Ch. 4, Sec. 6: Optimization Problems

1. **Quote.**

> *"The more wit we have, the less satisfied we are with it."*
>
> — Jean le Rond d'Alembert.

2. **Learning Objectives.**

3. **Procedure. Steps in Solving Optimization Problems**

   (a) Understand the problem.

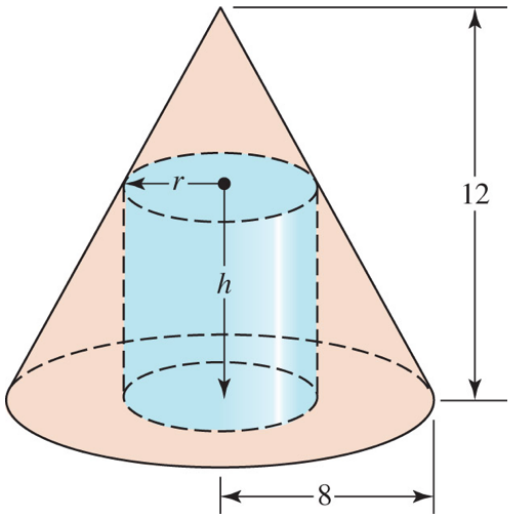
   (b) Identify the target quantity you are trying to optimize. Eliminate as needed to make it univariate, and write its domain.

   (c) Use the Closed Interval Method (perhaps modified) to find the maximum or minimum.

4. **Reminder.** The Closed Interval Method.

5. **Square and rectangles.** Find the maximum possible area of a rectangle of perimeter P.

6. **Shape optimization.** Find the dimensions of the right circular cylinder with greatest volume that can be inscribed in a right circular cone of radius 8 cm and height 12 cm.

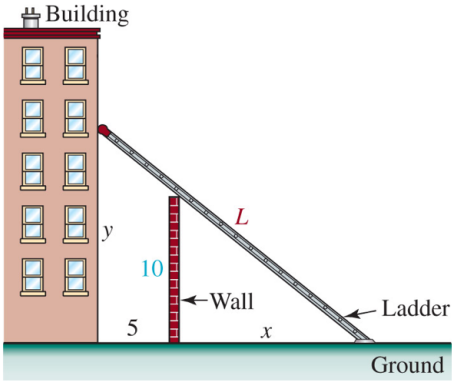


7. **Finding range.** A 10-ft wall stands 5 ft from a building and a ladder of variable length L, supported by the wall, is placed so it reaches from the ground to the building. Let y denote the vertical distance from the ground to where the tip of the ladder touches the building, and let x denote the horizontal distance from the wall to the base of the ladder.

   (a) Find an expression for the height y as a function of x.

   (b) Find an expression for the length L as a function of x.

   (c) Determine the domain and range of the function $L(x)$ found in part (b).



8. **Dimensional analysis.** A farmer wants to hire workers to pick 900 bushels of beans. Each worker can pick 5 bushels per hour and is paid $\$1.00$ per bushel. The farmer must also pay a supervisor $\$10$ per hour while the picking is in progress, and he has additional miscellaneous expenses of $\$8$ per worker. How many workers should he hire to minimize the total cost? What will be the cost per bushel picked?

9. **I'm on a boat.** Maya is 2 km offshore on a boat and wishes to reach a coastal village which is 6 km down the straight shoreline form the point on the shore nearest to the boat. She can row at 2 km/hour and run 5 km/hour. Where she should land her boat to reach the village in the least amount of time?

10. **Regiomontanus' angle maximization problem.** A painting is hung on a wall in such a way that its upper and lower edges are 10 ft and 7 ft above the floor. An observer whose eyes are 5 ft above the floor stands x feet away from the wall. How far away from the wall should she stand to maximize the angle θ subtended by the painting? Note that this angle is the best viewing-angle.