

Ch. 2, Sec. 1: The Tangent and Velocity Problems

1. Quote.

“I can calculate the motion of heavenly bodies, but not the madness of people.”

— Sir Isaac Newton.

2. Learning Objectives.

3. **Motivating problem.** Taro the (useless) Shiba Inu is running around on a field. I'm playing "fetch" with her, but she doesn't return the ball, and so I need to go catch her. At any given point, determine Taro's direction of travel.



4. **Problem. The tangent problem.** Our goal is to find an equation of the tangent line ℓ to a curve with equation $y = f(x)$ at a given point P . Three questions:

- (a) What is the tangent line ℓ to a curve with equation $y = f(x)$ at a given point P ?
- (b) If a curve with equation $y = f(x)$ and a point P on the curve are given, does the tangent ℓ exist?
- (c) If a curve with equation $y = f(x)$ and a point $P = (x_0, f(x_0))$ are given and if the tangent line ℓ exists then an equation of ℓ is given by

$$y - f(x_0) = m(x - x_0) .$$

How to calculate the slope m ?

5. **Example.** Find the slopes of the secant lines to the parabola $y = x^2$ through the points $(1, 1)$ and:

(a) $x = 2$

(b) $x = 1.5$

(c) $x = 1.1$

(d) $x = 1.001$

(e) $x = a$

6. **BIG QUESTION.** What if the second point is **VERY, VERY** close to the point $(1, 1)$?

7. **Definition.** Given a displacement of an object, it's **average velocity** is given by

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

8. **Example.** The position of the car is given by the values in the table.

t	0	1	2	3	4	5
s	0	10	32	70	119	178

where t is in seconds and s is in feet.

Find the average velocity for the time beginning when $t = 2$ and lasting

(a) 3 seconds

(b) 2 seconds

(c) 1 second

9. **Example.** Revisit the previous question by graphing s vs t , and interpret the average velocities graphically.

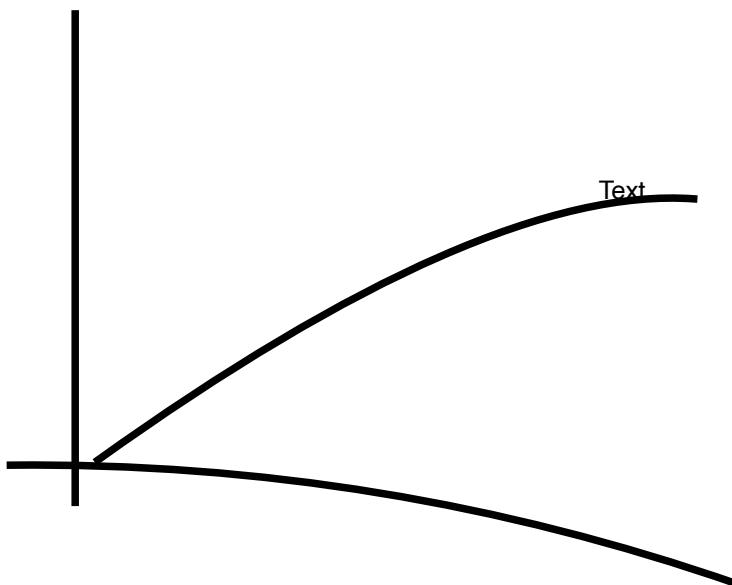
t	0	1	2	3	4	5
s	0	10	32	70	119	178

10. **Homework.** Recall the data from above.

t	0	1	2	3	4	5
s	0	10	32	70	119	178

Compute the average velocity starting at $t = 1$ and lasting...

- (a) 3 seconds
- (b) 2 seconds
- (c) 1 second



11. **Homework.** A rock travels up and down on a vertical line, and its displacement can be described by $s(t) = -16t^2 + 96t$.

- (a) What is its domain?
- (b) Compute the rock's average velocity over the intervals $[1, 3]$, $[1, 2]$, $[1, 1.1]$, and $[1, 1.01]$.
- (c) Use your result(s) from part (b) to estimate the instantaneous velocity at $t = 1$.