

Ch. 3, Sec. 7: Derivatives of Logarithms and Logarithmic Differentiation

1. Quote.

“Divide each difficulty into as many parts as is feasible and necessary to resolve it.”

— René Descartes.

2. Learning Objectives.

3. Theorem.

$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

4. Corollary.

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

5. **Examples.** Differentiate the following.

(a) $y = \log_2(3x^2 + e^x)$

(b) $y = \ln(x + \sqrt{x^2 - 1})$

6. **Examples.** Differentiate the following.

(a) $y = \sqrt{\ln x}$

(b) $y = \ln \sqrt{x}$

7. **Example.** Differentiate $y = \ln \left(\frac{x^2}{(x+3)^4} \right)$.

8. **Examples.** Differentiate

$$(a) \ y = \frac{\sqrt[4]{x^3} \sqrt[5]{x^3 + 1}}{(2x + 1)^3}$$

$$(b) \ y = x^{x^2}$$

$$(c) \ y = \ln |x|$$

1. **Homework.** Use logarithmic differentiation to find y' .

(a) $y = \frac{x^2 \sqrt{x+2}}{\sqrt[5]{\cos x}}$

(b) $y = \frac{x^3 \sqrt[3]{x-5} \cos x}{(2x+7)^5 (x-1)}$

2. **Homework.** Differentiate

(a) $y = x^{\cos x}$

(b) $y = (\tan x)^{\ln x}$

3. **Homework.** If $f(x) = \ln x$, find a formula for $f^{(k)}(x)$, the k th derivative of $f(x)$.