Telecommunicatie B (EE2T21)

Lecture 10 overview:

Angle modulation $\Rightarrow$ non-linear modulation techniques for analog signals:

- **Phase Modulation (PM)**
- **Frequency Modulation (FM)**

Signal-to-Noise Ratio after detection for analog signals

- **Coherent detection of AM, DSB, SSB**
- **Non-coherent detection of AM**

EE2T21 Telecommunicatie B

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Colleges en Instructies
Telecommunicatie B

Colleges:

Maandag	2-5, 9-5, 30-5, 6-6	5e+6e uur, EWI-CZ Chip
Dinsdag	10-5	7e+8e uur, EWI-CZ Pi

Instructies:

Dinsdag	17-5	5e+6e uur, EWI-CZ Boole
Dinsdag	31-5	7e+8e uur, EWI-CZ Pi
Maandag	13-6	5e+6e uur, EWI-CZ Chip

Bandpass Signals

Mathematical description of bandpass signals:

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\}$$
$$= R(t)\cos[\omega_c t + \theta(t)]$$
$$= x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

Complex envelope or complex equivalent baseband signal:

$$g(t) = f(m(t)) = x(t) + jy(t) \quad = \quad R(t)e^{j\theta(t)}$$

Modulation function

In-phase and Quadrature-phase component

Amplitude: AM

Phase: PM, FM

Angle modulation

The transmitted signal for angle modulation is:

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{A_c e^{j\theta(t)} e^{j\omega_c t}\} = A_c \cos[\omega_c t + \theta(t)]$$

with:

$$g(t) = f(m(t)) = A_c e^{j\theta(t)}$$

$\Rightarrow$ - angle modulation
- constant amplitude

$\Rightarrow$ - efficient power generation using class-C amplifiers

Average signal power:

$$P_s = \langle s^2(t) \rangle = \tfrac{1}{2} \langle |g(t)|^2 \rangle = \tfrac{1}{2} \langle R^2(t) \rangle = \tfrac{1}{2} A_c^2$$

Independent of the selected type of angle modulation!

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Phase- and Frequency modulation (1)

Phase Modulation (PM):

$$\theta(t) = D_p m(t)$$

$\Rightarrow$ so $\theta(t)$ is linearly proportional with the information signal $m(t)$.

$\Rightarrow$ D_p = phase deviation constant [rad/V]

Frequency Modulation (FM):

$$\theta(t) = D_f \int_{-\infty}^{t} m(\lambda) d\lambda$$

$\Rightarrow$ so $\theta(t)$ is proportional with the integral of the information signal $m(t)$.

$\Rightarrow$ D_f = frequency deviation constant [rad/V.s] or [kHz/V].

Phase- and Frequency modulation (2)

For PM and FM: $\theta(t) = \mathfrak{L}\{m(t)\}$, however, for $g(t)$ we find:

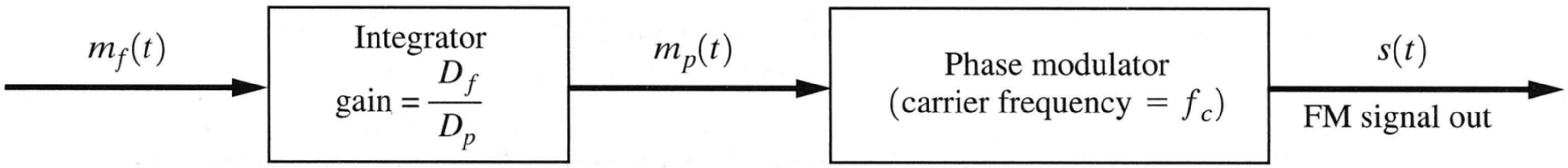
$$\begin{aligned} g(t) &= A_c e^{j\theta(t)} = x(t) + jy(t) = A_c \cos\theta(t) + jA_c \sin\theta(t) \\ &= A_c \cos\left(\mathfrak{L}\{m(t)\}\right) + jA_c \sin\left(\mathfrak{L}\{m(t)\}\right) \\ &= \text{ a non-linear function of } m(t) \end{aligned}$$

Therefore, PM and FM are called non-linear modulation techniques

Phase- and Frequency modulation (2)

Let $m_p(t)$ and $m_f(t)$ be the input signals of a PM and FM modulator, respectively. What is the relation between $m_p(t)$ and $m_f(t)$?

1. Frequency modulation with an PM-modulator:

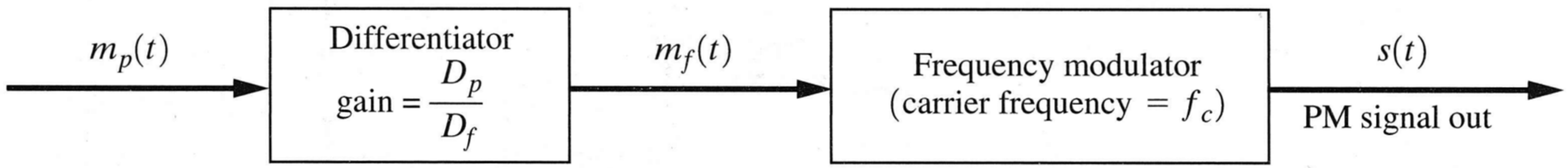


(a) Generation of FM Using a Phase Modulator

$m_p(t) = \frac{D_f}{D_p} \int_{-\infty}^{t} m_f(\lambda) d\lambda$ is the corresponding $m_p(t)$ to obtain FM for $m_f(t)$ with a frequency deviation constant D_f using a PM modulator with phase deviation constant D_p.

Phase- and Frequency modulation (3)

2. Phase modulation with an FM-modulator:

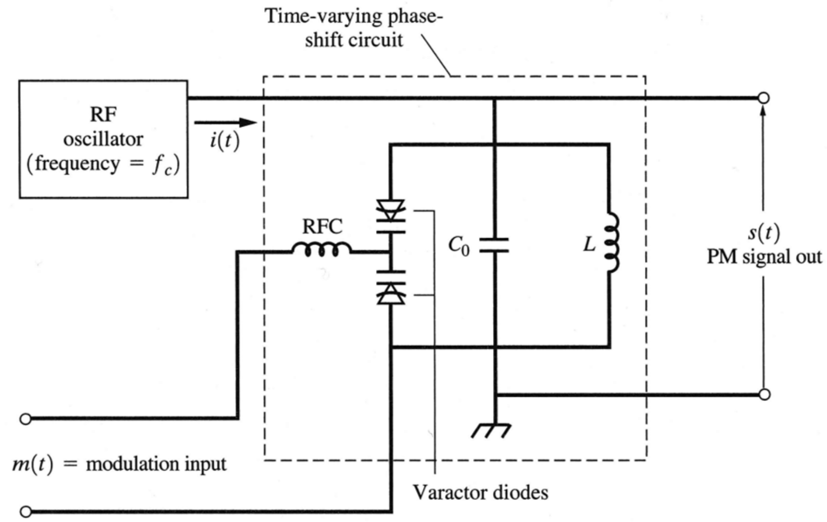


(b) Generation of PM Using a Frequency Modulator

Figure 5–7 Generation of FM from PM, and vice versa.

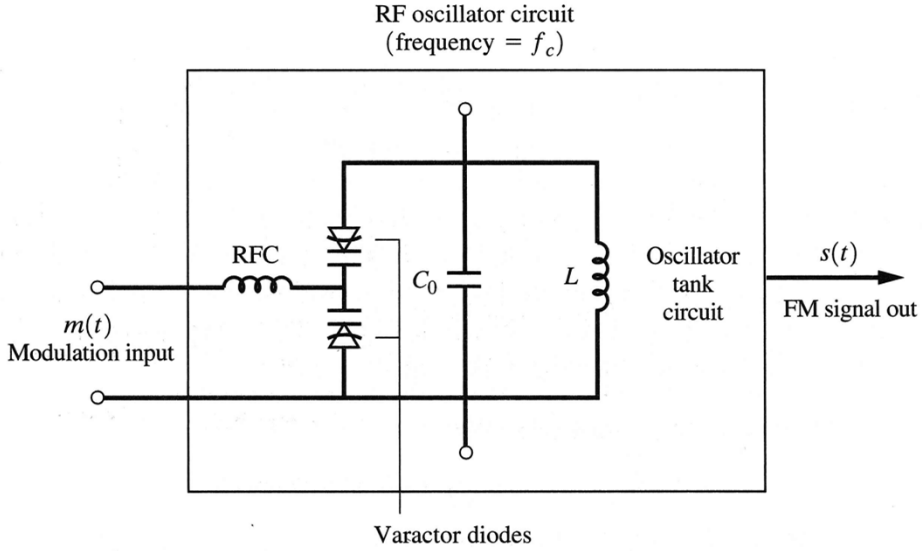
$m_f(t) = \frac{D_p}{D_f}\frac{dm_p(t)}{dt}$ is the corresponding $m_f(t)$ to obtain PM for $m_p(t)$ with phase deviation constant D_p using an FM modulator with frequency deviation constant D_f.

Direct phase- and frequency modulation



(a) A Phase Modulator Circuit

Voltage dependent frequency determining circuit of an oscillator.



(b) A Frequency Modulator Circuit

Figure 5–8 Angle modulator circuits. RFC = radio-frequency choke.

Voltage dependent phase shifting circuit.

Definition instantaneous frequency (1)

Instantaneous frequency $f_i(t)$:

Assume:

$$s(t) = R(t)\cos\psi(t) = R(t)\cos(\omega_c t + \theta(t))$$

$$\Rightarrow f_i(t) \triangleq \frac{1}{2\pi}\omega_i(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

So for FM we find with: $\theta(t) = D_f \int_{-\infty}^{t} m(\lambda)d\lambda$

$$f_i(t) = f_c + \frac{1}{2\pi}D_f m(t)$$

Therefore, D_f is called the frequency deviation constant
and the instantaneous frequency is linearly proportional with $m(t)$.

Definition instantaneous frequency (2)

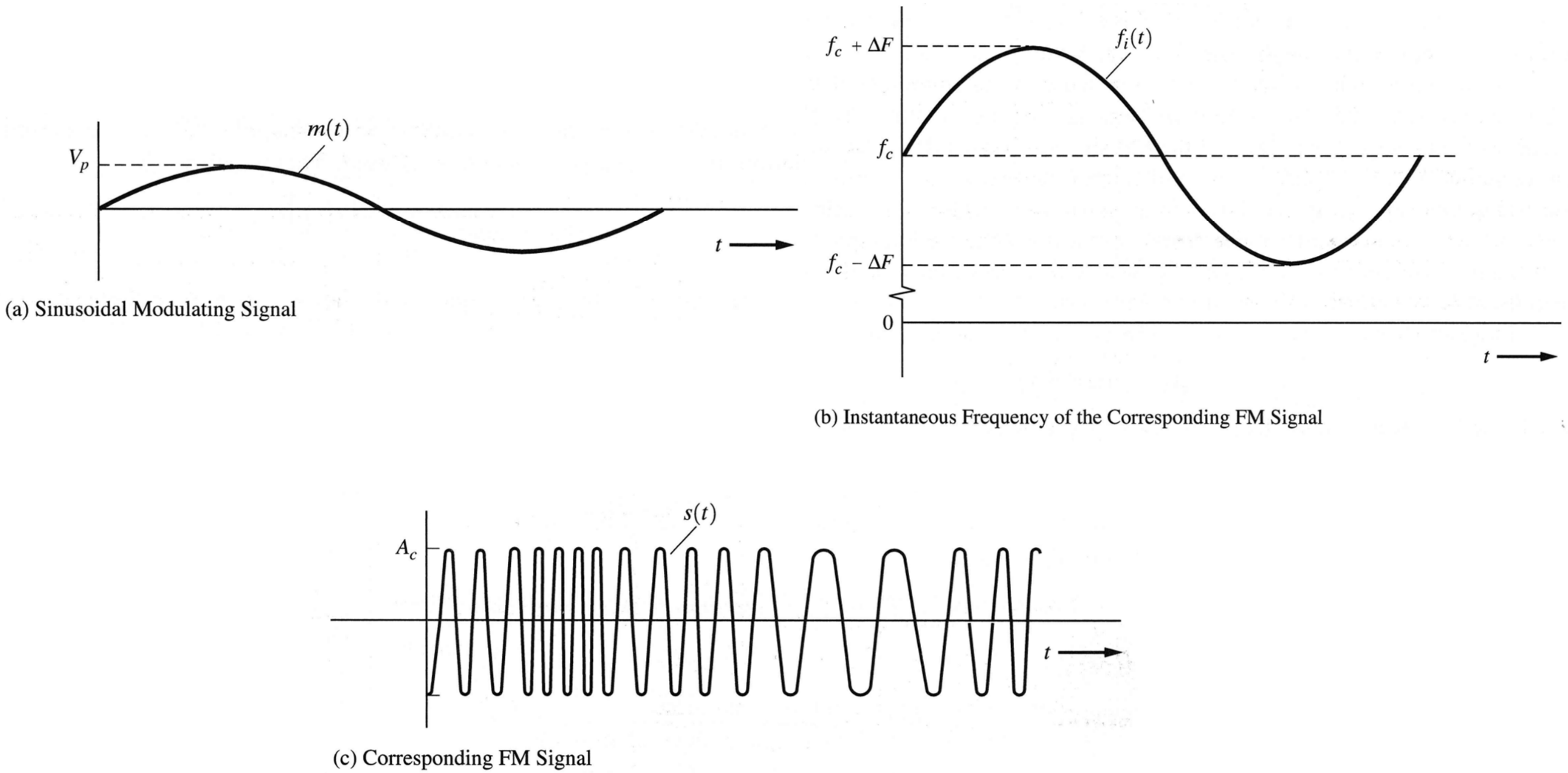


Figure 5–9 FM with a sinusoidal baseband modulating signal.

Definition frequency deviation

The frequency deviation is closely related to the instantaneous frequency:

$$\Rightarrow f_d(t) \triangleq f_i(t) - f_c = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \overset{FM}{=} \frac{1}{2\pi} D_f m(t)$$

Peak-frequency deviation ("frequentiezwaai"): $\Delta F \triangleq \max\{|f_d(t)|\} \geq 0$

Peak-peak deviation: $\Delta F_{pp} = \max\{f_d(t)\} - \min\{f_d(t)\} \simeq 2\Delta F$

For FM: $\Delta F = \frac{1}{2\pi} D_f V_p$ [Hz] with $V_p = \max\{m(t)\}$

For PM: $\Delta\theta = \max\{\theta(t)\} = D_p V_p$ (usually < π) [rad].

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PM and FM with sine-wave modulation

PM with $m_p(t) = A_m \sin\omega_m t \;\Rightarrow\; \theta(t) = D_p A_m \sin\omega_m t$

results in: $s(t) = A_c \cos\left(\omega_c t + D_p A_m \sin\omega_m t\right)$

$$= A_c \cos\left(\omega_c t + \Delta\theta \sin\omega_m t\right)$$

For both, PM and FM, we find a similar signal description.

FM with $m_f(t) = A_m \cos\omega_m t \;\Rightarrow\; \theta(t) = D_f \int_{-\infty}^{t} A_m \cos\omega_m \lambda \, d\lambda$

$$= \frac{A_m D_f}{\omega_m} \sin\omega_m t$$

results in: $s(t) = A_c \cos\left(\omega_c t + \frac{A_m D_f}{\omega_m} \sin\omega_m t\right) = A_c \cos\left(\omega_c t + \frac{\Delta F}{f_m} \sin\omega_m t\right)$

Definition of the modulation index

For PM the modulation index is defined as: $\beta_p \triangleq \Delta\theta = D_p V_p$

with $V_p = \max\{m(t)\}$. Now:

For a simple detector, $\Delta\theta_{\max} \leq \pi$, otherwise ambiguity in phase.

$$s(t) = A_c \cos\left(\omega_c t + \Delta\theta \sin\omega_m t\right) = A_c \cos\left(\omega_c t + \beta_p \sin\omega_m t\right)$$

For FM the modulation index is defined as: $\beta_f \triangleq \dfrac{\Delta F}{B}$ where B is the bandwidth of the modulation signal $m(t)$: for tone modulation $B = f_m$.

Now: $s(t) = A_c \cos\left(\omega_c t + \dfrac{\Delta F}{f_m}\sin\omega_m t\right) = A_c \cos\left(\omega_c t + \beta_f \sin\omega_m t\right)$

For PM and FM with sine-modulation we find equal modulation index:
$\beta_p = \beta_f$ when $\Delta\theta = \Delta F / f_m$.

Signal spectrum (1)

The frequency spectra of PM and FM modulated signals are found according to the definitions:

$$S(f) = \tfrac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$$

with: $G(f) = \mathfrak{F}\{g(t)\} = A_c \mathfrak{F}\{e^{j\theta(t)}\}$

Problem: for angle-modulation, $g(t)$ is a non-linear function of $m(t)$.
Therefore, a simple relation between $M(f)$ and $G(f)$ does not exist like for the linear modulation schemes (AM, DSB and SSB).
Also superposition cannot be applied in PM and FM.

Only for a few cases it is possible to derive the signal spectrum analytically for PM and FM. One of these cases is tone-modulation.

Signal spectrum (2)

For the signal spectrum with tone-modulation we take:

1. For PM: $m_p(t) = A_m \sin \omega_m t$

$$\Rightarrow \quad \theta(t) = D_p m(t) = \underbrace{D_p A_m}_{\beta_p} \sin \omega_m t = \beta \sin \omega_m t$$

2. For FM: $m_f(t) = A_m \cos \omega_m t$

$$\Rightarrow \quad \theta(t) = D_f \int_{-\infty}^{t} m(\lambda) d\lambda = D_f \int_{-\infty}^{t} A_m \cos \omega_m \lambda d\lambda$$

$$= \underbrace{\frac{A_m D_f}{\omega_m}}_{\beta_f} \sin \omega_m t = \beta \sin \omega_m t$$

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Signal spectrum (3)

So for both, PM and FM, we can now write: $g(t) = A_c e^{j\theta(t)} = A_c e^{j\beta \sin \omega_m t}$

Since $g(t)$ is a periodic function with period $T_m = 1/f_m$, $g(t)$ can be rewritten as a Fourier series:

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t} \quad \text{with} \quad c_n = \frac{A_c}{T_m} \int_{-T_m/2}^{T_m/2} e^{j\beta \sin \omega_m t} e^{jn\omega_m t} dt$$

Substitute $u = \omega_m t \quad \Rightarrow \quad t = \frac{uT_m}{2\pi} = \frac{u}{\omega_m} \quad \Rightarrow \quad dt = \frac{T_m}{2\pi} du$

$$\Rightarrow \quad c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin u - nu)} du \triangleq A_c J_n(\beta) \qquad J_{-n}(\beta) = (-1)^n J_n(\beta)$$

Bessel function of the 1st kind and order n

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Signal spectrum (4)

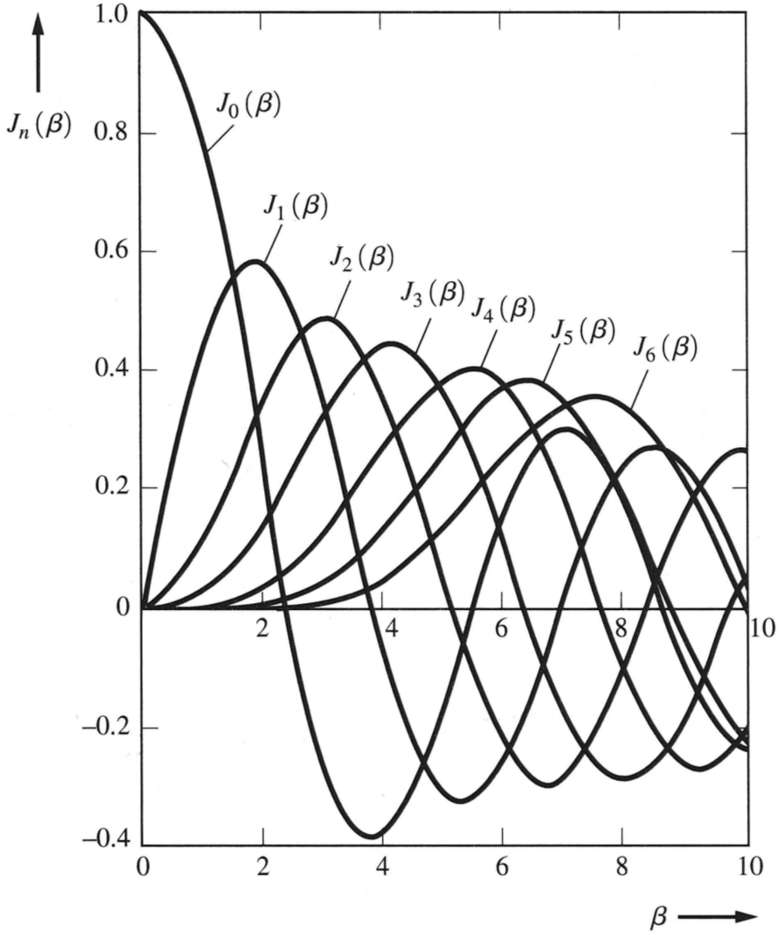

Figure 5–10 Bessel functions for $n = 0$ to $n = 6$.

Signal spectrum (3)

Using this, we find for the signal $s(t)$:

$$s(t) = \mathrm{Re}\{g(t)e^{j\omega_c t}\} = \mathrm{Re}\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta)e^{jn\omega_m t}e^{j\omega_c t}\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c t + n\omega_m t)$$

Frequency components at $f_c + nf_m$

For the frequency spectra of $g(t)$ and $s(t)$ we find now:

$$G(f) = \mathfrak{F}\{\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t}\} = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_m) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f - nf_m)$$

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - nf_m - f_c) + \delta(-f - nf_m - f_c)]$$

Transmission bandwidth infinite, in principle.

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Signal spectrum (4)

Now we find for the power spectral density:

$$P_s(f) = \frac{A_c^2}{4} \sum_{n=-\infty}^{\infty} J_n^2(\beta)[\delta(f - nf_m - f_c) + \delta(-f - nf_m - f_c)]$$

Signal spectrum (5)

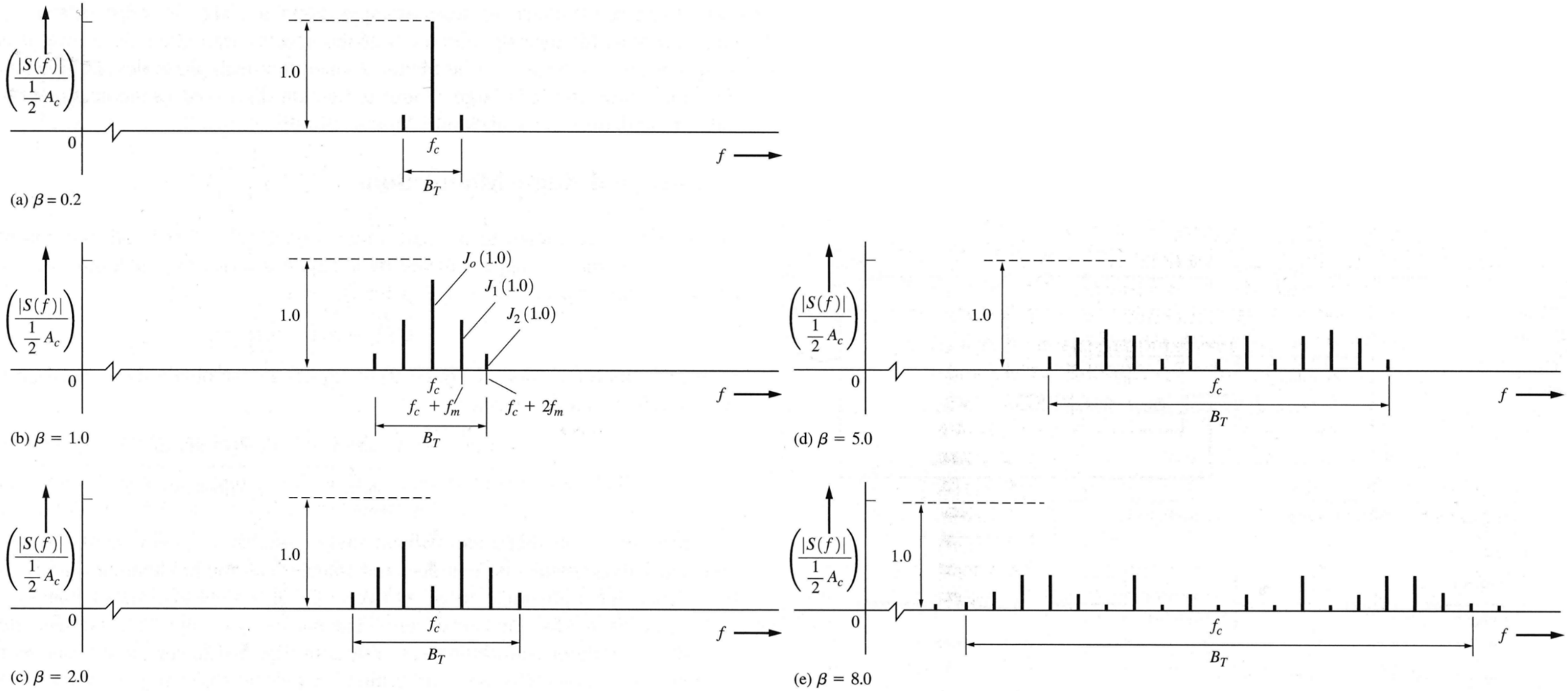

Figure 5–11 Magnitude spectra for FM or PM with sinusoidal modulation for various modulation indexes.

Signal spectrum (6)

Table 5–2 FOUR-PLACE VALUES OF THE BESSEL FUNCTIONS $J_n(\beta)$

n \ β:	0.5	1	2	3	4	5	6	7	8	9	10
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	−0.09033	−0.2459
1	0.2423	0.4401	0.5767	0.3391	−0.06604	−0.3276	−0.2767	−0.004683	0.2346	0.2453	0.04347
2	0.03060	0.1149	0.3528	0.4861	0.3641	0.04657	−0.2429	−0.3014	−0.1130	0.1448	0.2546
3	0.002564	0.01956	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809	0.05838
4		0.002477	0.03400	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655	−0.2196
5			0.007040	0.04303	0.1321	0.2611	0.3621	0.3479	0.1858	−0.05504	−0.2341
6			0.001202	0.01139	0.04909	0.1310	0.2458	0.3392	0.3376	0.2043	−0.01446
7				0.002547	0.01518	0.05338	0.1296	0.2336	0.3206	0.3275	0.2167
8					0.004029	0.01841	0.05653	0.1280	0.2235	0.3051	0.3179
9						0.005520	0.02117	0.05892	0.1263	0.2149	0.2919
10						0.001468	0.006964	0.02354	0.06077	0.1247	0.2075
11							0.002048	0.008335	0.02560	0.06222	0.1231
12								0.002656	0.009624	0.02739	0.06337
13									0.003275	0.01083	0.02897
14									0.001019	0.003895	0.01196
15										0.001286	0.004508
16											0.001567

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Signal spectrum (7)

Table 5–3 ZEROS OF BESSEL FUNCTIONS: VALUES FOR β WHEN $J_n(\beta) = 0$

	Order of Bessel Function, n						
	0	**1**	**2**	**3**	**4**	**5**	**6**
β for 1st zero	2.40	3.83	5.14	6.38	7.59	8.77	9.93
β for 2nd zero	5.52	7.02	8.42	9.76	11.06	12.34	13.59
β for 3rd zero	8.65	10.17	11.62	13.02	14.37	15.70	17.00
β for 4th zero	11.79	13.32	14.80	16.22	17.62	18.98	20.32
β for 5th zero	14.93	16.47	17.96	19.41	20.83	22.21	23.59
β for 6th zero	18.07	19.61	21.12	22.58	24.02	25.43	26.82
β for 7th zero	21.21	22.76	24.27	25.75	27.20	28.63	30.03
β for 8th zero	24.35	25.90	27.42	28.91	30.37	31.81	33.23

Transmission bandwidth

In principle, the absolute transmission bandwidth B_T of PM and FM is infinite!! B_T is determined by β and B (or f_m).

Carson's rule: 98% of the transmitted power is contained in the bandwidth $2(\beta+1)B$ for arbitrary $m(t)$.

Carson bandwidth: $B_T \simeq 2(\beta+1)B$

B is the bandwidth of $m(t)$.

For $2<\beta<10$ a better approximation for B_T is: $B_T \simeq 2(\beta+2)B$

For tone modulation with frequency f_m : $B_T \simeq 2(\beta+1)f_m$

Narrowband angle modulation (1)

For small angle variations: $|\theta(t)| < 0.2 \text{ rad}$

$e^{z} \simeq 1 + z \quad |z| << 1$

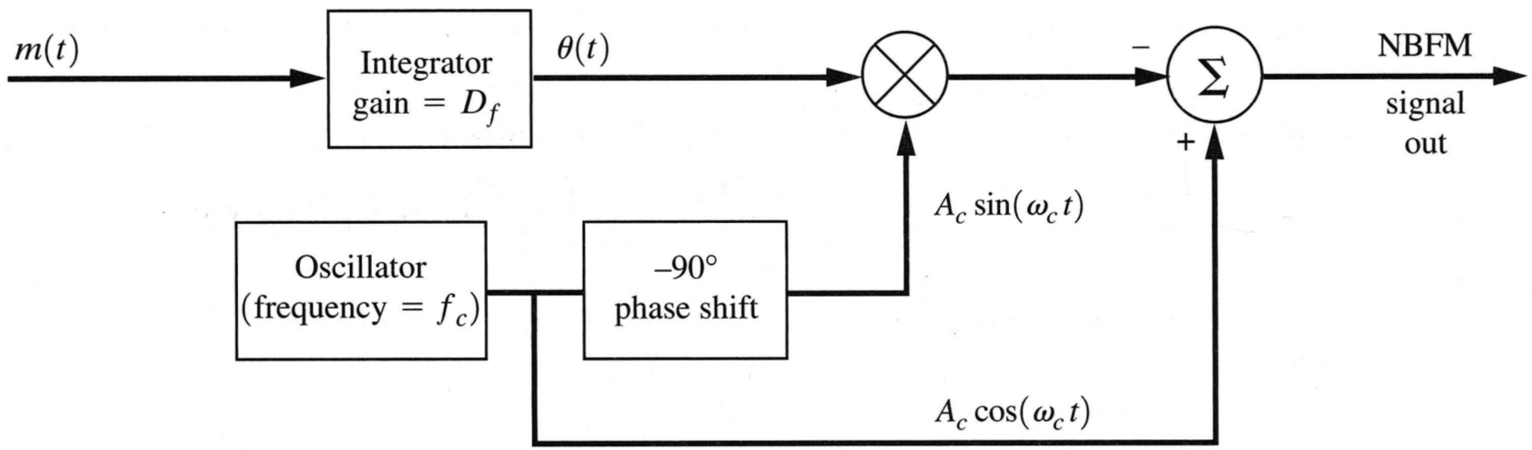
$g(t)$ can be approximated as: $g(t) = A_c e^{j\theta(t)} \simeq A_c[1 + j\theta(t)]$

Now we find for $s(t)$: $s(t) = \underbrace{A_c \cos\omega_c t}_{\text{carrier}} - \underbrace{A_c\theta(t)\sin\omega_c t}_{\text{sideband: 90° out of phase}}$

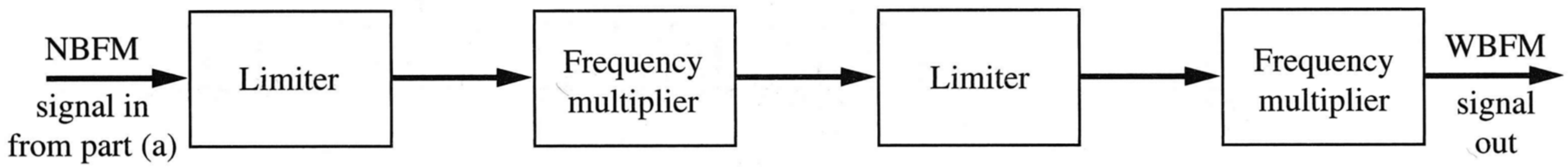
with: $$S(f) = \frac{A_c}{2}\{[\delta(f - f_c) + \delta(f + f_c)] + j[\Theta(f - f_c) - \Theta(f + f_c)]\}$$

where: $$\Theta(f) = \mathfrak{F}\{\theta(t)\} = \begin{cases} D_p M(f) & \text{for PM} \\ \dfrac{D_f}{2\pi j f} M(f) & \text{for FM} \end{cases}$$

Narrowband angle modulation (2)



(a) Generation of NBFM Using a Balanced Modulator



(b) Generation of WBFM From a NBFM Signal

Figure 5–12 Indirect method of generating WBFM (Armstrong method).

Detection performance for analogue modulations

Signal quality after detection is determined by:
- received signal power
- noise- and interference power at the receiver input

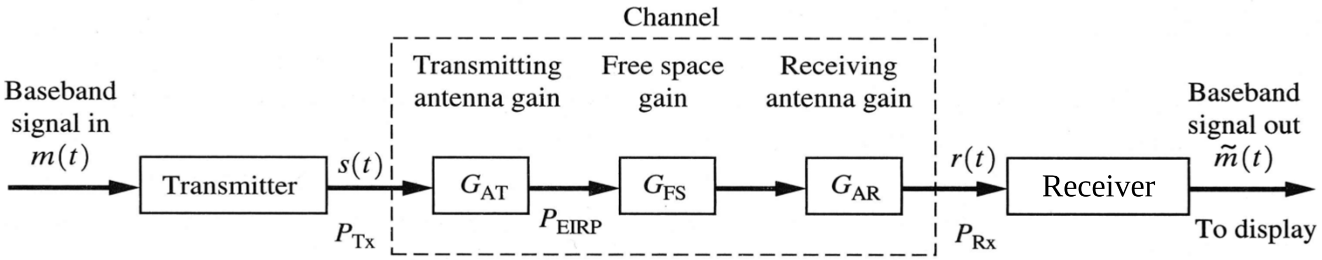


Figure 8–16 Block diagram of a communication system with a free-space transmission channel.

Carrier-to-Noise ratio at the detector input:

$$\frac{C}{N} \triangleq \frac{\text{received signal power}}{\text{noise power in } B_T}$$

Determines: - SNR_{out} after detection for analogue signals
- bit error probability (BER) for digital signals

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Signal detection quality

Analog modulation: after detection, there is a linear relation between S/N_{out} and S/N_{in}.

- AM, DSB, SSB: $S/N_{out} \leq 2\ S/N_{in}$ $(B_T \approx 2B)$

- FM, PM: $S/N_{out} >> S/N_{in}$ is possible
if $B_T >> 2B$, and $S/N_{in} > S/N_{th}$

A better S/N_{out} can be obtained at the cost of a larger B_T.

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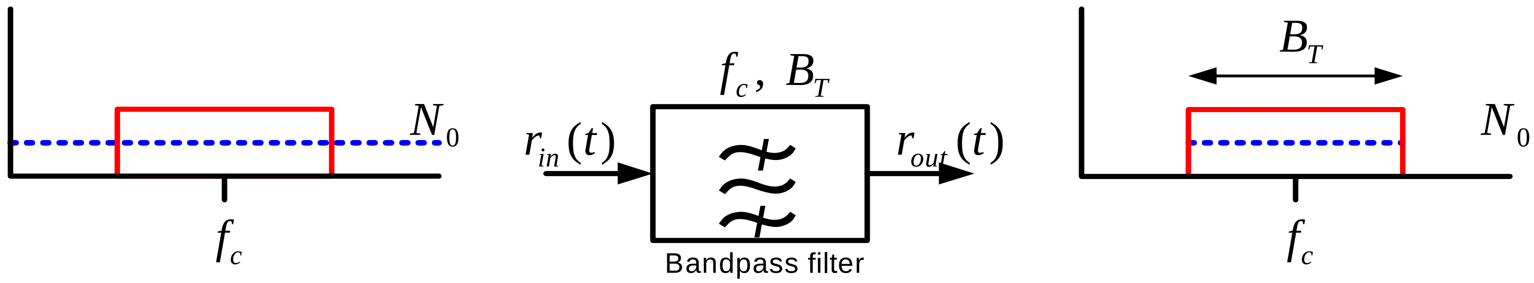
SNR_{out} for analog systems (1)

Additive noise in bandpass systems:



$$
\begin{aligned}
r(t) &= s(t) + n(t) \\
&= \mathrm{Re}\{[g_s(t) + g_n(t)]e^{j(\omega_c t + \theta_c)}\}
\end{aligned}
$$

n(t) is additive white Gaussian noise (AWGN), with PSD $P_n(f) = N_0/2$ (double sided).

Baseband equivalent complex envelope:

$$
\begin{aligned}
g_T(t) &\hat{=} g_s(t) + g_n(t) \\
&= [x_s(t) + x_n(t)] + j[y_s(t) + y_n(t)]
\end{aligned}
$$

in-phase   quadrature-phase

SNR_{out} for analog systems (2)

Reference $\Rightarrow$ baseband system (without modulation): standard

$$\left(\frac{S}{N}\right)_{baseband} = \frac{P_s}{N_0 B} = \frac{P_s}{(N_0/2)\cdot 2B}$$

SNR at detector input:

$$\left(\frac{S}{N}\right)_{in} = \frac{P_s}{N_0 B_T}$$
$$= \left(\frac{S}{N}\right)_{baseband} \frac{B}{B_T}$$

In the following, we assume that the bandwidth used for noise power calculations refers to the equivalent noise bandwidth.

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Coherent detection (AM, DSB and SSB)

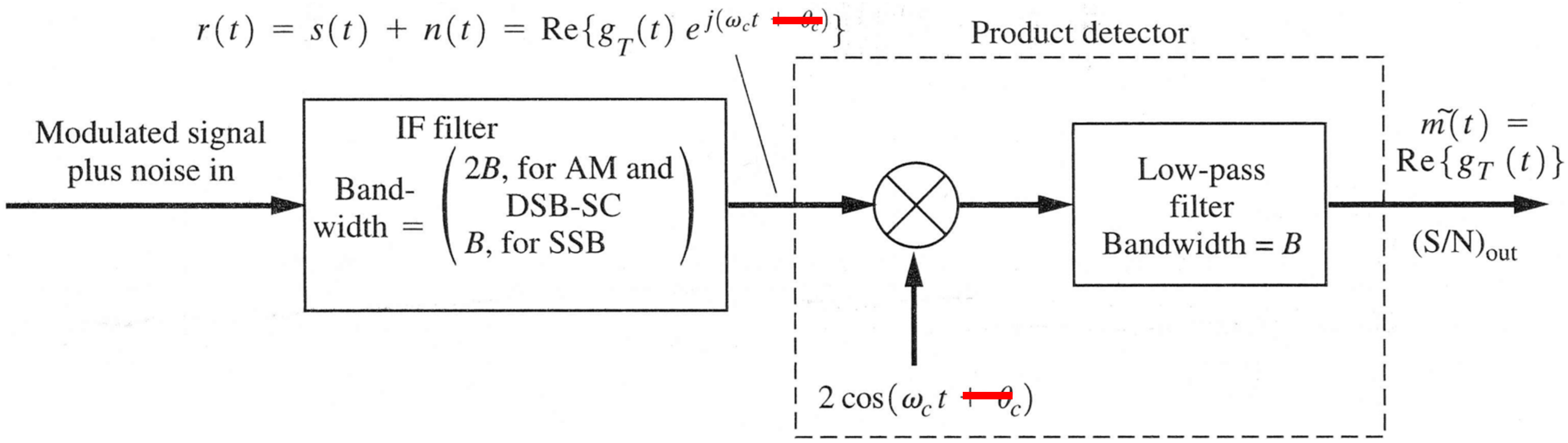


Figure 7–19 Coherent receiver.

Coherent detection without phase error ($\theta_{lo} = \theta_c$) gives:

$$\tilde{m}(t) = \mathrm{Re}\{g_T(t)\} = x_s(t) + x_n(t)$$

Coherent detection (AM, DSB and SSB)

Let: $r(t) = s(t) + n(t) = [x_s(t) + x_n(t)]\cos\omega_c t - [y_s(t) + y_n(t)]\sin\omega_c t$

then we find after coherent detection without phase error and $\theta_c = 0$:

$$
\begin{aligned}
2r(t)\cos\omega_c t &= 2[x_s(t) + x_n(t)]\cos^2\omega_c t - 2[y_s(t) + y_n(t)]\sin\omega_c t\cos\omega_c t \\
&= 2[x_s(t) + x_n(t)](\tfrac{1}{2} + \tfrac{1}{2}\cos 2\omega_c t) - [y_s(t) + y_n(t)]\sin 2\omega_c t \\
&\overset{LPF}{=} [x_s(t) + x_n(t)]
\end{aligned}
$$

See also: Product detector circuit § 4.13

SNR: coherent detection AM (1)

AM-modulation: $g_T(t) = A_c[1 + m(t)] + x_n(t) + jy_n(t)$

$\overline{x_n^2(t)} = \overline{y_n^2(t)} = 2N_0B$

and $\widetilde{m}(t) = A_c + A_c m(t) + x_n(t)$

Input signal power: $P_s = \dfrac{A_c^2}{2}\overline{[1 + m(t)]^2} = \dfrac{A_c^2}{2}[1 + \overline{m^2(t)}]$

Now we find for SNR_{out}:

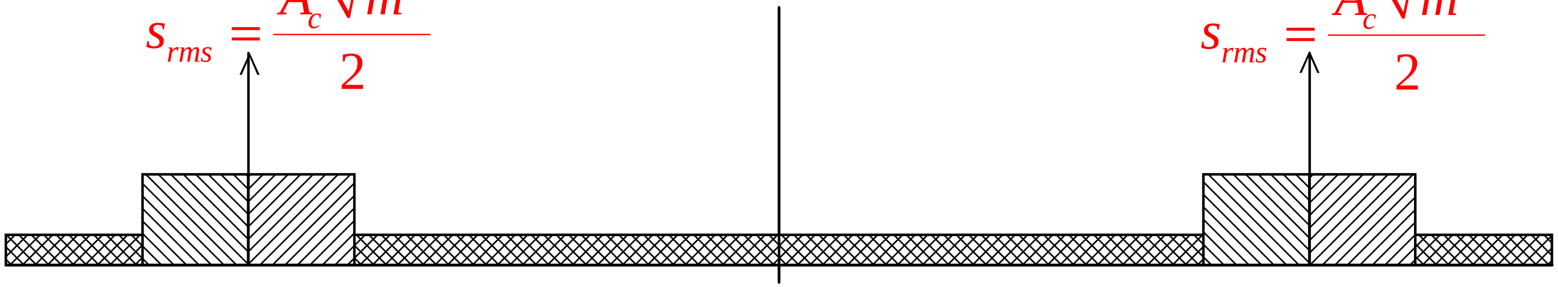
What happens when there is a phase error $\theta_{lo} \neq \theta_c$ in the coherent detector?

$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2\,\overline{m^2(t)}}{\overline{x_n^2(t)}} = \frac{A_c^2\,\overline{m^2(t)}}{2N_0B}$$

§6.7, equation 6.133
Stoch. Proc.

SNR: coherent detection AM (2)

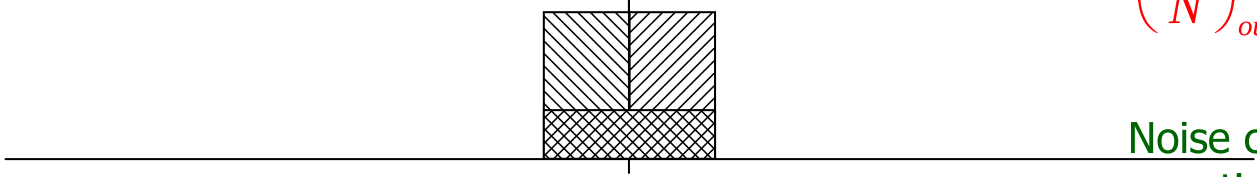
$s_{rms} = \frac{A_c \sqrt{\overline{m^2}}}{2}$ $\qquad$ $s_{rms} = \frac{A_c \sqrt{\overline{m^2}}}{2}$



$\overline{x_n^2} = 2B \cdot (N_0 / 2)$ $\qquad$ $\overline{x_n^2} = 2B \cdot (N_0 / 2)$

After coherent detection: based on sideband power

$s_{rms} = A_c \sqrt{\overline{m^2}}$



$\overline{x_n^2} = 2B \cdot N_0$

$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2 \overline{m^2(t)}}{\overline{x_n^2(t)}} = \frac{A_c^2 \overline{m^2(t)}}{2N_0 B}$$

Noise components at positive and negative frequencies are uncorrelated: their powers add up. The signal components are correlated: their amplitudes add up.

Based on sideband power $SNR_{out} = 2SNR_{in}$.

SNR: coherent detection AM (3)

With the input signal power $P_s = \frac{A_c^2}{2}(1+\overline{m^2(t)})$ and noise power $P_n = 2N_0B$ we find:

$$\left(\frac{S}{N}\right)_{in} = \frac{\frac{A_c^2}{2}(1+\overline{m^2})}{2N_0B} \quad \Rightarrow \quad \frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1+\overline{m^2}} \leq 1$$

Comparison to baseband:

$$\left(\frac{S}{N}\right)_{bb} = \frac{P_s}{N_0B} = \frac{\frac{A_c^2}{2}(1+\overline{m^2})}{N_0B} \quad \Rightarrow \quad \frac{(S/N)_{out}}{(S/N)_{bb}} = \frac{\overline{m^2}}{1+\overline{m^2}} \leq 0.5$$

SNR: coherent detection AM (4)

For sine wave modulation: $\overline{m^2} = 0.5$ and

$$\frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2}{3}, \qquad \frac{(S/N)_{out}}{(S/N)_{baseband}} = \frac{1}{3}$$

So AM performs at least 4.8 dB worse than baseband (AM is sub-standard) $\Rightarrow$ loss in carrier power $\geq$ 3 dB.

SNR: coherent detection DSB

DSB-modulation: $g_T(t) = A_c m(t) + x_n(t) + jy_n(t)$

and $\tilde{m}(t) = A_c m(t) + x_n(t)$

Using the signal power: $P_s = \frac{A_c^2}{2}\overline{m^2(t)}$, $\left(\frac{S}{N}\right)_{out} = \frac{A_c^2\overline{m^2(t)}}{2N_0 B}$

we find: $\frac{(S/N)_{out}}{(S/N)_{in}} = 2, \quad \frac{(S/N)_{out}}{(S/N)_{baseband}} = 1 \Rightarrow \text{standard}$

3 dB improvement is obtained by coherent addition of the sidebands.

SNR: coherent detection SSB

SSB-modulation: $g_s(t) = A_c[m(t) \mp j\hat{m}(t)] \quad \begin{matrix} USSB \\ LSSB \end{matrix}$

$$\Rightarrow \quad g_T(t) = [A_c m(t) + x_n(t)] + j[\mp A_c \hat{m}(t) + y_n(t)]$$

Coherent detection: $\tilde{m}(t) = \mathrm{Re}\{g_T(t)\} = A_c m(t) + x_n(t), \quad \overline{x_n^2(t)} = N_0 B$

$$\Rightarrow \quad \left(\frac{S}{N}\right)_{out} = \frac{A_c^2 \overline{m^2}}{\overline{x_n^2}} = \frac{A_c^2 \overline{m^2}}{N_0 B}$$

Signal- and noise power at detector input:

$$P_s = \tfrac{1}{2}\overline{|g_s(t)|^2} = \tfrac{A_c^2}{2}[\overline{m^2} + \overline{\hat{m}^2}] = A_c^2 \overline{m^2} \qquad P_n = N_0 B$$

$$\Rightarrow \quad \frac{(S/N)_{out}}{(S/N)_{in}} = 1 \qquad \frac{(S/N)_{out}}{(S/N)_{basisb.}} = 1$$

The performance of SSB is identical to baseband: standard.

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Coherent detection (SSB)

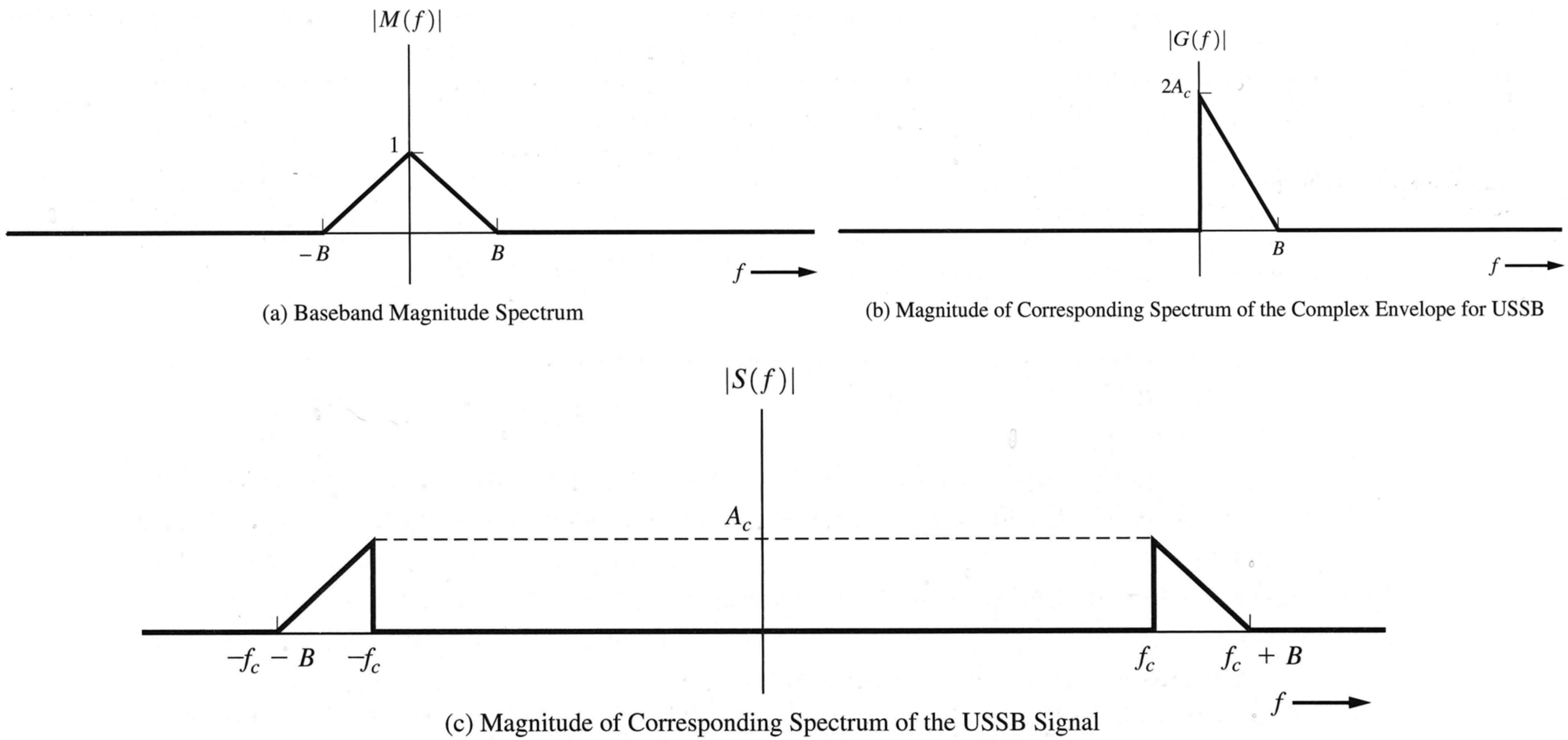
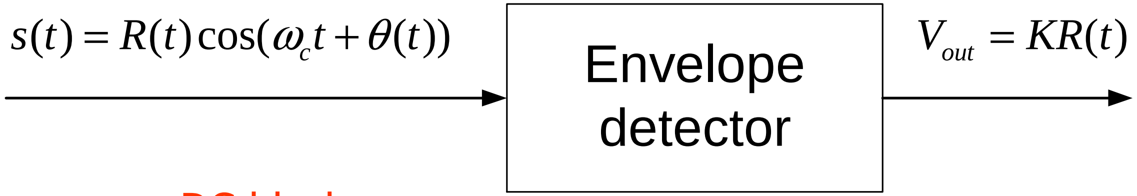
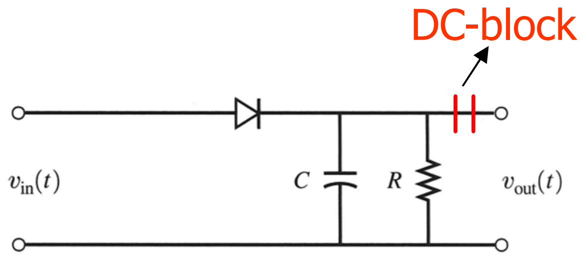


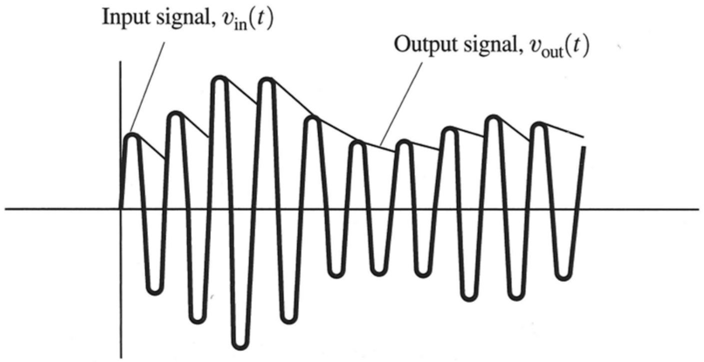
Figure 5–4 Spectrum for a USSB signal.

Non-coherent AM-detection: envelope detector





(a) A Diode Envelope Detector



(b) Waveforms Associated with the Diode Envelope Detector

Figure 4–13 Envelope detector.

$$V_{out} = KR(t) = KA_c[1 + m(t)] = KA_c + KA_c m(t)$$

$$\frac{1}{f_c} << \tau = RC << \frac{1}{B}$$

Note: the rectifying envelope detector is a non-linear component: $R(t) = \sqrt{x^2(t) + y^2(t)}$!

But ... , it is cheap!!!

SNR: AM with envelope detection (1)

The output signal of an envelope detector with gain K is:

non-linear

$$r_0(t) = KR_T(t) = K\,|\,g_T(t)\,| = K\,|\,g_s(t) + g_n(t)\,|$$
$$= K\,|\,[A_c(1+m(t)) + x_n(t)] + jy_n(t)\,|$$

with

$$g_n(t) = x_n(t) + jy_n(t) = R_n(t)e^{j\theta_n(t)}$$

The output signal of the non-linear envelope detector is given by:

$$r_0(t) = KR_T(t) = K\sqrt{[A_c(1+m(t)) + x_n(t)]^2 + y_n^2(t)}$$

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Noise and Rayleigh distribution (1)

In the complex envelope of a noise signal:

$$g_n(t) = x_n(t) + jy_n(t) = R_n(t)e^{j\theta_n(t)}$$

x_n and y_n are normally distributed with $\overline{x_n} = \overline{y_n} = 0,$ $\overline{x_n^2} = \overline{y_n^2} = \sigma^2 = N_0 B_n$

and $P_{x_n}(f) = P_{y_n}(f) = \dfrac{N_0}{2}$ $\Rightarrow$ white Gaussian noise

The amplitude $R_n(t)$ of $g_n(t)$ has a Rayleigh distribution and its phase $\theta_n(t)$ is uniformly distributed over $[0,\ 2\pi)$ (pag. 442 - 444).

$$f_R(R) = \begin{cases} \dfrac{R}{\sigma^2} e^{-R^2/2\sigma^2} & ,R \geq 0 \\ 0 & ,R < 0 \end{cases}$$

met $\overline{R} = \sigma\sqrt{\dfrac{\pi}{4}}$ and $\sigma_R^2 = \left(1 - \frac{\pi}{4}\right)\sigma^2$

Rayleigh distribution (2)

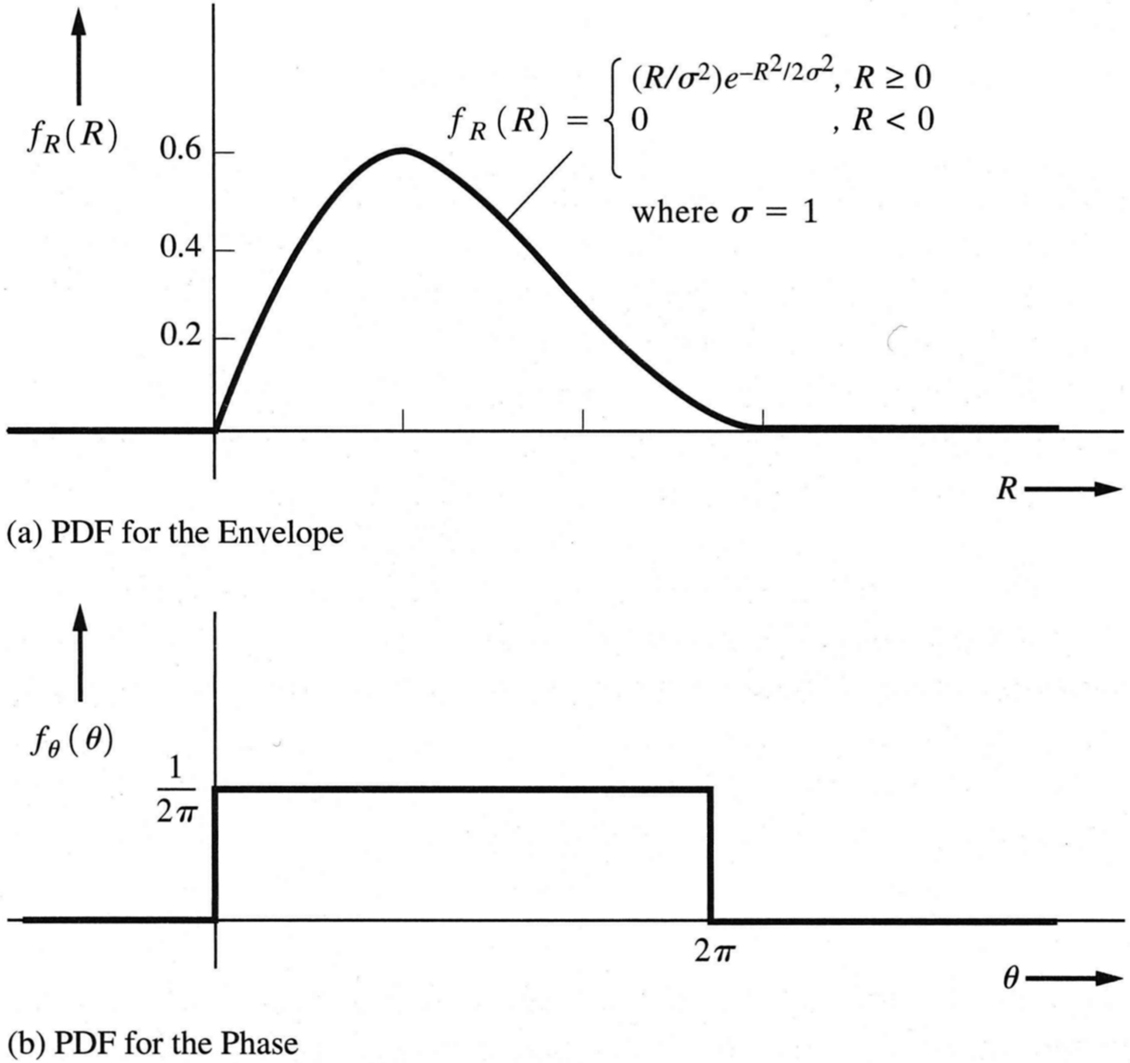

Figure 6–14 PDF for the envelope and phase of a Gaussian process.

SNR: AM with envelope detection (2)

For $SNR_{in} >> 1$, thus $(x_n / A_c)^2, (y_n / A_c)^2 << 1$ the envelope can be approximated by:

$$r_0(t) = K\left\{A_c^2[1+m(t)]^2 + 2A_c[1+m(t)]x_n(t) + x_n^2(t) + y_n^2(t)\right\}^{\frac{1}{2}}$$

$$= KA_c[1+m(t)]\left\{1 + \frac{2x_n(t)}{A_c[1+m(t)]} + \cancel{\frac{x_n^2(t)}{A_c^2[1+m(t)]^2}} + \cancel{\frac{y_n^2(t)}{A_c^2[1+m(t)]^2}}\right\}^{\frac{1}{2}}$$

$$\approx KA_c[1+m(t)] + Kx_n(t)$$

$$\sqrt{(1+x)} \approx 1 + \frac{x}{2} \quad \text{if } x << 1$$

$$\Rightarrow \overline{r_0^2(t)} = K^2A_c^2 + K^2A_c^2\overline{m^2(t)} + K^2\overline{x_n^2(t)}$$

Signal and noise are additive!

Thus for SNR_{out} we find:

$$SNR_{out} = \frac{A_c^2\overline{m^2}}{\overline{x_n^2}} = \frac{A_c^2\overline{m^2}}{2N_0B}$$

identical to the coherent product detector, **if** $SNR_{in} >> 1$.

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SNR: AM with envelope detection (3)

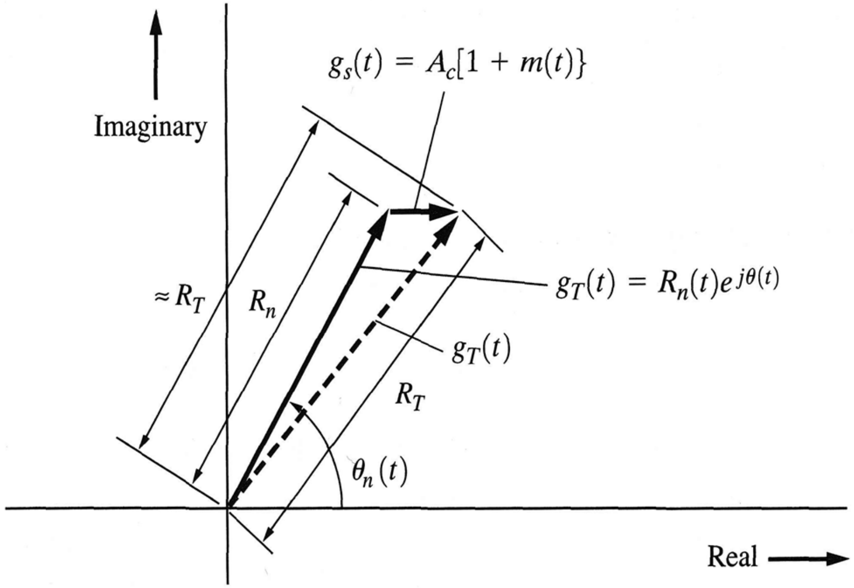

Figure 7–20 Vector diagram for AM, $(S/N)_{\text{in}} \ll 1$.

For $SNR_{in} \cancel{\gg} 1$, the output signal of the envelope detector (non-linear) will follow the noise amplitude in stead of the signal amplitude:

Signal and noise are multiplicative!

$$r_0(t) = KR_T(t) = K\,|\,A_c[1+m(t)] + R_n(t)e^{j\theta_n(t)}\,|$$
$$\approx K\{A_c[1+m(t)]\cos\theta_n(t) + R_n(t)\}$$

For small SNR_{in} , SNR_{out} decreases very fast: *threshold effect*.

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Non-coherent detection of AM

Usually, an envelope detector (much cheaper) is used for detecting AM signals in stead of a coherent detector:

- for $SNR_{in} >> 1$ $\Rightarrow$ signal and noise at the output are additive
  $\Rightarrow$ performance equal to the coherent detector
- for $SNR_{in} \not\gg 1$ $\Rightarrow$ noise becomes multiplicative!!
  $\Rightarrow$ very bad performance.

Somewhere, a threshold occurs between linear and non-linear detection behavior. This threshold is found around SNR_{in} = 10 dB.

In practice this threshold is not very relevant for audio/voice signals, because the SNR for these signals has to be better than 20 – 25 dB anyway. For AM-data transmission, however, this threshold is dramatic, and usually a coherent detector has to be used.