

Ch. 1, Sec. 1: Four Ways To Represent a Function

1. Quote.

“Logic is the foundation of the certainty of all the knowledge we acquire.”

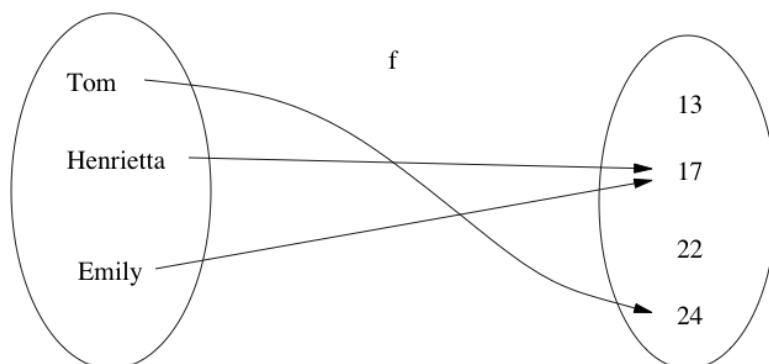
— Leonhard Euler.

2. Learning Objectives.

3. **Discussion.** State some words related to *functions*.

- 3

6. **Example.** The following function maps each person to their age. What is its domain and range?



7. **Examples.** Investigate how the following two quantities are related.

(a) **Energy bill vs energy consumption** – <https://energyrates.ca/british-columbia/explaining-your-british-columbia-electric>

(b) **Revenue vs price.**

8. **Examples.** Consider the following two quantities. Which is a function of which?
- (a) A student at Langara college and their student number.
 - (b) An email address and its user.
 - (c) A Pokémon and its trainer.

9. **Remark.** Functions can be described in many ways. Here are four important way:

- (a) verbally
- (b) algebraically
- (c) visually (a graph)
- (d) numerically (a table of values)

10. **Example.** The area of a circle A is a function of its radius r . Describe this function in the four ways.

11. **Reminder.** To find the domain of a function, be a pessimist. In mathematics, what causes *bad things* to happen?

i.

ii.

iii.

iv.

12. **Homework.** Find the domains of the following functions.

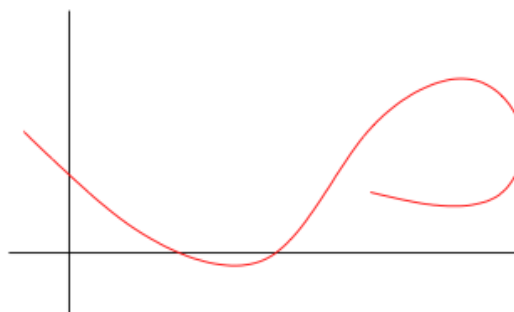
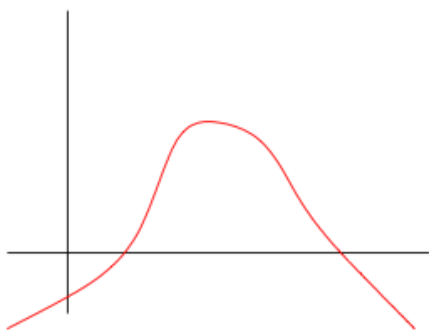
(a) $f(x) = \sqrt{1 - 5x}$

(b) $g_1(x) = \frac{1}{x^2 - x}$

(c) $g_2(x) = \frac{\sqrt{x}}{x - 3}$

13. **Theorem.** A curve in the xy -plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

14. **Example.** Which curve is the graph of a function?



15. **Questions.**

- (a) Is $y = x^2$ a function?
- (b) Is $y = \sqrt{x}$ a function?

16. **Motivating problem. Piecewise defined functions** (or **piecewise functions**). Suppose you launch a small rocket from the ground. It goes up, then comes back down. Discuss what is a function of what, and sketch its graph.

17. **Definition.** A **piecewise defined function** (or **piecewise function**) is a function that behaves differently depending on the domain.

18. **Remark.** The first thing you should ask yourself whenever approaching piece-wise functions:

19. **Notation.** How to read piece-wise defined functions:

20. **Example.** Compute the following function values, where

$$f(x) = \begin{cases} x + 5 & \text{if } x \leq -3 \\ 1 & \text{if } -3 < x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$$

Then sketch the graph of $y = f(x)$.

(a) $f(-4)$

(b) $f(-3)$

(c) $f(0)$

(d) $f(2)$

(e) $f(3)$

21. **Example. Human age vs dog age.** The widely held belief “each dog year is seven human years” turns out to be inaccurate. The American Veterinary Medical Association (AVMA) presents a different model (which I’ve simplified a bit):

- (a) The first two years of a dog’s life is equivalent to 24 human years.
- (b) Subsequently, each human year is equivalent to 5 dog years.

Describe AVMA’s model by representing a dog’s age D as a piecewise function of a human’s age H .

Hint. Sketch a graph.

22. **Definition.** A function f is called **increasing** on an interval I if for any a or b in I ,

$$f(a) < f(b) \text{ whenever } a < b.$$

It is **decreasing** on I if

$$f(a) > f(b) \text{ whenever } a < b.$$