Ch. 1, Sec. 1: Four Ways To Represent a Function

1. **Quote.**

   "Logic is the foundation of the certainty of all the knowledge we acquire."

   — Leonhard Euler.

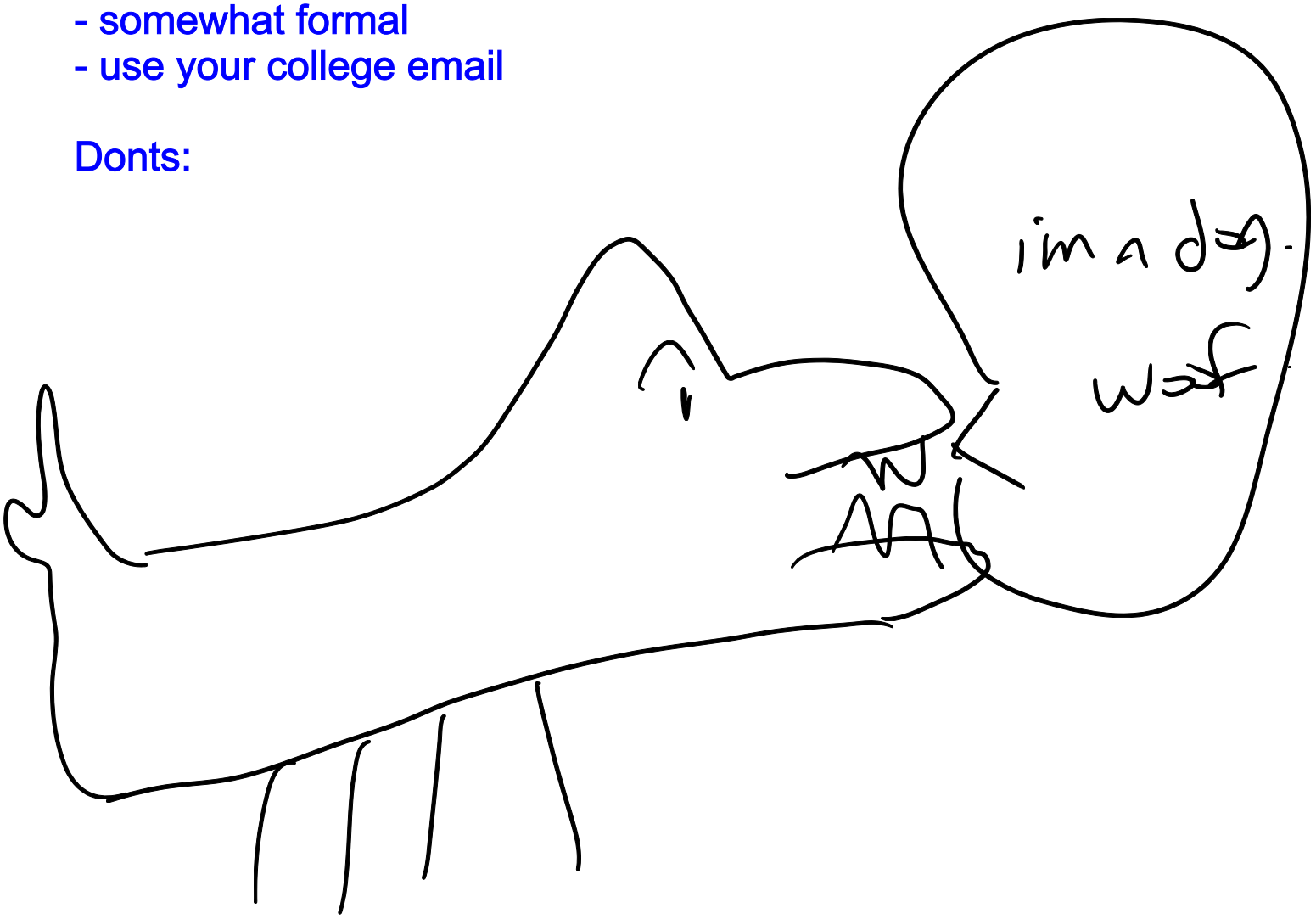
2. **Learning Objectives.**

Emails dos and do nots

Dos:

- formal greeting salutation
  Hello Jeremy
  To whom it may concern
  Dear Dr__
- course number / and section
- subject: start with coursenumber (section) subject
- signature identifiable name / other form of contact / dept or degree you're in enrolled in / langara's logo
- optional: preferred gender pronoun
- somewhat formal
- use your college email

Donts:



3. **Discussion.** State some words related to *functions*.

domain
range
x-axis
y-axis
relations
variables
one-to-one
onto
vertical line test
composite
inverse
logarithmics
reciprocal
slope
exponential
tangent line
average
limits
secant line
asymptotes
derivative
integral
differentiate
trigonometric
extreme value
application

what is the def. of a function?

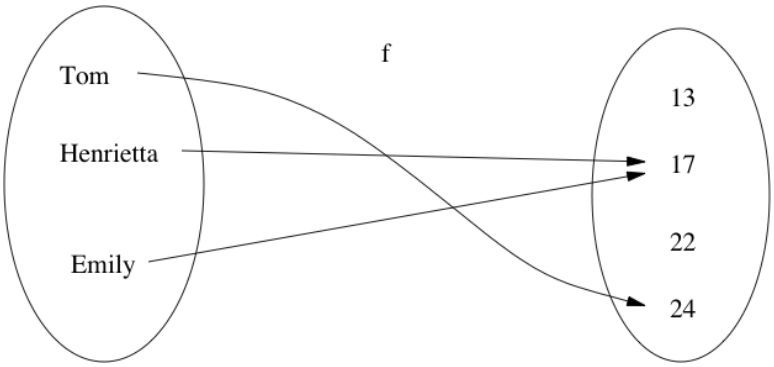
A function is a relationship that that has a unique output

are the two quantites related?
is there a unique output?

4. **Definition.** (Informal) A **function** is a relationship where for every input, there is a unique output.

5. **Definition.** (Formal) A **function** f is a rule that assigns to each element x in a set D exactly one element, called $y = f(x)$, in a set R.

6. **Example.** The following function maps each person to their age. What is its domain and range?



input: person

output: range

dom = { Tom, Hen., Em. }

range = { 13, 17, 22, 24 }

{ 17, 24 }.

Range: given a function and its domain, the range is the set of values actually attained

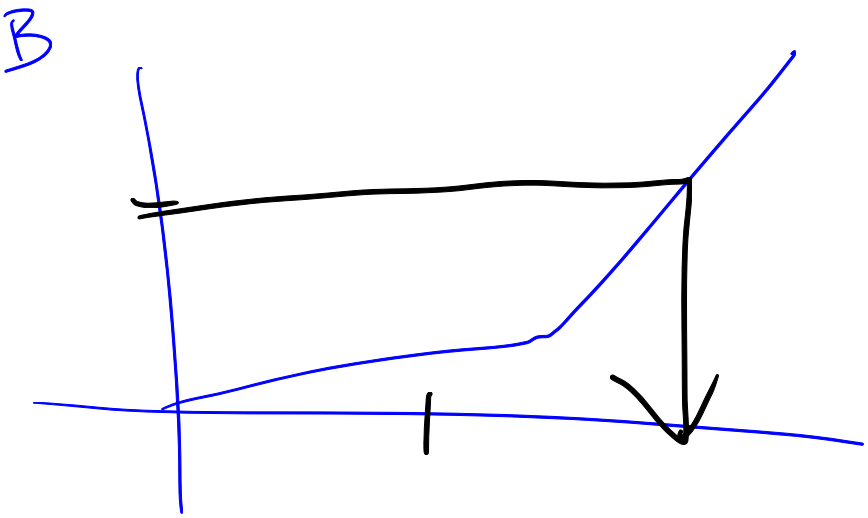
y vs x

7. **Examples.** Investigate how the following two quantities are related.

   (a) Energy bill vs energy consumption – https://energyrates.ca/british-columbia/explaining-your-british-columbia-electric

   (b) Revenue vs price.

a) let B: energy bill $
   C: energy consumption kwh



Is B a func. of C?
1) related? yes
2) unique output?
yes.
⇒ B = f(C)
f: C → B
f kwh → $

Is C also a function of B?
related? yes
unique output? yes
C is a function of B

in real world, does B depend on C or does C depend on B?

B depends on C

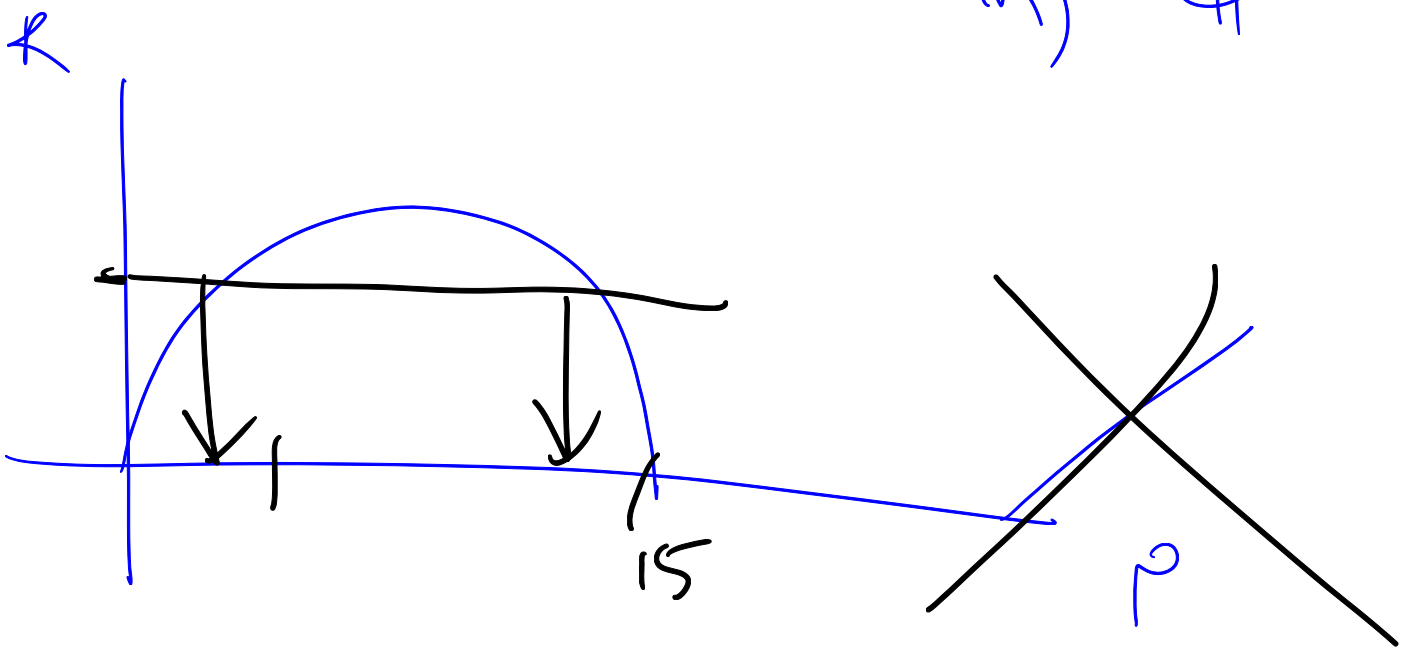
(C doesn't really depwend on B)

Revenue vs price

Let

p: price of coffee $\frac{\$}{\text{item}}$

R: avg revenue (in a month) $\$$



Is R a function of p?
1) related? yes
2) unique output?

given a p, is there only one R? yes $\Rightarrow R = g(p)$

$g: p \to R$

is p a function of R?
related? yes
unique output? No

if $R=0$
$p=0$ $p=15$

$g: \frac{\$}{\text{item}} \to \$$

even though p is related to R, p is NOT a function of R.

given one R, there are multiple corresponding prices

8. **Examples.** Consider the following two quantities. Which is a function of which?

   (a) A student at Langara college and their student number.

   (b) An email address and its user.

   (c) A Pokémon and it's trainer.

y is a function of x . y=f(X)

a) functions of one another

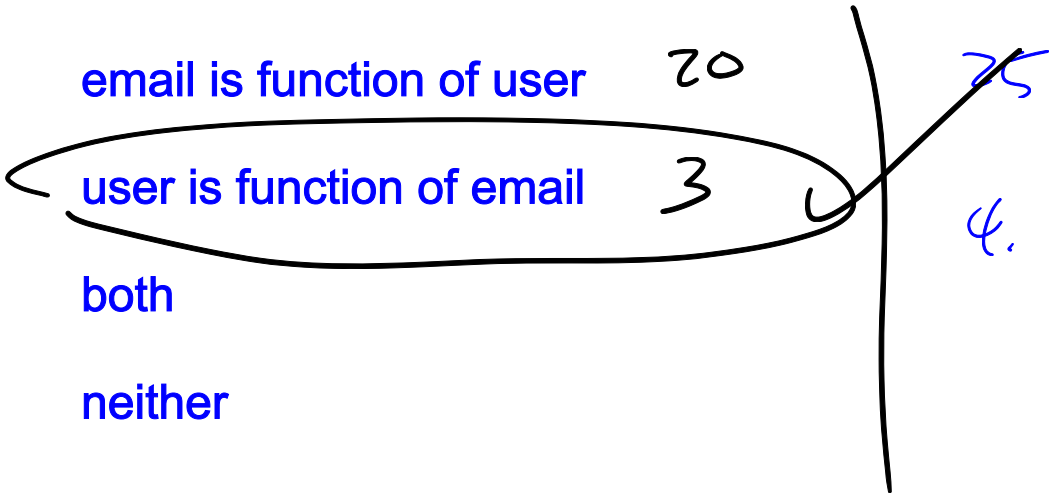
b)

email is function of user

user is function of email

both

neither



input user output. @gmail

· Spam @gmail.com

· @outlook.c

input personal email output. one user

Why study functions?

we can learn relations between two quantities

applicaitons

If i open a coffee shop, i cna't set revenue at start of month.
but i can set price,
with the function, i can now control revenue at start of month by controlling price

area = side length ^2,

^ ^ ^ examples of the power of RELATIONS, not necessarily functions

unique output is for OUR CONVENIENCE.

when there are multiple output, we need to decide which one to use. we don't want to deal with this

9. **Remark.** Functions can be described in many ways. Here are four important way:
   (a) verbally
   (b) algebraically
   (c) visually (a graph)
   (d) numerically (a table of values)

10. **Example.** The area of a circle A is a function of its radius r. Describe this function in the four ways.

a) area is pi * radius squared

b) $A = \pi r^2$

c) 

d)

r	1	2	3
A	π	$\pi \cdot 2^2$	$\pi \cdot 3^2$

11. **Reminder.** To find the domain of a function, be a pessimist. In mathematics, what causes *bad things* to happen?

i. $\div 0$

ii. $\sqrt{\text{neg \#}}$

iii. log non-pos.

negative numbers do not include 0
non-pos numbers include 0

iv. physical

12. **Homework.** Find the domains of the following functions.

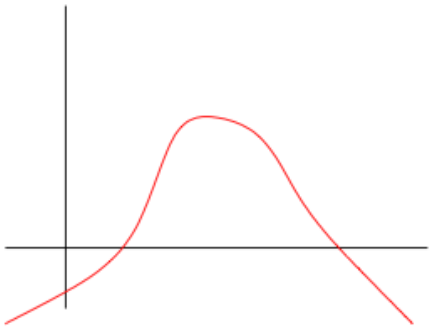
(a) $f(x) = \sqrt{1 - 5x}$

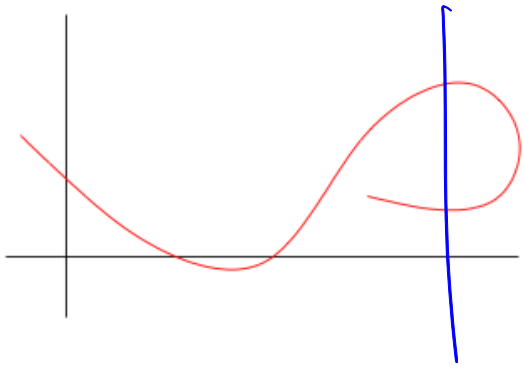
(b) $g_1(x) = \dfrac{1}{x^2 - x}$

(c) $g_2(x) = \dfrac{\sqrt{x}}{x - 3}$

13. **Theorem.** A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

14. **Example.** Which curve is the graph of a function?





fails VLT.

15. **Questions.**

(a) Is $y = x^2$ a function? yes

(b) Is $y = \sqrt{x}$ a function?

yes 15 4

no 7 15

what is 3 squared? 9

what is square root of 9? +3 or -3

$y = \sqrt{x}$ this radical notation is defined to be the positive square root

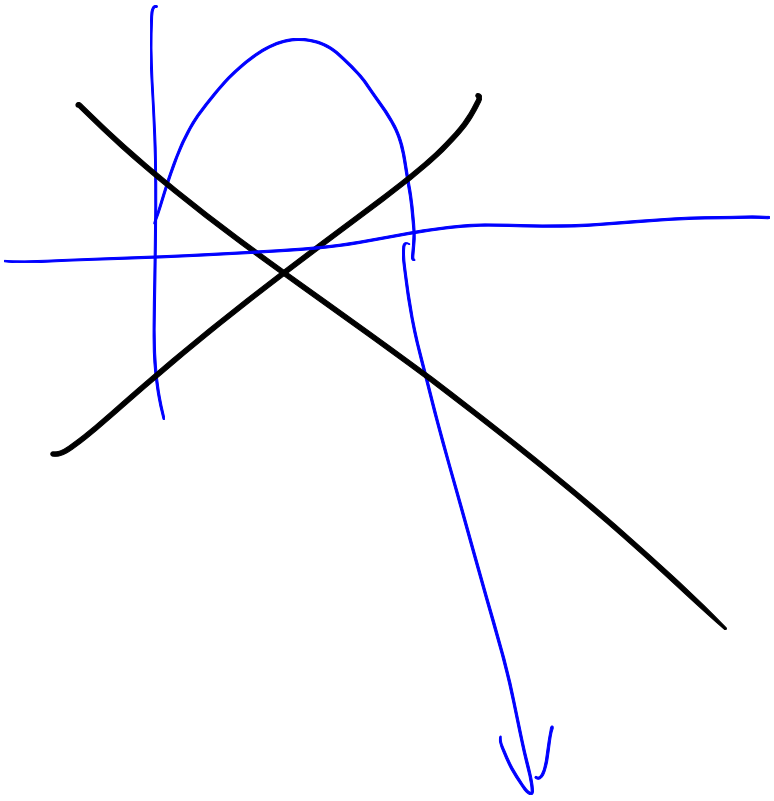
- square root of 16, ± 4

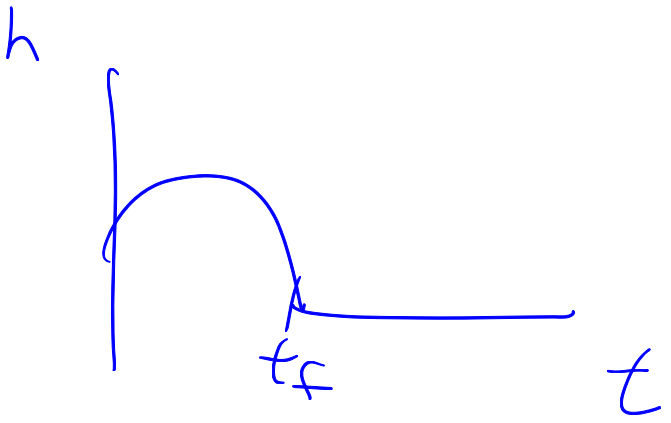
- $\sqrt{16} = 4$

16. **Motivating problem.** **Piecewise defined functions** (or **piecewise functions**). Suppose you launch a small rocket from the ground. It goes up, then comes back down. Discuss what is a function of what, and sketch its graph.

Let h: height of rocket

t: time after launch.





to describe h as a function of t, it depends on the domain we're in

if $t \le t_f$ $h = +(t_f - t)$

if $t > t_f$, $h = 0$

$$h = \begin{cases} t(t_f - t) & \text{if } t \leq t_f \\ 0 & \text{if } t > t_f \end{cases}$$

a piecewise function is a function that changes behaviour depending on the domain you're in

17. **Definition.** A **piecewise defined function** (or **piecewise function**) is a function that behaves differently depending on the domain.

18. **Remark.** The first thing you should ask yourself whenever approaching piece-wise functions:

    which domain are we in

19. **Notation.** How to read piece-wise defined functions:

20. **Example.** Compute the following function values, where

$$f(x) = \begin{cases} x+5 & \text{if} \quad x \leq -3 \\ 1 & \text{if} \quad -3 < x \leq 2 \\ x^2 & \text{if} \quad x > 2 \end{cases}$$

Then sketch the graph of $y = f(x)$.

(a) $f(-4)$

(b) $f(-3)$

(c) $f(0)$

(d) $f(2)$

(e) $f(3)$

21. **Example.** **Human age vs dog age.** The widely held belief "each dog year is seven human years" turns out to be inaccurate. The American Veterinary Medical Association (AVMA) presents a different model (which I've simplified a bit):

    (a) The first two years of a dog's life is equivalent to 24 human years.

    (b) Subsequently, each human year is equivalent to 5 dog years.

    Describe AVMA's model by representing a dog's age D as a piecewise function of a human's age H.

    Hint. Sketch a graph.

22. **Definition.** A function f is called **increasing** on an interval I if for any a or b in I,

$$f(a) < f(b) \text{ whenever } a < b.$$

It is **decreasing** on I if

$$f(a) > f(b) \text{ whenever } a < b.$$