

SCHOOL OF COMPUTATION, INFORMATION  
AND TECHNOLOGY — INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Bachelor's Thesis, Master's Thesis, ... in Informatics

**Evaluating learning algorithms: An efficient  
way/Efficient ways to find Regular Inductive  
Statements**

Author

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## **Evaluating learning algorithms: An efficient way/Efficient ways to find Regular Inductive Statements**

### **Titel der Abschlussarbeit**

Author:	Author
Supervisor:	Supervisor
Advisor:	Advisor
Submission Date:	Submission date

I confirm that this bachelor's thesis, master's thesis, . . . is my own work and I have documented all sources and material used.

Munich, Submission date

Author

# **Abstract**

## **Acknowledgments**

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# 1 Introduction

As software systems grow in size and permeate more and more areas of our lives. Individuals and organizations use the majority of software in their systems. Thus the reliability and stability of the software testing are of major importance. Simulation and testing can detect bugs but not prove their absence. Such reactive systems, when no function is being computed, termination is usually undesirable. For this reason, we are interested in *property checking* or *model checking*. It has found a wide range of applications spanning from adaptive model checking.

*Model checking* is a powerful technique for automatic verification of finite state concurrent systems [CES09]. In this thesis, we only focus on *regular model checking*, which is a important framework for infinite state model-checking. The model is typically a regular transition system. It describes the potential behavior of discrete systems by representing it in finite state automaton. In [CES09] the author introduces an approach, that uses the inductive statements for the regular transition for checking safety conditions. “Statement  $\psi$  is inductive if the transition relation only relates a state  $\nu$  satisfying  $\psi$  with states that also satisfy  $\psi$ . Thus, the set of all states that satisfy  $\psi$  over-approximates the set of all states reachable from  $\nu$ ”.

Based on automata learning, one can learn a set of inductive statements that are powerful enough to establish a given safety property. The learned language of inductive statements is a certificate of the correctness of the property. The purpose of this thesis is to collect and analyze empirical data on the performance of learning algorithms such as  $L^*$ ,  $NL^*$ , Kearns-Vazirani, Rivest-Schapire.

## Structure of the thesis

In the first part of our thesis, we consider the regular transition system and the set of all inductive statements. In the second part of the thesis,

## 2 Preliminaries

In this section, we introduce some basic notions that we use throughout this thesis.

### Finite automata

We use standard notions of finite automata. We distinguish between deterministic and non-deterministic automata to recognize regular languages of finite words.

**Definition 2.1:** *Deterministic finite automaton (DFA).*

A DFA is a quintuple  $\mathcal{M} = (Q, q_0, \Sigma, \delta, F)$  where  $Q$  is a finite set of states with a initial state  $q_0 \in Q$ . A set of input symbols called the alphabet  $\Sigma$ . A transition  $\delta : Q \times \Sigma \rightarrow Q$  and a set of final states  $F$ . Let  $w = a_1a_2...a_n$  be a string over the alphabet  $\Sigma$ . The automaton  $\mathcal{M}$  accepts  $w$  if a sequence of states,  $r_0, r_1, ...r_n$  exist in  $Q$ :

- $r_0 = q_0$
- $r_{i+1} = \delta(r_i, a_{i+1})$ , for  $i = 0, ..., n - 1$
- $r_n \in F$

**Definition 2.2:** *Nondeterministic finite automaton (NFA).*

A NFA is a quintuple  $\mathcal{N} = (Q, q_0, \Sigma, \Delta, F)$  where  $Q$ ,  $\Sigma$  and  $F$  are as for a DFA. Let  $w = a_1a_2...a_n$  be a string over the alphabet  $\Sigma$ . The automaton  $\mathcal{N}$  accepts  $w$  if a sequence of states,  $r_0, r_1, ...r_n$  exist in  $Q$ :

- $r_0 = q_0$
- $r_{i+1} \in \Delta(r_i, a_{i+1})$ , for  $i = 0, ..., n - 1$
- $r_n \in F$

## 3 Inductive statements for regular transition system

### Related Work

Inductive statements for regular transition system was introduced by M.Sc. Welzel-Mohr.

*Model checking* is a potent technique that automatically verifies finite state concurrent systems [CES09]. In this thesis, we aim to tackle the model-checking problem for a particular class of *parameterized systems* - those that consist of an arbitrary number of components. Consider some systems  $\mathcal{S}$  that is parameterized by some value  $n$ .

### Regular transition system

*Regular transition system* is a commonly used model for representing parameterized systems in *regular model checking*. It describes all reachable configurations from initial configurations through a transducer.

**Definition 3.1:** *Transducer.*

A  $\Sigma$ - $\Gamma$ -transducer is an NFA  $\langle Q, Q_0, \Sigma \times \Gamma, \Delta, F \rangle$ , it represents a list of pairs  $\langle u_1, v_1 \rangle \dots \langle u_n, v_n \rangle$  where  $\langle u_1 \dots u_n, v_1 \dots v_n \rangle \in \bigcup_{n \geq 0} \Sigma^n \times \Gamma^n$

**Definition 3.2:** *Regular transition system (RTS).*

An RTS is a triple  $\mathcal{R} = \langle \Sigma, \mathcal{I}, \mathcal{T} \rangle$  where  $\Sigma$  is finite alphabet while  $\mathcal{I}$  is an NFA, which represents initial configurations. And  $\mathcal{T}$  is a  $\Sigma$ - $\Sigma$ -transducer.

### Inductive statements

**Definition 3.3:** *Interpretation.*



For any RTS  $\mathcal{R} = \langle \Sigma, \mathcal{I}, \mathcal{T} \rangle$ , we call a pair  $\langle \Gamma, \mathcal{V} \rangle$  an  $\Gamma$ -interpretation where  $\Gamma$  is a finite alphabet and  $\mathcal{V}$  is a deterministic  $\Sigma$ - $\Sigma$ -transducer. In the following, we denote  $u \models I$  to indicate  $\langle u, I \rangle \in [[\mathcal{V}]]$

**Definition 3.4:** *Inductive statements.*

For any given  $\Gamma$ -interpretation for  $\mathcal{R} = \langle \Sigma, \mathcal{I}, \mathcal{T} \rangle$ , we define

$$\begin{aligned} \text{Inductive}_{\mathcal{V}}(\mathcal{R}) &= \{I \in \Gamma^* \mid \forall u \rightsquigarrow_{\mathcal{T}} . \text{if } \langle u, I \rangle \in [[\mathcal{V}]] \text{ then } \langle v, I \rangle \in [[\mathcal{V}]]\} \\ &= \{I \in \Gamma^* \mid \forall u \rightsquigarrow_{\mathcal{T}} . \text{if } u \models I \text{ then } v \models I\} \end{aligned}$$

## 4 Algorithmic Learning of Finite Automata

Learning automata is a computational model for solving problems, where an agent learns to optimize its behavior by interacting with an unknown environment. The agent, also known as a learner, observes the feedback from the teacher, updates its internal state, and adjusts its actions accordingly. The *Teacher* This interaction process between the *Learner* and the *Teacher* is the primary mechanism of learning automata. In the field of automata learning, there are generally two distinct settings: active and passive learning. Passive algorithms are provided with a fixed set of examples consisting of strings that the automaton should either accept or reject. Active algorithms, unlike passive ones, have the ability to expand the set of examples as needed by asking further queries. However, in this thesis, our focus is solely on active learning. We do not introduce passive learning here but refer the interested reader to [CES09].

This chapter aims to provide a deeper understanding of the process of learning automata, including the roles and responsibilities of the *Teacher* and *Learner* in Section 4.1. In Section 4.2, we will introduce several of active algorithms that use for our experiment.

### 4.1 The oracles

In this learning scenario, the *Teacher* is proficient in the language being taught and is responsible for answering any questions posed by the learner. The *Learner* is given the opportunity to ask two types of queries - membership and equivalence. Membership queries are used to classify a word based on whether it belongs to the language being taught or not. Equivalence queries, on the other hand, are used to determine whether an assumed automaton is equivalent to the language the *Teacher* has in mind. The learning process continues until the *Teacher* answers an equivalence query positively.

**Membership oracle** The *Learner* provides a word  $w \in \Sigma^*$ , the *Teacher* replies "yes" or "no" depending on whether  $w \in \mathcal{L}$  or not.

**Equivalent oracle** The *Learner* conjectures a regular language, typically given as a DFA  $\mathcal{M}$ , and the *Teacher* checks whether  $\mathcal{M}$  is an equivalent description of the target language  $\mathcal{L}$

and return "yes", otherwise return a counterexample  $u \in \Sigma^*$  with  $u \in \mathcal{L}(\mathcal{M}) \iff u \notin \mathcal{L}$  or  $u \in \mathcal{L} \iff u \notin \mathcal{L}(\mathcal{M})$ .

On equivalent oracle, the *Teacher* can return a positive counterexample or a negative counterexample. A positive counterexample is a missing word in the conjecture but present in the target. The negative one is defined similarly.

It is crucial for the *Teacher* to have a clear and specific understanding of the correct hypothesis. Since we know how to implement the *Teacher* to answer the oracles, it is now simple to apply different of learning algorithms.

## 4.2 Algorithms

A learning algorithm—often called learner—learns a regular target language  $\mathcal{L} \subset \Sigma^*$  over an a priori fixed alphabet  $\Sigma$  by actively querying a teacher. We apply several of these algorithms in the course of this thesis.

### 4.2.1 $L^*$

$L^*$  learning automata was introduced by Angluin in 1987.

### 4.2.2 $NL^*$

In general, a nondeterministic finite automata *NFA* is often preferable to a deterministic finite automata *DFA* due to potentially exponential differences in their sizes (REFERENCE FOR COMPARISON OF NFA AND DFA). As such, learning algorithms for *NFA* are needed. In this section, we will introduce another active learning algorithm called the  $NL^*$  algorithm, patterned after  $L^*$ . The  $NL^*$  infers a canonical residual finite-state automata, a subclass of nondeterministic finite automata was introduced in the seminar work [CES09].

### 4.2.3 Kearns-Vazirani

### 4.2.4 Rivest-Schapire

## 5 Implementation

We apply automata learning algorithms to solve the regular model checking problems as well as finding an inductive statements for regular transition system.

**Membership query** On a membership oracle, the learner provides a statement and asks the teacher if this statement whether inductive or not. As we described in ??, a statement  $I$  is *inductive* if, for any transition  $v \rightsquigarrow u$  where  $u$  satisfies  $I$ ,  $v$  also satisfies the statement. REF TO ABOVE DEFINITION. Therefore, one can simply implement the Membership Oracle by checking the acceptance of  $\mathcal{M}$  while  $\mathcal{M}$  is an automaton for  $\overline{Inductive_{\mathcal{V}}(\mathcal{R})}$  and negating the answer. The  $\overline{Inductive_{\mathcal{V}}(\mathcal{R})}$  is defined by:

$$\overline{Inductive_{\mathcal{V}}(\mathcal{R})} = \{I \in \Gamma^* \mid \exists u \rightsquigarrow_{\mathcal{T}} w. u \models I \text{ and } w \not\models I\}$$

Let  $\mathcal{T} = \langle P, \Sigma \times \Sigma, \Delta, p_0, E \rangle$  is a transducer and  $\mathcal{V} = \langle P, \Sigma \times \Gamma, \delta, q_0, F \rangle$  is an interpretation. The automaton of  $\overline{Inductive_{\mathcal{V}}(\mathcal{R})}$  is defined by  $\langle Q \times P \times Q, \Gamma, \Delta, \langle q_0, p_0, q_0 \rangle, E \times F \times (Q \setminus F) \rangle$  where

$$\Delta(\langle q_1, p, q_2 \rangle, I) = \delta()$$

---

### Algorithm 1 Membership query

---

**Input:** Statement  $I$

**Output:** True or False

begin

$\mathcal{M} \leftarrow \text{getAutomaton}(\overline{Inductive_{\mathcal{V}}(\mathcal{R})})$

**if**  $I \in \mathcal{L}(\mathcal{M})$  **then**

return *false*;

**else**

return *true*;

**end if**

end

---

---

**Algorithm 2** Equivalent query

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**Require:**  $n \geq 0$

**Ensure:**  $y = x^n$

$y \leftarrow 1$

$X \leftarrow x$

$N \leftarrow n$

**while**  $N \neq 0$  **do**

**if**  $N$  is even **then**

$X \leftarrow X \times X$

$N \leftarrow \frac{N}{2}$

**else if**  $N$  is odd **then**

$y \leftarrow y \times X$

$N \leftarrow N - 1$

**end if**

**end while**

---

► This is a comment

## **6 Experiments**

## 7 Conclusion

We studied Regular Model Checking of safety properties. We evaluated the performance of our algorithms based on a prototype implementation.

### **Open Questions and Future Research**

## **Abbreviations**



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# Bibliography

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