

The Name of the Title Is Hope

tbd

ABSTRACT

tbd.

CCS CONCEPTS

• Theory of computation → Cryptographic primitives.

KEYWORDS

tbd

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1 INTRODUCTION

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2 PRELIMINARY

2.1 Basic Notations

Point and multi-point functions. Given a domain size N and Abelian group \mathbb{G} , a point function $f_{\alpha,\beta} : [N] \rightarrow \mathbb{G}$ for $\alpha \in [N]$ and $\beta \in \mathbb{G}$ evaluates to β on input α and to $0 \in \mathbb{G}$ on all other inputs. We denote by $\hat{f}_{\alpha,\beta} = (N, \mathbb{G}, \alpha, \beta)$ the representation of such a point function. A multi-point function $f_{A,B} : [N] \rightarrow \mathbb{G}$ for $A = (\alpha_1, \dots, \alpha_t) \in [N]^t$ and $B = (\beta_1, \dots, \beta_t) \in \mathbb{G}^t$ evaluates to β_i on input α_i for $1 \leq i \leq t$ and to 0 on all other inputs. Denote $\hat{f}_{A,B}(N, \mathbb{G}, A, B)$ the representation of such a point function.

Enote: MPF. Also representation of groups.

2.2 Distributed Multi-Point Functions

Enote: should directly adapt to multi-point function case

We begin by defining a slightly generalized notion of distributed point functions (DPFs), which accounts for the extra parameter \mathbb{G}' .

DEFINITION 1 (DPF [1, 3]). A (2-party) distributed point function (DPF) is a triple of algorithms $\Pi = (\text{Gen}, \text{Eval}_0, \text{Eval}_1)$ with the following syntax:

- $\text{Gen}(1^\lambda, \hat{f}_{\alpha,\beta}) \rightarrow (k_0, k_1)$: On input security parameter $\lambda \in \mathbb{N}$ and point function description $\hat{f}_{\alpha,\beta} = (N, \mathbb{G}, \alpha, \beta)$, the (randomized) key generation algorithm Gen returns a pair of keys $k_0, k_1 \in \{0, 1\}^*$. We assume that N and \mathbb{G} are determined by each key.

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- $\text{Eval}_i(k_i, x) \rightarrow y_i$: On input key $k_i \in \{0, 1\}^*$ and input $x \in [N]$ the (deterministic) evaluation algorithm of server i , Eval_i returns $y_i \in \mathbb{G}$.

We require Π to satisfy the following requirements:

- **Correctness:** For every λ , $\hat{f} = \hat{f}_{\alpha,\beta} = (N, \mathbb{G}, \alpha, \beta)$ such that $\beta \in \mathbb{G}$, and $x \in [N]$, if $(k_0, k_1) \leftarrow \text{Gen}(1^\lambda, \hat{f})$, then

$$\Pr \left[\sum_{i=0}^1 \text{Eval}_i(k_i, x) = f_{\alpha,\beta}(x) \right] = 1$$

- **Security:** Consider the following semantic security challenge experiment for corrupted server $i \in \{0, 1\}$:

- (1) The adversary produces two point function descriptions $(\hat{f}^0 = (N, \mathbb{G}, \alpha_0, \beta_0), \hat{f}^1 = (N, \mathbb{G}, \alpha_1, \beta_1)) \leftarrow \mathcal{A}(1^\lambda)$, where $\alpha_i \in [N]$ and $\beta_i \in \mathbb{G}$.
- (2) The challenger samples $b \leftarrow \{0, 1\}$ and $(k_0, k_1) \leftarrow \text{Gen}(1^\lambda, \hat{f}^b)$.
- (3) The adversary outputs a guess $b' \leftarrow \mathcal{A}(k_i)$.

Denote by $\text{Adv}(1^\lambda, \mathcal{A}, i) = \Pr[b = b'] - 1/2$ the advantage of \mathcal{A} in guessing b in the above experiment. For every non-uniform polynomial time adversary \mathcal{A} there exists a negligible function ν such that $\text{Adv}(1^\lambda, \mathcal{A}, i) \leq \nu(\lambda)$ for all $\lambda \in \mathbb{N}$.

We will also be interested in applying the evaluation algorithm on all inputs. Given a DPF $(\text{Gen}, \text{Eval}_0, \text{Eval}_1)$, we denote by FullEval_i an algorithm which computes Eval_i on every input x . Hence, FullEval_i receives only a key k_i as input.

2.3 Batch Codes

combinatorial/probabilistic batch codes, with cuckoo hashing a concrete instantiation

2.4 Oblivious Key-Value Stores

DEFINITION 2 (OKVS[2, 4]). An Oblivious Key-Value Stores (OKVS) scheme is a pair of randomized algorithms $(\text{Encode}_r, \text{Decode}_r)$ with respect to a statistical security parameter λ_{stat} and a computational security parameter λ , a randomness space $\{0, 1\}^\kappa$, a key space \mathcal{K} , a value space \mathcal{V} , input length n and output length $m(n)$. The algorithms are of the following syntax:

- $\text{Encode}_r(\{(k_1, v_1), (k_2, v_2), \dots, (k_n, v_n)\}) \rightarrow P$: On input n key-value pairs with distinct keys, the encode algorithm with randomness r in the randomness space outputs an encoding $P \in \mathcal{V}^m \cup \perp$.
- $\text{Decode}_r(P, k) \rightarrow v$: On input an encoding from \mathcal{V}^m and a key $k \in \mathcal{K}$, output a value v .

We require the scheme to satisfy

- **Correctness:** For every $S \in (\mathcal{K} \times \mathcal{V})^n$, $\Pr_{r \leftarrow \{0, 1\}^\kappa} [\text{Decode}_r(S) = \perp] \leq 2^{-\lambda_{\text{stat}}}$.
- **Obliviousness:** Given any distinct key sets $\{k_1^0, k_2^0, \dots, k_n^0\}$ and $\{k_1^1, k_2^1, \dots, k_n^1\}$ that are different, if they are paired with

random values then their encodings are computationally indistinguishable, i.e.,

$$\{r, \text{Encode}_r(\{(k_1^0, v_1), \dots, (k_n^0, v_n)\})\}_{v_1, \dots, v_n \leftarrow \mathcal{V}, r \leftarrow \{0,1\}^\kappa} \\ \approx_c \{r, \text{Encode}_r(\{(k_1^1, v_1), \dots, (k_n^1, v_n)\})\}_{v_1, \dots, v_n \leftarrow \mathcal{V}, r \leftarrow \{0,1\}^\kappa}$$

One can obtain a linear OKVS if in addition require:

- **Linearity:** There exists a function family $\{\text{row}_r : \mathcal{K} \rightarrow \mathcal{V}^m\}_{r \in \{0,1\}^\kappa}$ such that $\text{Decode}_r(P, k) = \langle \text{row}_r(k), P \rangle$.

The Encode process for a linear OKVS is the process of sampling a random P from the set of solutions of the linear system $\{\langle \text{row}_r(k_i), P \rangle = v_i\}_{1 \leq i \leq n}$.

We evaluate an OKVS scheme by its encoding size (output length m), encoding time and decoding time. We stress the following two (linear) OKVS constructions:

CONSTRUCTION 1 (POLYNOMIAL). Suppose $\mathcal{K} = \mathcal{V} = \mathbb{F}$ is a field. Set

- $\text{Encode}(\{(k_i, v_i)\}_{1 \leq i \leq n}) \rightarrow P$ where P is the coefficients of a $(n-1)$ -degree \mathbb{F} -polynomial g_P that $g_P(k_i) = v_i$ for $1 \leq i \leq n$.
- $\text{Decode}(P, k) \rightarrow g_P(k)$.

The polynomial OKVS possesses an optimal encoding size $m = n$, but the Encode process is a polynomial interpolation which is only known to be achieved in time $O(n \log^2 n)$. The time for a single decoding is $O(n)$ and that for batched decodings is (amortized) $O(\log^2 n)$.

An alternative construction that has near optimal encoding size but much better running time is as follows.

CONSTRUCTION 2 (3-HASH GARBLED CUCKOO TABLE (3H-GCT)[2, 4]). Suppose $\mathcal{V} = \mathbb{F}$ is a field. Set $\text{row}_r(k) := \text{row}_r^{\text{sparse}}(k) \parallel \text{row}_r^{\text{dense}}(k)$ where $\text{row}_r^{\text{sparse}}$ outputs a uniformly random weight- w vector in $\{0, 1\}^{m_1}$, and $\text{row}_r^{\text{dense}}(k)$ outputs a short dense vector in \mathbb{F}^{m_2} .

- $\text{Encode}(\{(k_i, v_i)\}_{1 \leq i \leq n}) \rightarrow P$ where P is solved from the system $\{\langle \text{row}_r(k_i), P \rangle = v_i\}_{1 \leq i \leq n}$ using the triangulation algorithm in [4].
- $\text{Decode}(P, k) \rightarrow \langle \text{row}_r(k), P \rangle$.

This OKVS construction features a linear encoding time, constant decoding time while having a linear encoding size.

TBD: Carefully(!) recompute the comparison table for OKVS and insert

We take $w = 3$, the most common option that outruns other choices of w in terms of running time. Restating the conclusion in [4]: given n and λ_{stat} , the choices of e and \hat{g} are $e = 1.223 + \frac{\lambda_{\text{stat}} + 9.2}{4.144n^{0.55}}$ and $\hat{g} = \frac{\lambda_{\text{stat}}}{\log_2(en)}$.

TBD: mention some connections to cuckoo hashing

3 NEW DMPF CONSTRUCTIONS

TBD: explain

3.1 Big-State DMPF

TBD: explain

Set $l \leftarrow t$, the upperbound of $|A|$.

procedure INITIALIZE($\{\text{seed}_b^{(0)}, \text{sign}_b^{(0)}\}_{b=0,1}$)

For $b = 0, 1$, let $\text{seed}_b^{(0)} = [r_b]$ where $r_b \xleftarrow{\$} \{0, 1\}^\lambda$.

For $b = 0, 1$, set $\text{sign}_b^{(0)} = [b|0^{t-1}]$.

end procedure

procedure GENCW($i, A, \{\text{seed}_b^{(i-1)}, \text{sign}_b^{(i-1)}\}_{b=0,1}$)

Let $\{A^{(i)}\}_{0 \leq i \leq n}$ be defined as in fig. 1.

Sample a list CW of t random strings from $\{0, 1\}^{\lambda+2t}$.

for $k = 1$ to $|A^{(i-1)}|$ **do**

Parse $G(\text{seed}_b^{(i-1)}[k]) = \text{seed}_b^0 \parallel \text{sign}_b^0 \parallel \text{seed}_b^1 \parallel \text{sign}_b^1$, for $b = 0, 1$, $\text{seed}_b^0, \text{seed}_b^1 \in \{0, 1\}^\lambda$ and $\text{sign}_b^0, \text{sign}_b^1 \in \{0, 1\}^t$.

Compute $\Delta \text{seed}^c = \text{seed}_0^c \oplus \text{seed}_1^c$ and $\Delta \text{sign}^c = \text{sign}_0^c \oplus \text{sign}_1^c$ for $c = 0, 1$.

Denote $\text{path} \leftarrow A^{(i-1)}[k]$.

if both $\text{path}||z$ for $z = 0, 1$ are in $A^{(i)}$ **then**

$d \leftarrow$ the index of $\text{path}||0$ in $A^{(i)}$.

$CW[d] \leftarrow r \parallel \Delta \text{sign}^0 \oplus e_d \parallel \Delta \text{sign}^1 \oplus e_{d+1}$ where $r \xleftarrow{\$} \{0, 1\}^\lambda$, $e_d = 0^{d-1}10^{t-d}$.

else

Let z be such that $\text{path}||z \in A^{(i)}$.

$d \leftarrow$ the index of $\text{path}||z$ in $A^{(i)}$.

$CW[d] \leftarrow \begin{cases} \Delta \text{seed}^1 \parallel \Delta \text{sign}^0 \oplus e_d \parallel \Delta \text{sign}^1 & z = 0 \\ \Delta \text{seed}^0 \parallel \Delta \text{sign}^0 \parallel \Delta \text{sign}^1 \oplus e_d & z = 1 \end{cases}$

end if

end for

return CW .

end procedure

procedure GENCONVCW($A, B, \{\text{seed}_b^{(n)}, \text{sign}_b^{(n)}\}$)

Sample a list CW of t random \mathbb{G} -elements.

for $k = 1$ to $|A|$ **do**

$\Delta g \leftarrow G_{\text{convert}}(\text{seed}_0^{(n)}[k]) - G_{\text{convert}}(\text{seed}_1^{(n)}[k])$.

$CW[k] \leftarrow (-1)^{\text{sign}_0^{(n)}[k][k]}(\Delta g - B[k])$.

end for

return CW .

end procedure

procedure CORRECT($\bar{x}, \text{seed}, \text{sign}, CW$)

Let z be the last bit of \bar{x} .

$C_{\text{seed}} \parallel C_{\text{sign}^0} \parallel C_{\text{sign}^1} \leftarrow \sum_{i=1}^t \text{sign}[i] \cdot CW[i]$, where C_{sign^0} and C_{sign^1} are t -bit.

return $G_z(\text{seed}) \oplus (C_{\text{seed}} \parallel C_{\text{sign}^z})$.

end procedure

procedure CONVCORRECT($x, \text{seed}, \text{sign}, CW$)

return $G_{\text{convert}}(\text{seed}) \oplus \sum_{i=1}^t \text{sign}[i] \cdot CW[i]$.

end procedure

Figure 2: The parameter l and methods' setting that turns the paradigm of DMPF in fig. 1 into the big-state DMPF.

Public parameters:

The multi-point function family $\{f_{A,B}\}$, an upperbound t of the number of nonzero points ($|A| \leq t$), input domain $[N] = \{0, 1\}^n$ and the output group \mathbb{G} .

Suppose there is a public PRG $G : \{0, 1\}^\lambda \rightarrow \{0, 1\}^{2\lambda+2l}$. Parse $G = G_0 || G_1$ to the left half and right half.

Suppose there is a public PRG $G_{\text{convert}} : \{0, 1\}^\lambda \rightarrow \mathbb{G}$.

procedure $\text{GEN}(1^\lambda, \hat{f}_{A,B})$

Denote $A = (\alpha_1, \dots, \alpha_t)$ in lexicographical order, $B = (\beta_1, \dots, \beta_t)$.

For $0 \leq i \leq n-1$, let $A^{(i)}$ denote the sorted and deduplicated list of i -bit prefixes of strings in A . Specifically, $A^{(0)} = [\epsilon]$.

For $0 \leq i \leq n-1$ and $b = 0, 1$, initialize empty lists $\text{seed}_b^{(i)}$ and $\text{sign}_b^{(i)}$.

Initialize($\{\text{seed}_b^{(0)}, \text{sign}_b^{(0)}\}_{b=0,1}$).

for $i = 1$ to n **do**

$CW^{(i)} \leftarrow \text{GenCW}(i, A, \{\text{seed}_b^{(i-1)}, \text{sign}_b^{(i-1)}\}_{b=0,1})$.

for $k = 1$ to $|A^{(i-1)}|$ and $z = 0, 1$ **do****if** $A^{(i-1)}[k] || z \in A^{(i)}$ **then**

For $b = 0, 1$, compute $\text{temp}_b \leftarrow \text{Correct}(A^{(i-1)}[k] || z, \text{seed}_b^{(i-1)}[k], \text{sign}_b^{(i-1)}[k], CW^{(i)})$.

Append the first λ bit of temp_b to $\text{seed}_b^{(i)}$ and the rest to $\text{sign}_b^{(i)}$.

end if**end for****end for**

$CW^{(n+1)} \leftarrow \text{GenConvCW}(A, B, \{\text{seed}_b^{(n)}, \text{sign}_b^{(n)}\}_{b=0,1})$.

Set $k_b \leftarrow (\text{seed}_b^{(0)}, \text{sign}_b^{(0)}, CW^{(1)}, CW^{(2)}, \dots, CW^{(n+1)})$.

return (k_0, k_1) .

end procedure**procedure** $\text{EVAL}_b(1^\lambda, k_b, x)$

Parse $k_b = ([\text{seed}^{(0)}], [\text{sign}^{(0)}], CW^{(1)}, CW^{(2)}, \dots, CW^{(n+1)})$.

Denote $x = x_1 x_2 \dots x_n$.

for $i = 1$ to n **do**

$\text{seed}^{(i)} || \text{sign}^{(i)} \leftarrow \text{Correct}(x_1 \dots x_i, \text{seed}^{(i-1)}, \text{sign}^{(i-1)}, CW^{(i)})$ where $\text{seed}^{(i)}$ is λ -bit.

end for

return $(-1)^b \cdot \text{ConvCorrect}(x, \text{seed}^{(n)}, \text{sign}^{(n)}, CW^{(n+1)})$.

end procedure

Figure 1: The paradigm of our DMPF schemes. We leave the PRG expand length l , methods Initialize, GenCW, GenConvCW, Correct, ConvCorrect to be determined by specific constructions.

3.2 Batch-Code DMPF

display the batch-code DMPF

3.3 OKVS-based DMPF

TBD: explain

3.4 Comparison

Comparison table dependent to PRG & F-MUL(list the formulas?)

analyze tradeoff

distributed gen advantage

3.5 Distributed Key Generation

4 APPLICATIONS

4.1 PCG for OLE from Ring-LPN

Characterize parameters

show nonregular optimization

plug in new DMPF and show overall optimization

4.2 PSI-WCA

plug in new DMPF and analyze advantage interval

plug in distributed gen

4.3 Heavy-hitters

private heavy-hitter

or parallel ORAM?

Set $l \leftarrow 1$.
 For $1 \leq i \leq n$, let OKVS_i be an OKVS scheme (definition 2) with key space $\mathcal{K} = \{0, 1\}^{i-1}$, value space $\mathcal{V} = \{0, 1\}^{\lambda+2}$ and input length t .
 let $\text{OKVS}_{\text{convert}}$ be an OKVS scheme with key space $\mathcal{K} = \{0, 1\}^n$, value space $\mathcal{V} = \mathbb{G}$ and input length t .

procedure INITIALIZE($\{\text{seed}_b^{(0)}, \text{sign}_b^{(0)}\}_{b=0,1}$)
 For $b = 0, 1$, let $\text{seed}_b^{(0)} = [r_b \xleftarrow{\$} \{0, 1\}^\lambda]$ and $\text{sign}_b^{(0)} = [b]$.
end procedure

procedure GENCW($i, A, \{\text{seed}_b^{(i-1)}, \text{sign}_b^{(i-1)}\}_{b=0,1}$)
 Let $\{A^{(i)}\}_{0 \leq i \leq n}$ be defined as in fig. 1.
 Sample a list V of t random strings from $\{0, 1\}^{\lambda+2}$.
for $k = 1$ to $|A^{(i-1)}|$ **do**
 Parse $G(\text{seed}_b^{(i-1)}[k]) = \text{seed}_b^0 || \text{sign}_b^0 || \text{seed}_b^1 || \text{sign}_b^1$, for
 $b = 0, 1$, $\text{seed}_b^0, \text{seed}_b^1 \in \{0, 1\}^\lambda$ and $\text{sign}_b^0, \text{sign}_b^1 \in \{0, 1\}$.
 Compute $\Delta \text{seed}^c = \text{seed}_0^c \oplus \text{seed}_1^c$ and $\Delta \text{sign}^c = \text{sign}_0^c \oplus \text{sign}_1^c$ for $c = 0, 1$.
 Denote $\text{path} \leftarrow A^{(i-1)}[k]$.
 if both $\text{path} || z$ for $z = 0, 1$ are in $A^{(i)}$ **then**
 $V[k] \leftarrow r || \Delta \text{sign}^0 \oplus 1 || \Delta \text{sign}^1 \oplus 1$, where $r \xleftarrow{\$} \{0, 1\}^\lambda$.
 else
 Let z be such that $\text{path} || z \in A^{(i)}$.
 $V[k] \leftarrow \Delta \text{seed}^1 || \Delta \text{sign}^0 \oplus (1 - z) || \Delta \text{sign}^1 \oplus z$.
 end if
end for
return $\text{OKVS}_i.\text{Encode}(\{A^{(i-1)}[k], V[k]\}_{1 \leq k \leq |A^{(i-1)}|})$.
end procedure

procedure GENCONVCW($A, B, \{\text{seed}_b^{(n)}, \text{sign}_b^{(n)}\}$)
 Sample a list V of t random \mathbb{G} -elements.
for $k = 1$ to $|A|$ **do**
 $\Delta g \leftarrow G_{\text{convert}}(\text{seed}_0^{(n)}[k]) - G_{\text{convert}}(\text{seed}_1^{(n)}[k])$.
 $V[k] \leftarrow (-1)^{\text{sign}_0^{(n)}[k][k]} (\Delta g - B[k])$.
end for
return $\text{OKVS}_{\text{convert}}(\{A[k], V[k]\}_{1 \leq k \leq t})$.
end procedure

procedure CORRECT($\bar{x}, \text{seed}, \text{sign}, CW$)
 Suppose $\bar{x} = x_1 x_2 \dots x_i$ and let $\bar{x}^- = x_1 \dots x_{i-1}$.
 $C_{\text{seed}} || C_{\text{sign}^0} || C_{\text{sign}^1} \leftarrow \text{OKVS}_i.\text{Decode}(CW, \bar{x}^-)$, where
 C_{sign^0} and C_{sign^1} are bits.
return $G_z(\text{seed}) \oplus (C_{\text{seed}} || C_{\text{sign}^z})$.
end procedure

procedure CONVCORRECT($x, \text{seed}, \text{sign}, CW$)
return $G_{\text{convert}}(\text{seed}) \oplus \text{OKVS}_{\text{convert}}.\text{Decode}(CW, x)$.
end procedure

Figure 3: The parameter l and methods' setting that turns the paradigm of DMPF in fig. 1 into the OKVS-based DMPF.

5 ACKNOWLEDGMENTS

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A BATCH-CODE DMPF SCHEME

B SECURITY PROOFS