Ch. 4, Sec. 1: Related Rates

1. **Quote.**

   "I have never met a man so ignorant that I couldn't learn something from him."

   — Galileo Galilei.

2. **Learning Objectives.**

3. **Procedure. The Method of Related Rates**

When two variables are related by an equation and both are functions of a third variable (such as time), we can find a relation between their rates of change. In this case, we say the rates are related, and we can compute one if we know the other.

We proceed as follows:

(a) Identify the independent variable (usually time) on which the other quantities depend and assign it a symbol, such as t. Also, assign symbols to the variable quantities that depend on t.

   i. Translate all known / given information into the math world. That is, use variables and equations to express what is known. Identify the target you are trying to compute (it could be a variable, a function, a derivative, or something else).

(b) Find an equation that relates the dependent variables.

(c) Differentiate both sides of the equation with respect to t (using the chain rule if necessary).

(d) Substitute the given information into the related rates equation and solve for the unknown rate.

4. **Example.** Suppose $x^2 + y^2 = 25$, where x and y are functions of t.

   (a) When $y = 4$, what is x?

   (b) Suppose $dy/dt = 6$, find dx/dt at the points found above.

5. **Example.** A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant. Hint: The solution is 2cm/s.

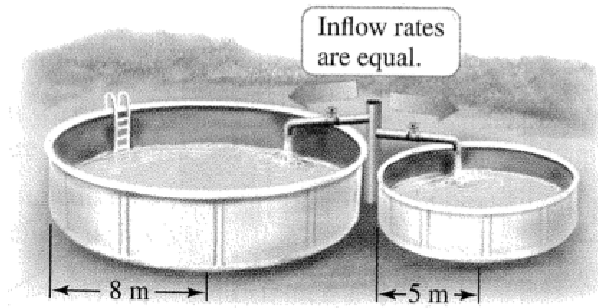
6. **Example.** A large spherical balloon is being inflated. The volume of the balloon is increasing at the rate of 0.2 m^3/s when the radius is 5 m. At what rate is the radius of the balloon increasing at that moment?

7. **Example.** Betty and Robert meet at a gas station. Betty then drives a racecar due North at a constant speed of 100km/h, while Robert cycles due East at a constant 8km/h. At what speed is the area of the triangle (found by connecting Betty, Robert, and the gas station) increasing when Betty is 25km from the gas station?

8. **Example.** A rocket is launched vertically and is tracked by a radar station located on the ground 5 km from the launch pad. Suppose that the elevation angle θ of the line of sight to the rocket is increasing at 3° per second when $\theta = 60^\circ$. What is the velocity of the rocket at that instant?

9. **Homework.** A man 6 ft tall walks with a speed of 8 ft/s away from a street light that is atop an 18-ft pole. How fast is the tip of his shadow moving along the ground when he is 100 ft from the light pole? Hint: The solution is 12ft/s.

10. **Homework.** Two cylindrical swimming pools are being filled simultaneously at the same rate (in m^3/min, see figure). The smaller pool has a radius of 5 m and the water rises at a rate of 0.5 m/min. The larger pool has a radius of 8 m.



(a) At what rate is water flowing into the smaller pool?

(b) At what rate is the water level rising in the larger pool?

Hint: The solutions are (a) $25\pi/2$ m^3/min and (b) 25/128 m/min.

11. **Example.** I'm dog-sitting Taro the shiba inu and Lucky the pomeranian. I'm holding them both to the ground, when suddenly Taro runs due north at 5 m/s, while Lucky runs due east at 12 m/s (they often run away from me). Two seconds later, at what rate is the distance between Taro and Lucky increasing? In your solution, define your variables properly, and include a labelled sketch.

1. **Homework.** A man 6 ft tall walks with a speed of 8 ft/s away from a street light that is atop an 18-ft pole. How fast is the tip of his shadow moving along the ground when he is 100 ft from the light pole? Hint: The solution is 12ft/s.

2. **Homework.** A particle moves along the curve $y = \sqrt{x}$. As the particle passes through the point $(9, 3)$, its y-coordinate decreases at a rate of 2 cm/s. How fast is the distance from the particle to the origin changing at this instant?

3. **Homework.** A kite 30 m above the ground moves horizontally at a speed of 1 m/s. At what rate is the angle between the string and the horizontal decreasing when 100 m of string has been released?

4. **Homework.** A plane flying with a constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later?

5. **Homework.** Consider a baseball diamond where the bases are a length ℓ apart. A player is sprinting from home plate to first base at 20km/h. If the player is $\frac{\ell}{4}$ away from home plate, at what rate is the distance between the player and second base changing? Hint: The solution is -12km/h.