

# Homework 2

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## Section 2.1

### Exercise 11

Determine whether each of these statements is true or false.

- (a)  $0 \in \emptyset$
- (b)  $\emptyset \in \{0\}$
- (c)  $\{0\} \subset \emptyset$
- (d)  $\emptyset \subset \{0\}$
- (e)  $\{0\} \in \{0\}$
- (f)  $\{0\} \subset \{0\}$
- (g)  $\{\emptyset\} \subseteq \{\emptyset\}$

### Solution

- (a)  $0 \in \emptyset$ : This statement is false because  $\emptyset$  has no elements so 0 can not be an element of the empty set.
- (b)  $\emptyset \in \{0\}$ : This statement is false because  $\emptyset$  is not an element in set  $\{0\}$ .
- (c)  $\{0\} \subset \emptyset$ : This statement is false because  $\emptyset$  has no elements so that  $\{0\}$  can not be a subset of  $\emptyset$ .
- (d)  $\emptyset \subset \{0\}$ : This statement is true because  $\emptyset$  is one of the two sets that every nonempty set is guaranteed to have.
- (e)  $\{0\} \in \{0\}$ : This statement is false because  $\{0\}$  is an element of  $\{\{0\}\}$  not an element of  $\{0\}$ .
- (f)  $\{0\} \subset \{0\}$ : This statement is false because the two set all have the same elements 0 so that is must be  $\subseteq$ .
- (g)  $\{\emptyset\} \subseteq \{\emptyset\}$ : This statement is true because both singleton set have the same element  $\emptyset$ . Therefore, this statement is true.

## Exercise 12

Determine whether these statements are true or false.

- (a)  $\emptyset \in \{\emptyset\}$
- (b)  $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- (c)  $\{\emptyset\} \in \{\emptyset\}$
- (d)  $\{\emptyset\} \in \{\{\emptyset\}\}$
- (e)  $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (f)  $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
- (g)  $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

## Solution

- (a)  $\emptyset \in \{\emptyset\}$ : This statement is true because  $\emptyset$  is an element of a singleton set contains element  $\emptyset$ .
- (b)  $\emptyset \in \{\emptyset, \{\emptyset\}\}$ : This statement is true because  $\emptyset$  is an element of the set  $\{\emptyset, \{\emptyset\}\}$ .
- (c)  $\{\emptyset\} \in \{\emptyset\}$ : This statement is false because  $\emptyset$  must be an element of  $\{\{\emptyset\}\}$ .
- (d)  $\{\emptyset\} \in \{\{\emptyset\}\}$ : This statement is true because the set  $\{\{\emptyset\}\}$  contains  $\{\emptyset\}$ .
- (e)  $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ : This statement is true because  $\emptyset$  is an element is the set  $\{\emptyset, \{\emptyset\}\}$  so the set contains  $\emptyset$  is a subset of  $\{\emptyset, \{\emptyset\}\}$ .
- (f)  $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ : This statement is true and its reason is the same as problem (e).
- (g)  $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ : We can see that the set  $\{\{\emptyset\}, \{\emptyset\}\}$  has two elements which are equal to each other. Therefore, we can simplify it to  $\{\{\emptyset\}\}$ . Therefore, this statement is false because these sets are equal to each other so it must be  $\subseteq$  instead of  $\subset$ .

## Exercise 13

Determine whether each of these statements is true or false.

- (a)  $x \in \{x\}$
- (b)  $\{x\} \subseteq \{x\}$
- (c)  $\{x\} \in \{x\}$
- (d)  $\{x\} \in \{\{x\}\}$
- (e)  $\emptyset \subseteq \{x\}$
- (f)  $\emptyset \in \{x\}$

**Solution**

- (a)  $x \in \{x\}$ : This statement is true because  $x$  is an element in set  $x$ .
- (b)  $\{x\} \subseteq \{x\}$ : This statement is true.
- (c)  $\{x\} \in \{x\}$ : This statement is false because  $x$  is an element of  $\{\{x\}\}$  not  $\{x\}$ .
- (d)  $\{x\} \in \{\{x\}\}$ : This statement is true due to the reason from problem(c).
- (e)  $\emptyset \subseteq \{x\}$ : This statement is true according to **Theorem 1**.
- (f)  $\emptyset \in \{x\}$ : This statement is false because  $\emptyset$  is not an element of set  $\{x\}$ .

**Exercise 26**

Determine whether each of these sets is the power set of a set, where  $a$  and  $b$  are distinct elements.

- 1.  $\emptyset$
- 2.  $\{\emptyset, \{a\}\}$
- 3.  $\{\emptyset, \{a\}, \{\emptyset, a\}\}$
- 4.  $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

**Solution**