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1, $y(x) = w^T\phi(x) + b$ $\quad t_n \in \{-1; 1\}$

find $w; b$: $y(x_n) > 0 \Rightarrow t_n = 1$

$y(x_n) < 0 \Rightarrow t_n = -1$

$$\Rightarrow \frac{t_n . y(x_n)}{\|w\|} = \frac{t_n(w^T\phi(x) + b)}{\|w\|}$$

$$\arg\max_{w,b} \left\{ \frac{1}{\|w\|} \min_n [t_n(w^T\phi(x_n) + b)] \right\}$$

optimization problem : $\arg\min_{w,b} \frac{1}{2}\|w\|^2$. with

constraints : $t_n(w^T\phi(x_n) + b) \geq 1$, $n = 1, \ldots, N$.

Larange function:

$$L(w, b, a) = \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N} a_n [t_n(w^T\phi(x_n) + b) - 1]$$

$a = (a_1, \ldots, a_N)^T$

$$\frac{dL}{dw} = 0 \Leftrightarrow w = \sum_{n=1}^{N} a_n t_n \phi(x_n)$$

$$\frac{dL}{db} = 0 \Leftrightarrow 0 = \sum_{n=1}^{N} a_n t_n$$

Eliminating W and b from $L(w, b, a)$, then gives the dual representation, we maximize:

$$\tilde{L}(a) = \sum_{n=1}^{N} a_n - \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} a_n . a_m . t_n . t_m \, k(x_n, x_m)$$

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With constraints: $a_n \geqslant 0 \qquad n = 1, \dots, N$

$$\sum_{n=1}^{N} a_n t_n = 0.$$

$$y(x) = \sum_{n=1}^{N} a_n t_n k(x, x_n) + b.$$

KKT:
$$\begin{cases} a_n \geqslant 0 \\ t_n \cdot y(x_n) - 1 \geqslant 0 \\ a_n \cdot \{ t_n y(x_n) - 1 \} = 0 \end{cases}$$

We have:
$$t_n \left(\sum_{m \in S} a_m t_m k(x_n, x_m) + b \right) = 1.$$

- S is the set of indices of the support vectors.
- Making use of $t_n^2 = 1$,
$$b = \frac{1}{N_S} \sum_{n \in S} \left(t_n - \sum_{m \in S} a_m t_m k(x_n, x_m) \right)$$