

# Problem Solving

Codility and Leetcode Practitioner



# “Brute Force” to “Dynamic Programming”





# Problems

## Description:

Write a function `fib(n)` that takes in a number as an argument and return the n-th number of the Fibonacci sequence.

## Example:

- `fib(1)` is 1
- `fib(2)` is 1
- `fib(3)` is 2
- `fib(4)` is 3
- `fib(n)` is `fib(n-1)+fib(n-2)`

# Brute-force

Costly but work

```
public long fib(int n) {  
    if (n <= 2) return 1;  
  
    return fib(n-1) + fib(n-2);  
}
```

Use stack memory  
Limited size (1-8Mb)

```
public long fibWithoutRecursion(int n) {  
    if (n <= 2) return 1;  
  
    Stack<Integer> stack = new Stack<>();  
    long result = 0;  
  
    stack.push(n);  
  
    while (!stack.isEmpty()) {  
        int current = stack.pop();  
  
        if (current <= 2) {  
            result += 1;  
        } else {  
            // Push both subproblems to stack  
            stack.push(current - 1);  
            stack.push(current - 2);  
        }  
    }  
  
    return result;  
}
```

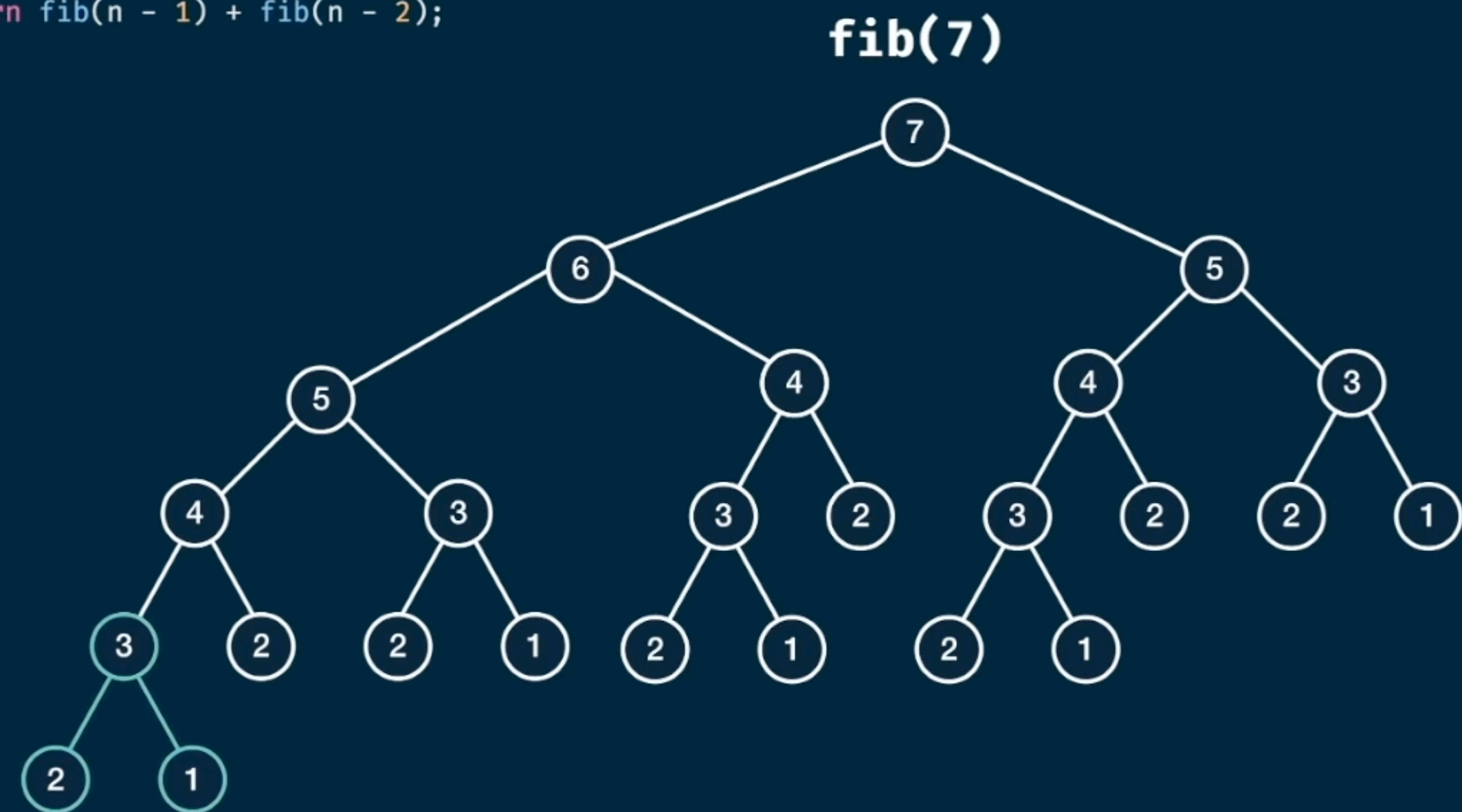
Use Heap memory  
Much larger than stack

# Brute-force

# Costly but work

```
1  const fib = (n) => {
2    if (n <= 2) return 1;
3    return fib(n - 1) + fib(n - 2);
4  };

```

 $O(2^N)$

# Memoization

Remove duplicate by Caching

```
public long fib(int n) { 2 usages new *
    if (n <= 2) return 1L;

    return fib(n - 1) + fib(n - 2);
}

public long fibWithMem(int n, Map<Integer, Long> mem) { 2 usages new *
    if (mem.containsKey(n)) {
        return mem.get(n);
    }

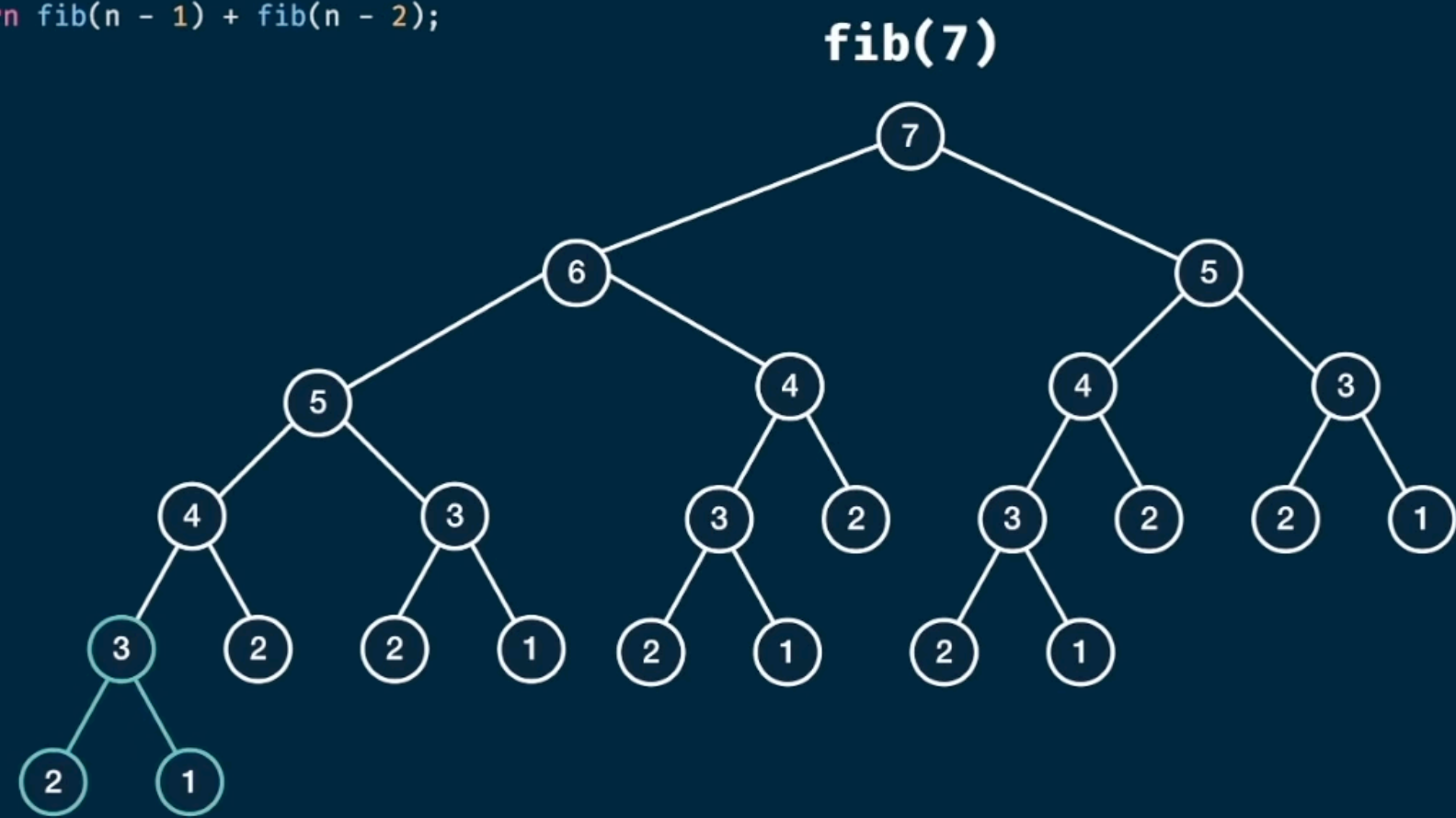
    if (n <= 2) {
        mem.put(n, 1L);
        return 1L;
    }

    long result = fibWithMem(n - 1, mem) + fibWithMem(n - 2, mem);
    mem.put(n, result);

    return result;
}
```

## Brute-force ( $2^N$ )

```
1 const fib = (n) => {  
2   if (n <= 2) return 1;  
3   return fib(n - 1) + fib(n - 2);  
4 };
```



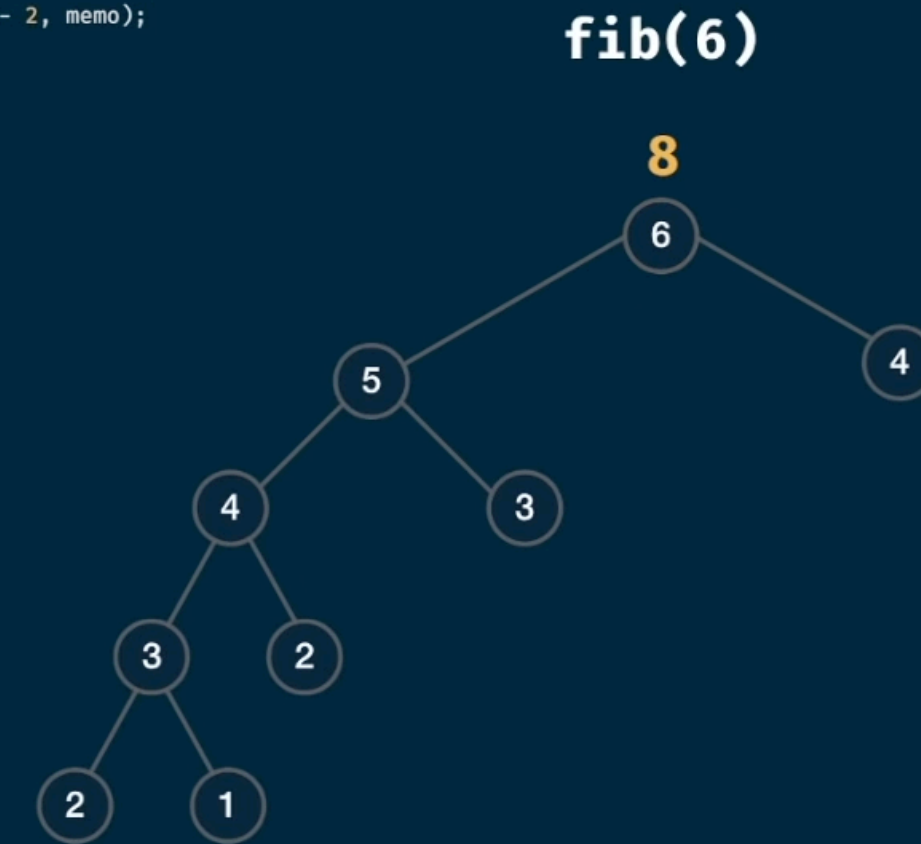
$O(2^N)$

## Memoization ( $2N$ )

```
1 const fib = (n, memo = {}) => {  
2   if (n in memo) return memo[n];  
3   if (n <= 2) return 1;  
4  
5   memo[n] = fib(n - 1, memo) + fib(n - 2, memo);  
6   return memo[n];  
7 };
```

memo

```
{  
  3: 2,  
  4: 3,  
  5: 5,  
  6: 8  
}
```



$O(2N) \sim O(N)$

# Dynamic Programming

Solving Big Problems by Reusing Small Ones

```
public long dynamicProgramming(int n) { 7 usages new *
    if (n <= 2) return 1;

    long[] dp = new long[n + 1];
    dp[1]=1;
    dp[2]=1;

    for (int i = 3; i <= n; i++) {
        dp[i] = dp[i-1] + dp[i-2];
    }

    return dp[n];
}

public long dynamicProgrammingWithSlidingWindow(int n) { 7 usages new *
    if (n <= 2) return 1;
    long n1 = 1L, n2 = 1L;

    for (int i = 3; i <= n; i++) {
        long temp = n1;
        n1 += n2;
        n2 = temp;
    }

    return n1;
}
```



# Problems

## Description:

Given a natural number  $n \leq 100$ . How many ways are there to partition  $n$  into a sum of positive integers? Permutations of each partition are considered the same way.

## Example:

If  $n = 4$ , there are 5 ways. Notice that  $3 + 1$  is the same as  $1 + 3$

- 4
- 3+1
- 2+2
- 2+1+1
- 1+1+1+1

# Brute-force

```
public long bruteForce(int n) { 4 usages new *
    if (n == 0) {
        return 1;
    }

    return bruteForce(n, n);
}

public long bruteForce(int n, int maxVal) { 3 usages new *
    if (n == 0) return 1;

    if (n < 0) return 0;

    if (maxVal == 0) {
        return 0;
    }

    long waysIncludingMaxVal = bruteForce(n: n - maxVal, maxVal);
    long waysExcludeMaxVal = bruteForce(n, maxVal: maxVal - 1);

    return waysIncludingMaxVal + waysExcludeMaxVal;
}
```

# Brute-force With Memo

```
public long bruteForceWithMemo(int n, int maxVal, Map<String, Long> memo) { 3 usages new *
    recursiveCount++;
    if (n == 0) return 1;

    if (n < 0) return 0;

    if (maxVal == 0) {
        return 0;
    }

    String memoKey = String.format("%d-%d", n, maxVal);
    if (memo.containsKey(memoKey)) {
        duplicateBranchCutted++;
        return memo.get(memoKey);
    }

    long waysIncludingMaxVal = bruteForceWithMemo(n - maxVal, maxVal, memo);
    long waysExcludeMaxVal = bruteForceWithMemo(n, maxVal - 1, memo);

    memo.put(memoKey, waysIncludingMaxVal + waysExcludeMaxVal);

    return waysIncludingMaxVal + waysExcludeMaxVal;
}
```



# Dynamic Programming

## With Tabulation

```
public long dynamicProgrammingWithTabulation(int n) { 4 usages new *
    long[][] tabulation = new long[n + 1][n + 1];
    tabulation[0][0] = 1;

    for (int i = 0; i <= n; i++) {
        for (int j = 0; j <= n; j++) {
            long waysIncludingMaxVal = (i - j < 0) ? 0 : tabulation[i - j][j];
            long waysExcludeMaxVal = (j - 1 < 0) ? 0 : tabulation[i][j - 1];
            tabulation[i][j] = waysIncludingMaxVal + waysExcludeMaxVal;
        }
    }

    return tabulation[n][n];
}
```

Evaluate expression (↵) or add a watch (⌘⌘↵)

```
> this = {AnalyticNumber@2166}
Ⓟ n = 5
v tabulation = {long[6][]@2165}
> 0 = {long[6]@2167} [1, 1, 1, 1, 1, 1] ... View
> 1 = {long[6]@2170} [0, 1, 1, 1, 1, 1] ... View
> 2 = {long[6]@2171} [0, 1, 2, 2, 2, 2] ... View
> 3 = {long[6]@2172} [0, 1, 2, 3, 3, 3] ... View
> 4 = {long[6]@2173} [0, 1, 3, 4, 5, 5] ... View
> 5 = {long[6]@2174} [0, 1, 3, 5, 6, 7] ... View
10
01 i = 5
> ∞ tabulation[0] = {long[6]@2167} [1, 1, 1, 1, 1, 1] ... View
∞ tabulation[0][0] = 1
```

# Dynamic Programming

From small to big

```
public long dynamicProgrammingWithTabulation(int n) { 4 usages new *
    long[][] tabulation = new long[n + 1][n + 1];
    tabulation[0][0] = 1;

    for (int i = 0; i <= n; i++) {
        for (int j = 0; j <= n; j++) {
            long waysIncludingMaxVal = (i - j < 0) ? 0 : tabulation[i - j][j];
            long waysExcludeMaxVal = (j - 1 < 0) ? 0 : tabulation[i][j - 1];
            tabulation[i][j] = waysIncludingMaxVal + waysExcludeMaxVal;
        }
    }

    return tabulation[n][n];
}
```

Start from smallest  
Scale to destination,  
Caching the result to  
tabulation

# Brute force

From big to small

```
public long bruteForce(int n) { 4 usages new *
    if (n == 0) {
        return 1;
    }

    return bruteForce(n, n);
}

public long bruteForce(int n, int maxVal) { 3 usages new *
    if (n == 0) return 1;

    if (n < 0) return 0;

    if (maxVal == 0) {
        return 0;
    }

    long waysIncludingMaxVal = bruteForce(n - maxVal, maxVal);
    long waysExcludeMaxVal = bruteForce(n, maxVal - 1);

    return waysIncludingMaxVal + waysExcludeMaxVal;
}
```

Start from destination  
Breakdown to smallest  
and return.  
Wasted time on rework

# Run Code

Run test and compare the run times



# Dynamic Programming

With Tabulation (Space complexity is  $O(n^2)$  - 2D)

```
public long dynamicProgrammingWithTabulation(int n) { 4 usages new *
    long[][] tabulation = new long[n + 1][n + 1];
    tabulation[0][0] = 1;

    for (int i = 0; i <= n; i++) {
        for (int j = 0; j <= n; j++) {
            long waysIncludingMaxVal = (i - j < 0) ? 0 : tabulation[i - j][j];
            long waysExcludeMaxVal = (j - 1 < 0) ? 0 : tabulation[i][j - 1];
            tabulation[i][j] = waysIncludingMaxVal + waysExcludeMaxVal;
        }
    }

    return tabulation[n][n];
}
```

Evaluate expression (🔗) or add a watch (🔗🔗)

```
> this = {AnalyticNumber@2166}
Ⓟ n = 5
v tabulation = {long[6][]@2165}
> 0 = {long[6]@2167} [1, 1, 1, 1, 1] ... View
> 1 = {long[6]@2170} [0, 1, 1, 1, 1] ... View
> 2 = {long[6]@2171} [0, 1, 2, 2, 2] ... View
> 3 = {long[6]@2172} [0, 1, 2, 3, 3] ... View
> 4 = {long[6]@2173} [0, 1, 3, 4, 5] ... View
> 5 = {long[6]@2174} [0, 1, 3, 5, 6, 7] ... View
10 01 i = 5
> ∞ tabulation[0] = {long[6]@2167} [1, 1, 1, 1, 1] ... View
> ∞ tabulation[0][0] = 1
```

Can we somehow convert 2D to 1D -  $O(n)$  for space complexity?

# Dynamic Programming

## Summary

1. How to define state: What parameters define a subproblem?
2. How to handle base case (smallest case)
3. How to write transition function
4. How to reduce space complexity (Use 1D instead of 2D)

# Learn more about DP

<https://youtu.be/oBt53YbR9Kk?si=07UnEOktFISjLKVV>



# Thank you

It is not the end, It's just the beginning