Ch. 4, Sec. 7: Newton's Method

1. **Quote.**

> *"Tact is the knack of making a point without making an enemy."*
>
> — Sir Isaac Newton.

2. **Learning Objectives.**

3. **Motivating problem.** Solve $5x + \cos x = 5$.

4. **Definition.** Suppose we want to find the roots of $f(x)$. Most methods fall into one of two categories:

   (a) **Direct method:** Solve directly.

   (b) **Iterative method:** Begin with a guess, and step-by-step get better approximations.

5. **Motivation.** Suppose we want to find the roots of $f(x)$ and are given an estimate x_1. Use a linear approximation to estimate another root.

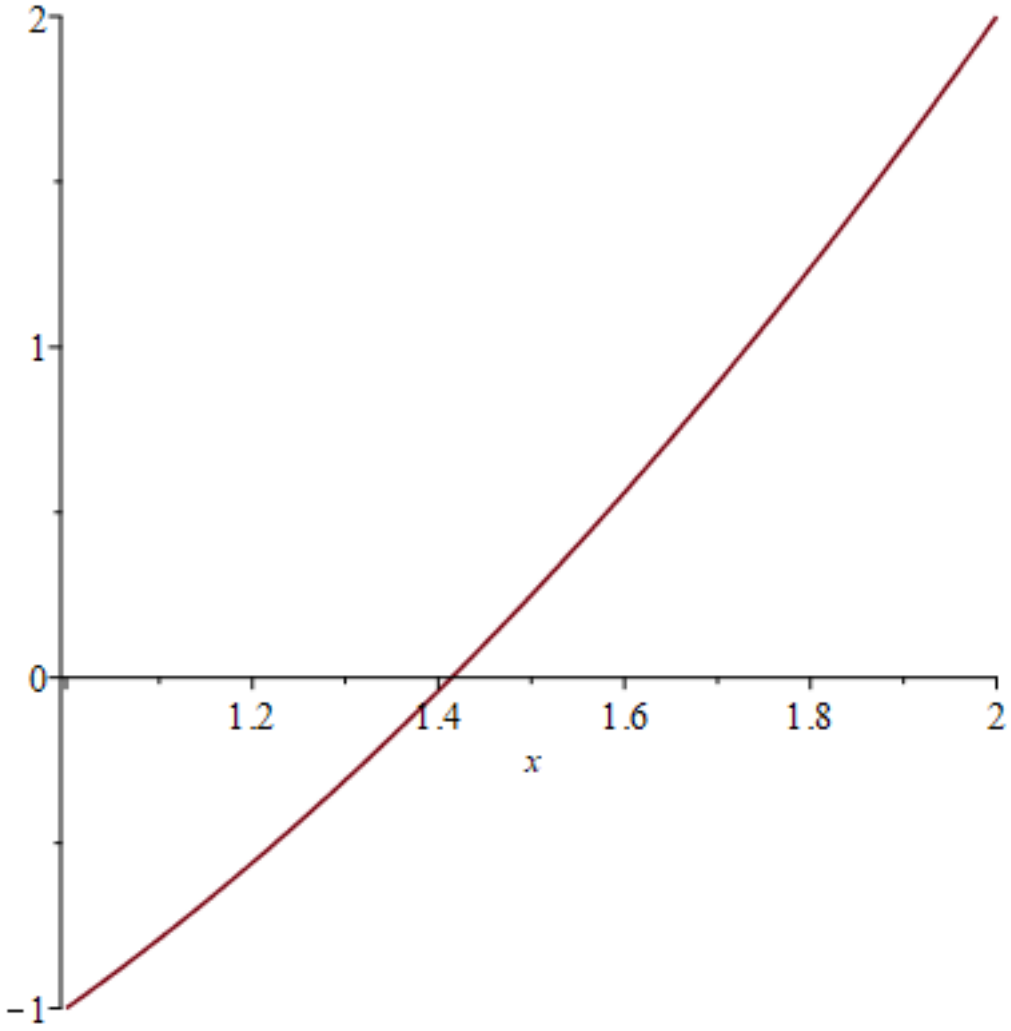
6. **Procedure. Newton's Method** or **Newton-Raphson Method.**

   0. Suppose you're after a number r. Choose $f(x)$ so that $f(r) = 0$.
   i. Find a "good" **initial guess** x_1.
   ii. Find the simplify the **update formula**,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

   iii. **Iterate** by successively applying the update formula.
   iv. **Terminate:** If x_n and $x_n + 1$ agree to k decimal places then x_n approximates the root r up to k decimal places and $f(x_n) \approx 0$.

7. **Example.** Graphically illustrate Newton's Method on the following function's graph.



8. **Example.** Set up Newton's method to solve

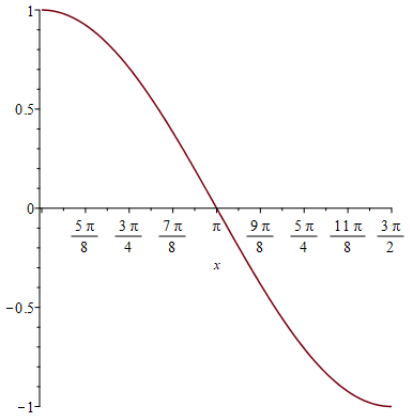
$$5x + \cos x = 5$$

for $x \in [0, 1]$. State a reasonable x_0, and write x_1 in calculator-ready form, and write x_2 in terms of x_1.

9. **Example.** Use Newton's method to find $\sqrt{2}$ accurate to two decimal places.

10. **Example.** Use Newton's method to solve $x^{1/3} = 0$ by taking $x_0 = 1$.

11. **Example.** Use Newton's method to approximate π.



12. **Example.** Let $f(x) = x^3 + 3x + 1$.

    (a) Show that f has at least one root in the interval $(-1/2, 0)$. Explain.

    (b) Use Newton's method to approximate the root that lies in the interval $(-1/2, 0)$. Stop when the next iteration agrees with the previous one at two decimal places.