Ch. 4, Sec. 8: Antiderivatives

1. **Quote.**

> *"Mathematical analysis compares the most diverse phenomena and discovers the secret analogies that unite them."*
>
> — Joseph Fourier.

2. **Learning Objectives.**

3. **Motivating problem.** Consider a function $f(x)$.

   (a) What equation does its derivative $f'(x)$ satisfy?

   (b) Suppose $F(x)$ is an *inverse derivative* of $f(x)$. What equation might $F(x)$ satisfy?

4. **Definition.** A function F is called an **antiderivative** of f on an interval I if

$$F'(x) = f(x)$$

for all $x \in I$.

5. **Example.** Find an antiderivative of $f(x) = x^2$.

6. **Example.** Find the most general antiderivative of $f(x) = x^n$.

7. **Theorem.** If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

8. **Theorem.** **(Simple) Inverse Chain Rule.**

Let $F(x)$ be an antiderivative of $f(x)$. Then $\dfrac{F(cx)}{c}$ is an antiderivative of $f(cx)$.

9. **Theorem. Antiderivative Formulas.**

Suppose $G(x)$ and $H(x)$ are antiderivatives of $g(x)$ and $h(x)$, respectively.

function	**antiderivative**	**function**	**antiderivative**
$cg(x)$	$cG(x)$	$\sin x$	$-\cos x$
$g(x)+h(x)$	$G(x)+H(x)$	$\sec^2 x$	$\tan x$
$x^n, n \neq -1$	$\frac{1}{n+1}x^{n+1}$	$\sec x \tan x$	$\sec x$
$1/x$	$\ln \lvert x \rvert$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\frac{1}{1+x^2}$	$\tan^{-1} x$
$\cos x$	$\sin x$		

10. **Example.** Find all antiderivatives of the following functions.

(a) $f(x) = x^3 + 3\sqrt{x} - \dfrac{4}{x^2}$

(b) $g(x) = (x+3)(x^2-5)$

(c) $h(x) = \dfrac{x^3 - \sqrt{x} + 7}{x}$

11. **Example.**

(a) Find f if $f'(t) = 2\cos(3t) + 5\sin(4t) + 3e^t$.

(b) Which of the functions in (a) satisfies $f(0) = 0$?

12. **Example.** Find f such that $f''(x) = 2e^{-2x} + 5$, $f(0) = 3/2$ and $f'(0) = 1$.

13. **Reminder. Rectilinear Motion.**

    (a) Position function - $s = f(t)$

    (b) Velocity function - $v(t) = s'(t)$

    (c) Acceleration function - $a(t) = v'(t)$

14. **Example.** A particle moves with acceleration function

$$a(t) = 5 + 4t - 2t^2.$$

    Its inital velocity is $v(0) = 3$ m/s and its initial displacement is $s(0) = 10$m. Find its position after t seconds.

15. **Example.** A car is travelling at 80 km/h when the brakes are fully applied, producing a constant deceleration of 7m/s^2. What is the distance traveled before the car comes to a stop?

1. **Homework.** The skid marks made by an automobile indicate that its brakes are fully applied for a distance of 50m before it came to a stop. Suppose that the car in question has a constant deceleration of 6 m/s^2 under the condition of the skid. How fast was the car travelling when its brakes were applied?

2. **Homework.** Consider $f(x) = \dfrac{1}{x}$.

   (a) Find a particular antiderivative of $f(x)$. Hint: Consider cases for $x > 0$ and $x < 0$.

   (b) Find the general antiderivative of $f(x)$.