PYTHON EXERCISES

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1. Where to start?

- · Where to find Python?
 - You can log in tarski.math.bme.hu the same way you log in to leibniz (if you are on leibniz, you just need to write ssh -Y tarski in a terminal) and you will find yourself in your home directory. There you can start spyder3 or ipython3 in a terminal. (There is spyder and ipython on leibniz, too, but avoid them because they are based on an old version of Python2.)
 - On your own windows machine you probably want this: http://wiki.math.bme.hu/view/AnacondaInstall With this, you'll get a graphical development environment called spyder that people seem to like.
 - https://colab.research.google.com/orhttps://cocalc.com. You need to register here, but get a nice a jupyter notebook.
 - https://sagecell.sagemath.org/, choose Python from the available languages. This is the worst choice.
- Lecture notes: http://math.bme.hu/~asimon/info2/python.pdf

2. SIMPLE THINGS

Exercise 2.1. Write code that prints in Celsius the temperature given in Fahrenheit stored in the variable f. The formula is $T_c = \frac{5}{9}(T_f - 32)$.

Exercise 2.2. Write code that prints all the numbers below the positive integer stored in the variable n. Do the same with the value of n included.

Exercise 2.3. Write code that prints the sum of the numbers not bigger than the positive integer stored in the variable n. Do the same with only the even numbers summed. (Do this without using if.)

Exercise 2.4. Write code that prints the factorial of the positive integer stored in the variable n.

Exercise 2.5. Write code that prints the (decimal) digits of the positive integer stored in n in reverse order. (Integer division is //)

Exercise 2.6. Write code that prints the sum of the (decimal) digits of the positive integer stored in n.

Exercise 2.7. Write code that prints the first n terms of the Fibonacci sequence. (It's not easy using only while. Hint: keep track of the last two terms.) Reminder: $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = f_n + F_{n+1}$.

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Exercise 2.8. Rewrite your solution to exercises 2.2-2.4 using for instead of while.

Exercise 2.9. Write a program that prints the sum of all the *positive* numbers in the list 1 of numbers. Use a for loop and do this with and without continue.

Exercise 2.10. Write a program using a for loop and break that prints the first proper divisor of the positive int stored in n if it has one, and does nothing otherwise.

Exercise 2.11. Write a function compare() of two arguments (both numbers), that prints The numbers are equal. if the two arguments are equal, The first number is bigger. if this is the case, and The second number is bigger. otherwise.

Exercise 2.12. Write a function divisible() of two arguments that returns True if its second argument is divisible by its first argument, and False otherwise.

Exercise 2.13. Write (using both the if statement and the if expression) a one-argument function absval() that returns the absolute value of the number given as its argument. (The built-in function abs() also does this.) Is else required in the version that uses the if statement?

Exercise 2.14. What does Python print after executing the following pieces of code?

```
x = 10

def f():
    x = 5

f()
    print(x)

x = 10

def f():
    x = 5
    return x

x = f()
    print(x)
```

Exercise 2.15. What are the values of the following expressions

- s[3]
- s[-3]
- s[3:6]
- s[3:-4]
- s[4::2]
- s[-4::2]
- s[3::2]
- s[5::-2]
- s[5:1:-2]

if s = 'abcdefghij'?

Exercise 2.16. Let numbers = list(range(5)). What's the difference between numbers[1] = [True] and numbers[1:2] = [True]? Is one of these equivalent to numbers[1] = True?

Exercise 2.17. Generate a list of the first 10 even numbers (a) using list comprehension (in two different ways) and (b) as a slice of a suitable list.

Exercise 2.18. Using nested list comprehensions, generate the following lists:

```
[[[0, 2], [0, 3], [0, 4]], [[1, 2], [1, 3], [1, 4]]]
```

és a

```
[[[0, 2], [1, 2]], [[0, 3], [1, 3]], [[0, 4], [1, 4]]]
```

Exercise 2.19. Suppose that a == 1, b == 2 and c == 3. Write *one* command that results in a == 2, b == 3 és c == 1.

Exercise 2.20. Generate a list of the first 10 even numbers using a loop and the . append() method.

Exercise 2.21. Write a function solve2(), that, given the three coefficients of a second order equation, returns the list of its *real* roots. For example,

```
>>> solve2(1,1,-2), solve2(1,0,2)
([1.0, -2.0], [])
```

because $x^2 + x - 2 = (x - 1)(x + 2)$, and $x^2 + 2$ has no real roots. To compute square roots, use the sqrt() function of the math module.

Exercise 2.22. Rewrite the following so it doesn;t use elif:

3. More challenging stuff

Exercise 3.1. Write a function of one argument, that, given a list of numbers $n_1, n_2, ..., n_k$ returns a new list containing the numbers $n_1^3 - 1, n_2^3 - 1, ..., n_k^3 - 1$. Do it both with and without using list comprehension.

Exercise 3.2. Write a function squares() of two arguments, so that squares(m,n) returns the list of squares between m and n. Do it both with and without using list comprehension.

Exercise 3.3. Generalize squares() of the previous exercise to every positive exponents! That is, using one of your implementations of squares() as your starting point, write powers(), so that powers(m,n,k) returns the list of the kth powers between m and n

Exercise 3.4. Write a function index_of(), which, given an object and a list, returns the index of the first occurrence of the object in the list, or None if it doesn't occur in the list. (Recall that if a function doesn't do return, it returns None.)

Exercise 3.5. Write a function indices_of(), which, given an object and a list, returns the list of indices of the occurrences of the object in the list. For example:

```
>>> indices_of(3,[4,1,3,2,3])
[2, 4]
>>> indices_of(0,[4,1,3,2,3])
[]
```

Exercise 3.6. Write a function substitute(), which, given a list and two objects, returns a new list with all occurrences of the first object replaced by the second. For example:

```
>>> substitute([1,2,3,4,2],2,'a')
[1, 'a', 3, 4, 'a']
```

Exercise 3.7. Write a version of substitute() which doesn't return anything, but does the substitution on its list argument. For example:

```
>>> mylist = [1,2,3,4,2]
>>> substitute(mylist,2,'a')
>>> mylist
[1, 'a', 3, 4, 'a']
```

Exercise 3.8. Define a function divisibles_by() of two arguments, a list and an integer, which returns the list of those members of the list that are divisible by the integer. For example,

```
>>> divisibles_by(list(range(30,50)),7)
[35, 42, 49]
```

Exercise 3.9. Define a function divisors() that returns the list of proper divisors of its only argument.

Exercise 3.10. Define sigma() of one argument, which computes the number theoretic function $\sigma(n) = \sum_{1 < d < n.d|n} d$. Check it for a few values. For example:

```
>>> [(n,sigma(n)) for n in range (6,60,11)]
[(6, 12), (17, 18), (28, 56), (39, 56), (50, 93)]
```

Exercise 3.11. Define the function is_perfect() which returns True if its argument is a perfect number (which means that it is equal to the sum of its proper divisors) and False otherwise. Using your function, print all perfect numbers below 10000.

Exercise 3.12. Define a function divisors_ival() such that divisors_ival(m,n) prints the list of proper divisors of all integers between m and n, like this:

```
>>> divisors_ival(30,35)
30 -> [1, 2, 3, 5, 6, 10, 15]
31 -> [1]
32 -> [1, 2, 4, 8, 16]
33 -> [1, 3, 11]
34 -> [1, 2, 17]
35 -> [1, 5, 7]
```

Exercise 3.13. Write a function is_prime which returns True if its argument is a prime number and False otherwise.

Exercise 3.14. Write a function primes_between() such that primes_between(m,n) returns the list of primes in the interval [m,n].

Exercise 3.15. Write a function prime_divisors() that returns the list of prime divisors of its argument.

Exercise 3.16. Write a function max_exp() such that max_exp(m,n) returns the biggest k such that $m^k \mid n$. You can assume m > 1.

Exercise 3.17. Write a function prime_decomp() that returns the prime decomposition of its argument as a list of pairs (or lists) whose first member is a prime divisor of the argument and the second is the exponent of the prime divisor in the prime decomposition. For example:

```
>>> prime_decomp(90)
[(2, 1), (3, 2), (5, 1)]
```

Exercise 3.18. Define a function lookup() of two arguments. The second is a list of 2-long tuples, and the first a "key". lookup(key, list) should return the second member of the first tuple in the list whose first member is equal to key, or None if there's no such member of the list.

Exercise 3.19. Using the functions in the two previous exercises, write a function gcd() that computes the greatest common divisor of its two arguments.

Exercise 3.20. Define a function that computes $\varphi(n)$ (the number of relative primes to n below n) for every natural number n.

Exercise 3.21. Write a function separate(), which, when called with a list l of numbers, returns a pair of lists, the first containing the negative, the second the non-negative members of l.

Exercise 3.22. Define a function is_sorted() of one argument, a list, which returns True if it is sorted in ascending order and False otherwise.

Hint: if you have trouble defining it, look up the definition of repeats() in the notes!

Exercise 3.23. Write a function which returns the minimal member of the nonempty list that is given as its only argument. Call it my_min(), because there is a built-in function min() which does this. Needless to say, don't use that. What other ways are there to cheat here? (It's good to be aware of the possibilities, but write a version without using any of them.)

Hint (for cheating): there's a built-in max() fuction.

Exercise 3.24. Write a function min_index() which returns the *index of* the first occurrence of the minimal member of the nonempty list of numbers that is given as its only argument.

Exercise 3.25. Write a function min_indices() which returns the list of the indices of the occurrences of the minimal member of the nonempty list of numbers that is given as its only argument. For example:

```
>>> min_indices([1,3,4,2,1,3,1,2])
[0, 4, 6]
```

Exercise 3.26. Write a function nearest_to_avg() that, given a list of numbers, returns the member closest to the average of the members, or None if the list is empty. You can use the function abs().

Exercise 3.27. Define a function has_duplicates() of one argument, a list, which returns True if there is an object which occurs at least twice in the list, and False otherwise.

Exercise 3.28. Write a function longest_run() of one argument, a list of numbers, which returns the length of the longest sequence of the same number. So, for example:

```
>>> longest_run([])
0
>>> longest_run([1])
1
>>> longest_run([1,2,3])
1
>>> longest_run([1,2,3,3,4])
2
>>> longest_run([1,2,2,2,1,2,3,3,4])
3
```

Exercise 3.29. Print the following ugly multiplication table:

```
1: 1 2 3 4 5 6 7 8 9
2: 2 4 6 8 10 12 14 16 18
3: 3 6 9 12 15 18 21 24 27
4: 4 8 12 16 20 24 28 32 36
5: 5 10 15 20 25 30 35 40 45
6: 6 12 18 24 30 36 42 48 54
7: 7 14 21 28 35 42 49 56 63
8: 8 16 24 32 40 48 56 64 72
9: 9 18 27 36 45 54 63 72 81
```

Hints: print(42, end='whatever') prints 42 followed by whatever instead of a newline. And print() simply prints a newline.

Exercise 3.30. Print all permutations (one by one) of 012!

```
0 1 2
0 2 1
1 0 2
1 2 0
2 0 1
2 1 0
```

Modify your program so that it prints the list of all such permutations as a list of list:

Exercise 3.31. Write a function that computes the factorial of a natural number! Do a version without recursion, too.

Exercise 3.32. Write a function lucas(n) which returns the nth Lucas-number. Lucas numbers are defined recursively as follows:

$$L_0 = 2,$$
 $L_2 = 1,$ $L_{n+2} = L_n + L_{n+1}$

Try writing the function following the pattern of this definition, and another version that is not recursive.

Homework 3.1. Using the lucas() function (preferably the non-recursive one) from the previous exercise, print out the first few members of the subsequence of Lucas numbers whose indices are divisible by 3. Can you observe some regularity? If yes, can you verify it for the first 100 or so members of the subsequence? (Do this with the iterative version; if you only have the recursive one, make it 10 instead of 100.) Does this regularity hold if we consider indices whose remainders modulo 3 is 1 or 2? Can you formulate a theorem? If yes, can you verify it for the first few hundred Lucas numbers?

Exercise 3.33. Goldbach's conjecture says that every even number greater than 2 is the sum of two primes. Write a function goldbach(), which, given $1 < n \in \mathbb{N}$, returns the number of ways 2n can be written as the sum of two primes. $(p_1 + p_2 \text{ and } p_2 + p_1 \text{ count as the same decomposition.}) For debugging your function, you may want to use print(), which, if given multiple arguments, will print them all, separated by a space.$

Exercise 3.34. Convince yourself that Goldbach's conjecture will not be easy to refute, by exploring various intervals using your goldbach() function from the previous exercise.

For example: what's the minimal number of "Goldbach decompositions" of the 100 even numbers from 2000? 200000?

Exercise 3.35. Write a function pascal() that, given a positive natural number n, returns, as a list of lists, the first n lines of Pascal's triangle. Use the recursive definition of Pascal's triangle (this does not mean that your function must be recursive, see for example 3.31!): the nth row is of length n; the first row consists of one 1, and each entry of each subsequent row is the sum of the number above and to the left and the number above and to the right, treating blank entries as 0 (Wikipedia). For example:

```
>>> pascal(5)
[[1], [1, 1], [1, 2, 1], [1, 3, 3, 1], [1, 4, 6, 4, 1]]
You might want to use n*[1] to create a list of 1s of length n.
```

Exercise 3.36. An associative array is a data structure that maps values to keys. (Python has such a data structure: it's called dict, and we will use it later – but in this exercise we roll our own.)

There are four activites related to an associative array: we need to be able to create one, add key-value pairs to it, look up the value associated to a key in it, and remove a key-value pair from it.

We have already written the third component in Exercise 3.18. The first could simply be

```
def make_new_aa(): return list()
```

Write the remaining two: insert(key,value,aa) should return a copy of aa, either extended with the tuple (key,value) or, if key is already present in aa, it should return a copy of aa in which (key,value') is replaced by (key,value).

Finally, delete(key,aa) should return a copy of aa with no tuple of the form (key,value).

For example:

```
>>> d = make_new_aa()
>>> d = insert('one',1,d); d = insert('two',2,d); d = insert('three',3,d)
>>> d
[('one', 1), ('two', 2), ('three', 3)]
```

or

```
>>> lookup('two',d)
2
>>> d = insert('two',22,d); d = insert('four',4,d); d
[('one', 1), ('three', 3), ('two', 22), ('four', 4)]
>>> d = delete('five',d); d
[('one', 1), ('three', 3), ('two', 22), ('four', 4)]
>>> d = delete('two',d); d
[('one', 1), ('three', 3), ('four', 4)]
```

Exercise 3.37. Write a function insert_sort() of one argument, a list of numbers, which returns the sorted (in ascending order) version of the list.

You should use the Insertion sort algorithm, which works as follows: we read the elements of the list from left to right, and any time we encounter a member e that is smaller than the previous one, we shift to the right all the members that are left of e and are bigger than e, and insert e where the leftmost of these members were.

In other words, we scan the list from left to right, and whenever we find that the first *k* element is not in order, we correct this.

```
def insert_sort(lst):
    def insrt(x, 1): #l is ordered
        if l == []: return [x]
        hd,*tl = l
        if x <= hd: return [x]+l
        return [hd] + insrt(x, tl)
    if lst == []: return lst
    return insrt(lst[0], insert_sort(lst[1:]))</pre>
```

Exercise 4.1. Write a program that asks for numbers, one after the other, and if the user presses RETURN without entering a number first, it returns the average of the numbers. So an interaction with your program should look something like this:

```
Enter a number: 1
Enter a number: 4
Enter a number: 12
Enter a number: 3
Enter a number:
The average is 5.0
```

Exercise 4.2. Write a program that asks for numbers separated by spaces, and prints their average. So an interaction with your program should look something like this:

```
>>> Enter some numbers separated by spaces: 1 4 13 2
The average is 5.0
>>>
```

Exercise 4.3. Write a function read_first_lines() of two arguments: the name of a file and an integer n. It should read and print the first n lines of the file (or all lines if there are no more than n lines in it).

Exercise 4.4. Write a function copy_first_lines() of three arguments: the name of an input and an output file, and an integer. It should do what the function in the

previous exercise did, except that the output should go to the file named by the second argument. Check with an editor, or with cat or less at the command line that your function did what it's supposed to do.

Exercise 4.5. Write a function count_lines() of one argument, the name of a file. It should return the number of lines in the file (that's what wc -1 does on the command line).

Exercise 4.6. Write a function read_to_string() that returns a string containing all the text in the text file named by its only argument. It shouldn't contain newline characters (\ns) but of course the last word of a line should not run into the first word of the next line.

Help: strings can be concatenated using +. And use .rstrip(), as in the lecture!

5. CONTAINERS

Exercise 5.1. Write a function list_diff() of two arguments, both lists, which returns the list of those members of the first list (in the original order) which are not in the second. For example:

```
>>> list_diff(list(range(10)),list(range(0,15,3)))
[1, 2, 4, 5, 7, 8]
```

Exercise 5.2. Print the following pattern:

```
1
22
333
4444
55555
666666
7777777
8888888
999999999
```

Exercise 5.3. Write a function wave () of two arguments, such that for example wave (5, 2) prints the pattern

00000

0000

Note that the maximum "height" is 5 and there are two waves.

If it works, write new_wave(), which is similar to wave(), except that there is only one row of maximum "height". For example, new_wave(5,1) should print

Exercise 5.4. Write a function merge() which accepts two lists as arguments, and returns a new list that contains the first member of the first list, then the first member of the second list, then the second member of the first list, etc. In the first version, you can assume that the lists are of equal length. But in the final one if one of the lists is longer, its remaining elements should come last, after the proper "merging" is done. For example:

```
>>> merge(list(range(5)),list(range(10,18)))
[0, 10, 1, 11, 2, 12, 3, 13, 4, 14, 15, 16, 17]
```

Exercise 5.5. Write a function cat() that prints in upper case the lines of the file whose name is given as its argument.

Exercise 5.6. Write a function that accepts one argument: the name of a text file. The file can be assumed to contain floating point numbers, one per line. It should return the average of these numbers.

Hint:Use the float() function to convert the string representation of a floating point number to a float. And enumerate() works for text files, too (see §2 of the lecture notes).

Exercise 5.7. Define a function transpose() of one argument, that, given a square matrix represented as a list of lists, returns its transpose. For example:

```
>>> transpose([[1,2,3],[4,5,6],[7,8,9]])
[[1, 4, 7], [2, 5, 8], [3, 6, 9]]
```

Make a new list of lists, don't change the argument!

Exercise 5.8. Define a function matrix_add() of two arguments that returns the sum of two matrices (of the same shape) represented as lists of rows, where each row is represented as a list of numbers. For example:

```
>>> m1 = [[1,0,0],[0,1,0],[0,0,1]]
>>> m2 = [[1,2,3],[4,5,6],[7,8,9]]
>>> matrix_add(m1,m2)
[[2, 2, 3], [4, 6, 6], [7, 8, 10]]
```

Make a new list of lists, don't change either of the arguments!

Exercise 5.9. Define a function matrix_mult() of two arguments that returns the product of two matrices (of the appropriate shape) represented as lists of rows, where each row is represented as a list of numbers. For example:

```
>>> matrix_mult([[1,2],[3,4],[5,6]],[[1,0,0],[0,1,0]])
[[1, 2, 0], [3, 4, 0], [5, 6, 0]]
>>> matrix_mult([[1,0,0],[0,1,0]],[[1,2],[3,4],[5,6]])
[[1, 2], [3, 4]]
```

Make a new list of lists, don't change either of the arguments!

Exercise 5.10. Define a function myzip2() of two arguments, both sequences (lists or strings), that returns a list of tuples: the *i*th member of the *j*th tuple in the list should be the *j*th member of the *i*th argument. The length of the list should be the length of its shortest argument. For example:

```
>>> myzip2('abcdefg', list(range(4)))
[('a', 0), ('b', 1), ('c', 2), ('d', 3)]
Do not use zip()!
```

Exercise 5.11. Write a function lindex(string, substring) that returns the smallest index in string where substring is found, or -1 if substring is not found. This is almost what the .index() method does, so don't use that! For example,

```
>>> lindex("At the turn of the century", "the")
3
>>> lindex("At the turn of the century", "them")
-1
```

Exercise 5.12. Write a function count_occurrences() of two arguments, both strings. It should return the number of times its second argument occurs as a substring in its first argument. For example:

```
>>> count_occurrences("At the turn of the century", "the")
2
>>> count_occurrences("At the turn of the century", "them")
0
```

Exercise 5.13. Suppose we have a text file inventory.csv, each line of which has the structure

```
good, amount, unit price
```

That is, within each line, the three "fields" are separated by commas. (This is a well known format called "comma separated values".) Write a short program that prints the content of each line in the format

```
Name: good
Amount: amount
Unit price: unit price
For example, if inventory.csv contains this:
>>> import os
>>> print(os.popen("cat inventory.csv").read())
then the output should be
Good: ball
Amount: 570
Unit price: 0.13
```

Good: table
Amount: 3
Unit price: 2000
Good: racket
Amount: 12
Unit price: 185
Good: net
Amount: 17
Unit price: 23
Use an f-string!

Exercise 5.14. Modify your solution to the previous exercise so that it prints lines in the following format:

```
Name: good
```

Total price: amount * unit price

Don't forget to convert amounts and unit prices into numbers first!

Exercise 5.15. Using f-strings, print the following multiplication table:

Exercise 5.16. Suppose we have the mid-season standing of a football league as a csv file. The fields are: the name of the club, games played and points won by it. Write a function whose only argument is the name of the csv file, and which prints a nicely formatted list of the clubs ordered by the number of points *lost* (the less the better, of course) or the average points/game so far. (For the former, we need one additional information: each win is worth 3 points.) The idea is that these orderings give a better picture of how things stand when different clubs have not played the same number of matches. For example, if the csv file looks like this:

```
>>> import os
>>> print(os.popen("cat pl.csv").read())
Man City,23,57
Liverpool,22,48
Chelsea,24,47
Man Utd,22,38
West Ham,23,37
Arsenal,21,36
Tottenham,20,36
Wolverhampton,21,34
Brighton,22,30
```

```
Leicester, 20, 26
Aston Villa, 21, 26
Southampton, 22, 25
Crystal Palace, 22, 24
Brentford, 23, 23
Leeds, 21, 22
Everton, 20, 19
Norwich, 22, 16
Newcastle, 21, 15
Watford, 20, 14
Burnley, 18, 12
```

then the function should print this:

>>> by_lost_points("pl.csv")

	Club	GP	Pts	-Pt
1	Man City	23	57	12
2	Liverpool	22	48	18
3	Tottenham	20	36	24
4	Chelsea	24	47	25
5	Arsenal	21	36	27
6	Man Utd	22	38	28
7	Wolverhampton	21	34	29
8	West Ham	23	37	32
9	Leicester	20	26	34
10	Brighton	22	30	36
11	Aston Villa	21	26	37
12	Southampton	22	25	41
13	Leeds	21	22	41
14	Everton	20	19	41
15	Crystal Palace	22	24	42
16	Burnley	18	12	42
17	Brentford	23	23	46
18	Watford	20	14	46
19	Newcastle	21	15	48
20	Norwich	22	16	50

Use f-strings!

Exercise 5.17. Define a function is_palindrome() that checks whether its argument, which is a sequence (a list, a tuple, or a string) is a palindrome, that is, it reads the same forwards and backwards. Don't reverse the sequence!

Exercise 5.18. Define a function palindromes() of one argument, the name of a text file (such as /usr/share/dict/words) with one word on each line, that prints all the palindromes in the file and returns the number of palindromes it has found.

5.1. dict.

Exercise 5.19. Define a function substitute() of two parameters: a string and a dictionary. It should return a new string which is a copy of the old one except that each character that is a key in the dictionary is replaced by the corresponding value (a string). For example:

```
>>> substitute("acbcade",{'a':'xyz','c':'zyx'})
'xyzzyxbzyxxyzde'
>>> substitute("acbcade",{'a':'c','c':'a'})
'cabacdo'
```

Could this be done with repeated use of the .replace() method of the str class? Would it do the second example correctly?

Exercise 5.20. Modify your substitute() function of the previous exercise so that if the value associated in the dictionary to a character is None, it is deleted (replaced by the empty string ''). For example:

```
>>> substitute("acbcade",{'a':'c','c': None})
'cbcde'
```

Exercise 5.21. Write a function to_dict() of one argument, a list of pairs (tuples). It should return a dictionary that has the first members of the tuples as keys, and the second members as corresponding values. For example:

```
>>> to_dict([('one',1), ('two',2), ('three',3)]) {'one': 1, 'two': 2, 'three': 3}
```

Exercise 5.22. Write a function word_count(), which, given the name of a text file, returns the number of words in it. Turn it into a standalone program that can be started from the command line, like this:

```
python word_count.py text.txt
```

This should return the same number as

```
wc -w text.txt
```

Try it on a longer text, such as this.

Exercise 5.23. Write a function $top_freq()$, which, given the name of a file and a natural number n, returns the list of the n most frequent words in the file, with their frequencies. For more realistic results, make sure "this" and "This" are treated as being the same word. Test it on a small text file where you know the word frequencies, and then try it on a longer text, such as this, for which it should return the list

```
[('the', 12436), ('and', 8311), ('of', 7327), ('a', 4997), ('to', 4412)]
```

Hints: the dictionary methods .items() returns the contents of the dictionary as a list of tuples of key-value pairs. You can sort a list of such tuples on their second members with

```
sorted(list_of_tuples, key=lambda x : x[1])
or
sorted(list_of_tuples, key=lambda x : x[1],reverse=True)
if you want descending order.
```

Exercise 5.24. Using your solutions to 5.21 and 5.23, write a function top_freq_dict() which does exactly what top_freq() does, but returns the result as a dictionary. For example, for this file, it should return the dictionary

```
{'the': 12436, 'and': 8311, 'of': 7327, 'a': 4997, 'to': 4412}
```

Hint: if the definition of your function is longer than two lines, then there's probably some misunderstanding.

Exercise 5.25. There is a datatype set in Python, but let's make a new one. Represent a set by a dict whose keys are the members of the set and whose values are all None. For example, union could be defined on this representation of sets like this:

```
>>> def union(s1,s2):
... return dict.fromkeys(list(s1.keys()) + list(s2.keys()))
...
>>> union({1: None, 2: None}, {1: None, 3: None})
{1: None, 2: None, 3: None}
```

Here dict.fromkeys() does the same as s1.fromkeys() or s2.fromkeys(), but since the result (a new dictionary whose keys come from the iterable that is given as its argument and all of whose values are None) has nothing to do with either s1 or s2, we can write dict.fromkeys(). (That is, .fromkeys() is a so called *class method*, not an *instance method*.)

Define intersection(), difference(), and also set_add(), which adds an element (its second argument) to the set (its first), and set_remove(), which removes an element (its second argument) to the set (its first) if it's present, and leaves it alone otherwise. For example:

```
>>> s1 = empty_set() ; set_add(s1,1) ; set_add(s1,2) ; s1
{1: None, 2: None}
>>> s2 = empty_set() ; set_add(s2,2) ; set_add(s2,3) ; s2
{2: None, 3: None}
>>> 1 in union(s1,s2)
True
>>> 1 in intersection(s1,s2)
False
>>> 1 in difference(s1,s2)
```

Fortunately, the in operator works automatically as expected, because if d is a dict, then k in d returns True iff k is a key in d.

Exceptions.

Exercise 5.26. Redo Exercise 5.11, but this time using the built in .index() method. The challenge is that string.index(substring) raises a ValueError exception when substring is not found in string, but lindex() should return -1. Other exceptions should go through, and *not* be hidden by returning -1 (or any other value, for that matter). For example,

```
>>> lindex("At the turn of the century", "the")
3
>>> lindex("At the turn of the century", "them")
-1
>>> lindex("At the turn of the century", 42)  #should raise an exception
Traceback (most recent call last):
   File "<stdin>", line 1, in <module>
   File "<stdin>", line 3, in lindex
TypeError: must be str, not int
```

Exercise 5.27. Modify your solution to the previous exercise in such a way that if an exception other than ValueError is raised by .index(), the message "Unknown error" is printed before the error is passed on to the caller.

```
For example,

>>> lindex("At the turn of the century", "the")

3

>>> lindex("At the turn of the century", "them")

-1

>>> #So far everything is as before. But note the (only) difference

>>> #in the following output!

>>> lindex("At the turn of the century", 42)

Unknown error

Traceback (most recent call last):

File "<stdin>", line 1, in <module>

File "<stdin>", line 3, in lindex

TypeError: must be str, not int
```

5.2. List comprehension.

Exercise 5.28. Write a function even_odd() that accepts two lists of integers. It should return a list with all even members of the first list followed by the odd members of the second list. Use list comprehension!

Exercise 5.29. Using list comprehension, write a function same_mod() that accepts two lists of natural numbers and a positive natural numbers as arguments. It should return a list of all pairs of numbers whose first component comes from the first list, second component from the second list, and such that their remainders modulo the third argument are equal. For example:

```
>>> same_mod(list(range(4)), list(range(2,8)),3)
[(0, 3), (0, 6), (1, 4), (1, 7), (2, 2), (2, 5), (3, 3), (3, 6)]
```

Exercise 5.30. Modify your solution to the previous exercise so that same_mod() returns only those pairs which not only satisfy the condition in the previous exercise but also the extra condition that the first component of each pair is not divisible by the third argument. For example:

```
>>> same_mod(list(range(4)), list(range(2,8)),3)
[(1, 4), (1, 7), (2, 2), (2, 5)]
```

Exercise 5.31. Write a function squares() of two arguments: a list of integers and a positive natural number. squares(lst,n) should return a list of lists: the *i*th list in this list should contain the squares of the first n numbers starting from the *i*th member of lst. For example:

```
>>> squares(list(range(1,8,3)),3)
[[1, 4, 9], [16, 25, 36], [49, 64, 81]]
Use list comprehension(s)!
```

Exercise 5.32. Write a function concatenate() using list comprehension, that concatenates the list of lists that it is passed. For example:

```
>>> concatenate([list(range(3)),list(range(3,6)),list(range(6,8))])
[0, 1, 2, 3, 4, 5, 6, 7]
```

Exercise 5.33. As in 5.8, define a function matrix_add() of two arguments that returns the sum of two matrices (of the same shape) represented as lists of rows, where each row is represented as a list of numbers. But this time do it with the help of list comprehension!

Hint: use zip()!

Exercise 5.34.* Define a function merge() which merges two sorted list (that is, returns a sorted list with members of both lists). For example,

```
>>> merge([2,7,9], [1,5,6])
[1, 2, 5, 6, 7, 9]
```

Hint: try to write a recursive function, and think about what possibilities are there for the arguments.

6. FUNCTIONS

Exercise 6.1. Let's call for the duration of this and the next exercise an object a tree if either it's a number, or a list of trees. So 0, 1 and 2 are trees, because they are numbers, and thus [0, 1, 2] is also a tree, because it's a list of trees. Consequently,

```
[0, 1, [0, 1, 2], 2]
```

being a list of trees, is also a tree. And so is

```
[0, [0, 1, 2], [0, [0, 1, 2], 1, [0, 1, 2], 2], 2]
```

for the same reason.

Write a function sumtree() which computes the sum of the numbers of a tree. For example, it should return 14 for the last tree above.

Exercise 6.2.* Write a function flatten() that, given a tree, returns the list of the numbers it encounters by traversing it in a depth-first manner. "Depth-first" means starting from the beginning of the list (if the tree is a list — if it's not, then it is a number and the return value should be the list which has this number as its only element) and that whenever we encounter a list, we process that list first. A few examples should make it clearer:

```
>>> flatten(1)
[1]
>>> flatten([1])
[1]
>>> flatten([1, 2])
[1, 2]
>>> flatten([1, 2, [3, 4]])
[1, 2, 3, 4]
>>> flatten([0, [0, 1, 2], [0, [0, 1, 2], 1, [0, 1, 2], 2], 2])
[0, 0, 1, 2, 0, 0, 1, 2, 1, 0, 1, 2, 2, 2]
>>> flatten([[0, 1, 2], -1, [-2, [3, 4], [5, 6, 7, 8],
... [9, 10, 11]], [13, 14, 15], [16, 17]])
[0, 1, 2, -1, -2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17]
```

Exercise 6.3.* Write a function sublists() that returns the list of all sublists (in any order) of its argument. (Here 11 is a sublist of 12 if it's a subsequence of 12 in the sense you learned in calculus. That is, all members of 11 occur in 12, and in the same order.) For example:

```
>>> sublists([])
[[]]
>>> sublists([1])
[[1], []]
```

```
>>> sublists([1,2])
[[1, 2], [1], [2], []]
>>> sublists([1,2,3])
[[1, 2, 3], [1, 2], [1, 3], [1], [2, 3], [2], [3], []]
```

Exercise 6.4. Redo 3.18 but this time around lookup() should accept a keyword only argument called default, whose value it should return if it cannot find the key. If it's not given, lookup() should, as before, return nothing in this situation.

Exercise 6.5. Modify concatenate() from Exercise 5.32 so that instead of a list of lists, it accepts any number of list as arguments. For example:

```
>>> concatenate(list(range(3)),list(range(3,6)),list(range(6,8)))
[0, 1, 2, 3, 4, 5, 6, 7]
```

Hint: apart from one character, the two definitions should be the same.

Exercise 6.6. Define a function myzip() of any number of arguments, all sequences (lists or strings), that returns a list of tuples: the *i*th member of the *j*th tuple in the list should be the *j*th member of the *i*th argument. The length of the list should be the length of its shortest argument. For example:

```
>>> myzip('abcdefg', list(range(3)), list(range(10,14)))
[('a', 0, 10), ('b', 1, 11), ('c', 2, 12)]
Do not use zip()!
Hint: start from Exercise 5.10!
```

Exercise 6.7. As in Exercise 5.7, define a function transpose() of one argument, that, given a (not necessarily) square matrix represented as a list of lists, returns its transpose, but this time computed with list comprehension. For example:

```
>>> transpose([[1,2],[3,4],[5,6]])
[[1, 3, 5], [2, 4, 6]]
Hint: use * and zip()!
```

Exercise 6.8.[★] Redo Exercise 5.9 but this time using only list comprehension. For simplicity, you can use transpose().

Exercise 6.9. Define a function apply() of two arguments, which returns the result of applying its first argument, which should be a one-argument function, to its second.

Exercise 6.10. Write a function self_compose of two arguments such that self_compose(fun, n)

where fun is a function of one argument and n is a natural number, returns fun composed with itself n times. In particular, self_compose(fun, 1) should return fun itself, self_compose(fun, 2) the composition of fun with itself, etc. What should self_compose(fun, 0) return? If you don't know, just assume (and check with assert) that the second argument is always positive.

For example:

```
>>> (self_compose(lambda x: x+1,3))(0)
3
```

Exercise 6.11. Write a function compose12() of two arguments, so that if f1 is a function of one argument and f2 is a function of two arguments, then compose12(f1,f2) returns the two argument function $f1(f2(_,_))$. For example,

```
>>> compose12(lambda x: 2*x,lambda x,y: x+y)(2,3)
```

Exercise 6.12. Write a function composeln() of two arguments, so that if f1 is a function of one argument and f2 is a function of n arguments, then composeln(f1,f2) returns the n argument function $fl(f2(_,_, \ldots, _))$. For example,

```
>>> compose1n(lambda x: 2*x,lambda x,y: x+y)(2,3)
10
>>> compose1n(lambda x: 2*x,lambda x,y,z: z*(x+y))(2,3,4)
40
```