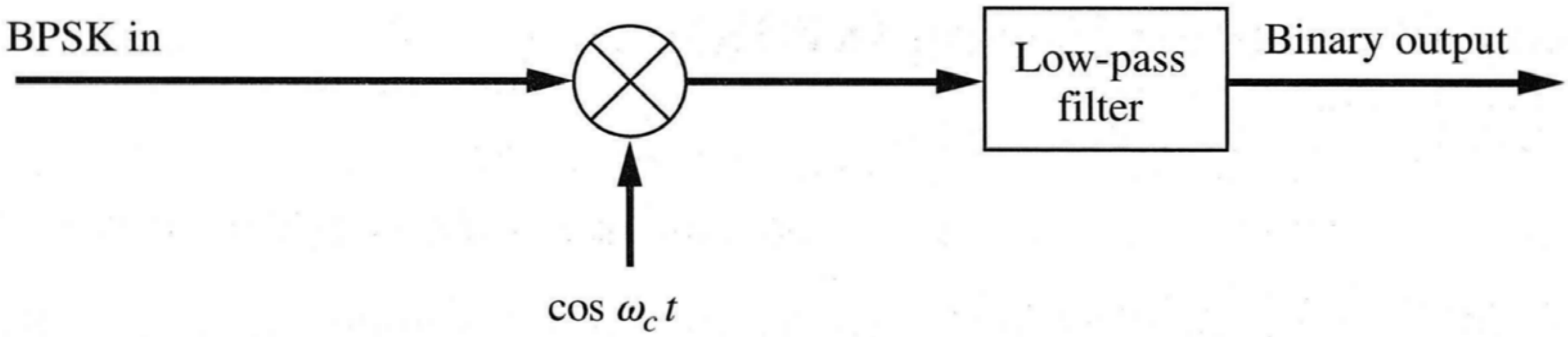


Figure 5–20 PSD of bandpass digital signals (positive frequencies shown).

Detection of BPSK signals



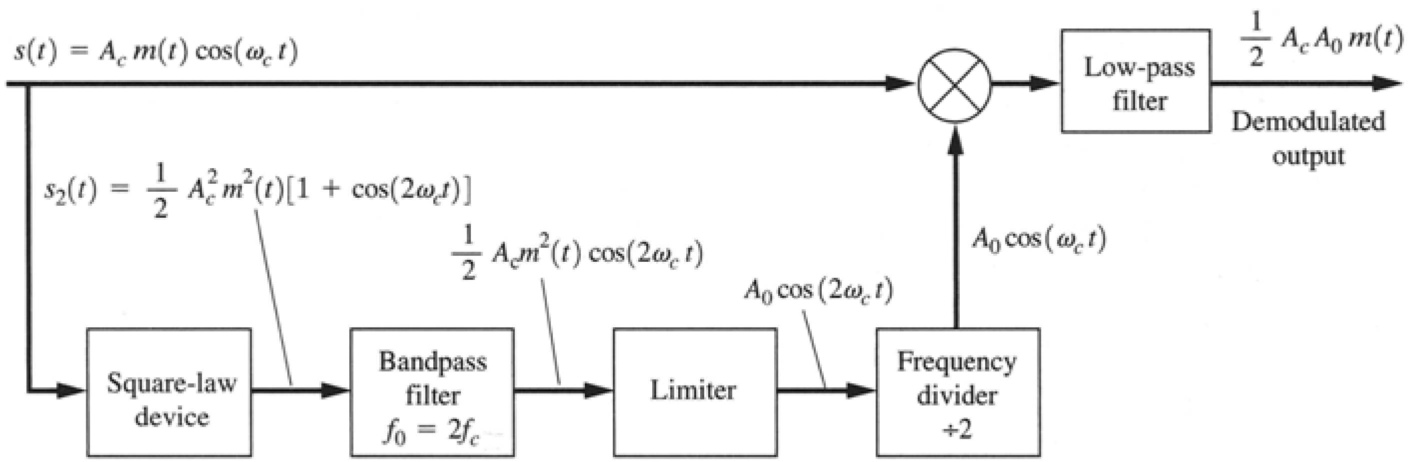
(a) Detection of BPSK (Coherent Detection)

Figure 5–22 Detection of BPSK and DPSK.

Binary Phase Shift Keying (4)

Since BPSK is a constant amplitude signal (and no carrier component is available), envelope detection is not possible. Product detection has to be used which first requires carrier recovery for phase synchronization.

1. Squaring-loop: see par. 5.4

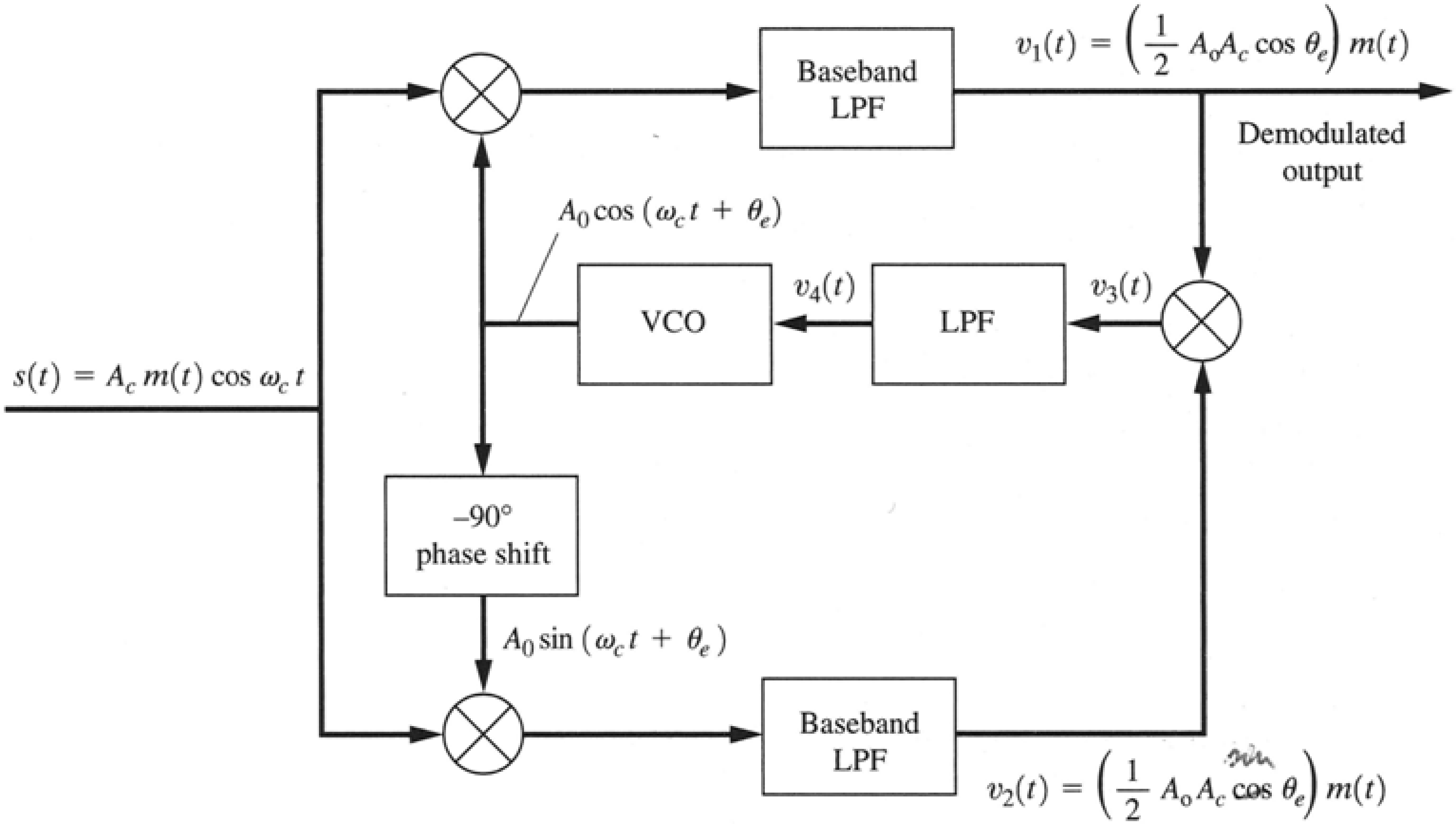


(b) Squaring Loop

Figure 5–3 Carrier recovery loops for DSB-SC signals.

Binary Phase Shift Keying (5)

2. Costas-loop



(a) Costas Phase-Locked Loop

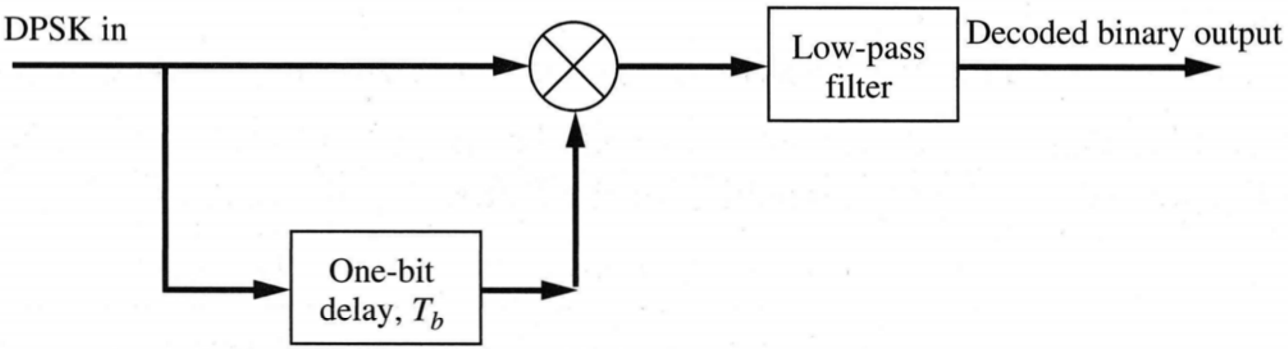
Differential Phase Shift Keying

In Differential Phase Shift Keying (DPSK), the data signal is first differentially encoded and successively transmitted using BPSK.

In the detector no carrier recovery is required, since we can use the phase of the previous symbol as a reference.

The DPSK-detector combines:

1. "coherent" detection
2. differential decoding



(b) Detection of DPSK (Partially Coherent Detection)

Figure 5–22 Detection of BPSK and DPSK.

Binary modulation schemes

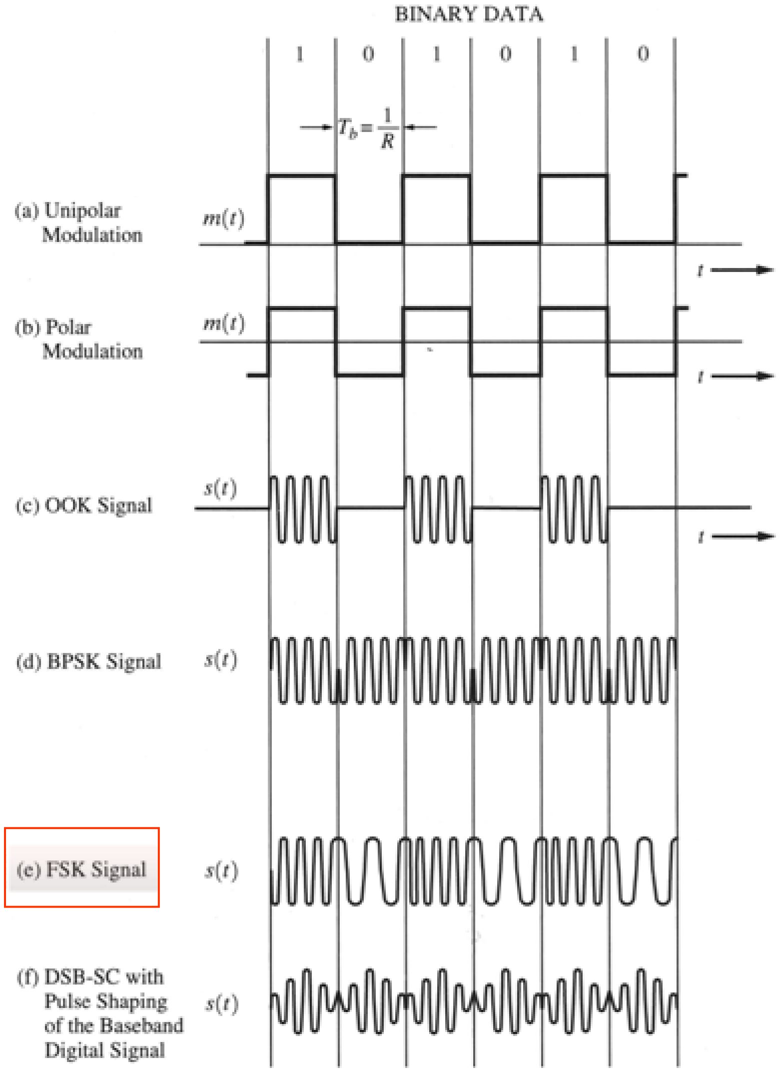


Figure 5–19 Bandpass digitally modulated signals.

Frequency Shift Keying (1)

In Frequency Shift Keying (FSK) the carrier frequency is switched between $\{ f_1, f_2 \}$ dependent of the bit to be sent.

Digital frequency modulation

Two cases:

1. Discontinuous phase FSK:

$$s(t) = \begin{cases} A_c \cos[\omega_1 t + \theta_1] & \text{for a "1"} \\ A_c \cos[\omega_2 t + \theta_2] & \text{for a "0"} \end{cases}$$

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Frequency Shift Keying (3)

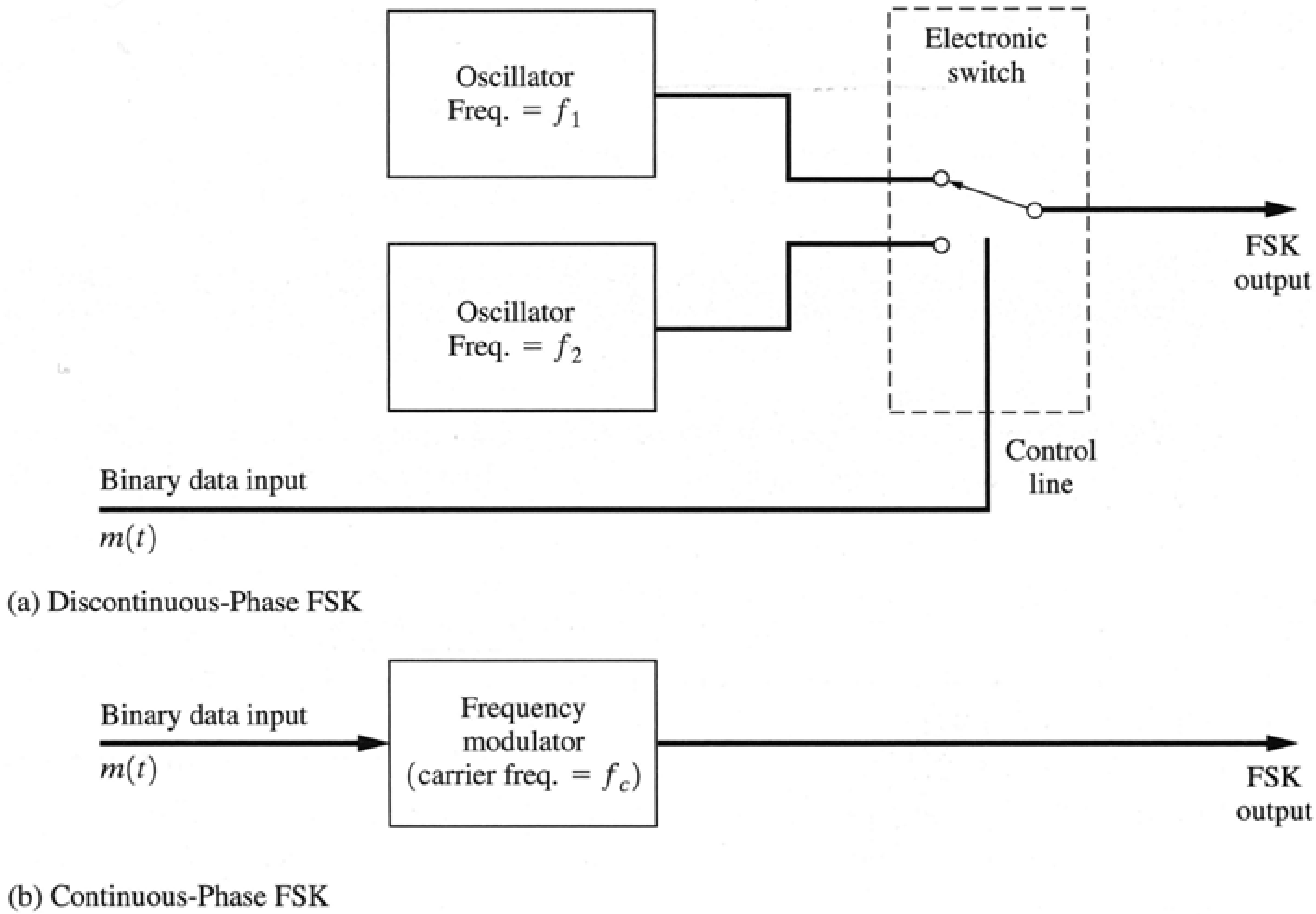

Figure 5–23 Generation of FSK.

Frequency Shift Keying (2)

2. Continuous phase FSK (CP-FSK):

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = A_c \cos[\omega_c t + D_f \int_{-\infty}^{t} m(\lambda)d\lambda]$$

with

$$g(t) = A_c e^{j\theta(t)} \quad \text{where} \quad \theta(t) \triangleq D_f \int_{-\infty}^{t} m(\lambda)d\lambda$$

Continuous, also when $m(t)$ discontinuous.

Digital baseband signal {-1, 1}

Example FSK: computer modem

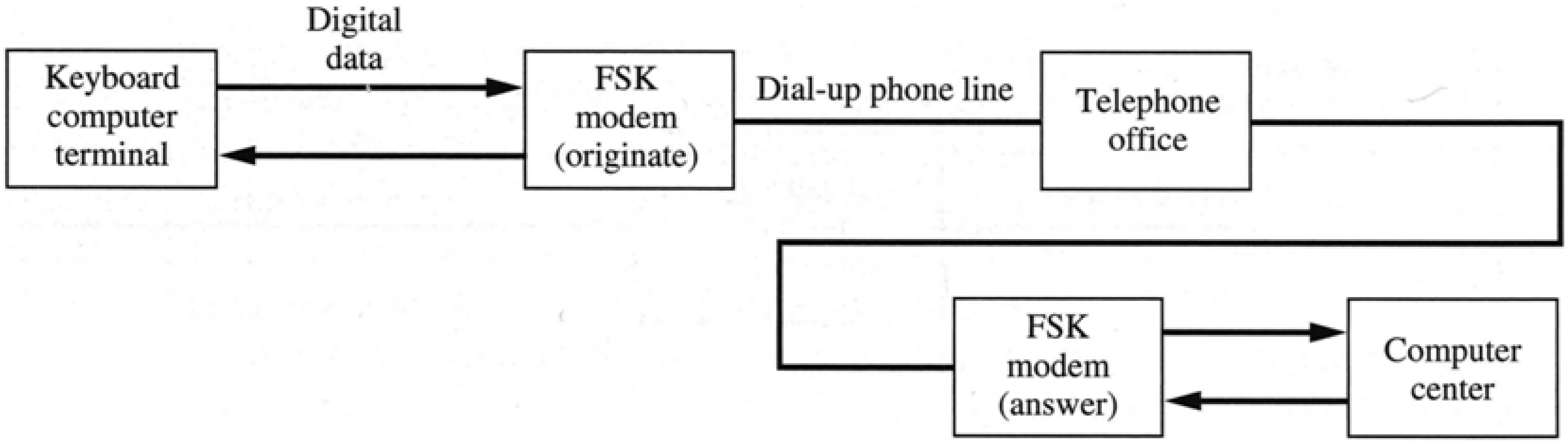

Figure 5–24 Computer communication using FSK signaling.

Table 5–5 MARK AND SPACE FREQUENCIES FOR THE BELL-TYPE 103 MODEM

	Originate Modem (Hz)	**Answer Modem (Hz)**
Transmit frequencies		
Mark (binary 1)	$f_1 = 1{,}270$	$f_1 = 2{,}225$
Space (binary 0)	$f_2 = 1{,}070$	$f_2 = 2{,}025$
Receive frequencies		
Mark (binary 1)	$f_1 = 2{,}225$	$f_1 = 1{,}270$
Space (binary 0)	$f_2 = 2{,}025$	$f_2 = 1{,}070$

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Frequency Shift Keying (3)

For FSK with $R_b = 1/T_b$ we find: $2\Delta\theta = 2\pi\Delta F T_b$

Digital modulation index:

$$\Rightarrow \; h = \frac{2\Delta\theta}{\pi} = 2\Delta F T_b = \frac{2\Delta F}{R_b}$$

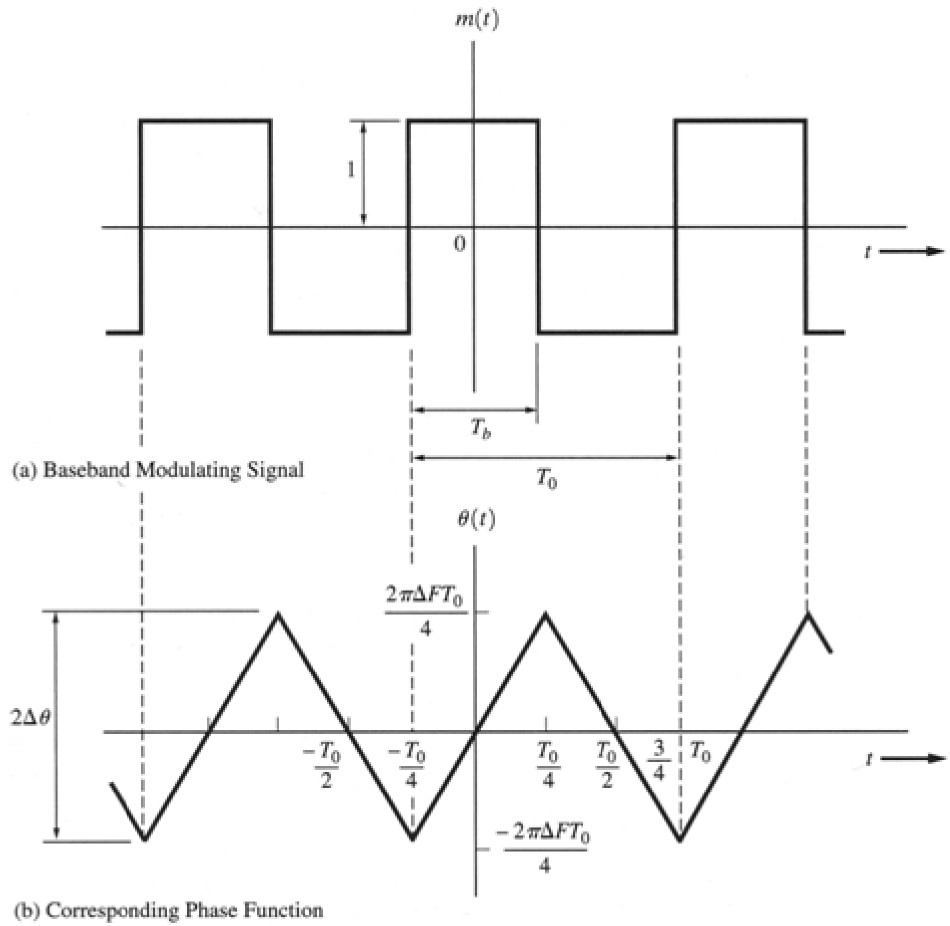

Figure 5–25 Input data signal and FSK signal phase function.

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Bandwidth of FSK signals (1)

The transmission bandwidth of FSK signals, based on the "null-to-null" bandwidth of the signal spectrum, is now found as:

- for rectangular pulses and $B = R_b$ is the 1st null bandwidth:

$$B_T = 2\Delta F + 2R_b$$

- for raised cosine pulses, the absolute baseband bandwidth for $B = \frac{1}{2}(1+r)R_b$ is given by:

$$B_T = 2\Delta F + R_b(1+r)$$

Bandwidth of FSK signals (2)

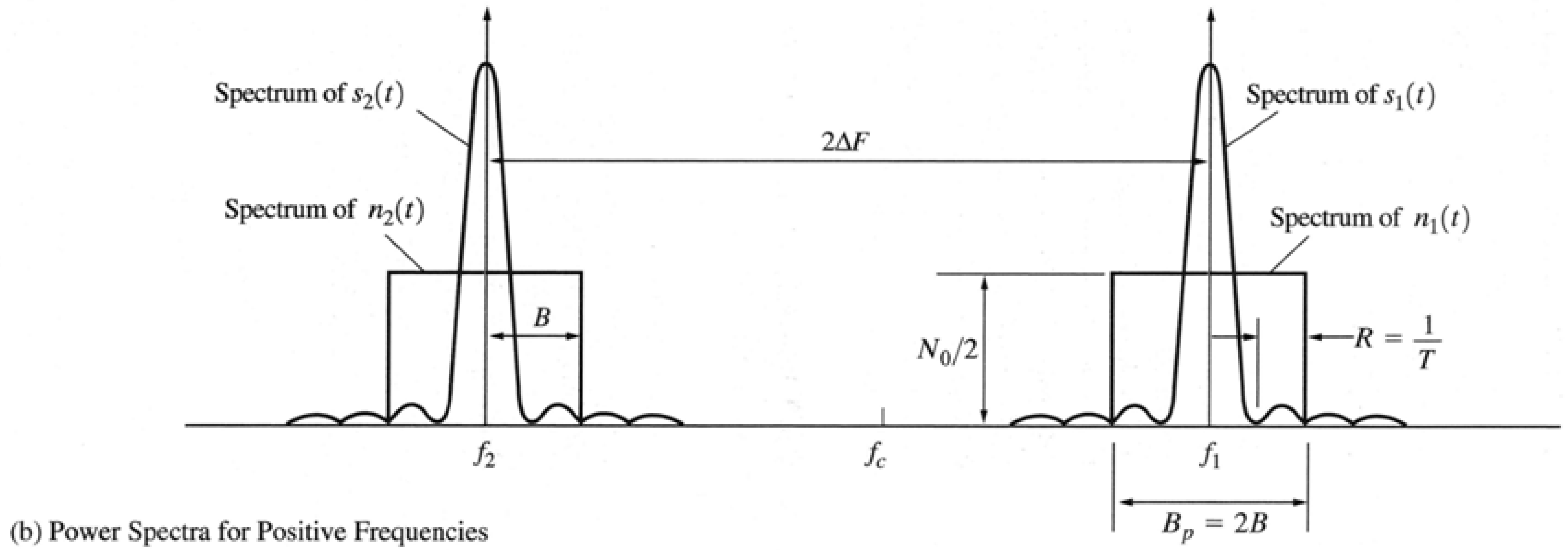
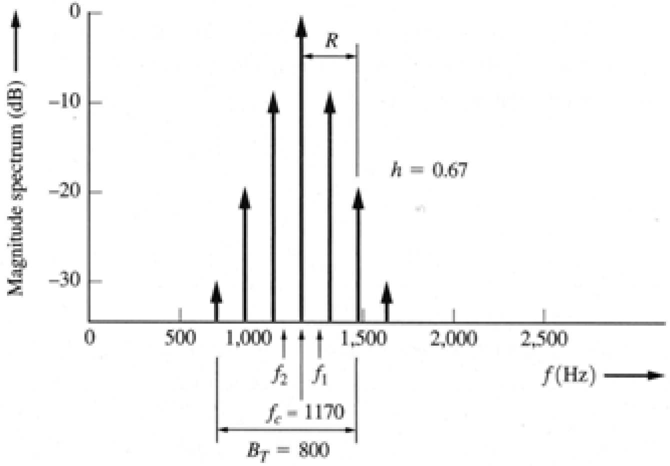
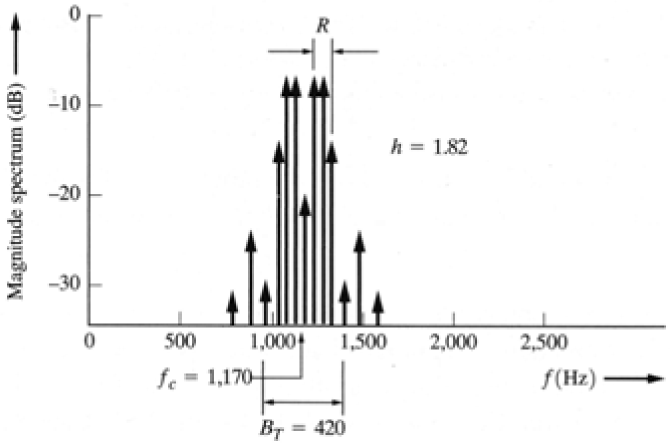


Figure 7–8 Coherent detection of an FSK signal.

Power spectrum of FSK signals (1)

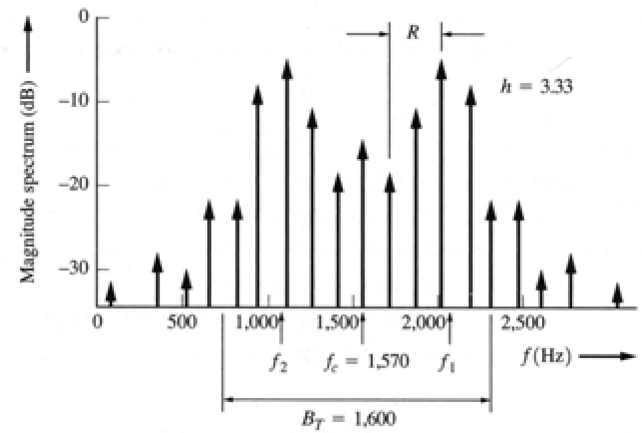


(a) FSK Spectrum with f_2 = 1,070 Hz, f_1 = 1,270 Hz, and R = 300 bits/sec (Bell 103 Parameters, Originate mode) for h = 0.67



(b) FSK Spectrum with f_2 = 1,070 Hz, f_1 = 1,270 Hz, and R = 110 bits/sec for h = 1.82

(b) FSK Spectrum with f_2 = 1,070 Hz, f_1 = 1,270 Hz, and R = 110 bits/sec for h = 1.82



(c) FSK Spectrum with f_2 = 1,070 Hz, f_1 = 2,070 Hz, and R = 300 bits/sec for h = 3.33

Figure 5–26 FSK spectra for alternating data modulation (positive frequencies shown with one-sided magnitude values).

The power spectral density of random data FSK is difficult to derive. Except for some special cases, a closed form expression cannot be obtained and we have to rely on simulation results.

Power spectrum of FSK signals (2)

$$\Delta\theta' = 2\pi \cdot \Delta F \cdot T_b = \pi \quad \text{for } h = 1$$

i.e. a π rad rotation during a symbol time, either clockwise or counter clockwise depending on the data symbol.

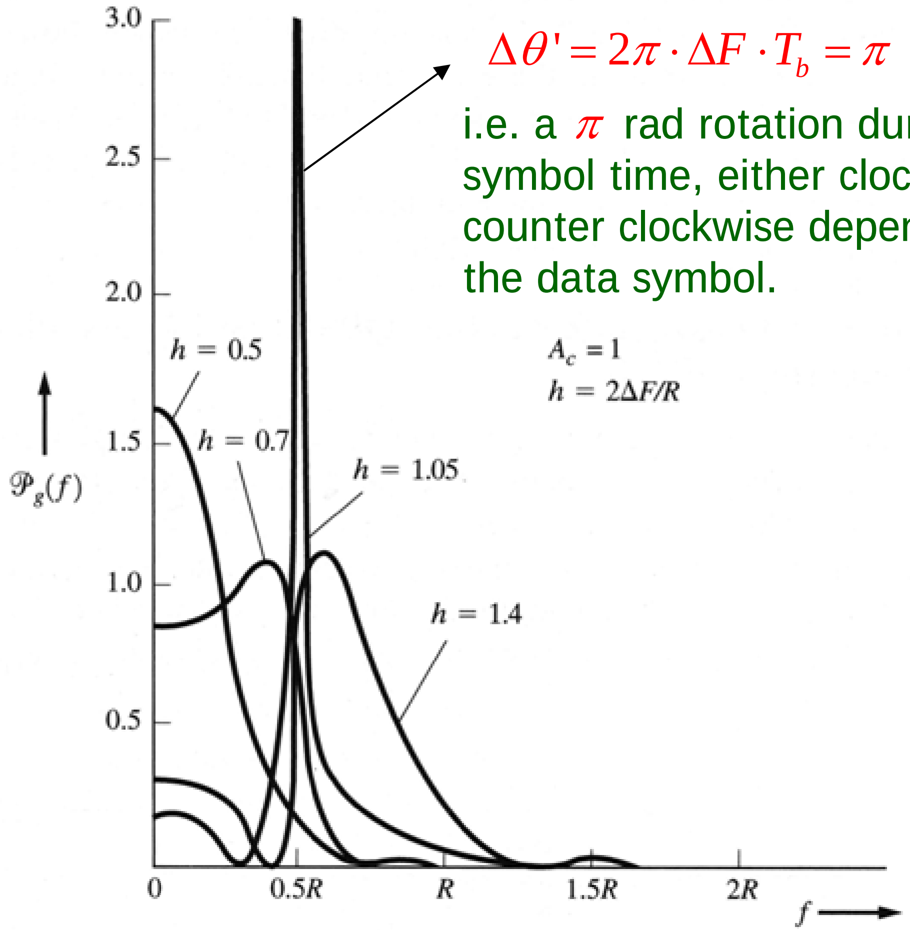


Figure 5–27 PSD for the complex envelope of FSK (positive frequencies shown).

Detection of FSK signals

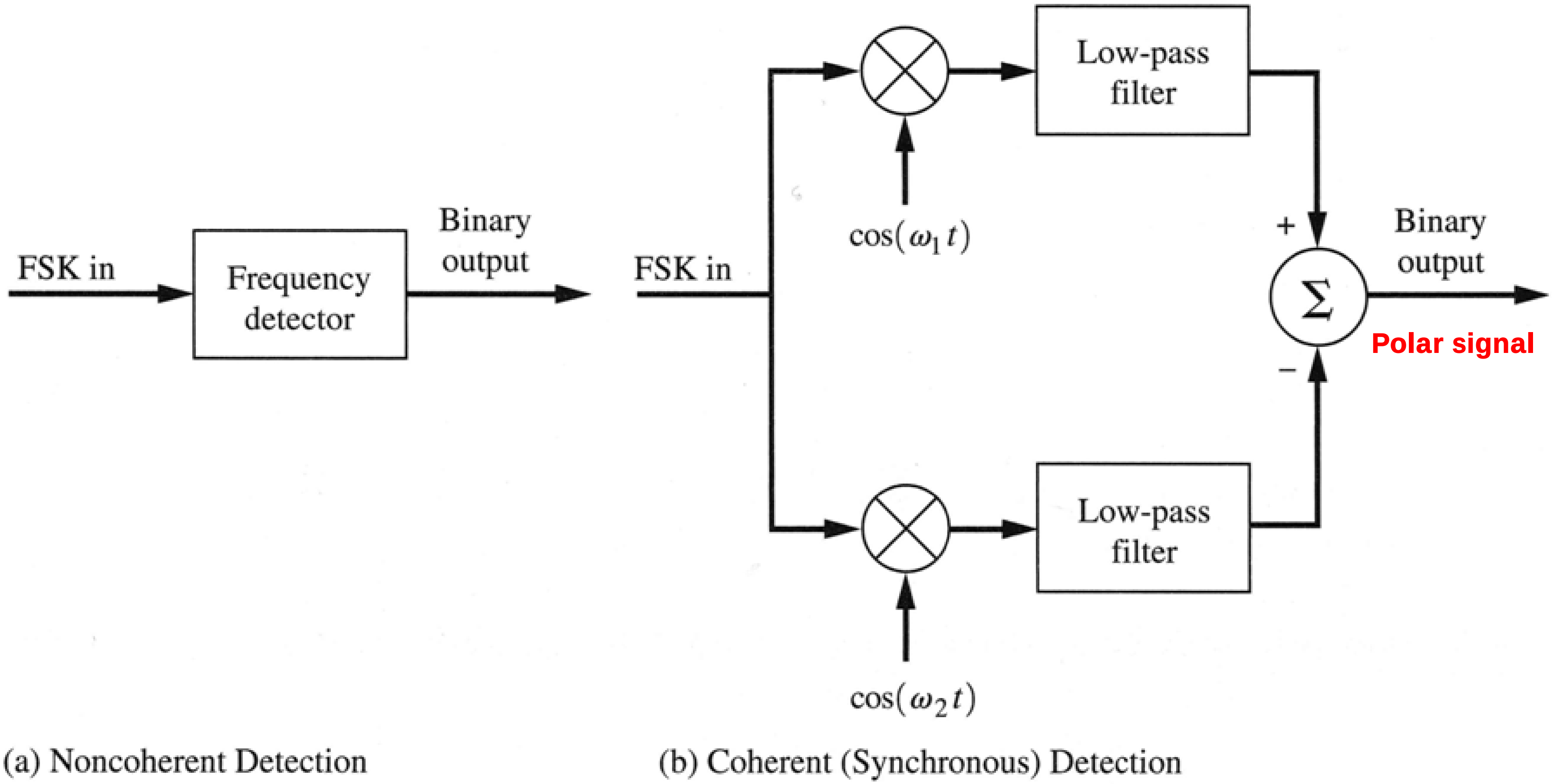


Figure 5–28 Detection of FSK.

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Basic techniques binary modulation

We have seen that the basic modulation techniques for binary signals have a direct analog counterpart:

1. OOK, ASK $\Longrightarrow$ AM
2. BPSK $\Longrightarrow$ DSB, PM
3. FSK $\Longrightarrow$ FM