

# The Name of the Title Is Hope

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## ABSTRACT

tbd.

## CCS CONCEPTS

• Theory of computation → Cryptographic primitives.

## KEYWORDS

tbd

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## 1 INTRODUCTION

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## 2 PRELIMINARY

### 2.1 Basic Notations

*Point and multi-point functions.* Given a domain size  $N$  and Abelian group  $\mathbb{G}$ , a *point function*  $f_{\alpha, \beta} : [N] \rightarrow \mathbb{G}$  for  $\alpha \in [N]$  and  $\beta \in \mathbb{G}$  evaluates to  $\beta$  on input  $\alpha$  and to  $0 \in \mathbb{G}$  on all other inputs. We denote by  $\hat{f}_{\alpha, \beta} = (N, \mathbb{G}, \alpha, \beta)$  the representation of such a point function. A *multi-point function*  $f_{A, B} : [N] \rightarrow \mathbb{G}$  for  $A = (\alpha_1, \dots, \alpha_t) \in [N]^t$  and  $B = (\beta_1, \dots, \beta_t) \in \mathbb{G}^t$  evaluates to  $\beta_i$  on input  $\alpha_i$  for  $1 \leq i \leq t$  and to  $0$  on all other inputs. Denote  $\hat{f}_{A, B}(N, \mathbb{G}, A, B)$  the representation of such a point function.

**Enote:** MPF. Also representation of groups.

### 2.2 Distributed Multi-Point Functions

**Enote:** should directly adapt to multi-point function case

We begin by defining a slightly generalized notion of distributed point functions (DPFs), which accounts for the extra parameter  $\mathbb{G}'$ .

**DEFINITION 1** (DPF [1, 3]). A (2-party) distributed point function (DPF) is a triple of algorithms  $\Pi = (\text{Gen}, \text{Eval}_0, \text{Eval}_1)$  with the following syntax:

- $\text{Gen}(1^\lambda, \hat{f}_{\alpha, \beta}) \rightarrow (k_0, k_1)$ : On input security parameter  $\lambda \in \mathbb{N}$  and point function description  $\hat{f}_{\alpha, \beta} = (N, \mathbb{G}, \alpha, \beta)$ , the (randomized) key generation algorithm  $\text{Gen}$  returns a pair of keys  $k_0, k_1 \in \{0, 1\}^*$ . We assume that  $N$  and  $\mathbb{G}$  are determined by each key.

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- $\text{Eval}_i(k_i, x) \rightarrow y_i$ : On input key  $k_i \in \{0, 1\}^*$  and input  $x \in [N]$  the (deterministic) evaluation algorithm of server  $i$ ,  $\text{Eval}_i$  returns  $y_i \in \mathbb{G}$ .

We require  $\Pi$  to satisfy the following requirements:

- **Correctness:** For every  $\lambda$ ,  $\hat{f} = \hat{f}_{\alpha, \beta} = (N, \mathbb{G}, \alpha, \beta)$  such that  $\beta \in \mathbb{G}$ , and  $x \in [N]$ , if  $(k_0, k_1) \leftarrow \text{Gen}(1^\lambda, \hat{f})$ , then

$$\Pr \left[ \sum_{i=0}^1 \text{Eval}_i(k_i, x) = f_{\alpha, \beta}(x) \right] = 1$$

- **Security:** Consider the following semantic security challenge experiment for corrupted server  $i \in \{0, 1\}$ :

- (1) The adversary produces two point function descriptions  $(\hat{f}^0 = (N, \mathbb{G}, \alpha_0, \beta_0), \hat{f}^1 = (N, \mathbb{G}, \alpha_1, \beta_1)) \leftarrow \mathcal{A}(1^\lambda)$ , where  $\alpha_i \in [N]$  and  $\beta_i \in \mathbb{G}$ .
- (2) The challenger samples  $b \leftarrow \{0, 1\}$  and  $(k_0, k_1) \leftarrow \text{Gen}(1^\lambda, \hat{f}^b)$ .
- (3) The adversary outputs a guess  $b' \leftarrow \mathcal{A}(k_i)$ .

Denote by  $\text{Adv}(1^\lambda, \mathcal{A}, i) = \Pr[b = b'] - 1/2$  the advantage of  $\mathcal{A}$  in guessing  $b$  in the above experiment. For every non-uniform polynomial time adversary  $\mathcal{A}$  there exists a negligible function  $\nu$  such that  $\text{Adv}(1^\lambda, \mathcal{A}, i) \leq \nu(\lambda)$  for all  $\lambda \in \mathbb{N}$ .

We will also be interested in applying the evaluation algorithm on all inputs. Given a DPF  $(\text{Gen}, \text{Eval}_0, \text{Eval}_1)$ , we denote by  $\text{FullEval}_i$  an algorithm which computes  $\text{Eval}_i$  on every input  $x$ . Hence,  $\text{FullEval}_i$  receives only a key  $k_i$  as input.

### 2.3 Batch Codes

combinatorial/probabilistic batch codes, with cuckoo hashing a concrete instantiation

### 2.4 Oblivious Key-Value Stores

**DEFINITION 2** (OKVS[2, 4]). An Oblivious Key-Value Stores (OKVS) scheme is a pair of randomized algorithms  $(\text{Encode}_r, \text{Decode}_r)$  with respect to a statistical security parameter  $\lambda_{\text{stat}}$  and a computational security parameter  $\lambda$ , a randomness space  $\{0, 1\}^\kappa$ , a key space  $\mathcal{K}$ , a value space  $\mathcal{V}$ , input length  $n$  and output length  $m(n)$ . The algorithms are of the following syntax:

- $\text{Encode}_r(\{(k_1, v_1), (k_2, v_2), \dots, (k_n, v_n)\}) \rightarrow P$ : On input  $n$  key-value pairs with distinct keys, the encode algorithm with randomness  $r$  in the randomness space outputs an encoding  $P \in \mathcal{V}^m \cup \perp$ .
- $\text{Decode}_r(P, k) \rightarrow v$ : On input an encoding from  $\mathcal{V}^m$  and a key  $k \in \mathcal{K}$ , output a value  $v$ .

We require the scheme to satisfy

- **Correctness:** For every  $S \in (\mathcal{K} \times \mathcal{V})^n$ ,  $\Pr_{r \leftarrow \{0, 1\}^\kappa} [\text{Decode}_r(S) = \perp] \leq 2^{-\lambda_{\text{stat}}}$ .
- **Obliviousness:** Given any distinct key sets  $\{k_1^0, k_2^0, \dots, k_n^0\}$  and  $\{k_1^1, k_2^1, \dots, k_n^1\}$  that are different, if they are paired with

random values then their encodings are computationally indistinguishable, i.e.,

$$\{r, \text{Encode}_r(\{(k_1^0, v_1), \dots, (k_n^0, v_n)\})\}_{v_1, \dots, v_n \leftarrow \mathcal{V}, r \leftarrow \{0,1\}^\kappa} \\ \approx_c \{r, \text{Encode}_r(\{(k_1^1, v_1), \dots, (k_n^1, v_n)\})\}_{v_1, \dots, v_n \leftarrow \mathcal{V}, r \leftarrow \{0,1\}^\kappa}$$

One can obtain a linear OKVS if in addition require:

- **Linearity:** There exists a function family  $\{\text{row}_r : \mathcal{K} \rightarrow \mathcal{V}^m\}_{r \in \{0,1\}^\kappa}$  such that  $\text{Decode}_r(P, k) = \langle \text{row}_r(k), P \rangle$ .

The Encode process for a linear OKVS is the process of sampling a random  $P$  from the set of solutions of the linear system  $\{\langle \text{row}_r(k_i), P \rangle = v_i\}_{1 \leq i \leq n}$ .

We evaluate an OKVS scheme by its encoding size (output length  $m$ ), encoding time and decoding time. We stress the following two (linear) OKVS constructions:

**CONSTRUCTION 1 (POLYNOMIAL).** Suppose  $\mathcal{K} = \mathcal{V} = \mathbb{F}$  is a field. Set

- $\text{Encode}(\{(k_i, v_i)\}_{1 \leq i \leq n}) \rightarrow P$  where  $P$  is the coefficients of a  $(n-1)$ -degree  $\mathbb{F}$ -polynomial  $g_P$  that  $g_P(k_i) = v_i$  for  $1 \leq i \leq n$ .
- $\text{Decode}(P, k) \rightarrow g_P(k)$ .

The polynomial OKVS possesses an optimal encoding size  $m = n$ , but the Encode process is a polynomial interpolation which is only known to be achieved in time  $O(n \log^2 n)$ . The time for a single decoding is  $O(n)$  and that for batched decodings is (amortized)  $O(\log^2 n)$ .

An alternative construction that has near optimal encoding size but much better running time is as follows.

**CONSTRUCTION 2 (3-HASH GARBLED CUCKOO TABLE (3H-GCT)[2, 4]).** Suppose  $\mathcal{V} = \mathbb{F}$  is a field. Set  $\text{row}_r(k) := \text{row}_r^{\text{sparse}}(k) \parallel \text{row}_r^{\text{dense}}(k)$  where  $\text{row}_r^{\text{sparse}}$  outputs a uniformly random weight- $w$  vector in  $\{0, 1\}^{m_1}$ , and  $\text{row}_r^{\text{dense}}(k)$  outputs a short dense vector in  $\mathbb{F}^{m_2}$ .

- $\text{Encode}(\{(k_i, v_i)\}_{1 \leq i \leq n}) \rightarrow P$  where  $P$  is solved from the system  $\{\langle \text{row}_r(k_i), P \rangle = v_i\}_{1 \leq i \leq n}$  using the triangulation algorithm in [4].
- $\text{Decode}(P, k) \rightarrow \langle \text{row}_r(k), P \rangle$ .

This OKVS construction features a linear encoding time, constant decoding time while having a linear encoding size.

TBD: Carefully(!) recompute the comparison table for OKVS and insert

We take  $w = 3$ , the most common option that outruns other choices of  $w$  in terms of running time. Restating the conclusion in [4]: given  $n$  and  $\lambda_{\text{stat}}$ , the choices of  $e$  and  $\hat{g}$  are  $e = 1.223 + \frac{\lambda_{\text{stat}} + 9.2}{4.144n^{0.55}}$  and  $\hat{g} = \frac{\lambda_{\text{stat}}}{\log_2(en)}$ .

TBD: mention some connections to cuckoo hashing

### 3 NEW DMPF CONSTRUCTIONS

TBD: explain

#### 3.1 Big-State DMPF

TBD: explain

Set  $l \leftarrow t$ , the upperbound of  $|A|$ .

**procedure** INITIALIZE( $\{\text{seed}_b^{(0)}, \text{sign}_b^{(0)}\}_{b=0,1}$ )

For  $b = 0, 1$ , let  $\text{seed}_b^{(0)} = [r_b]$  where  $r_b \xleftarrow{\$} \{0, 1\}^\lambda$ .

For  $b = 0, 1$ , set  $\text{sign}_b^{(0)} = [b|0^{t-1}]$ .

**end procedure**

**procedure** GENCW( $i, A, \{\text{seed}_b^{(i-1)}, \text{sign}_b^{(i-1)}\}_{b=0,1}$ )

Let  $\{A^{(i)}\}_{0 \leq i \leq n}$  be defined as in fig. 1.

Sample a list  $CW$  of  $t$  random strings from  $\{0, 1\}^{\lambda+2t}$ .

**for**  $k = 1$  to  $|A^{(i-1)}|$  **do**

Parse  $G(\text{seed}_b^{(i-1)}[k]) = \text{seed}_b^0 \parallel \text{sign}_b^0 \parallel \text{seed}_b^1 \parallel \text{sign}_b^1$ , for  $b = 0, 1$ ,  $\text{seed}_b^0, \text{seed}_b^1 \in \{0, 1\}^\lambda$  and  $\text{sign}_b^0, \text{sign}_b^1 \in \{0, 1\}^t$ .

Compute  $\Delta \text{seed}^c = \text{seed}_0^c \oplus \text{seed}_1^c$  and  $\Delta \text{sign}^c = \text{sign}_0^c \oplus \text{sign}_1^c$  for  $c = 0, 1$ .

Denote  $\text{path} \leftarrow A^{(i-1)}[k]$ .

**if** both  $\text{path}||z$  for  $z = 0, 1$  are in  $A^{(i)}$  **then**

$d \leftarrow$  the index of  $\text{path}||0$  in  $A^{(i)}$ .

$CW[d] \leftarrow r \parallel \Delta \text{sign}^0 \oplus e_d \parallel \Delta \text{sign}^1 \oplus e_{d+1}$  where  $r \xleftarrow{\$} \{0, 1\}^\lambda$ ,  $e_d = 0^{d-1}10^{t-d}$ .

**else**

Let  $z$  be such that  $\text{path}||z \in A^{(i)}$ .

$d \leftarrow$  the index of  $\text{path}||z$  in  $A^{(i)}$ .

$CW[d] \leftarrow \begin{cases} \Delta \text{seed}^1 \parallel \Delta \text{sign}^0 \oplus e_d \parallel \Delta \text{sign}^1 & z = 0 \\ \Delta \text{seed}^0 \parallel \Delta \text{sign}^0 \parallel \Delta \text{sign}^1 \oplus e_d & z = 1 \end{cases}$

**end if**

**end for**

**return**  $CW$ .

**end procedure**

**procedure** GENCONVCW( $A, B, \{\text{seed}_b^{(n)}, \text{sign}_b^{(n)}\}$ )

Sample a list  $CW$  of  $t$  random  $\mathbb{G}$ -elements.

**for**  $k = 1$  to  $|A|$  **do**

$\Delta g \leftarrow G_{\text{convert}}(\text{seed}_0^{(n)}[k]) - G_{\text{convert}}(\text{seed}_1^{(n)}[k])$ .

$CW[k] \leftarrow (-1)^{\text{sign}_0^{(n)}[k][k]}(\Delta g - B[k])$ .

**end for**

**return**  $CW$ .

**end procedure**

**procedure** CORRECT( $\bar{x}, \text{seed}, \text{sign}, CW$ )

Let  $z$  be the last bit of  $\bar{x}$ .

$C_{\text{seed}} \parallel C_{\text{sign}^0} \parallel C_{\text{sign}^1} \leftarrow \sum_{i=1}^t \text{sign}[i] \cdot CW[i]$ , where  $C_{\text{sign}^0}$  and  $C_{\text{sign}^1}$  are  $t$ -bit.

**return**  $G_z(\text{seed}) \oplus (C_{\text{seed}} \parallel C_{\text{sign}^z})$ .

**end procedure**

**procedure** CONVCORRECT( $x, \text{seed}, \text{sign}, CW$ )

**return**  $G_{\text{convert}}(\text{seed}) \oplus \sum_{i=1}^t \text{sign}[i] \cdot CW[i]$ .

**end procedure**

Figure 2: The parameter  $l$  and methods' setting that turns the paradigm of DMPF in fig. 1 into the big-state DMPF.

**Public parameters:**

The multi-point function family  $\{f_{A,B}\}$ , an upperbound  $t$  of the number of nonzero points ( $|A| \leq t$ ), input domain  $[N] = \{0, 1\}^n$  and the output group  $\mathbb{G}$ .

Suppose there is a public PRG  $G : \{0, 1\}^\lambda \rightarrow \{0, 1\}^{2\lambda+2l}$ . Parse  $G = G_0 || G_1$  to the left half and right half.

Suppose there is a public PRG  $G_{\text{convert}} : \{0, 1\}^\lambda \rightarrow \mathbb{G}$ .

**procedure**  $\text{GEN}(1^\lambda, \hat{f}_{A,B})$ 

Denote  $A = (\alpha_1, \dots, \alpha_t)$  in lexicographical order,  $B = (\beta_1, \dots, \beta_t)$ .

For  $0 \leq i \leq n-1$ , let  $A^{(i)}$  denote the sorted and deduplicated list of  $i$ -bit prefixes of strings in  $A$ . Specifically,  $A^{(0)} = [\epsilon]$ .

For  $0 \leq i \leq n-1$  and  $b = 0, 1$ , initialize empty lists  $\text{seed}_b^{(i)}$  and  $\text{sign}_b^{(i)}$ .

Initialize  $(\{\text{seed}_b^{(0)}, \text{sign}_b^{(0)}\}_{b=0,1})$ .

**for**  $i = 1$  to  $n$  **do**

$CW^{(i)} \leftarrow \text{GenCW}(i, A, \{\text{seed}_b^{(i-1)}, \text{sign}_b^{(i-1)}\}_{b=0,1})$ .

**for**  $k = 1$  to  $|A^{(i-1)}|$  and  $z = 0, 1$  **do**

**if**  $A^{(i-1)}[k] || z \in A^{(i)}$  **then**

For  $b = 0, 1$ , compute  $\text{temp}_b \leftarrow \text{Correct}(A^{(i-1)}[k] || z, \text{seed}_b^{(i-1)}[k], \text{sign}_b^{(i-1)}[k], CW^{(i)})$ .

Append the first  $\lambda$  bit of  $\text{temp}_b$  to  $\text{seed}_b^{(i)}$  and the rest to  $\text{sign}_b^{(i)}$ .

**end if**

**end for**

**end for**

$CW^{(n+1)} \leftarrow \text{GenConvCW}(A, B, \{\text{seed}_b^{(n)}, \text{sign}_b^{(n)}\}_{b=0,1})$ .

Set  $k_b \leftarrow (\text{seed}_b^{(0)}, \text{sign}_b^{(0)}, CW^{(1)}, CW^{(2)}, \dots, CW^{(n+1)})$ .

**return**  $(k_0, k_1)$ .

**end procedure**

**procedure**  $\text{EVAL}_b(1^\lambda, k_b, x)$ 

Parse  $k_b = ([\text{seed}], [\text{sign}], CW^{(1)}, CW^{(2)}, \dots, CW^{(n+1)})$ .

Denote  $x = x_1 x_2 \dots x_n$ .

**for**  $i = 1$  to  $n$  **do**

$\text{seed} || \text{sign} \leftarrow \text{Correct}(x_1 \dots x_i, \text{seed}, \text{sign}, CW^{(i)})$ .

**end for**

**return**  $(-1)^b \cdot \text{ConvCorrect}(x, \text{seed}, \text{sign}, CW^{(n+1)})$ .

**end procedure**

**procedure**  $\text{FULEVAL}_b(1^\lambda, k_b)$ 

Parse  $k_b = (\text{seed}^{(0)}, \text{sign}^{(0)}, CW^{(1)}, CW^{(2)}, \dots, CW^{(n+1)})$ .

For  $1 \leq i \leq n$ ,  $\text{Path}^{(i)} \leftarrow$  the lexicographical ordered list of  $\{0, 1\}^i$ .  $\text{Path}^{(0)} \leftarrow [\epsilon]$ .

**for**  $i = 1$  to  $n$  **do**

**for**  $k = 1$  to  $2^{i-1}$  and  $z = 0, 1$  **do**

$\text{seed}^{(i)}[2k+z] || \text{sign}^{(i)}[2k+z] \leftarrow \text{Correct}(\text{Path}[k] || z, \text{seed}^{(i-1)}[k], \text{sign}^{(i-1)}[k], CW^{(i)})$ .

**end for**

**end for**

**for**  $k = 1$  to  $2^n$  **do**

$\text{Output}[k] \leftarrow \text{ConvCorrect}(\text{Path}[k], \text{seed}^{(n)}[k], \text{sign}^{(n)}[k], CW^{(n+1)})$ .

**end for**

**return**  $\text{Output}$ .

**end procedure**

**Figure 1: The paradigm of our DMPF schemes. We leave the PRG expand length  $l$ , methods Initialize, GenCW, GenConvCW, Correct, ConvCorrect to be determined by specific constructions.**

Set  $l \leftarrow 1$ .  
 For  $1 \leq i \leq n$ , let  $\text{OKVS}_i$  be an OKVS scheme (definition 2) with key space  $\mathcal{K} = \{0, 1\}^{i-1}$ , value space  $\mathcal{V} = \{0, 1\}^{\lambda+2}$  and input length  $t$ .  
 let  $\text{OKVS}_{\text{convert}}$  be an OKVS scheme with key space  $\mathcal{K} = \{0, 1\}^n$ , value space  $\mathcal{V} = \mathbb{G}$  and input length  $t$ .

**procedure** INITIALIZE( $\{\text{seed}_b^{(0)}, \text{sign}_b^{(0)}\}_{b=0,1}$ )  
 For  $b = 0, 1$ , let  $\text{seed}_b^{(0)} = [r_b \xleftarrow{\$} \{0, 1\}^\lambda]$  and  $\text{sign}_b^{(0)} = [b]$ .  
**end procedure**

**procedure** GENCW( $i, A, \{\text{seed}_b^{(i-1)}, \text{sign}_b^{(i-1)}\}_{b=0,1}$ )  
 Let  $\{A^{(i)}\}_{0 \leq i \leq n}$  be defined as in fig. 1.  
 Sample a list  $V$  of  $t$  random strings from  $\{0, 1\}^{\lambda+2}$ .  
**for**  $k = 1$  to  $|A^{(i-1)}|$  **do**  
   Parse  $G(\text{seed}_b^{(i-1)}[k]) = \text{seed}_b^0 || \text{sign}_b^0 || \text{seed}_b^1 || \text{sign}_b^1$ , for  
    $b = 0, 1$ ,  $\text{seed}_b^0, \text{seed}_b^1 \in \{0, 1\}^\lambda$  and  $\text{sign}_b^0, \text{sign}_b^1 \in \{0, 1\}$ .  
   Compute  $\Delta \text{seed}^c = \text{seed}_0^c \oplus \text{seed}_1^c$  and  $\Delta \text{sign}^c = \text{sign}_0^c \oplus \text{sign}_1^c$  for  $c = 0, 1$ .  
   Denote  $\text{path} \leftarrow A^{(i-1)}[k]$ .  
   **if** both  $\text{path} || z$  for  $z = 0, 1$  are in  $A^{(i)}$  **then**  
      $V[k] \leftarrow r || \Delta \text{sign}^0 \oplus 1 || \Delta \text{sign}^1 \oplus 1$ , where  $r \xleftarrow{\$} \{0, 1\}^\lambda$ .  
   **else**  
     Let  $z$  be such that  $\text{path} || z \in A^{(i)}$ .  
      $V[k] \leftarrow \Delta \text{seed}^1 || \Delta \text{sign}^0 \oplus (1 - z) || \Delta \text{sign}^1 \oplus z$ .  
   **end if**  
**end for**  
**return**  $\text{OKVS}_i.\text{Encode}(\{A^{(i-1)}[k], V[k]\}_{1 \leq k \leq |A^{(i-1)}|})$ .  
**end procedure**

**procedure** GENCONVCW( $A, B, \{\text{seed}_b^{(n)}, \text{sign}_b^{(n)}\}$ )  
 Sample a list  $V$  of  $t$  random  $\mathbb{G}$ -elements.  
**for**  $k = 1$  to  $|A|$  **do**  
    $\Delta g \leftarrow G_{\text{convert}}(\text{seed}_0^{(n)}[k]) - G_{\text{convert}}(\text{seed}_1^{(n)}[k])$ .  
    $V[k] \leftarrow (-1)^{\text{sign}_0^{(n)}[k][k]} (\Delta g - B[k])$ .  
**end for**  
**return**  $\text{OKVS}_{\text{convert}}(\{A[k], V[k]\}_{1 \leq k \leq t})$ .  
**end procedure**

**procedure** CORRECT( $\bar{x}, \text{seed}, \text{sign}, CW$ )  
 Suppose  $\bar{x} = x_1 x_2 \dots x_i$  and let  $\bar{x}^- = x_1 \dots x_{i-1}$ .  
 $C_{\text{seed}} || C_{\text{sign}^0} || C_{\text{sign}^1} \leftarrow \text{OKVS}_i.\text{Decode}(CW, \bar{x}^-)$ , where  
 $C_{\text{sign}^0}$  and  $C_{\text{sign}^1}$  are bits.  
**return**  $G_z(\text{seed}) \oplus (C_{\text{seed}} || C_{\text{sign}^z})$ .  
**end procedure**

**procedure** CONVCORRECT( $x, \text{seed}, \text{sign}, CW$ )  
**return**  $G_{\text{convert}}(\text{seed}) \oplus \text{OKVS}_{\text{convert}}.\text{Decode}(CW, x)$ .  
**end procedure**

Figure 3: The parameter  $l$  and methods' setting that turns the paradigm of DMPF in fig. 1 into the OKVS-based DMPF.

### 3.2 Batch-Code DMPF

display the batch-code DMPF

### 3.3 OKVS-based DMPF

TBD: explain

### 3.4 Comparison

Comparison table dependent to PRG & F-MUL(list the formulas?)  
 analyze tradeoff  
 distributed gen advantage

### 3.5 Distributed Key Generation

## 4 APPLICATIONS

### 4.1 PCG for OLE from Ring-LPN

Characterize parameters  
 show nonregular optimization  
 plug in new DMPF and show overall optimization

### 4.2 PSI-WCA

plug in new DMPF and analyze advantage interval  
 plug in distributed gen

### 4.3 Heavy-hitters

private heavy-hitter  
 or parallel ORAM?

## 5 ACKNOWLEDGMENTS

tbd

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## A BATCH-CODE DMPF SCHEME

## B SECURITY PROOFS