Supplement – Determining the Sign of a Function

1. **Example.** Suppose we have the following graph of $f(x)$. Determine
   (a) where $f(x) > 0$.
   (b) where $f(x) \leq 0$.

2. To determine the sign of a function $f(x)$, use the following steps.

   (a) Determine the domain of $f(x)$.

   (b) Find the roots and the points of discontinuity of $f(x)$. Split the domain into open subintervals at these points.

   (c) **Sign-Line:** In each subinterval, determine if $f(x)$ is positive or negative by using a sample point. That is, pick a number $x = a$ in that subinterval, and if $f(a) > 0$, then $f(x)$ is positive on that subinterval; if $f(a) < 0$, then $f(x)$ is negative on that subinterval.

      - **Sign-Chart:** If $f(x)$ has many pieces, you may consider the following longer method:
        Write $f(x)$ as a product and quotient of subfunctions. Draw a table, and determine if each subfunction is positive or negative in each subinterval by evaluating the subfunction at a sample point in the subinterval.

   (d) Conclude if $f(x)$ is positive or negative in each subinterval, while worrying much about the end points which must be handled case-by-case.

3. **Question:** Why can we use a single point to determine sign on an entire subinterval?

4. **Example.** State where $h(x) > 0$ and $h(x) < 0$, where

$$h(x) = \frac{x^2 - 1}{x^2 + 4x - 5}$$

5. **Example.** State where $g(x) \geq 0$ and where $g(x) \leq 0$, given

$$g(x) = e^x \ln\left(\frac{7}{2} + x\right) \frac{x^2 + 2x - 3}{x^2 - 4x + 3}$$