Ch. 1, Sec. 1: Four Ways To Represent a Function

1. **Quote.**

   "Logic is the foundation of the certainty of all the knowledge we acquire."

   — Leonhard Euler.

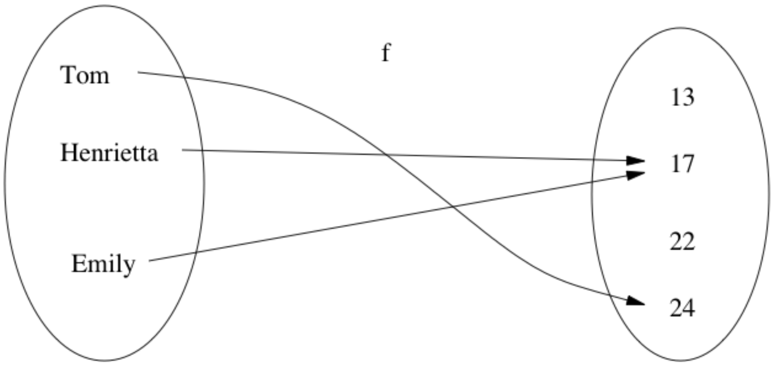
2. **Learning Objectives.**

3. **Discussion.** State some words related to *functions*.

4. **Definition.** (Informal) A **function** is a relationship where for every input, there is a unique output.

5. **Definition.** (Formal) A **function** f is a rule that assigns to each element x in a set D exactly one element, called $y = f(x)$, in a set R.

6. **Example.** The following function maps each person to their age. What is its domain and range?



want: dom

range

domain = { Tom, Hen., Em }

range = { 17, 24 }

7. **Examples.** Investigate how the following two quantities are related.

   (a) Energy bill vs energy consumption – https://energyrates.ca/british-columbia/explaining-your-british-columbia-electric

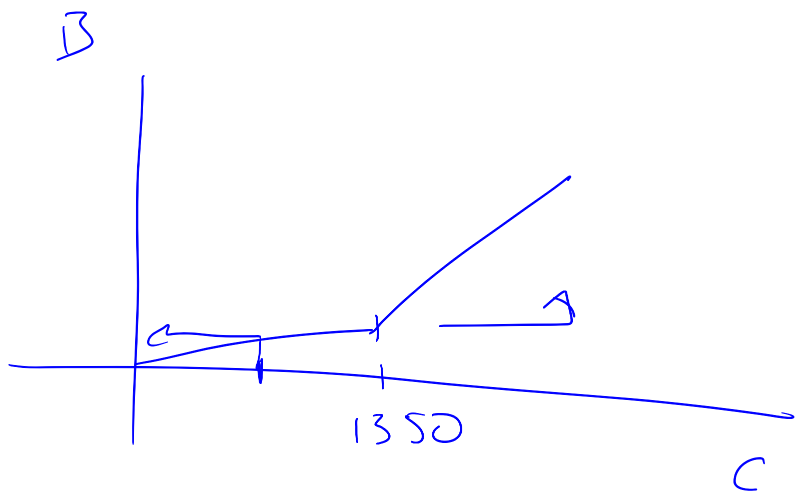
   (b) Revenue vs price.

y vs x

Given

B: energy bill [$]

C: " consumption [kWh]



function

- relationship
- unique output

$\Rightarrow B = f(C)$

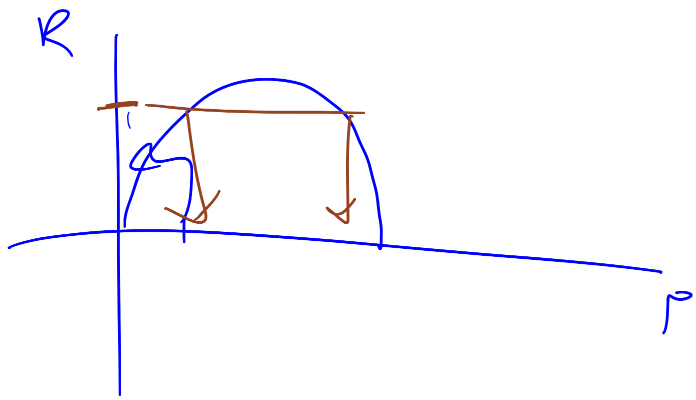
$f: C \to B$ $\quad$ $f: kWh \to \$$

Revenue vs price

Let

R: avg. revenue $

p: price $



function:

· related?

· unique output? ✓

$R = g(p)$ $g: p \to R,$ $g: \$ \to \$$

Question: is p a func of R?

related? yes

unique output? No!

8. **Examples.** Consider the following two quantities. Which is a function of which?

   (a) A student at Langara college and their student number.

   (b) An email address and its user.

   (c) A Pokémon and it's trainer.

9. **Remark.** Functions can be described in many ways. Here are four important way:

   (a) verbally

   (b) algebraically

   (c) visually (a graph)

   (d) numerically (a table of values)

10. **Example.** The area of a circle A is a function of its radius r. Describe this function in the four ways.

11. **Reminder.** To find the domain of a function, be a pessimist. In mathematics, what causes *bad things* to happen?

   i.

   ii.

   iii.

   iv.

12. **Homework.** Find the domains of the following functions.

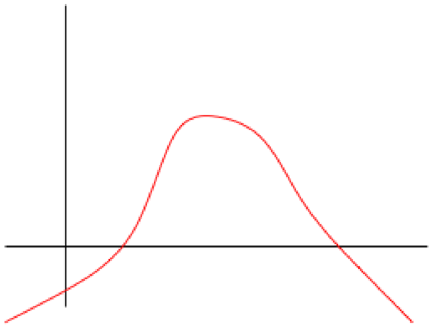
   (a) $f(x) = \sqrt{1-5x}$

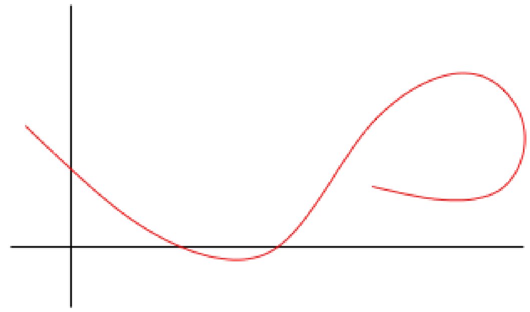
   (b) $g_1(x) = \dfrac{1}{x^2 - x}$

   (c) $g_2(x) = \dfrac{\sqrt{x}}{x-3}$

13. **Theorem.** A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

14. **Example.** Which curve is the graph of a function?





15. **Questions.**

(a) Is $y = x^2$ a function?

(b) Is $y = \sqrt{x}$ a function?

16. **Motivating problem.** **Piecewise defined functions** (or **piecewise functions**). Suppose you launch a small rocket from the ground. It goes up, then comes back down. Discuss what is a function of what, and sketch its graph.

17. **Definition.** A **piecewise defined function** (or **piecewise function**) is a function that behaves differently depending on the domain.

18. **Remark.** The first thing you should ask yourself whenever approaching piece-wise functions:

19. **Notation.** How to read piece-wise defined functions:

20. **Example.** Compute the following function values, where

$$f(x) = \begin{cases} x+5 & \text{if} \quad x \leq -3 \\ 1 & \text{if} \quad -3 < x \leq 2 \\ x^2 & \text{if} \quad x > 2 \end{cases}$$

Then sketch the graph of $y = f(x)$.

(a) $f(-4)$

(b) $f(-3)$

(c) $f(0)$

(d) $f(2)$

(e) $f(3)$

21. **Example.** **Human age vs dog age.** The widely held belief "each dog year is seven human years" turns out to be inaccurate. The American Veterinary Medical Association (AVMA) presents a different model (which I've simplified a bit):

    (a) The first two years of a dog's life is equivalent to 24 human years.

    (b) Subsequently, each human year is equivalent to 5 dog years.

    Describe AVMA's model by representing a dog's age D as a piecewise function of a human's age H.

    Hint. Sketch a graph.

22. **Definition.** A function f is called **increasing** on an interval I if for any a or b in I,

$$f(a) < f(b) \text{ whenever } a < b.$$

It is **decreasing** on I if

$$f(a) > f(b) \text{ whenever } a < b.$$