Ch. 1, Sec. 1: Four Ways To Represent a Function

1. **Quote.**

   "Logic is the foundation of the certainty of all the knowledge we acquire."

   — Leonhard Euler.

2. **Learning Objectives.**

3. **Discussion.** State some words related to *functions*.

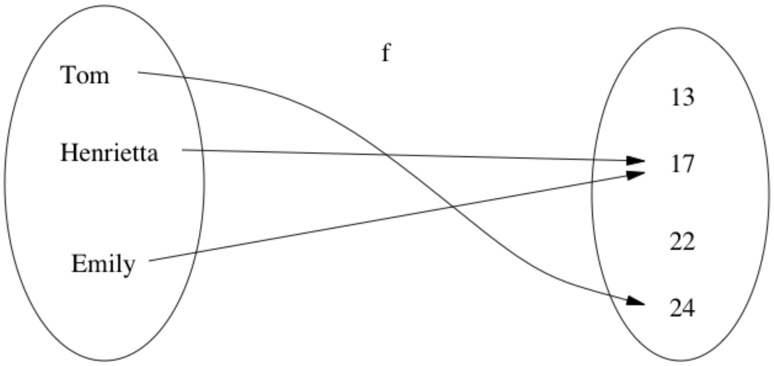
4. **Definition.** (Informal) A **function** is a relationship where for every input, there is a unique output.

5. **Definition.** (Formal) A **function** f is a rule that assigns to each element x in a set D exactly one element, called $y = f(x)$, in a set R.

6. **Example.** The following function maps each person to their age. What is its domain and range?

want: dom

range



domain = { Tom, Hen., Em. }

range = { 17, 24 }

7. **Examples.** Investigate how the following two quantities are related.

   (a) **Energy bill vs energy consumption** – https://energyrates.ca/british-columbia/explaining-your-british-columbia-electric

   (b) **Revenue vs price.**

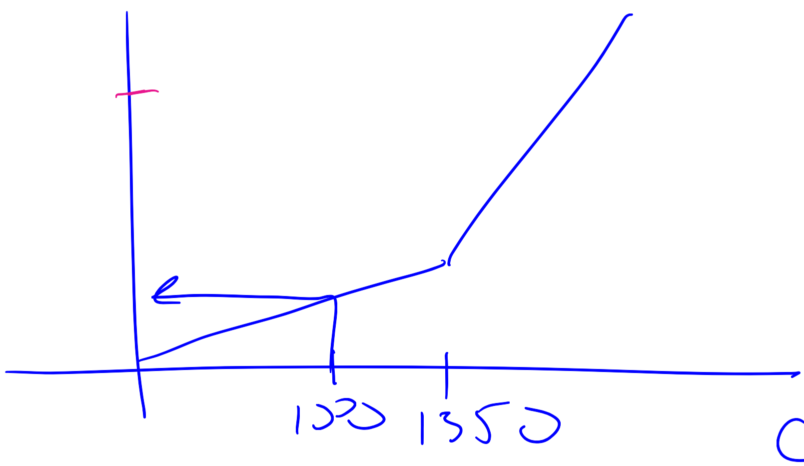
y vs x

Given:

B: energy bill [$]

C: " consumption [kWh]

B: $



C: kWh

Is C a func of B?

related? yes

uniq. output? yes

yes, $C = h(B)$

function:

- relationship ✓
- unique output ✓

$\Rightarrow$ $B = f(C)$

$f: C \to B$ $\qquad$ $f: \text{kWh} \to \$$

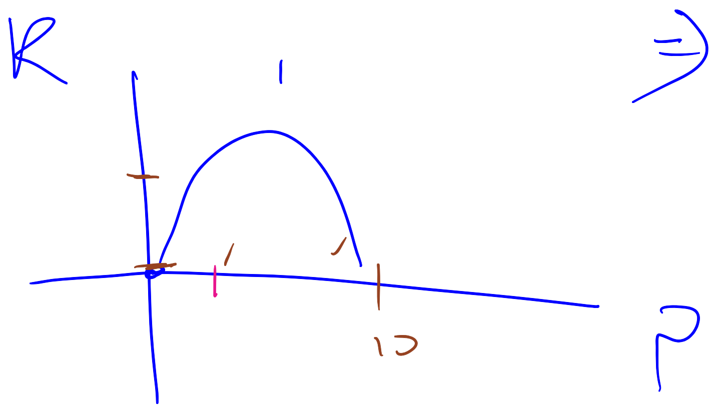
Revenue vs Price

Given

- R: average revenue $\$$
- p: price $\frac{\$}{\text{item}}$

related? yes

unique output? yes.



$\Rightarrow$ $R = g(p)$

$g: p \to R$

$g: \frac{\$}{\text{item}} \longrightarrow \$$

Is p a func of R? No!

related? yes

unique output? No!

if $R=0$, $p=0$ or $p=10$

8. **Examples.** Consider the following two quantities. Which is a function of which?

   (a) A student at Langara college and their student number.

   (b) An email address and its user.

   (c) A Pokémon and it's trainer.

· input: user output email addresses

email address not func. of user

· user is " " email address

c) input: ~~Charmander~~ Charizard output Ash

input: Ash output Butterfree/ Charizard/ Pikachu/ Taurus

trainer is func of Pokémon

y is func of x

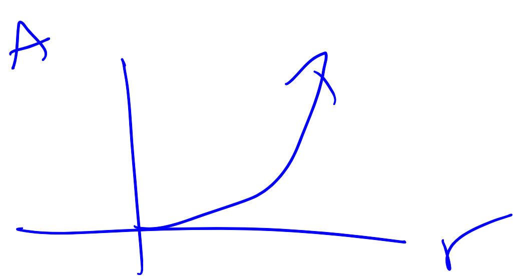
9. **Remark.** Functions can be described in many ways. Here are four important way:
   (a) verbally
   (b) algebraically
   (c) visually (a graph)
   (d) numerically (a table of values)

10. **Example.** The area of a circle A is a function of its radius r. Describe this function in the four ways.

b) $A = \pi r^2$

a) area is π times radius squared

c)



d)

r	1	2	3
A	π	4π	9π

11. **Reminder.** To find the domain of a function, be a pessimist. In mathematics, what causes *bad things* to happen?

i. $\div 0$

ii. $\sqrt{}$ neg. #

iii. log non-pos.

iv. physical constants

12. **Homework.** Find the domains of the following functions.

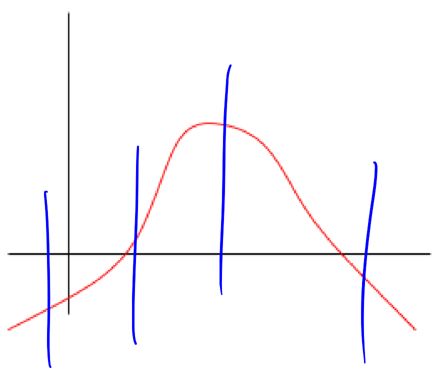
(a) $f(x) = \sqrt{1-5x}$

(b) $g_1(x) = \dfrac{1}{x^2 - x}$

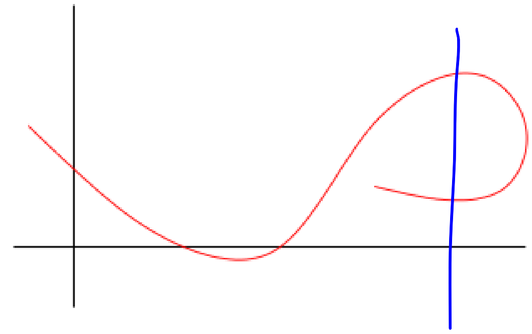
(c) $g_2(x) = \dfrac{\sqrt{x}}{x-3}$

13. **Theorem.** A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

14. **Example.** Which curve is the graph of a function?



passes VLT



fails VLT

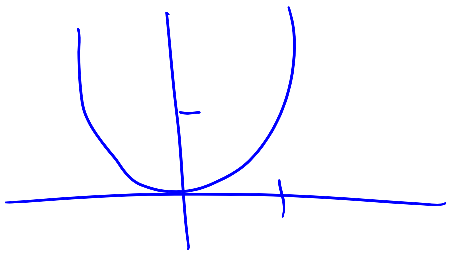
y not be func. x

15. **Questions.**

(a) Is $y = x^2$ a function? yes

(b) Is $y = \sqrt{x}$ a function?

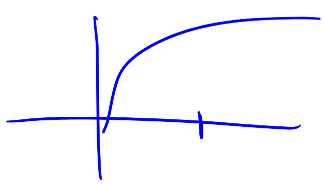
a)



b)

- $\sqrt{4} = 2$ $\sqrt{x}$: positive square root of x.
- what is sq. root of 4? ± 2
- $\sqrt{4} = 2$

$y = \sqrt{x}$



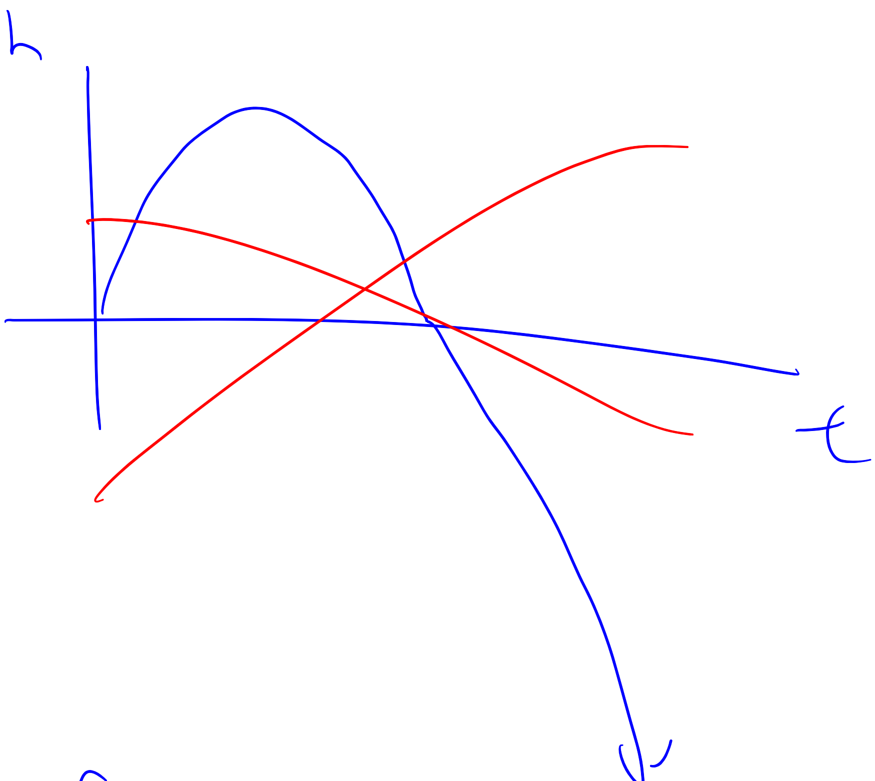
16. **Motivating problem.** **Piecewise defined functions** (or **piecewise functions**). Suppose you launch a small rocket from the ground. It goes up, then comes back down. Discuss what is a function of what, and sketch its graph.

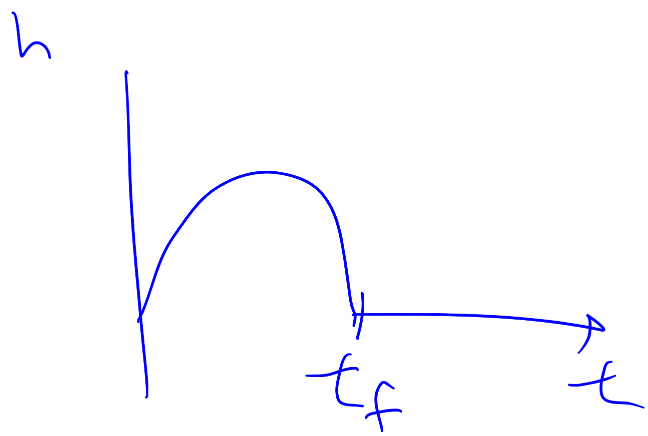
Let

h: height of rocket

t: launch time.

$h = f(t)$





if $t \le t_f$,

$h = t \cdot (1 - t_f)$

if $t \ge t_f$

$h = 0$

$$h = f(t) = \begin{cases} t(1-t_f) & \text{if } t \leq t_f \\ 0 & \text{if } t > t_f \end{cases}$$

17. **Definition.** A **piecewise defined function** (or **piecewise function**) is a function that behaves differently depending on the domain.

18. **Remark.** The first thing you should ask yourself whenever approaching piece-wise functions:

which domain are we in

19. **Notation.** How to read piece-wise defined functions:

$$h = f(t) = \begin{cases} h = t(1-t_f) & \text{if } t \le t_f \\ h = 0 & \text{if } t > t_f \end{cases}$$

20. **Example.** Compute the following function values, where

$$f(x) = \begin{cases} x+5 & \text{if} \quad x \le -3 \\ 1 & \text{if} \quad -3 < x \le 2 \\ x^2 & \text{if} \quad x > 2 \end{cases}$$

1 2 3

Then sketch the graph of $y = f(x)$.

(a) $\boxed{f(-4)}$ $= (-4)+5 = 1$

(b) $f(-3)$

(c) $f(0)$

(d) $f(2)$

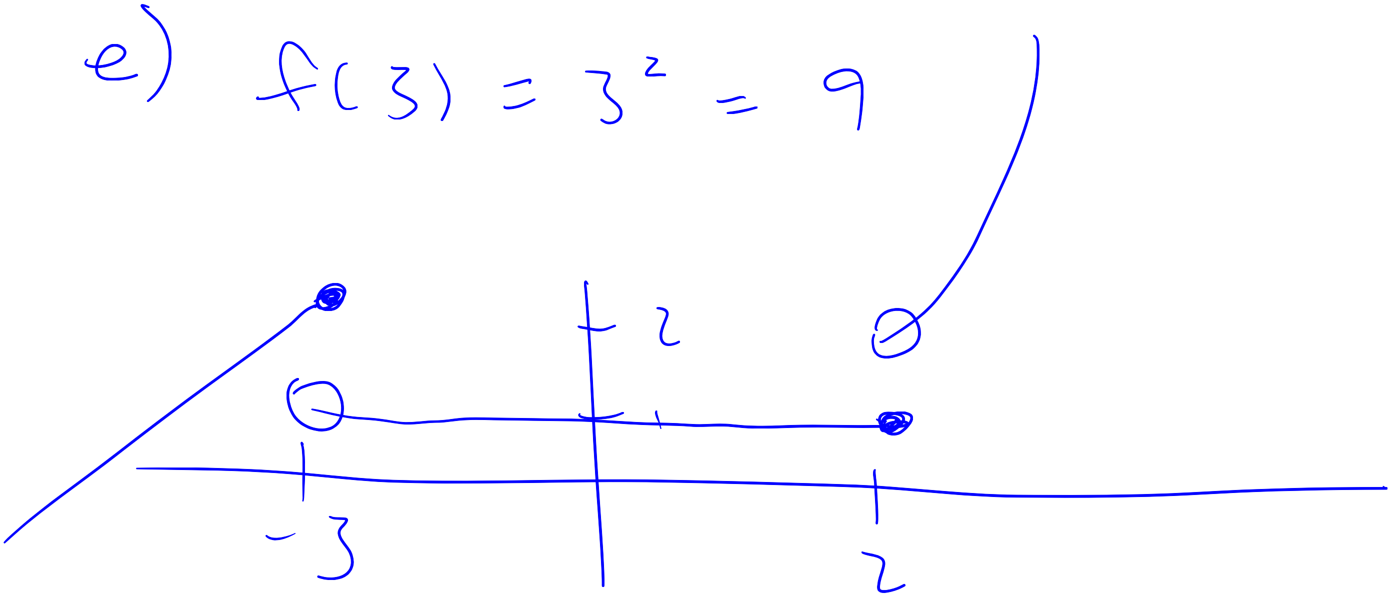
(e) $f(3)$

b) $f(-3) = -3+5 = 2$

c) $f(0) = 1$

d) $f(2) = 1$

e) $f(3) = 3^2 = 9$



21. **Example. Human age vs dog age.** The widely held belief "each dog year is seven human years" turns out to be inaccurate. The American Veterinary Medical Association (AVMA) presents a different model (which I've simplified a bit):

(a) The first two years of a dog's life is equivalent to 24 human years.

(b) Subsequently, each human year is equivalent to 5 dog years.

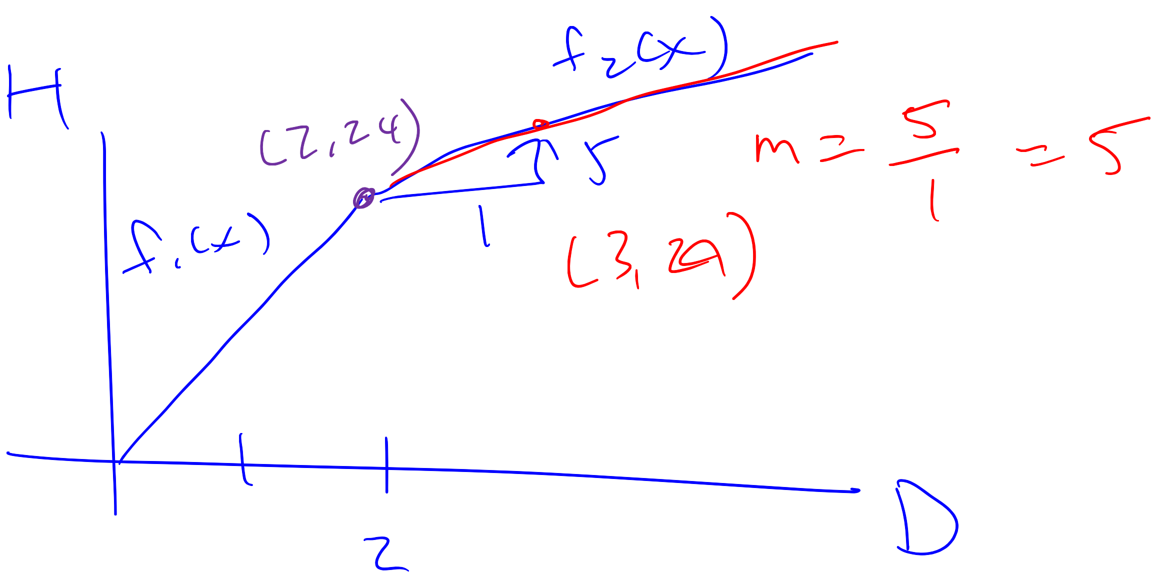
Describe AVMA's model by representing a human's age H as a piecewise function of a dog's age D.

Hint. Sketch a graph.

want $\quad H = f(D)$

Given

- D: dog's age
- H: human equivalent



$$H = f(D) = \begin{cases} f_1(x) & \text{if } 0 \le D \le 2 \\ f_2(x) & \text{if } D > 2 \end{cases}$$

notice $f_1(x)$ is a linear func.

recall, def of lin. func

$$y = m(x - x_1) + y_1$$

$x_1 = 2, \quad y_1 = 24$

$$m = \frac{24}{2} = 12$$

$\Rightarrow$

$$f_1(x) = y = 12(x - 2) + 24$$

$$f_1(D) = H = 12 \cdot (D - 2) + 24$$

$$f_2(x) = y = m(x - x_2) + y_2$$

$$= 5 \cdot (x - 3) + 29$$

$$H = \begin{cases} 12(D-2)+24 & \text{if } D \le 2 \\ 5 \cdot (D-3) + 29 & \text{if } D > 2 \end{cases}$$

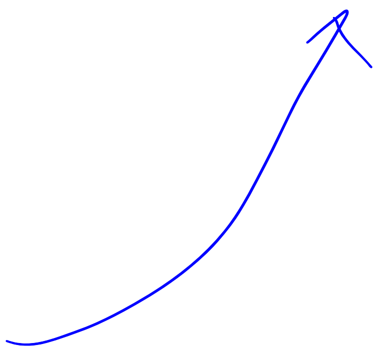
22. **Definition.** A function f is called **increasing** on an interval I if for any a or b in I,

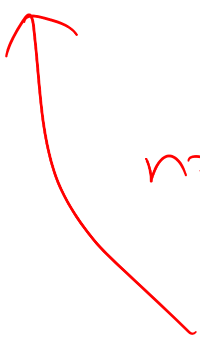
$$f(a) < f(b) \text{ whenever } a < b.$$

It is **decreasing** on I if

$$f(a) > f(b) \text{ whenever } a < b.$$

if $a < b$, $f(a) < f(b)$





not incr.