# 3.7 Supplement – Exponential and Logarithmic Functions

1. **Question.** What is an exponential function?

2. **Example.** Use a table of values to sketch $y = 2^x$.

3. **Definition.** A function of the form $f(x) = b^x$ where $b$ is a fixed real number, $b > 0$, $b \neq 1$ is called a **base *b* exponential function**. Its domain is $\mathbb{R}$, and its range is $(0, \infty)$.

4. **Theorem – Properties of Exponential Functions.** Consider an exponential function $f(x) = b^x$.

   Suppose $f(x) = b^x$.

   - The domain of $f$ is:

   - The range of $f$ is:

   - A point always on the graph is:

   - There is always a horizontal asymptote:

   - $f$ is one-to-one, continuous and smooth

   - If $b > 1$:
     - $f$ is always increasing
     - As $x \to -\infty$, $f(x) \to 0^+$
     - As $x \to \infty$, $f(x) \to \infty$
     - The graph of $f$ resembles:

   - If $0 < b < 1$:
     - $f$ is always decreasing
     - As $x \to -\infty$, $f(x) \to \infty$
     - As $x \to \infty$, $f(x) \to 0^+$
     - The graph of $f$ resembles:

5. **Theorem – Algebraic Properties of Exponential Functions.** If $x, y \in \mathbb{R}$, then

   (i) $b^x \cdot b^y = b^{x+y}$

   (ii) $\dfrac{b^x}{b^y} = b^{x-y}$

   (iii) $(b^x)^y = b^{xy}$

6. **Question.** What is a logarithm?

7. **Reminder.** Suppose $f(x)$ is one-to-one. Then its inverse $f^{-1}(x)$ satisfies

   (i) $f^{-1}(f(x)) = x$

   (ii) $f(f^{-1}(y)) = y$

   (iii) $\text{Dom}_{f^{-1}}$=$\text{Ran}_f$ and $\text{Ran}_{f^{-1}}$=$\text{Dom}_f$.

8. **Definition.** The inverse of the exponential function $f(x) = b^x$ is called the **base *b* logarithm function**, and is denoted $f^{-1}(x) = \log_b(x)$ The expression $\log_b(x)$ is read 'log base $b$ of $x$.'

9. **The most important log property:**

10. **Two important logarithms:**

    (a) The **common logarithm** of a real number $x$ is $\log_{10}(x)$ and is usually written $\log(x)$.

    (b) The **natural logarithm** of a real number $x$ is $\log_e(x)$ and is usually written $\ln(x)$.

11. **Theorem – Properties of Logarithmic Functions.**

    Suppose $f(x) = \log_b(x)$.

    - The domain of $f$ is: $(0, \infty)$
    - The range of $f$ is" $(-\infty, \infty)$.
    - $(1, 0)$ is on the graph of $f$ & $x = 0$ is a vertical asymptote of the graph of $f$.
    - $f$ is one-to-one, continuous and smooth
    - $b^a = c$ if and only if $\log_b(c) = a$. So $\log_b(c)$ is the exponent you put on $b$ to obtain $c$.
    - $\log_b(b^x) = x$ for all $x$ and $b^{\log_b(x)} = x$ for all $x > 0$

    - If $b > 1$:
        - $f$ is always increasing
        - As $x \to 0^+$, $f(x) \to -\infty$
        - As $x \to \infty$, $f(x) \to \infty$
        - The graph of $f$ resembles:

    - If $0 < b < 1$:
        - $f$ is always decreasing
        - As $x \to 0^+$, $f(x) \to \infty$
        - As $x \to \infty$, $f(x) \to -\infty$
        - The graph of $f$ resembles:

12. **Theorem – Algebraic Properties of Logarithm Functions.**

    Let $g(x) = \log_b(x)$ be a logarithmic function ($b > 0$, $b \neq 1$) and let $u > 0$ and $w > 0$ be real numbers, and $r$ any real number.

    (i) **[Log] Product Rule:** $\log_b(uw) = \log_b(u) + \log_b(w)$

    (ii) **[Log] Quotient Rule:** $\log_b\left(\frac{u}{w}\right) = \log_b(u) - \log_b(w)$

    (iii) **[Log] Power Rule:** $\log_b(u^r) = r\log_b(u)$

13. **Theorem. – Change of base formula.** $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$.

14. **Homework.** Create a table of values for the following logarithms, then try plotting them.

    (a) $f(x) = \log_2(x)$

    (b) $f(x) = \log(x)$

    (c) $f(x) = \log_{\frac{1}{2}}(x)$