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Question 1:

a) The function that maps unsigned 32-bit number to the corresponding integer is:

$$\text{map1}(a) = \sum_{i=0}^{31} a_i \cdot 2^i$$

where a_i is the i -th least significant bit.

The function, which maps a 32-bit signed number in two's complement to its integer is:

$$\text{map2}(a) = \sum_{i=0}^{30} a_i \cdot 2^i - a_{31} \cdot 2^{31}$$

where again a_i is the i -th least significant bit.

b) The add function for two 32-bit unsigned numbers is going to be:

$$\text{map1}(a \oplus b) = \text{map1}(a) + \text{map1}(b) \pmod{2^{32}}$$

We can only do the mod of this binary addition, because of the limitations of the output of `map1`. Each 32-bit number is uniquely specified modulus 2^{32} , so this addition function is uniquely specified. This adder can be used for signed numbers as well, because $\text{map2}(a) = \text{map1}(a)$ modulus 2^{32} . So:

$$\text{map2}(a \oplus b) = \text{map2}(a) + \text{map2}(b) \pmod{2^{32}}$$

c) To negate a 32-bit signed number, we can flip every bit and then add 1. So if we wanted $a-b$, then we can add a and $(-b)$. Therefore, to subtract the two numbers, we can add a and c , then add 1, where c is the 32-bit number, where the bits correspond to those in b , but are flipped.

d) Suppose we have to compare $3(000000000000000011)$ and $-72(1111111110111000)$. If we use either `blt` or `blo`, the compiler would subtract them and look at the flags to see which one is bigger. If we use `blt`, in order not to overflow we will need a 33rd bit and then subtract them. Then the result would be positive, so $3 > -72$. If we use `blo`, the compiler would think we are comparing 3 and 65464. So the result would be $3 < -72$.

e) When we want to compare two signed numbers, we subtract them and look at the result. If the result of $a-b$ is negative, then it could be either that b is larger than a , or that a is positive, b is negative and the subtraction overflowed. Suppose $a = 100$ and $b = -100$ (in 8-bit two's complement), the sign bit is going to be negative, which is not the desired result. To avoid this, we can look at the `V` flag to see if the subtraction overflowed. To implement `blt`, we have to check if the `N` flag is different from the `V` flag.

f) Because `blo` treats both numbers as unsigned, we can just look at the `C` flag, and see whether it is 0 or 1.

Question 2:

a) The calling convention is that the argument for a function is in `r0`, and the result is put in `r0`. If we want to use `r4-r7`, then we need to save the values in those registers, and then restore them back. We need to save `lr` as well.

b) The push instruction saves the values of r4, r5 and lr. When a pop instruction is executed, those registers will have the old values. This way we can use r4, r5 and lr without corrupting caller-saved registers.

c) instruction 1: blt foo_ret1
instruction 2: subs r0, r0, #3
instruction 3: adds r0, r0, r5

d) blank 1: $x \leq 2$
blank 2: return $x - \text{foo}(x+1)$