Introduction to

# Machine Learning and Data Mining

(Học máy và Khai phá dữ liệu)

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# Outline

- Introduction to Machine Learning & Data Mining
- Unsupervised learning
- **Supervised learning**
  - **Artificial neural network**
- Practical advice

# Artificial neural network: introduction (1)

- Artificial neural network (ANN) (mạng nơron nhân tạo)
  - Simulates the biological neural systems (human brain)
  - ANN is a structure/network made of interconnection of artificial neurons

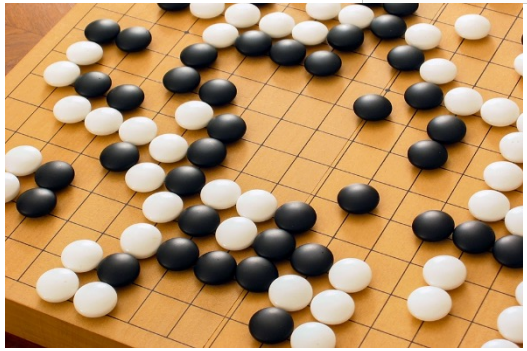
- Neuron
  - Has input/output
  - Executes a local calculation (local function)
- Output of a neuron is charactorized by
  - In/out characteristics
  - Connections between it and other neurons
  - (Possible) other inputs

# Artificial neural network: introduction (2)

- ANN can be thought of as a highly decentralized and parallel information processing structure
- ANN has the ability to learn, recall and generalize from the training data
- The ability of an ANN depends on
  - Network architecture
  - Input/output characteristics
  - Learning algorithm
  - Training data

# ANN: a huge breakthrough

- AlphaGo of Google the world champion at Go, 3/2016
  - Go is a 2500 year-old game.
  - Go is one of the most complex games
- AlphaGo learns from 30 millions human moves, and plays itself to find new moves



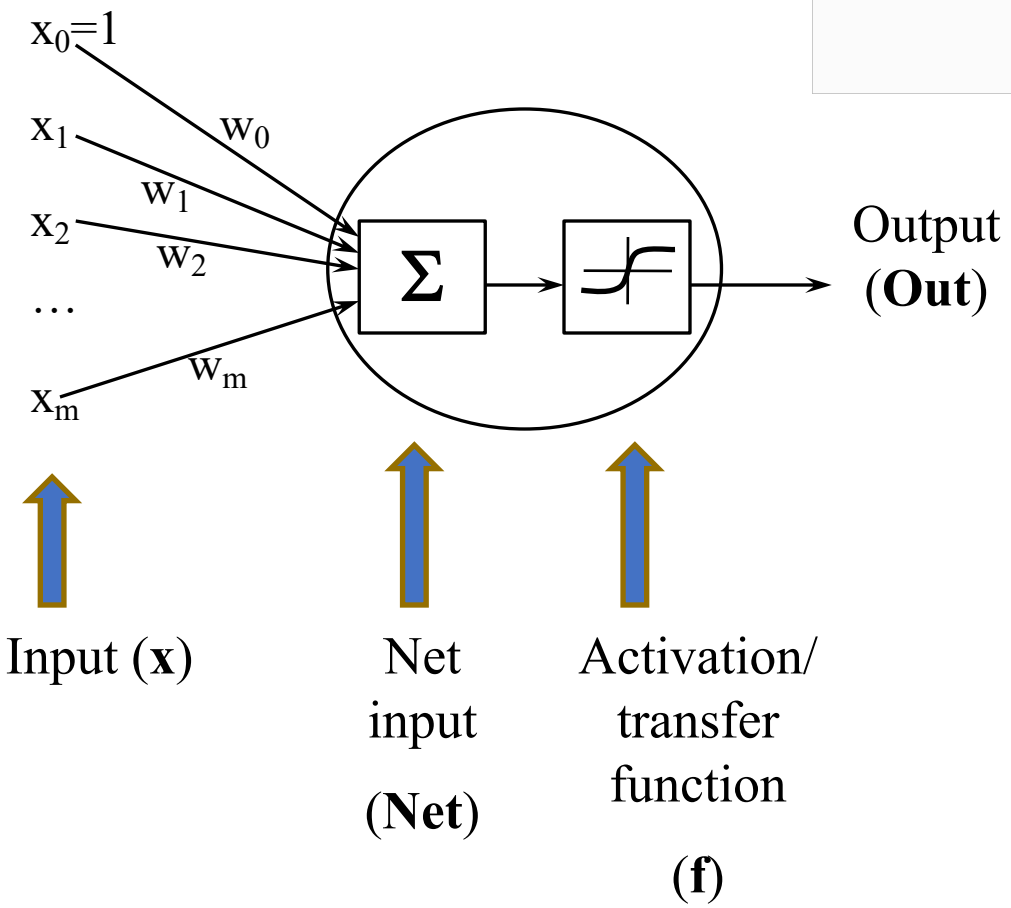
- It beat Lee Sedol (World champion)

  http://www.wired.com/2016/03/two-move future/

  http://www.nature.com/news/google-ai-al go-1.19234

# Structure of a neuron

- Input signals of a neuron
  $\{x_i, i = 1 \ldots m\}$
  - Each input signal $x_i$ is associated with a weight $w_i$
- Bias $w_0$ (with $x_0 = 1$)
- Net input is a combination of the input signals
  $Net(\boldsymbol{w}, \boldsymbol{x})$
- Activation/transfer function $f(\cdot)$ computes the output of a neuron
- Output
  $Out = f\,(Net\,(\boldsymbol{w}, \boldsymbol{x}))$

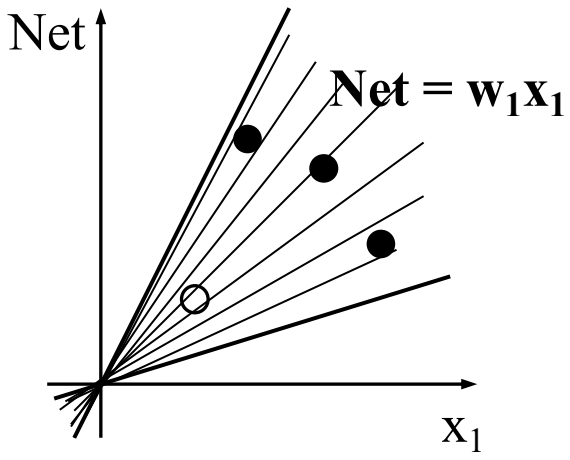


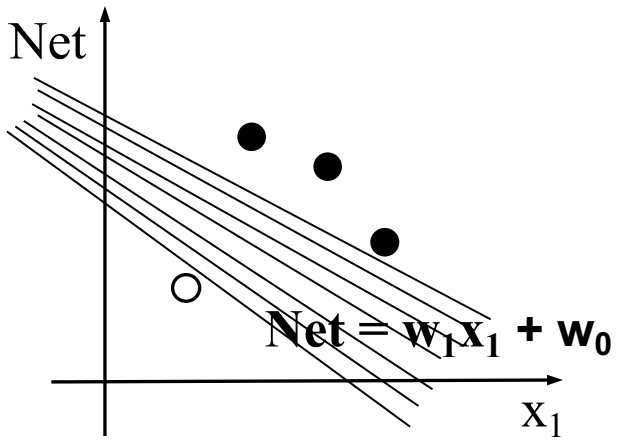
# Net Input

- Net input is usually calculated by a function of linear form

$$Net = w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_m x_m = w_0 .1 + \sum_{i=1}^{m} w_i x_i = \sum_{i=0}^{m} w_i x_i$$

- Role of bias:
  - $Net = w_1 x_1$ may not separate well the classes
  - $Net = w_1 x_1 + w_0$ is able to do better



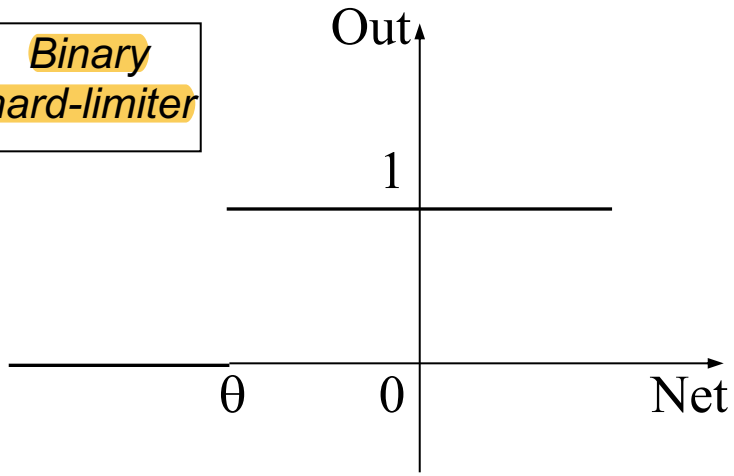
# Activation function: hard-limited

- Also know as a threshold function
- The output takes one of the two values
- $\theta$ is the threshold value
- **Properties:** discontinuous, non-smoothed (không trơn)

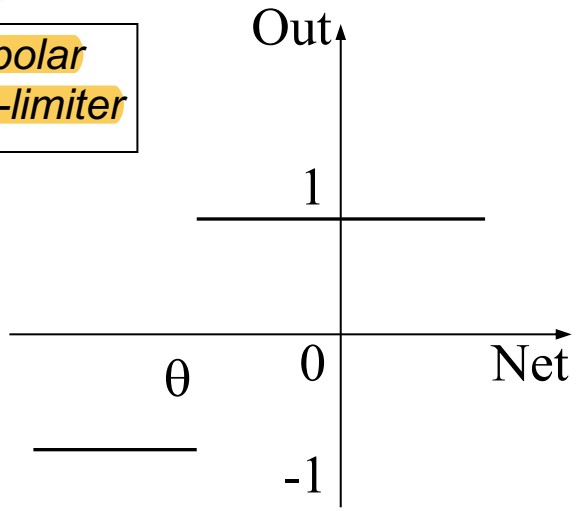
$$Out(Net) = HL(Net,\theta) = \begin{cases} 1, & \text{if } Net \geq \theta \\ 0, & \text{otherwise} \end{cases}$$

$$Out(Net) = HL2(Net,\theta) = sign(Net,\theta)$$

*Binary hard-limiter*




*Bipolar hard-limiter*

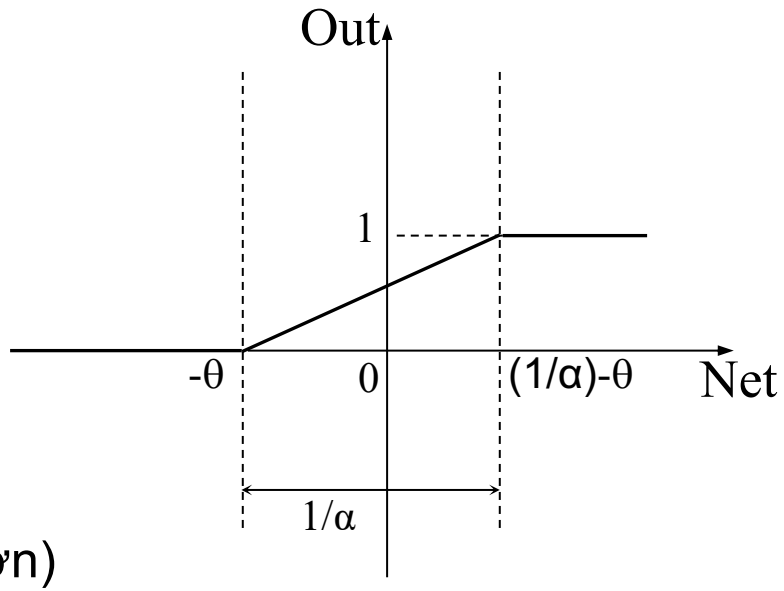


# Activation function: threshold logic

$$Out(Net) = tl(Net, \alpha, \theta) = \begin{cases} 0, & \text{if } Net < -\theta \\ \alpha(Net + \theta), & \text{if } -\theta \le Net \le \frac{1}{\alpha} - \theta \\ 1, & \text{if } Net > \frac{1}{\alpha} - \theta \end{cases} \qquad (\alpha > 0)$$

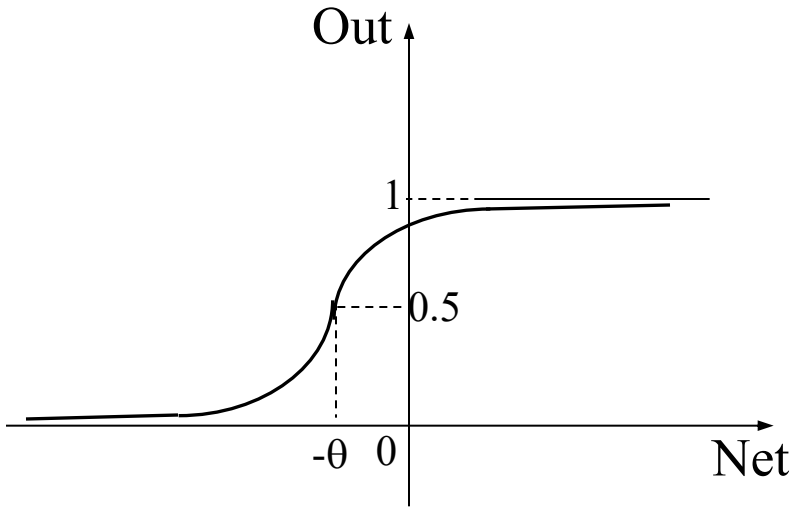
$$= \max(0, \min(1, \alpha(Net + \theta)))$$

- Also known as a saturating linear function
- Combination of 2 activation functions: linear and tight limits
- $\alpha$ determines the slope of the linear range
- **Properties:** continuous, non-smoothed (liên tục, nhưng không trơn)



# Activation function: Sigmoid

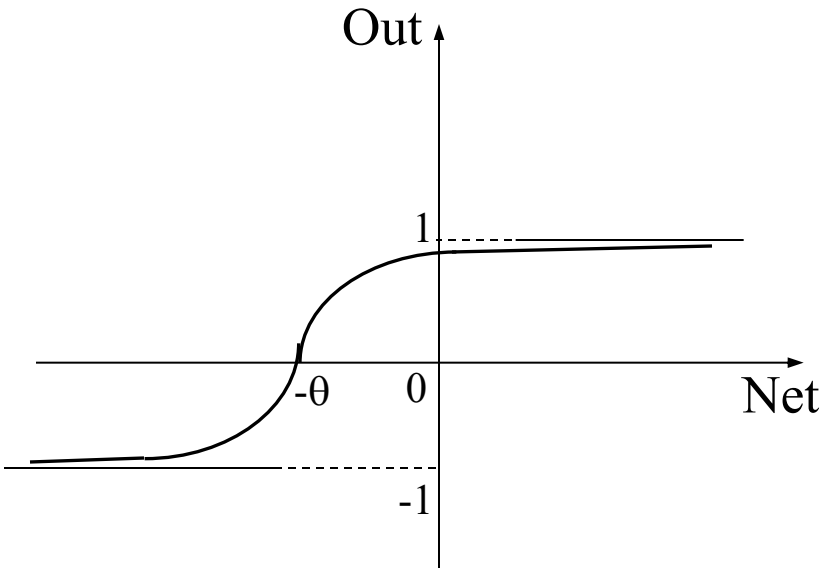
$$Out(Net) = sf(Net, \alpha, \theta) = \frac{1}{1 + e^{-\alpha(Net+\theta)}}$$




- Popular
- The parameter $\alpha$ determines the slope
- Output in the range of 0 and 1
- **Advantages**
  - Continuous, smoothed
  - Gradient of a sigmoid function is represented by a function of itself

# Activation function: Hyperbolic tangent

$$Out(Net) = \tanh(Net, \alpha, \theta) = \frac{1 - e^{-\alpha(Net+\theta)}}{1 + e^{-\alpha(Net+\theta)}} = \frac{2}{1 + e^{-\alpha(Net+\theta)}} - 1$$

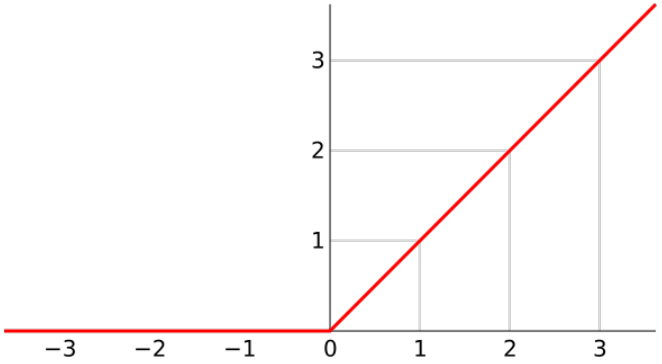



- Popular
- The parameter $\alpha$ determines the slope
- Output in the range of -1 and 1
- Advantages
  - Continuous, continuous derivative
  - Gradient of a tanh function is represented by a function of itself

# Act. function: Rectified linear unit (ReLU)

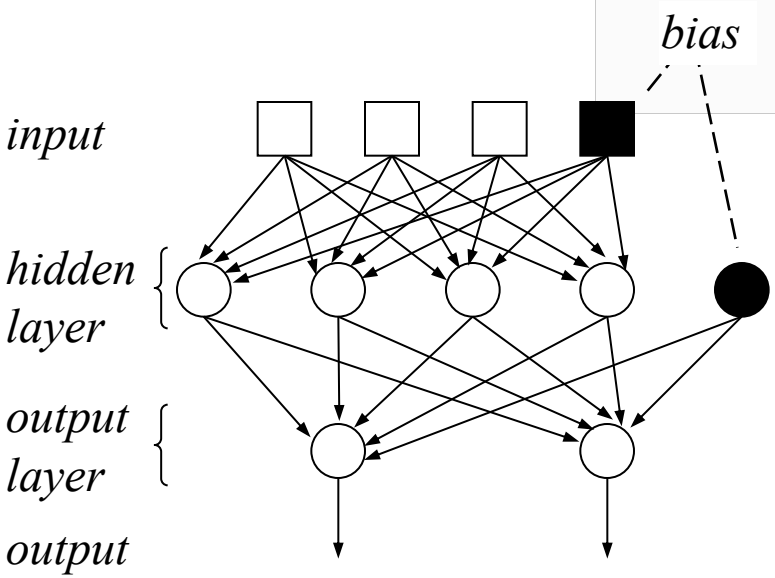
$$Out(net) = \max(0, net)$$

- Most popular
- Output is non-negative
- Advantages
  - Continuous
  - No derivative at point 0
  - Easy to calculate



# ANN: Architecture (1)

- ANN's architecture is determined by
  - Number of input and output signals
  - Number of layers
  - Number of neurons in each layer
  - Number of connection for each neuron
  - How neurons (with in a layer, or between layers) are connected
- An ANN must have
  - An input layer
  - An output layer
  - No, single, or multiple hidden layers

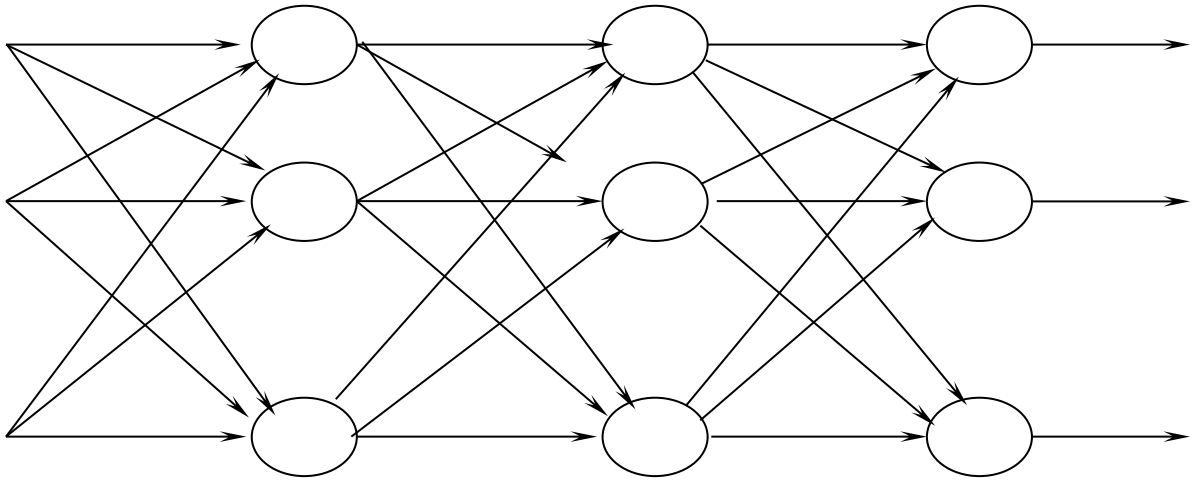



E.g: An ANN with single hidden layer

- Input: 3 signals
- Output: 2 signals
- Total, have 6 neurons
  - 4 neurons at hidden layer
  - 2 neurons at output layer

# ANN: Architecture (2)

- A layer (tầng) contains a set of neurons
- Hidden layer (tầng ẩn) is a layer between input layer and output layer
- Hidden nodes do not interact directly with external environment of the neural network
- An ANN is called a **fully connected** if outputs of a layer are connected to all neurons of the next layer

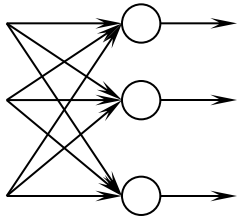


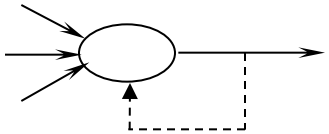
# ANN: Architecture (3)

- An ANN is called a **feed-forward network** (mạng lan truyền tiến) if there is not any output of a node being input of another node of the same layer or a previous layer
- When the output of a node is the input of the node the same layer or a previous layer, it is called a **feedback network** (mạng phản hồi)
  - If feedback connects to the input of nodes of the same layer, then it is called a **lateral feedback**.
- Feedback networks with closed loops are called **recurrent networks** (mạng hồi quy)

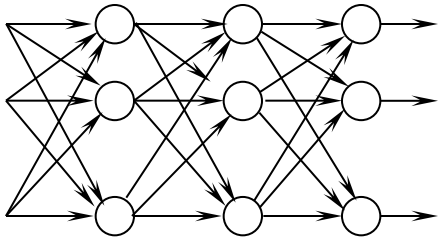
# ANN: Architecture (4)

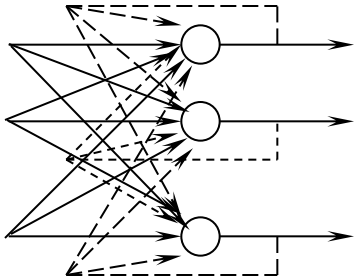
Feed-forward network





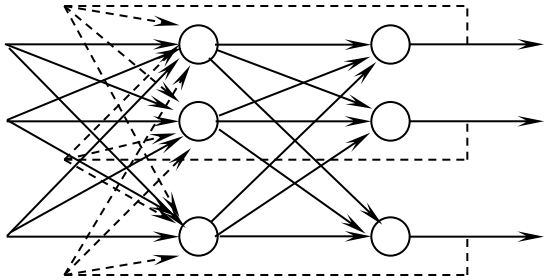
A neuron with feedback to itself





Recurrent network with single layer

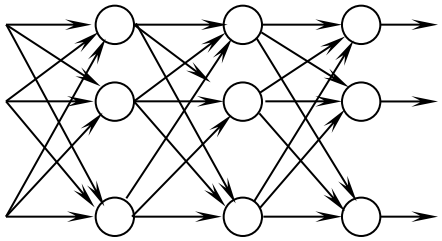
Feed-forward network with multiple layers



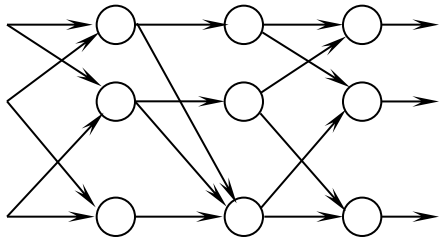
Recurrent network with multiple layers

# ANN: Training

- 2 types of learning in ANNs
  - ***Parameter learning***: The goal is to adapt the weights of the connections in the ANN, given a fixed network structure
  - ***Structure learning***: The goal is to learn the network structure, including the number of neurons and the types of connections between them, and the weights



Or



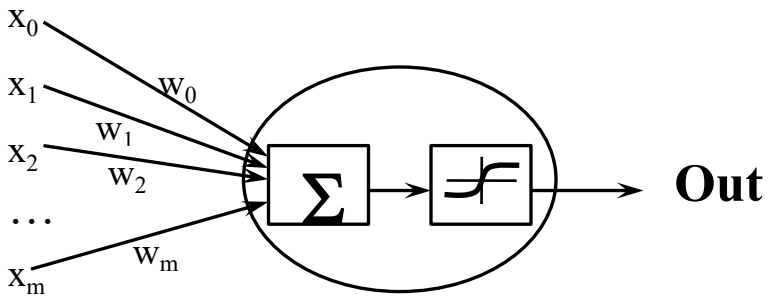
- Those two types can be done simultaneously or separately
- In this lecture, we will only consider parameter learning

# ANN: Idea for training

- Training a neural network (when fixing the architecture) is learning the weights **w** of the network from training data **D**
- Learning can be done by minimizing an empirical error function

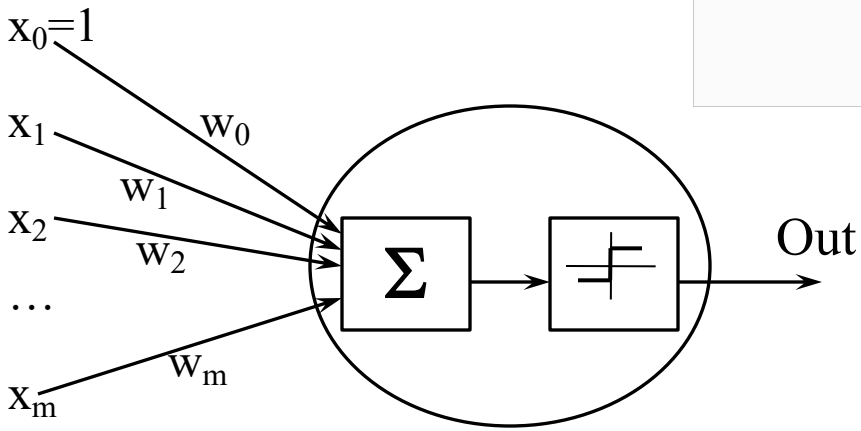
$$\mathrm{L}(\boldsymbol{w}) = \frac{1}{|\boldsymbol{D}|}\sum_{\boldsymbol{x}\in\boldsymbol{D}} loss(d_x, \mathrm{out}(\boldsymbol{x}))$$

  - Where out(**x**) is the output of the network, with the input **x** labeled accordingly as $d_x$; loss is a function for measuring prediction error
- Many gradient-based methods:
  - Backpropagation
  - Stochastic gradient decent (SGD)
  - Adam
  - AdaGrad



# Perceptron

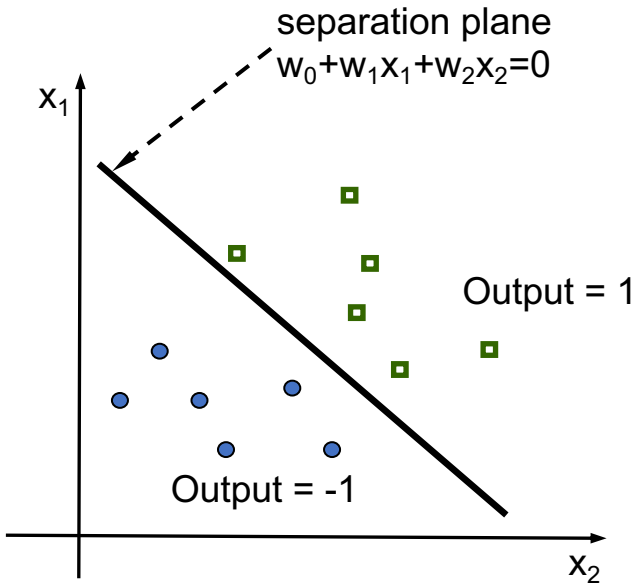
- A perceptron is the simplest type of ANNs (consists of only one neuron).




- Use the hard-limited activation function

$$Out = sign\big(Net(w,x)\big) = sign\left(\sum_{j=0}^{m} w_j x_j\right)$$

- For input **x**, the output value of perceptron
  - 1 if Net(**w**,**x**)>0
  - -1 otherwise



# Perceptron: Algorithm

- Training data **D** = **{(x**, d**)}**
  - **x** is input vector
  - d is output (1 or -1)
- The goal of perceptron learning (training) process determines a weight vector that allows the perceptron to produce the correct output value (-1 or 1) for each data point
- For data point **x** correctly classified by perceptron, the weight vector **w** unchanged
- If d = 1 but the perceptron produces -1 (Out = -1), then **w** needs to be changed so that the value of Net (**w, x**) increases
- If d = -1 but the perceptron produces 1 (Out = 1), then **w** needs to be changed so that the value of Net (**w, x**) decreases

# Perceptron: Algorithm

**Perceptron_batch**(**D**, η)

Initialize **w** ($w_i \leftarrow$ an initial (small) random value)

do

  $\Delta\mathbf{w} \leftarrow 0$

  for each training instance $(\mathbf{x},d) \in \mathbf{D}$

    Compute the real output value Out

    if (Out≠d)

      $\Delta\mathbf{w} \leftarrow \Delta\mathbf{w} + \eta(d\text{-Out})\mathbf{x}$

  end for

  $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$

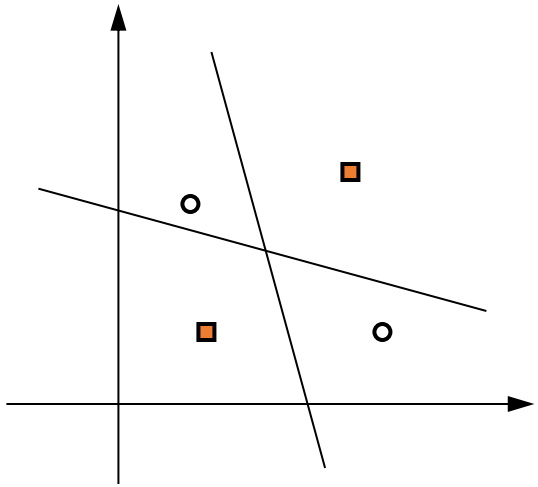
until all the training instances in **D** are correctly classified

return **w**

# Perceptron: Limitation

- The training algorithm for perceptron is proved to converge if:
  - Data points are linearly separable
  - Use a learning rate η small enough
- The training algorithm for perceptron may not converge if data points are not linearly separable

A perceptron cannot classify correctly for this case!



# Loss function

- Consider an ANN that has n output neurons
- For data point (**x**, d), the **training error** value caused by the (current) weight vector **w**:

$$E_{\mathbf{x}}(\mathbf{w}) = \frac{1}{2}\sum_{i=1}^{n}\left(d_i - Out_i\right)^2$$

- **Training error** for the training data **D** is

$$E_D(\mathbf{w}) = \frac{1}{|D|}\sum_{\mathbf{x}\in D} E_{\mathbf{x}}(\mathbf{w})$$

# Minimize errors with gradients

- Gradient of E (denoted by ∇E) is a vector

$$\nabla E(\mathbf{w}) = \left( \frac{\partial E}{\partial w_1}, \frac{\partial E}{\partial w_2}, \ldots, \frac{\partial E}{\partial w_N} \right)$$

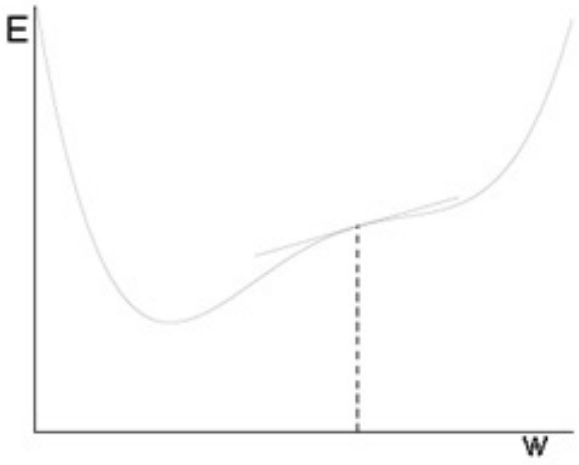
  - where N is the total number of weights (connections) in the ANN
- The gradient ∇E determines the direction that causes the **steepest increase** for the error value E
- Therefore, the direction that causes the **steepest decrease** is opposite to the gradient of E

$$\Delta \boldsymbol{w} = -\eta . \nabla E(\boldsymbol{w}); \quad \Delta w_i = -\eta \frac{\partial E}{\partial w_i} \text{ for } i = 1 \ldots N$$

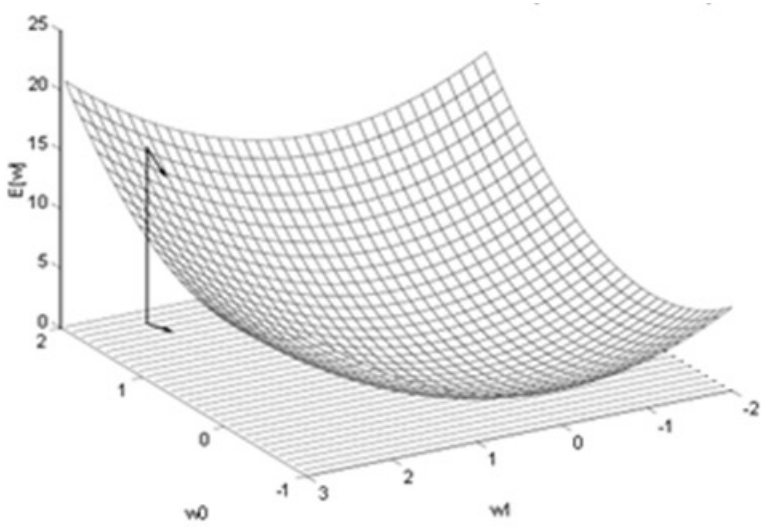
- Requirement: all the activation functions must be smoothed

# Gradient descent: Illustration

One-dimensional space
$E(w)$




2-dimensional space
$E(w_1,w_2)$



# Algorithm

**Gradient_descent_incremental** (**D**, $\eta$)

Initialize **w** ($w_i \leftarrow$ an initial (small) random value)

do

  for each training instance ($\mathbf{x}$, $d$) $\in$ **D**

    Compute the network output

    for each weight component $w_i$

      $w_i \leftarrow w_i - \eta(\partial E_{\mathbf{x}}/\partial w_i)$

    end for

  end for

until (stopping criterion satisfied)

return **w**

Stopping criterion: epochs, threshold error, ...

If we take a small *subset* (mini-batch) *randomly* from **D** to update the weights, we will have mini-batch training.

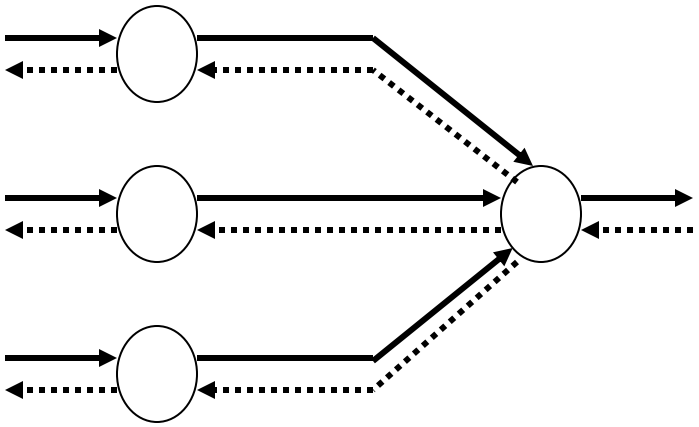
# Backpropagation algorithm

- A perceptron can only represent a linear function
- A multi-layer NN learned by the **Backpropagation** (BP) algorithm can represent a highly non-linear function
- The BP algorithm is used to learn the weights of an ANN
  - Fixed network structure (một cấu trúc mạng đã chọn trước)
  - For each neuron, the activation function must be differentiable
- The BP algorithm applies a *gradient descent* strategy to the rules for updating weights
  - To minimize errors between actual output values and desired output values, for training data

# Backpropagation algorithm (1)

- Back propagation algorithm seeks a vector of weights that minimizes the net errors on the training data
- The BP algorithm consists of 2 phases:
  - **Forward pass:** The input signals (input vector) are forwarded from the input layer to the output layer (passing through hidden layers).
  - **Error backward:**
    - Based on the desired output value of the input vector, calculate the error value
    - From the output layer, the error value is backward-propagated across the network, from a layer to previous layer, to the input layer.
    - Error back-propagation is executed by calculating (regressively) the local gradient values of each neuron

# Backpropagation algorithm (2)



Signal forward phase:

- Forward signals via the network

Error backward phase:

- Calculate the error at the output
- Error back-propagation

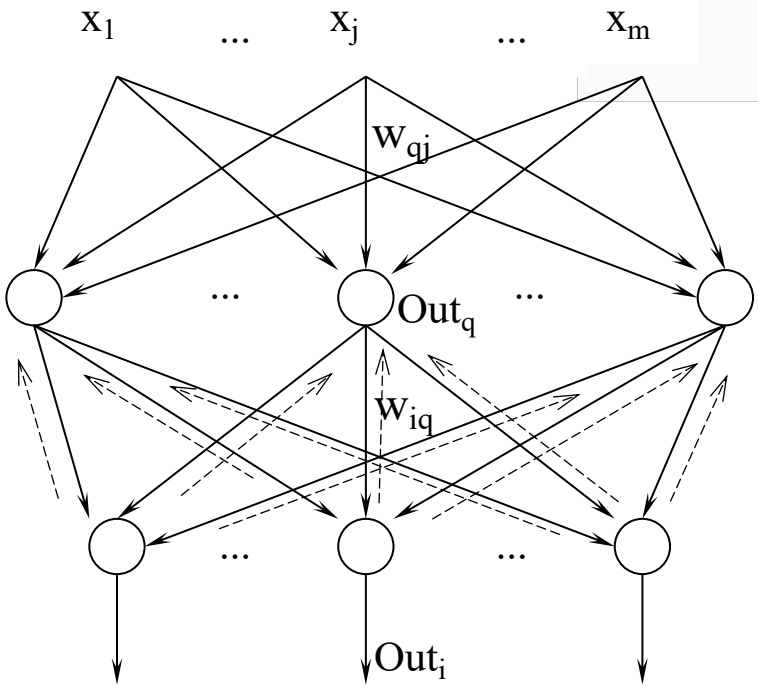
# Network structure

- Consider the 3-layer neural network (in the figure) to illustrate the BP algorithm
- $m$ input signals $x_j$ ($j$=1..$m$)
- $l$ hidden neurons $z_q$ ($q$=1..$l$)
- $n$ output neurons $y_i$ ($i$=1..$n$)
- $w_{qj}$ is the weight of the connection from the input signal $x_j$ to the hidden neuron $z_q$

Input $x_j$ ($j$=1..$m$)

Hidden neuron $z_q$ ($q$=1..$l$)

Output neuron $y_i$ ($i$=1..$n$)




- $w_{iq}$ is the weight of the connection from the hidden neuron $z_q$ to the output $y_i$
- $Out_q$ is the (local) output value of the hidden neuron $z_q$
- $Out_i$ is the output value of the network corresponding to the output neuron $y_i$

# BP algorithm: Forward (1)

- For each data point **x**
  - Input vector **x** is forwarded from the input layer to the output layer
  - The network will generate an actual output value **Out** (a vector with value $Out_i$, i = 1..n)
- For an input vector **x**, a neuron $z_q$ at the hidden layer receives the value of net input:

$$Net_q = \sum_{j=1}^{m} w_{qj} x_j$$

then produces a (local) output value

$$Out_q = f(Net_q) = f\left( \sum_{j=1}^{m} w_{qj} x_j \right)$$

where *f(.)* is a activation function of neuron $z_q$

# BP algorithm: Forward (2)

- Net input value of the neuron $y_i$ at the output layer

$$Net_i = \sum_{q=1}^{l} w_{iq} Out_q = \sum_{q=1}^{l} w_{iq} f\left( \sum_{j=1}^{m} w_{qj} x_j \right)$$

- Neuron $y_i$ produces output value (is an output value of network)

$$Out_i = f(Net_i) = f\left( \sum_{q=1}^{l} w_{iq} Out_q \right) = f\left( \sum_{q=1}^{l} w_{iq} f\left( \sum_{j=1}^{m} w_{qj} x_j \right) \right)$$

- Vector of the output values $Out_i$ ($i$=1..$n$) is the actual output value of the network, for the input vector **x**

# BP algorithm: Backward (1)

- For each data point **x**
  - Error signals due to the difference between the desired output value d and the actual output value **Out** are calculated
  - These error signals are **back-propagated** from the output layer to the front layers, to update weights
- To consider the error signals and their back-propagated ones, an error function needs to be defined

$$E(w) = \frac{1}{2}\sum_{i=1}^{n}(d_i - Out_i)^2 = \frac{1}{2}\sum_{i=1}^{n}[d_i - f(Net_i)]^2$$

$$= \frac{1}{2}\sum_{i=1}^{n}\left[d_i - f\left(\sum_{q=1}^{l} w_{iq} Out_q\right)\right]^2$$

# BP algorithm: Backward (2)

- According to the gradient-descent method, the weights of the connections from the hidden layer to the output layer are updated by

$$\Delta w_{iq} = -\eta \frac{\partial E}{\partial w_{iq}}$$

- Using the derivative chain rule for $\partial E/\partial w_{iq}$, we have

$$\Delta w_{iq} = -\eta \left[\frac{\partial E}{\partial Out_i}\right]\left[\frac{\partial Out_i}{\partial Net_i}\right]\left[\frac{\partial Net_i}{\partial w_{iq}}\right] = \eta\left[d_i - Out_i\right]\left[f'\left(Net_i\right)\right]\left[Out_q\right] = \eta\delta_i Out_q$$

- $\delta_i$ is **error signals** of neuron $y_i$ at output layer

$$\delta_i = -\frac{\partial E}{\partial Net_i} = -\left[\frac{\partial E}{\partial Out_i}\right]\left[\frac{\partial Out_i}{\partial Net_i}\right] = \left[d_i - Out_i\right]\left[f'\left(Net_i\right)\right]$$

where $Net_i$ is the net input of the neuron $y_i$ at the output layer, and $f'(Net_i)=\partial f(Net_i)/\partial Net_i$

# BP algorithm: Backward (3)

- To update the weights of the connections from the input layer to the hidden layer, we also apply the gradient-descent method and the derivative chain rule

$$\Delta w_{qj} = -\eta \frac{\partial E}{\partial w_{qj}} = -\eta \left[ \frac{\partial E}{\partial Out_q} \right] \left[ \frac{\partial Out_q}{\partial Net_q} \right] \left[ \frac{\partial Net_q}{\partial w_{qj}} \right]$$

- From the formula for calculating the error function E(**w**), we see that each error component ($d_i$-$y_i$) ($i$=1..$n$) is a function of $Out_q$

$$E(\mathbf{w}) = \frac{1}{2} \sum_{i=1}^{n} \left[ d_i - f\left( \sum_{q=1}^{l} w_{iq} Out_q \right) \right]^2$$

# BP algorithm: Backward (4)

- Apply the derivation chain rule, we have

$$\Delta w_{qj} = \eta \sum_{i=1}^{n} \left[ \left( d_i - Out_i \right) f'\left( Net_i \right) w_{iq} \right] f'\left( Net_q \right) x_j$$

$$= \eta \sum_{i=1}^{n} \left[ \delta_i w_{iq} \right] f'\left( Net_q \right) x_j = \eta \delta_q x_j$$

- $\delta_q$ is **error signals** of neuron $z_q$ at hidden layer

$$\delta_q = -\frac{\partial E}{\partial Net_q} = -\left[ \frac{\partial E}{\partial Out_q} \right] \left[ \frac{\partial Out_q}{\partial Net_q} \right] = f'\left( Net_q \right) \sum_{i=1}^{n} \delta_i w_{iq}$$

where $Net_g$ is the net input of the neuron $z_q$ at the hidden layer, and $f'(Net_q)=\partial f(Net_q)/\partial Net_q$

# BP algorithm: Backward (5)

- According to the formulas for calculating the error signals $\delta_i$ and $\delta_q$, the error signal of a neuron in the hidden layer is different from the error signal of a neuron in the output layer
- Because of this difference, the weight update procedure in BP algorithm is also known as general **delta learning rule**
- Error signals $\delta_q$ of neuron $z_q$ at hidden layer determined by:
  - Error signals $\delta_i$ of neuron $y_i$ at output layer (to which neuron $z_q$ are connected)
  - The weights $w_{iq}$

# BP algorithm: Backward (6)

- The process of calculating the error signals as above can be extended (generalized) easily for neural networks with more than 1 hidden layer
- The general form of the weighting update rule in BP algorithm

$$\Delta w_{ab} = \eta \delta_a x_b$$

  - b and a are 2 indices corresponding to the two ends of the connection (b → a) (from a neuron (or input signal) b to neuron a)
  - $\boldsymbol{x}_b$ is the output value of the neuron at the hidden layer (or input signal) b
  - $\delta_a$ is error signal of neuron a

# BP algorithm

**Back_propagation_incremental**(D, η)

Neural network consists of Q layer, $q = 1,2,...,Q$

${}^{q}Net_i$ and ${}^{q}Out_i$ are net input and output value of neuron $i$ at the layer $q$

*Network has m* input signals and $n$ output neuron

${}^{q}w_{ij}$ is the weight of the connection from neuron $j$ at the layer $(q-1)$ to the neuron $i$ at the layer $q$

**Step 0** (Initialization)

Select the error threshold $E_{threshold}$ (the error value is acceptable)

Initialize the initial value of the weights with random small values

Assign E=0

**Step 1** (Start a training cycle)

Apply the input vector of the data point k to the input layer ($q$=1)

${}^{q}Out_i = {}^{1}Out_i = x_i^{(k)}, \forall i$

**Step 2** (Forward)

Forward the input signals over the network, until the network output values (at the output layer) are received ${}^{Q}Out_i$

$$ {}^{q}Out_i = f\left({}^{q}Net_i\right) = f\left(\sum_j {}^{q}w_{ij}\,{}^{q-1}Out_j\right) $$

# BP algorithm

**Step 3** (Calculate the output error)

Calculate network output error and error signal ${}^{Q}\delta_i$ of each neuron at output layer

$$E = E + \frac{1}{2}\sum_{i=1}^{n}(d_i^{(k)} - {}^{Q}Out_i)^2$$

$${}^{Q}\delta_i = (d_i^{(k)} - {}^{Q}Out_i) f'({}^{Q}Net_i)$$

**Step 4** (Error backward)

Backpropagation the error to update the weights and calculate the error signals ${}^{q-1}\delta_i$ for the front layers

$$\Delta^{q}w_{ij} = \eta.({}^{q}\delta_i).({}^{q-1}Out_j); \qquad {}^{q}w_{ij} = {}^{q}w_{ij} + \Delta^{q}w_{ij}$$

$${}^{q-1}\delta_i = f'({}^{q-1}Net_i)\sum_j {}^{q}w_{ji}\,{}^{q}\delta_j; \quad \text{for all } q = Q, Q-1, \ldots, 2$$

**Step 5** (Check stopping criterion satisfied)

Check if the entire training data has been used yet

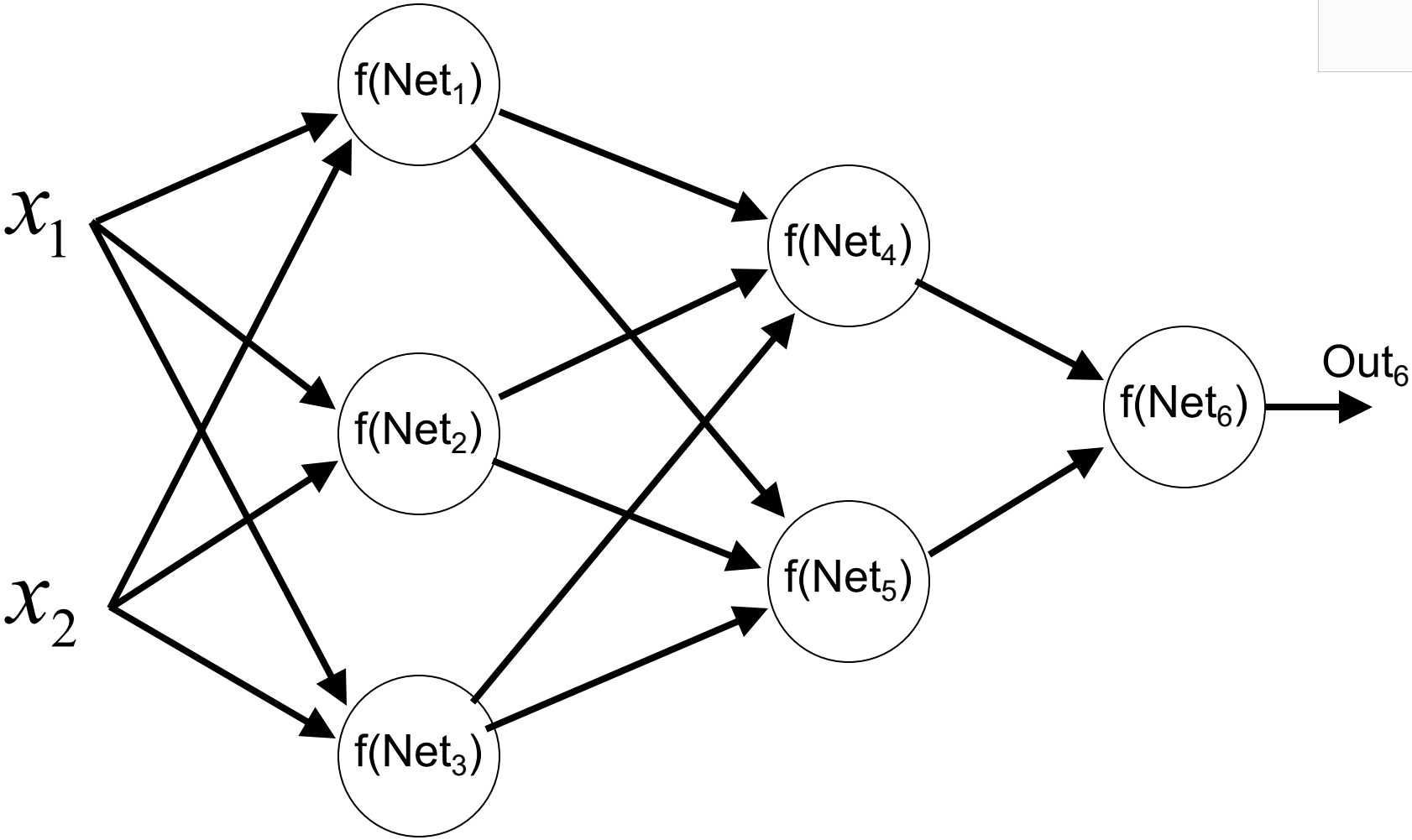
If the entire training data has used, go to Step 6, otherwise go to Step 1

**Step 6** (Check net error)

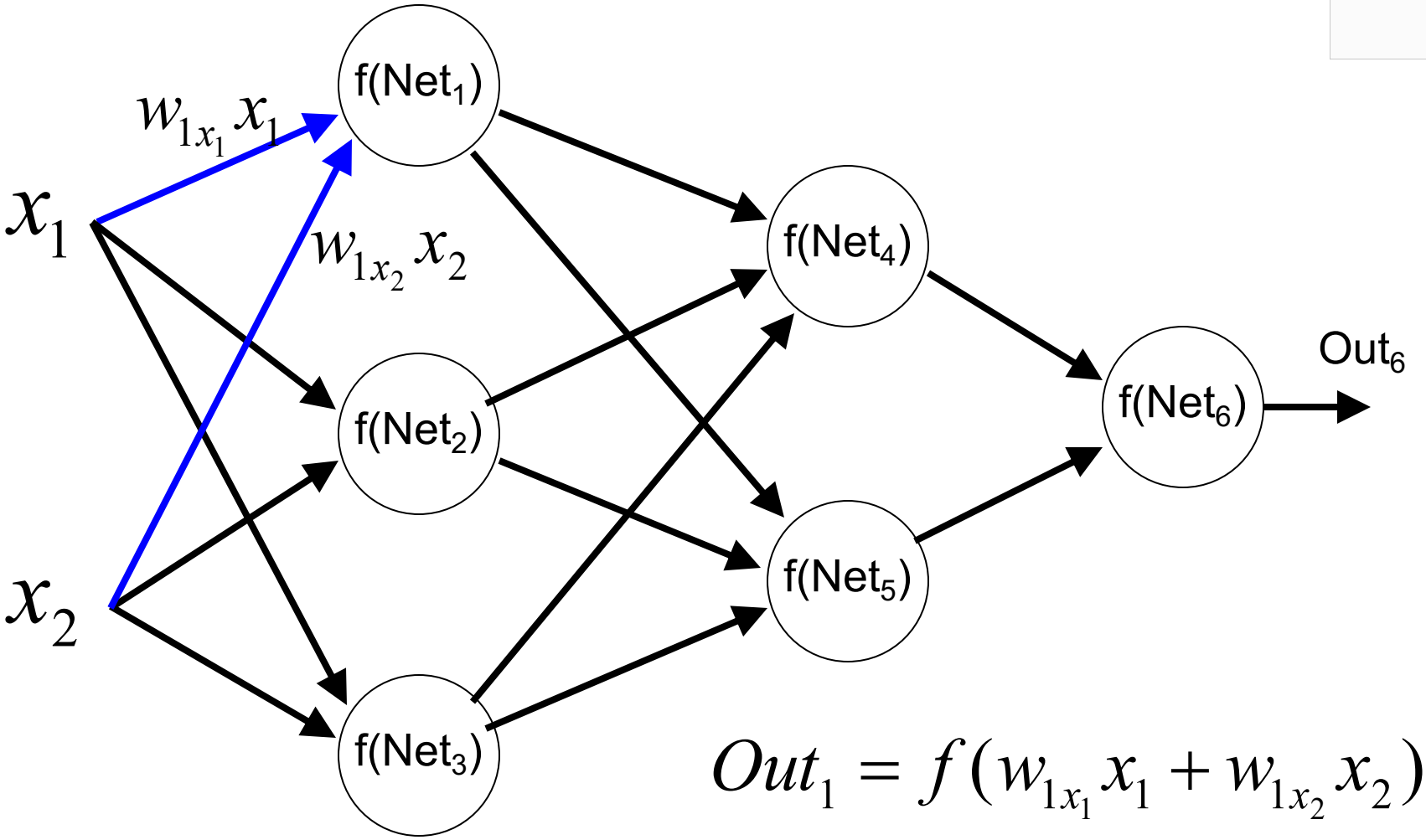
If net error E is less than the acceptable threshold ($<E_{threshold}$), then training is completed and returns the learned weights;

otherwise, assign E=0, and start new training cycle (go back to Step 1)

# BP algorithm: Forward (1)

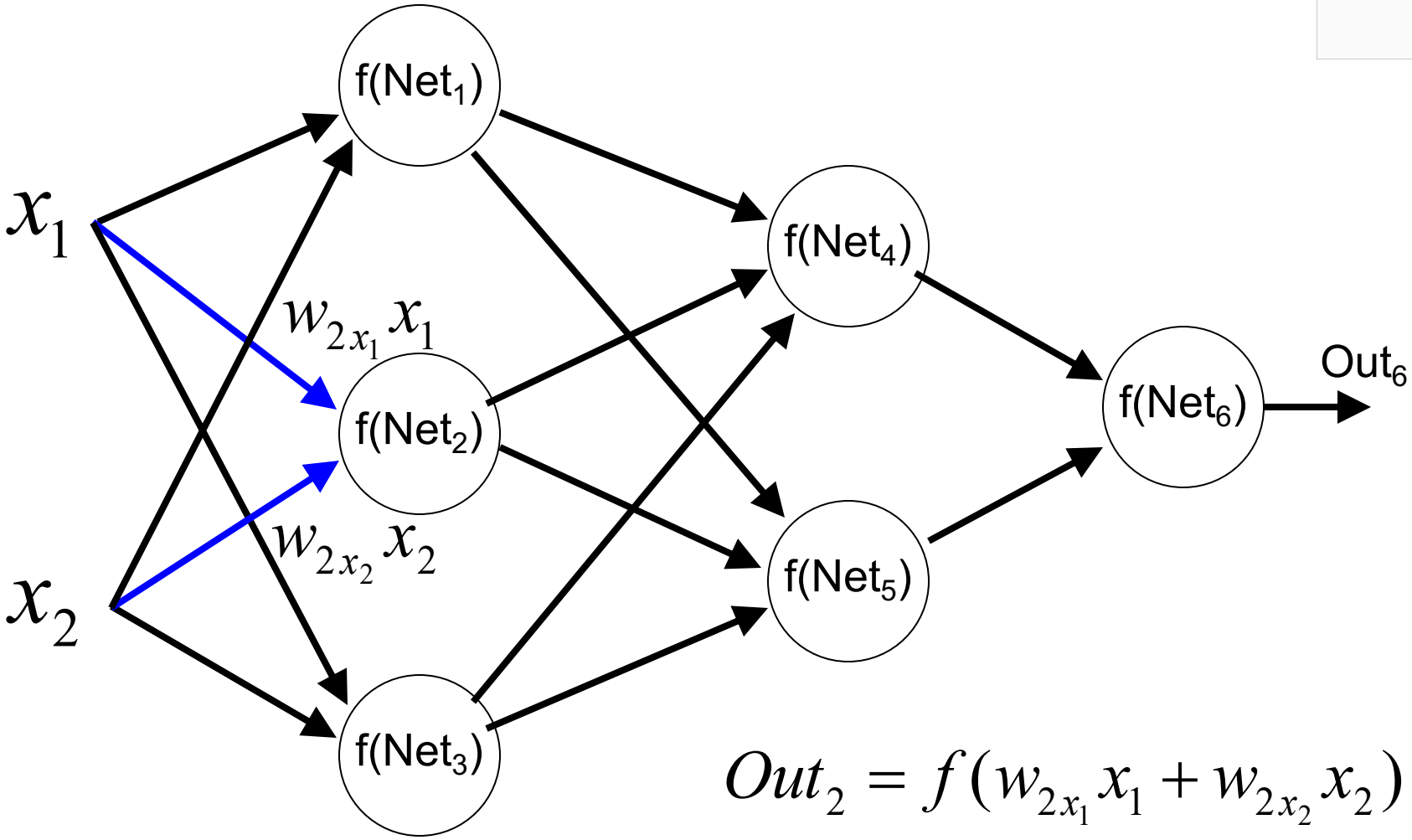


# BP algorithm: Forward (2)



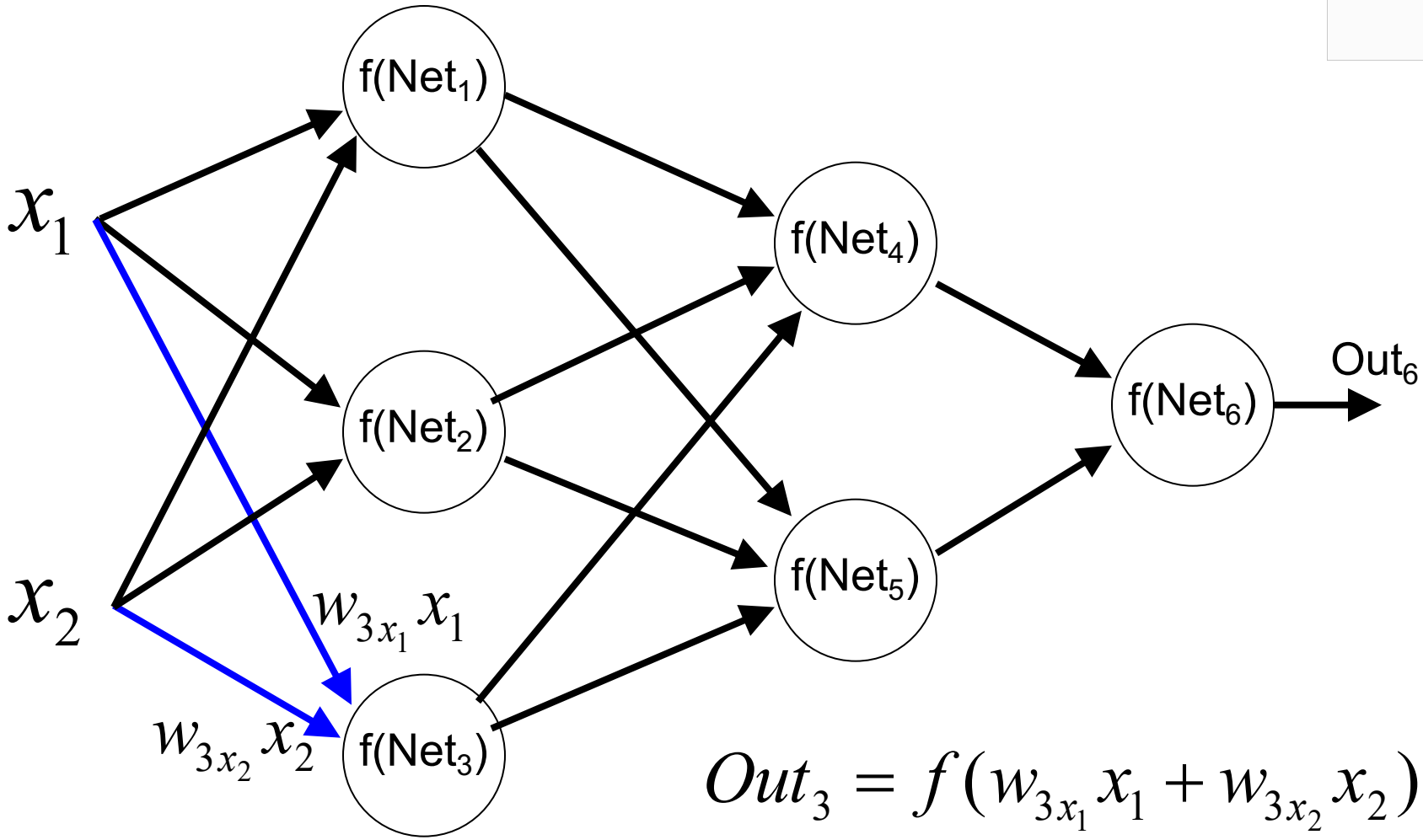

$$Out_1 = f(w_{1x_1} x_1 + w_{1x_2} x_2)$$

# BP algorithm: Forward (3)



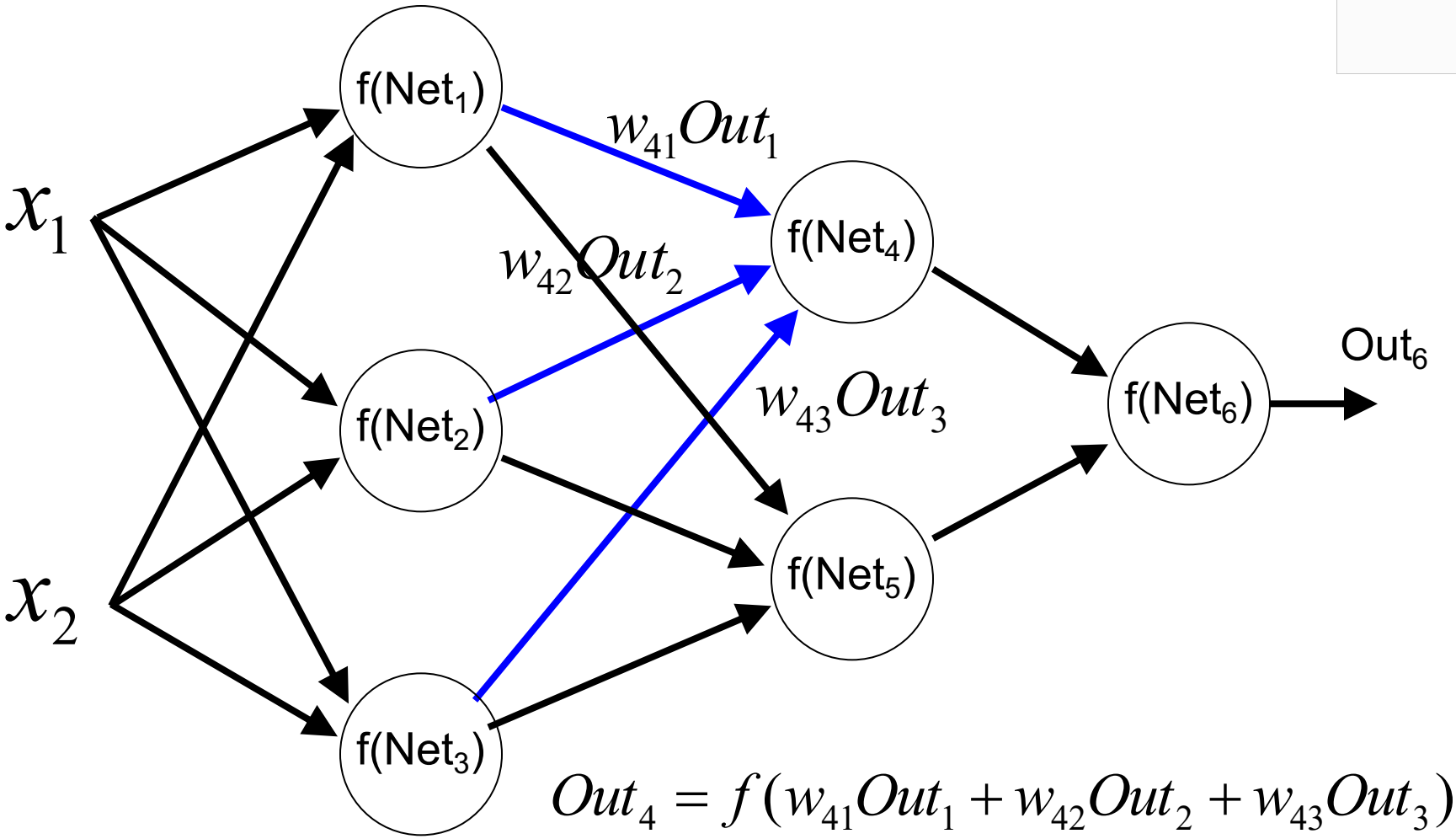

$$Out_2 = f(w_{2x_1} x_1 + w_{2x_2} x_2)$$

# BP algorithm: Forward (4)



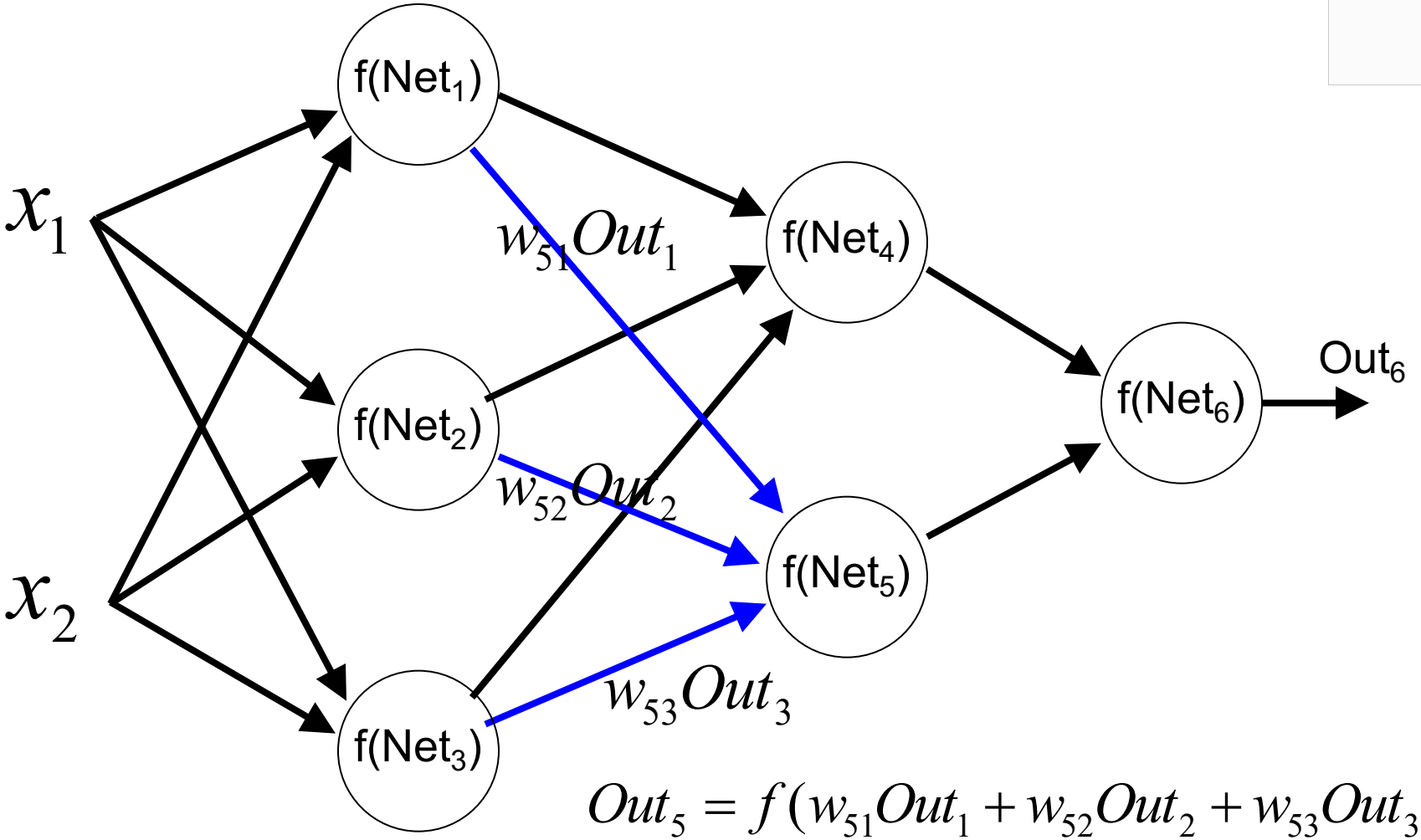

$$Out_3 = f(w_{3x_1} x_1 + w_{3x_2} x_2)$$

# BP algorithm: Forward (5)



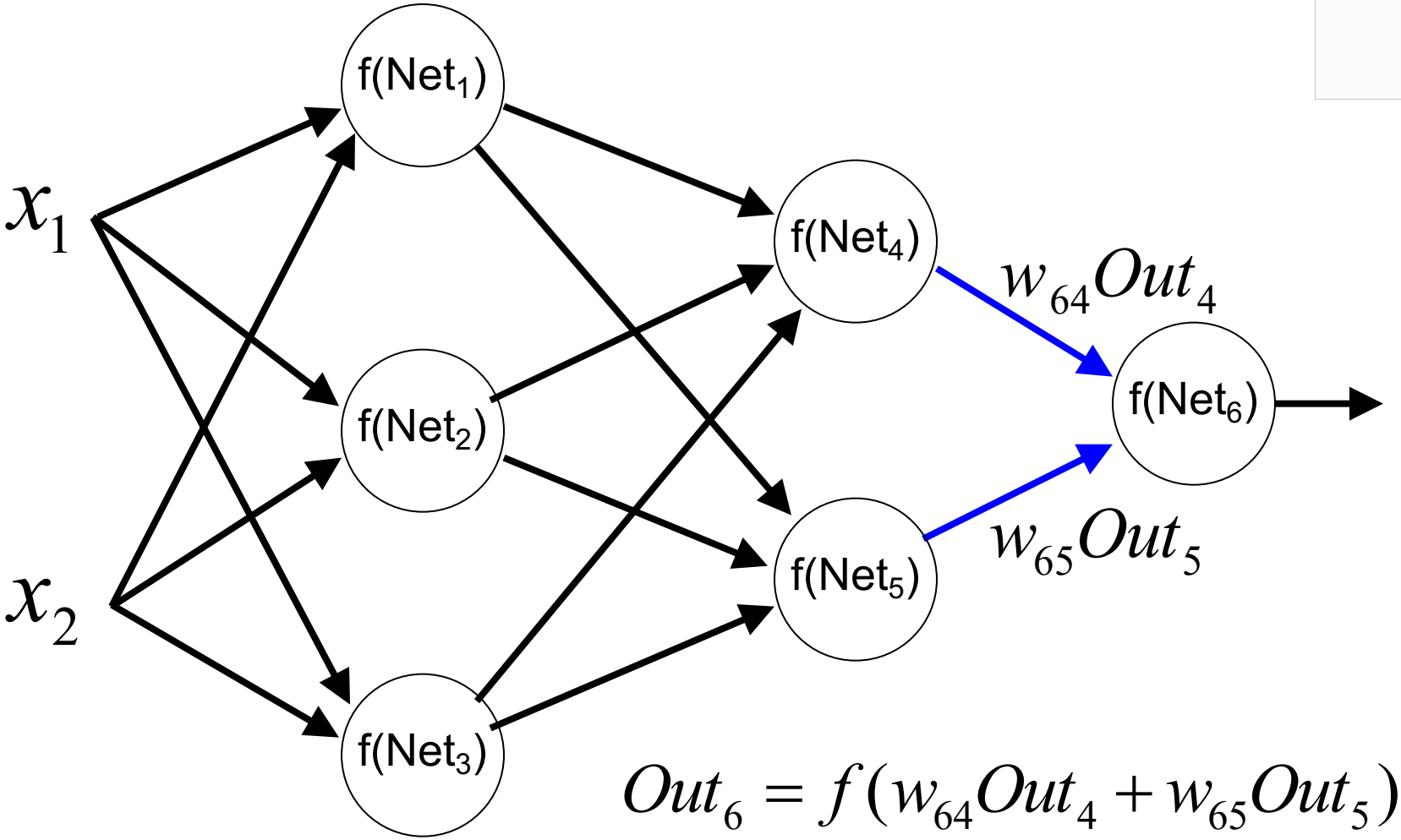

$$Out_4 = f(w_{41}Out_1 + w_{42}Out_2 + w_{43}Out_3)$$

# BP algorithm: Forward (6)



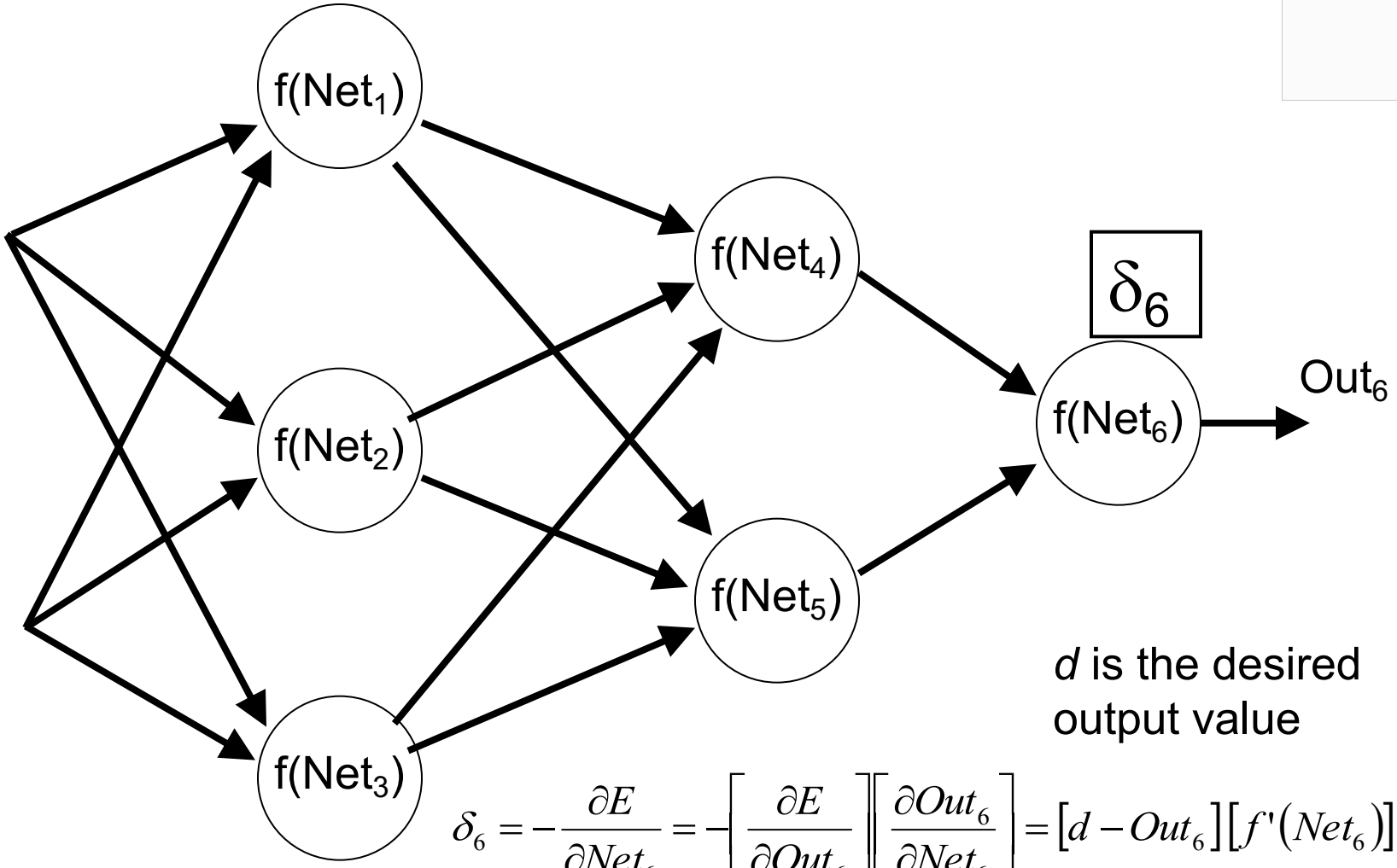

$$Out_5 = f(w_{51}Out_1 + w_{52}Out_2 + w_{53}Out_3)$$

# BP algorithm: Forward (7)

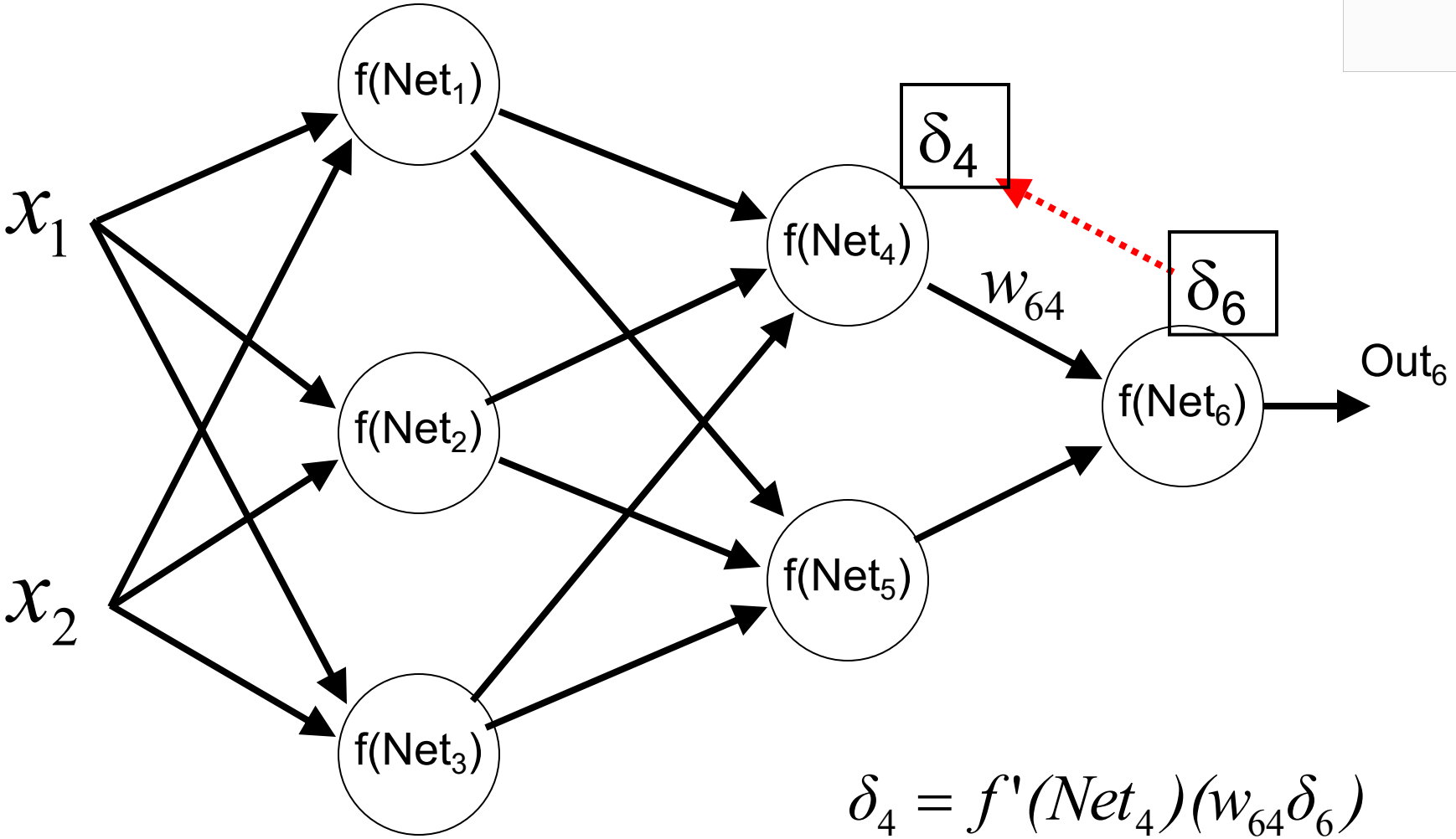



$$Out_6 = f(w_{64}Out_4 + w_{65}Out_5)$$

# BP algorithm: Calculate error

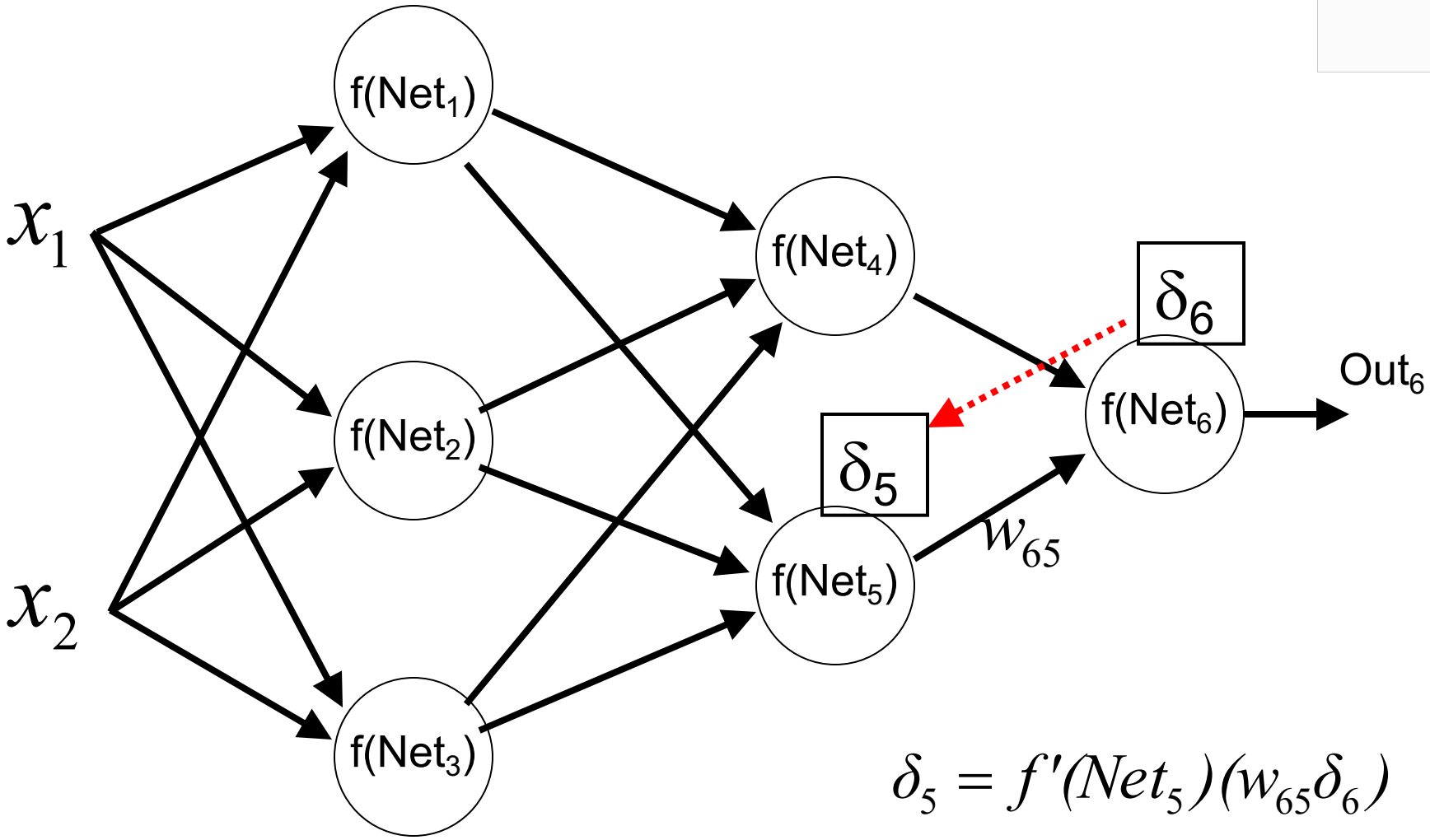


# BP algorithm: Backward(1)



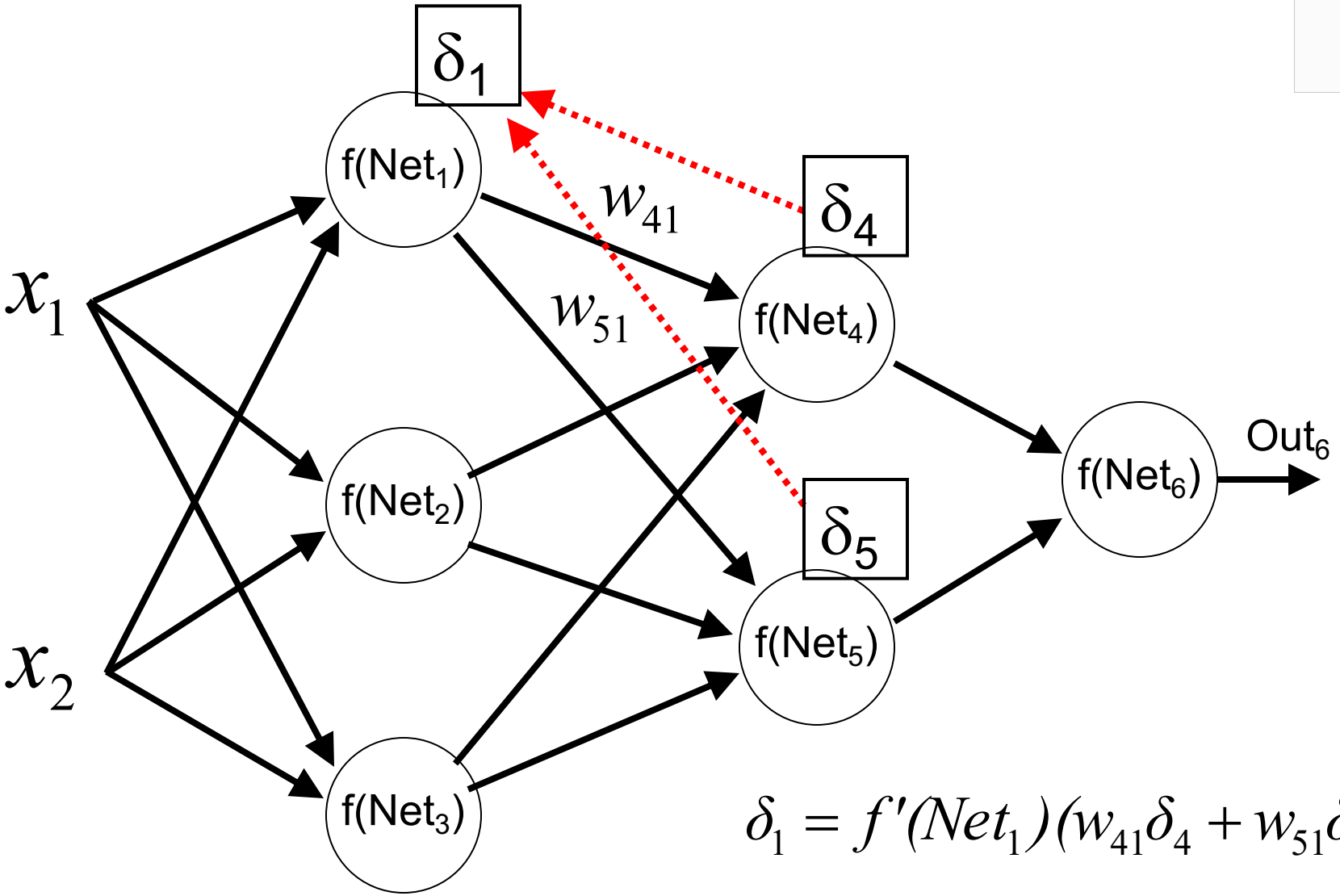

$$\delta_4 = f'(Net_4)(w_{64}\delta_6)$$

# BP algorithm: Backward(2)



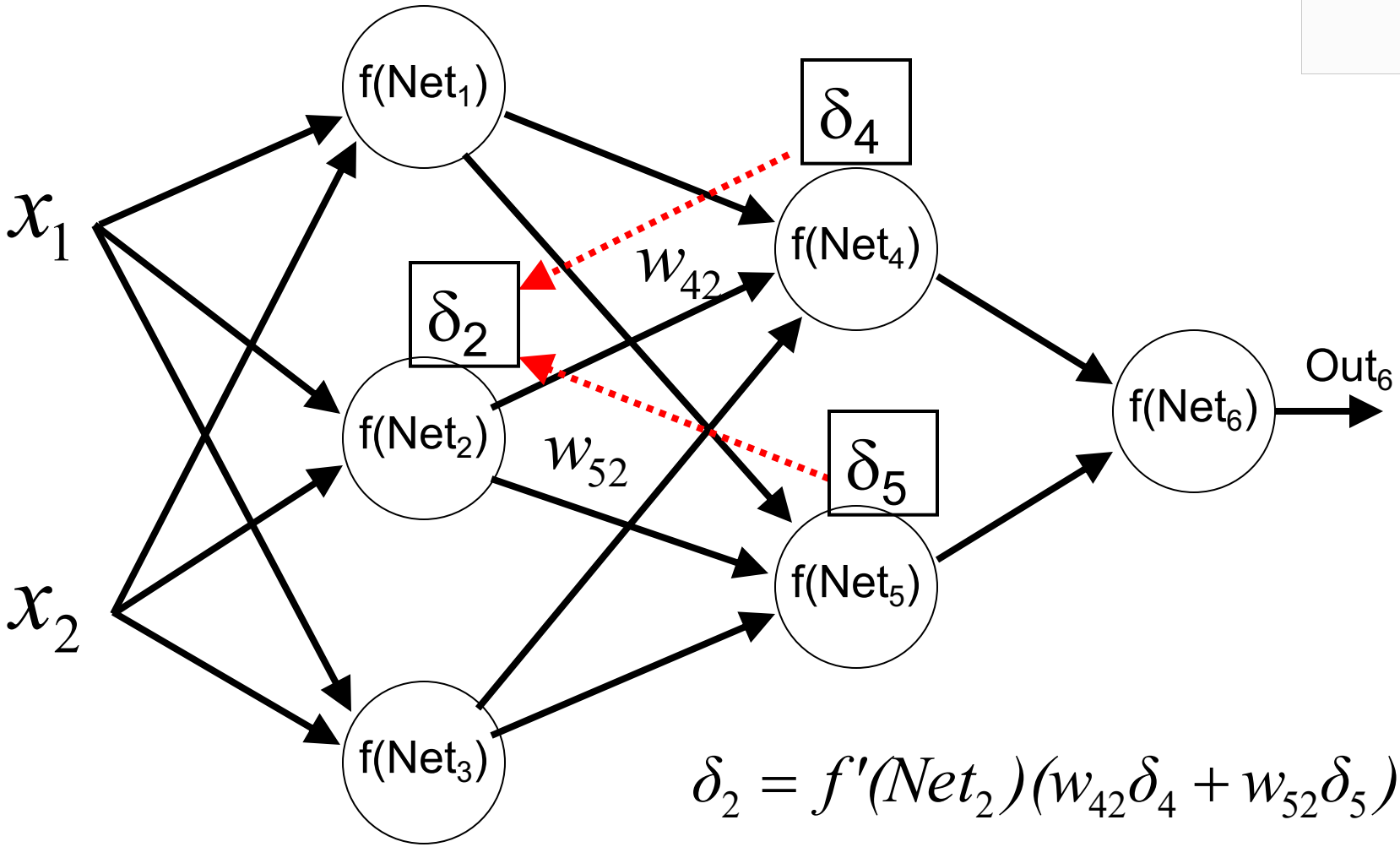

$$\delta_5 = f'(Net_5)(w_{65}\delta_6)$$

# BP algorithm: Backward(3)



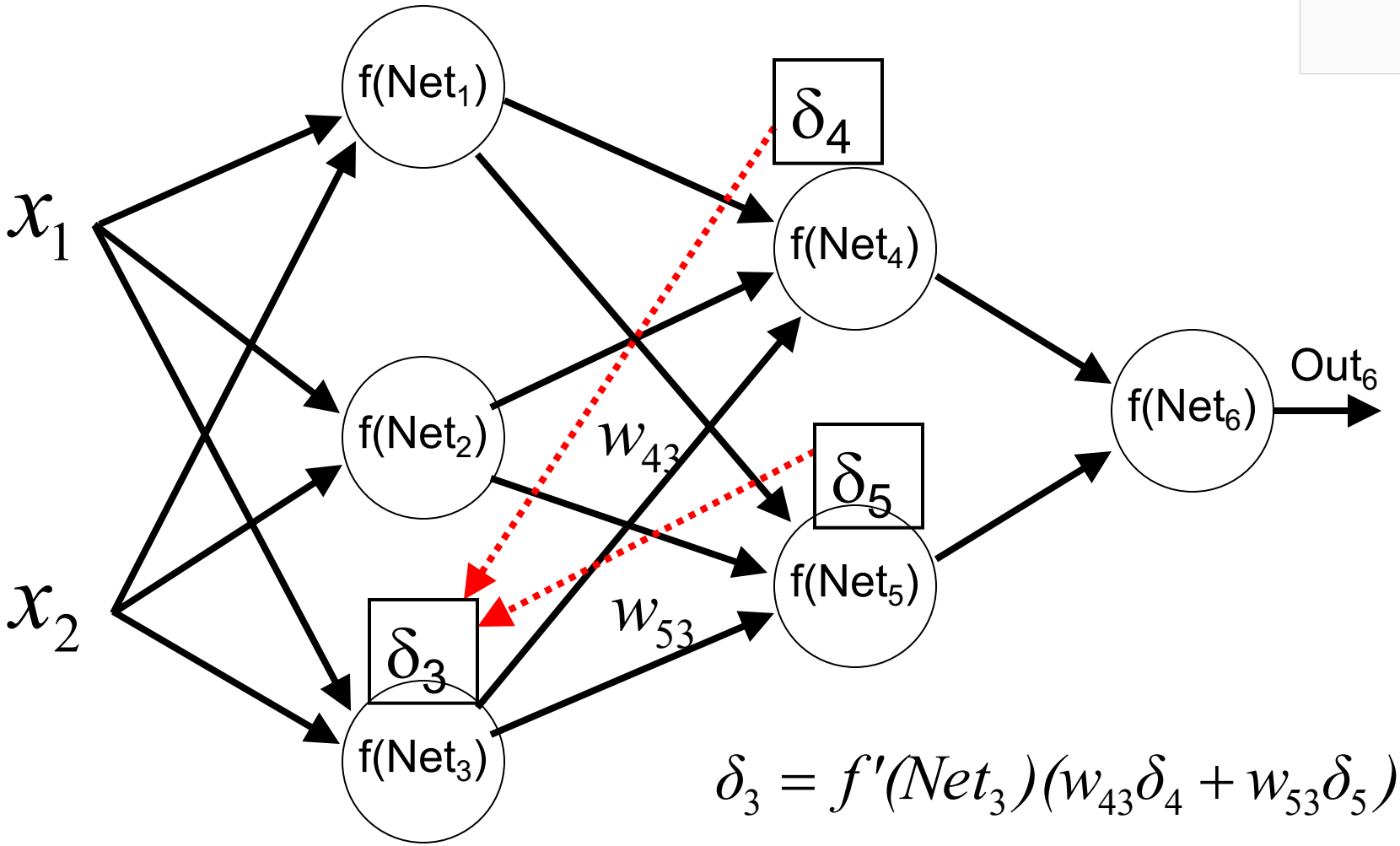

$$\delta_1 = f'(Net_1)(w_{41}\delta_4 + w_{51}\delta_5)$$

# BP algorithm: Backward(4)



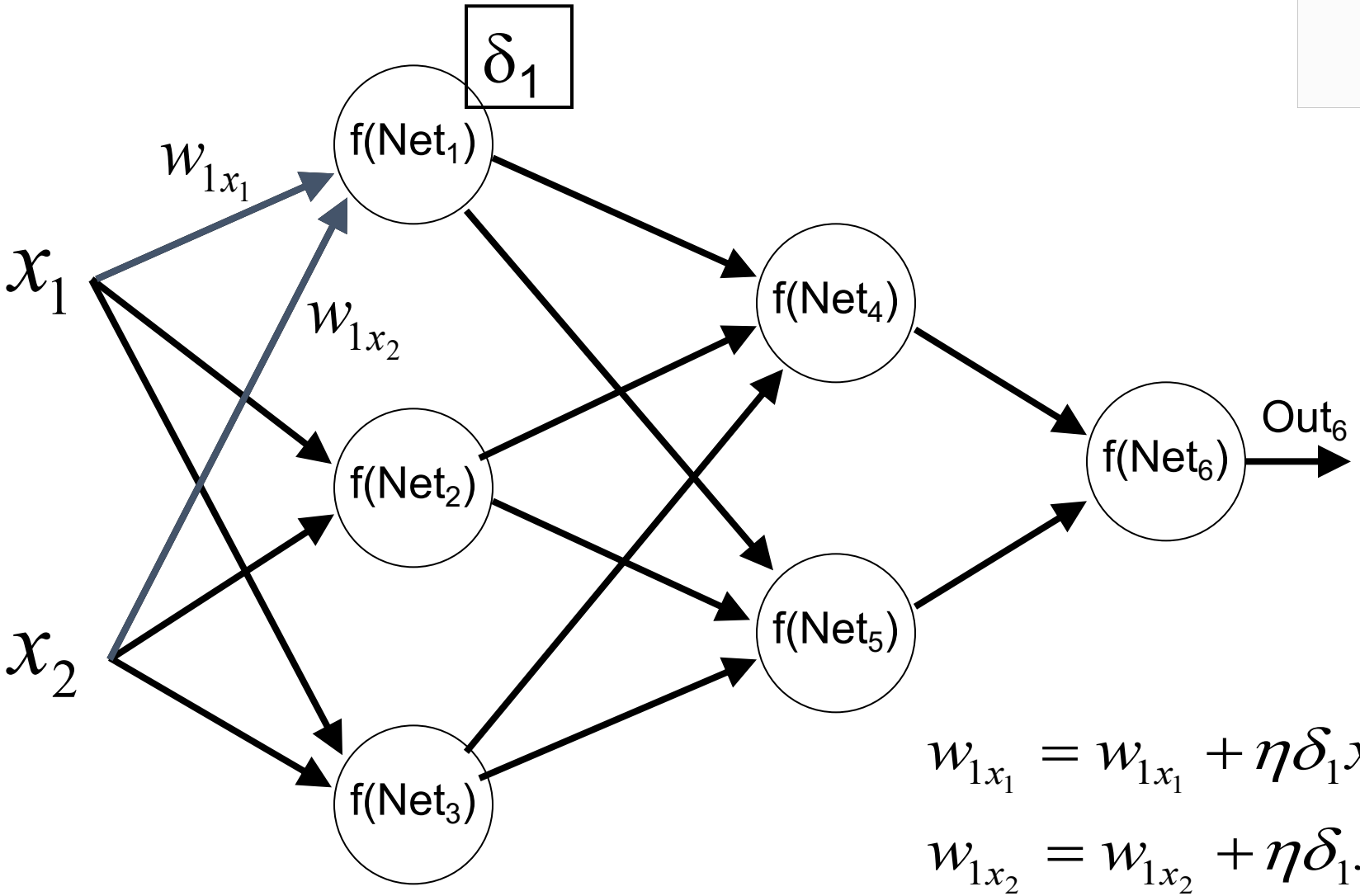

$$\delta_2 = f'(Net_2)(w_{42}\delta_4 + w_{52}\delta_5)$$

# BP algorithm: Backward(5)




$$\delta_3 = f'(Net_3)(w_{43}\delta_4 + w_{53}\delta_5)$$

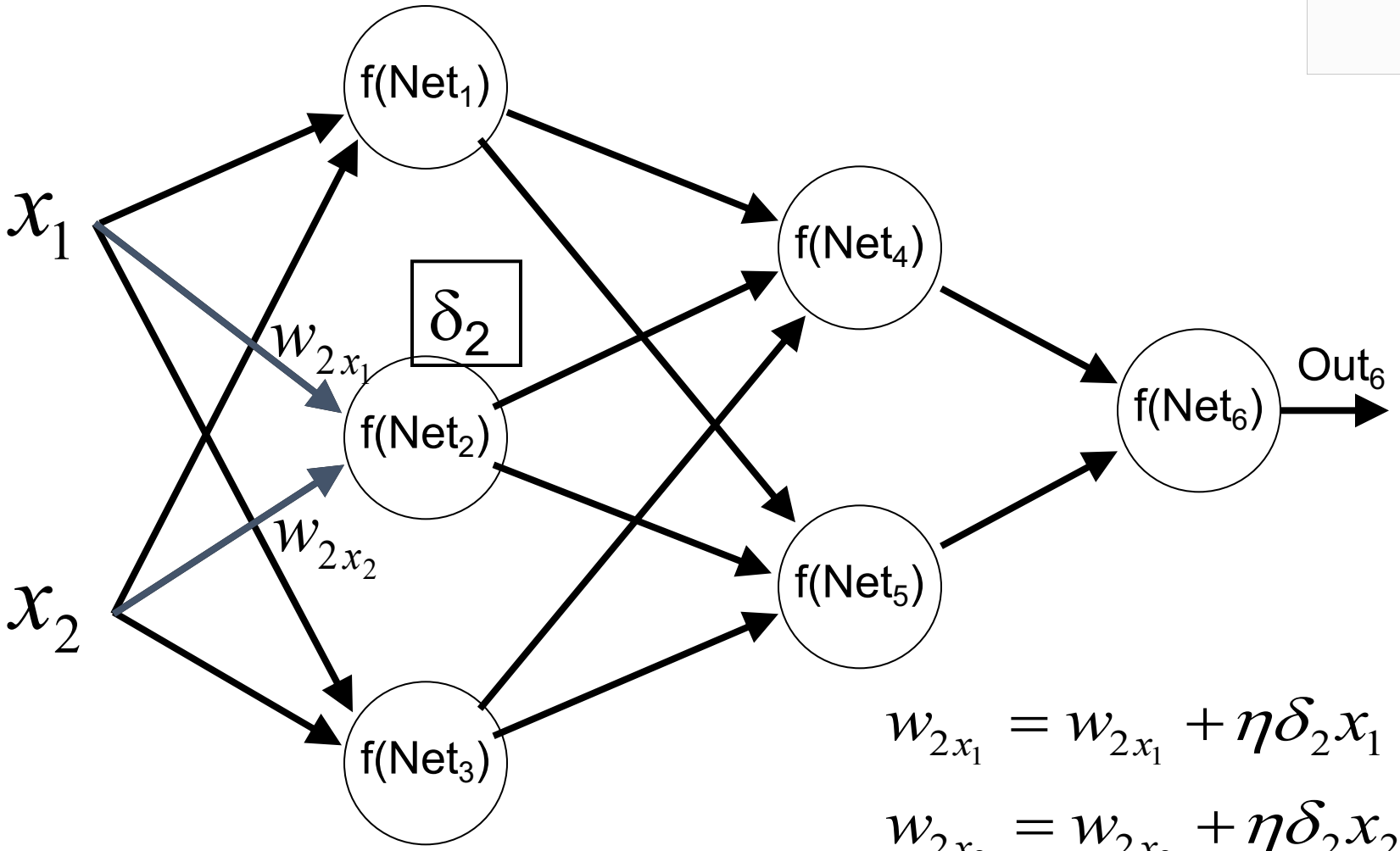
# BP algorithm: Update weight(1)




$$w_{1x_1} = w_{1x_1} + \eta\delta_1 x_1$$

$$w_{1x_2} = w_{1x_2} + \eta\delta_1 x_2$$

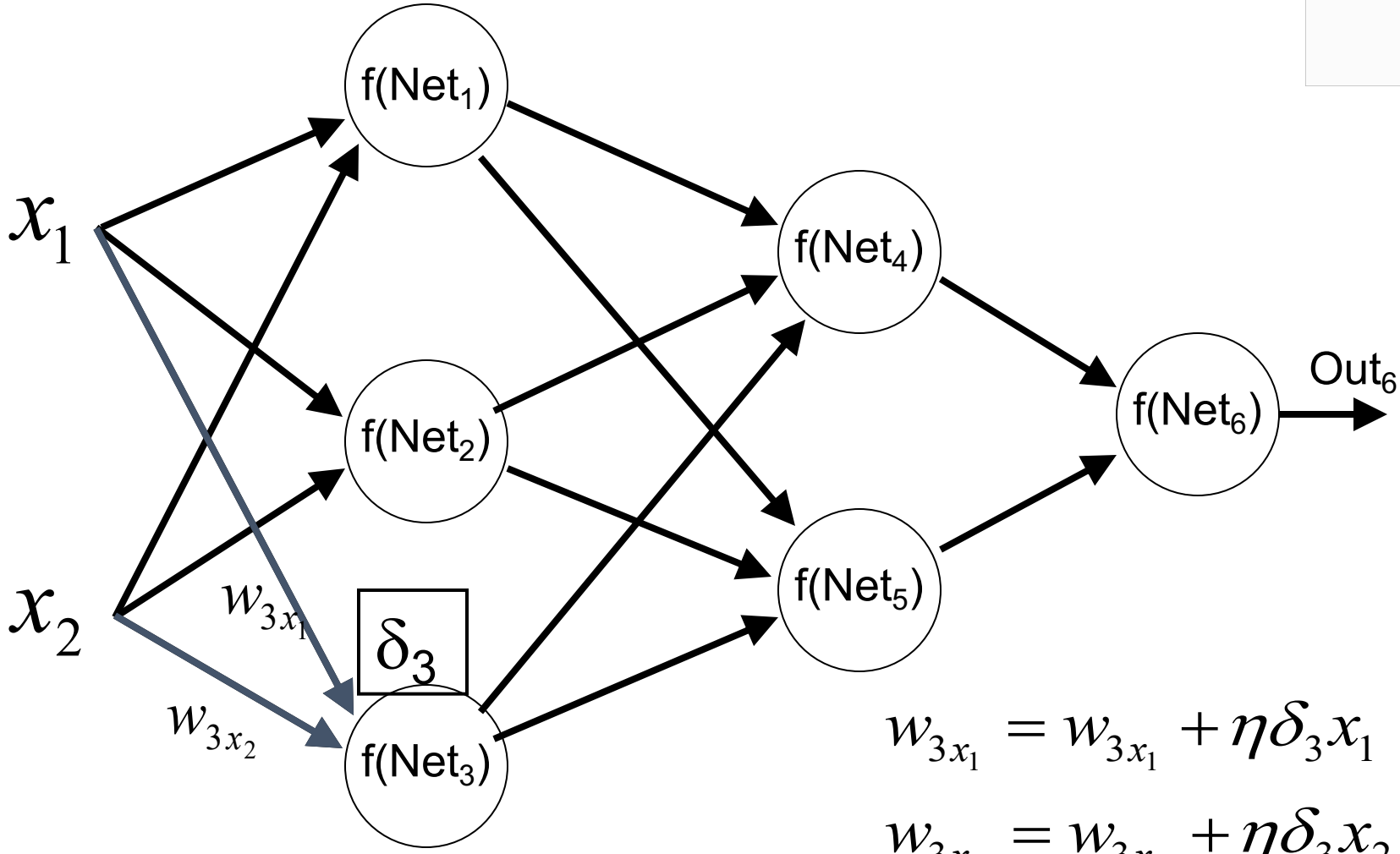
# BP algorithm: Update weight(2)




$$w_{2x_1} = w_{2x_1} + \eta\delta_2 x_1$$

$$w_{2x_2} = w_{2x_2} + \eta\delta_2 x_2$$

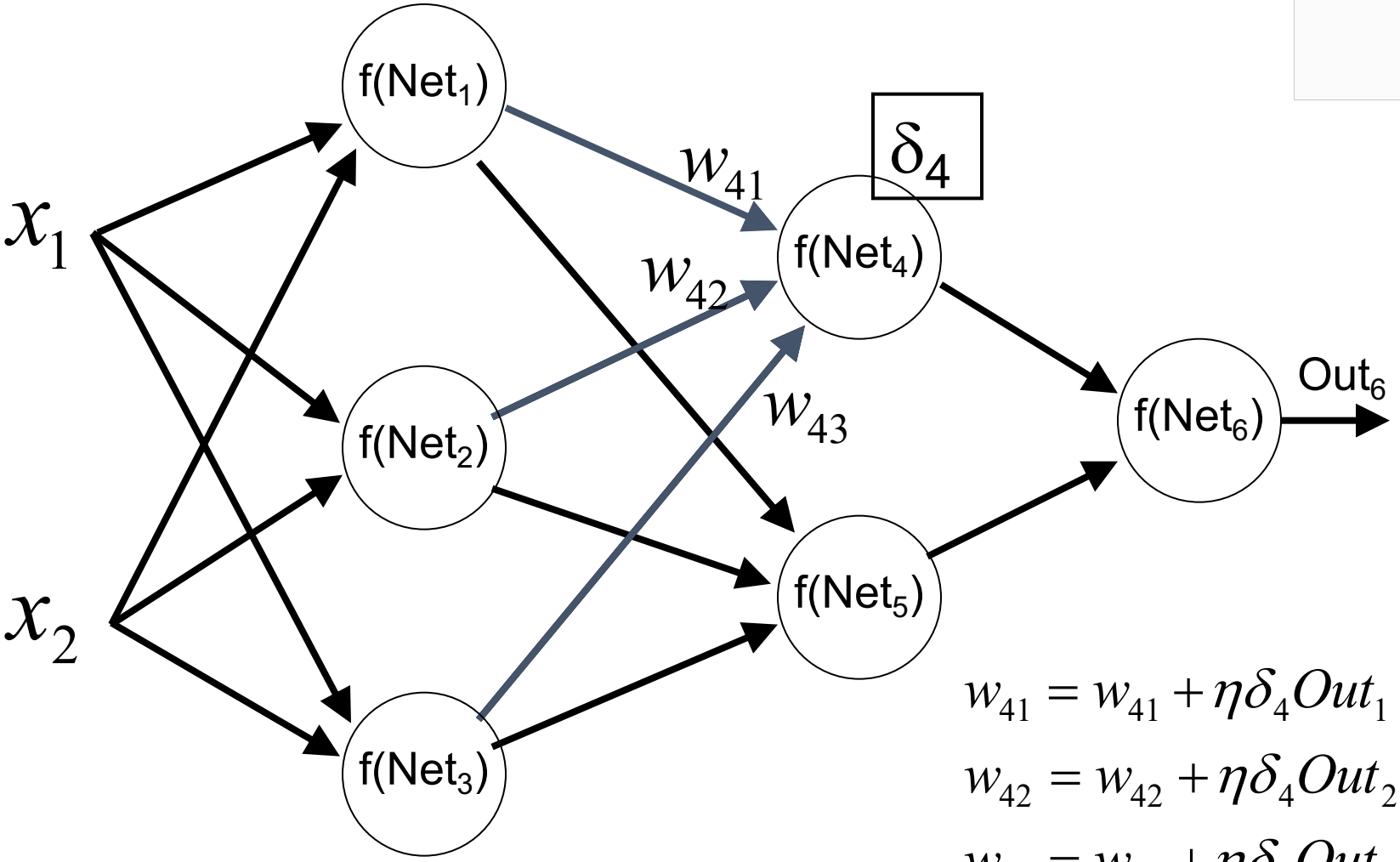
# BP algorithm: Update weight(3)




$$w_{3x_1} = w_{3x_1} + \eta\delta_3 x_1$$

$$w_{3x_2} = w_{3x_2} + \eta\delta_3 x_2$$

# BP algorithm: Update weight(4)

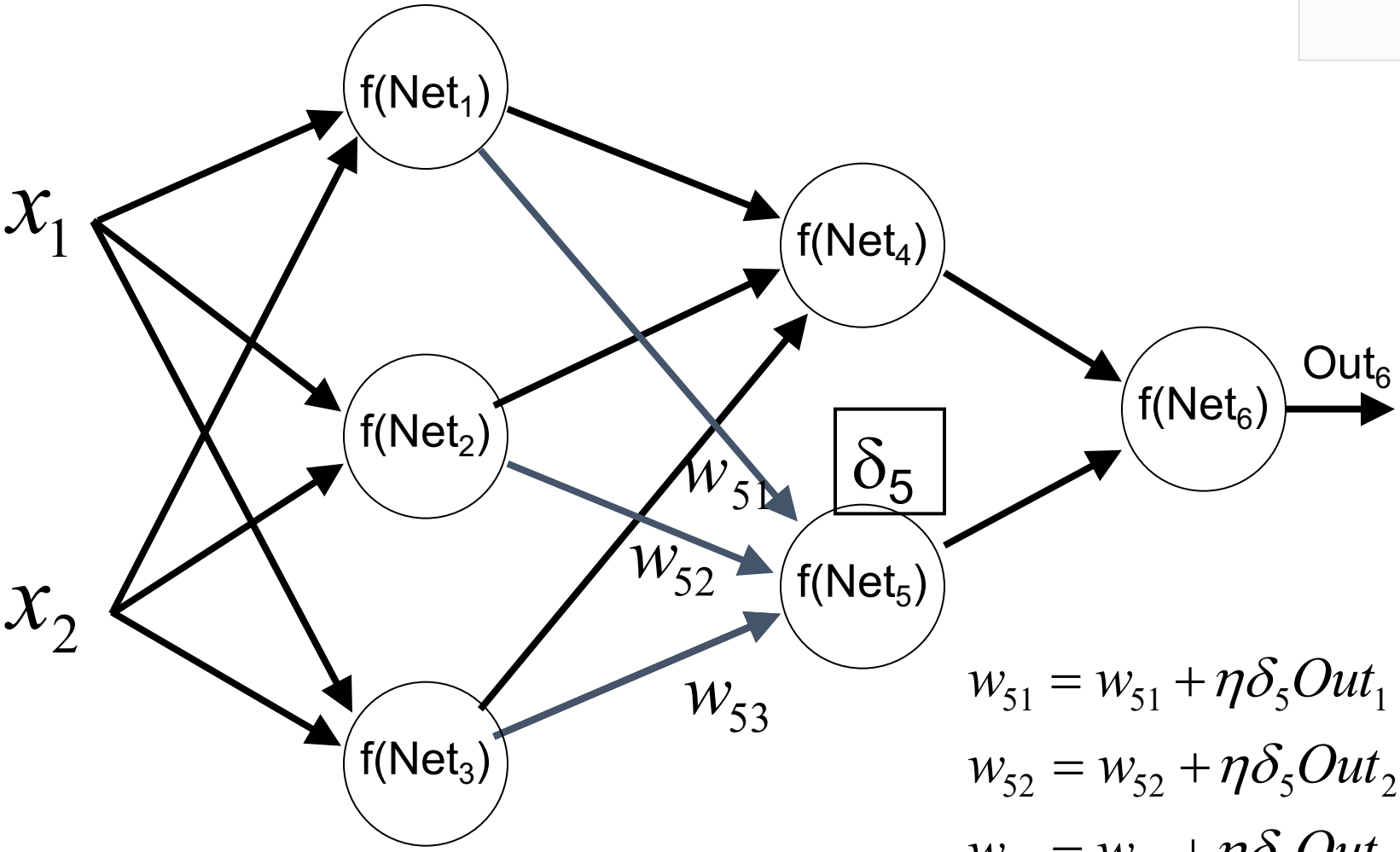



$$w_{41} = w_{41} + \eta\delta_4 Out_1$$

$$w_{42} = w_{42} + \eta\delta_4 Out_2$$

$$w_{43} = w_{43} + \eta\delta_4 Out_3$$

# BP algorithm: Update weight(5)

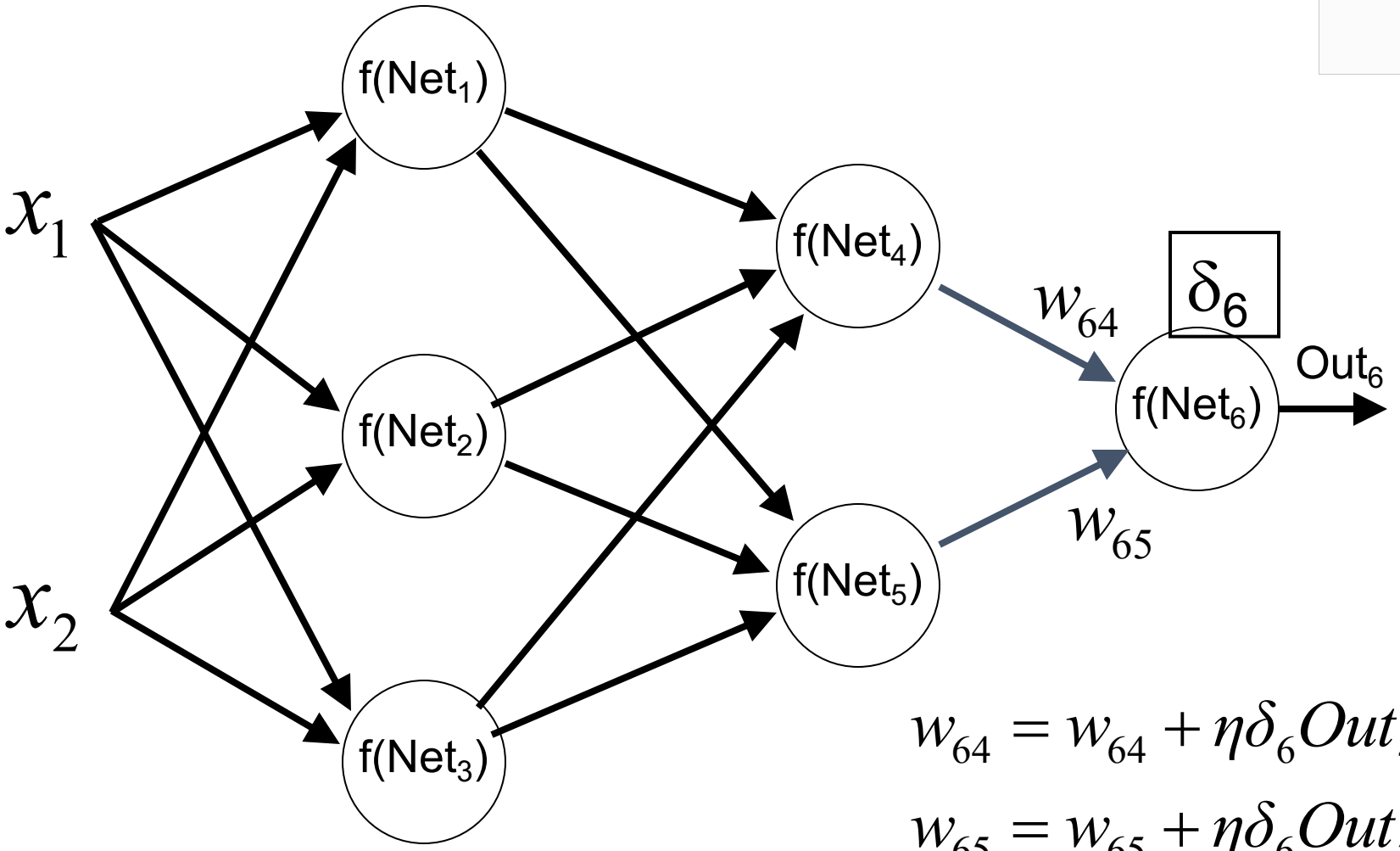



$$w_{51} = w_{51} + \eta\delta_5 Out_1$$

$$w_{52} = w_{52} + \eta\delta_5 Out_2$$

$$w_{53} = w_{53} + \eta\delta_5 Out_3$$

# BP algorithm: Update weight(6)




$$w_{64} = w_{64} + \eta\delta_6 Out_4$$

$$w_{65} = w_{65} + \eta\delta_6 Out_5$$

# BP algorithm: Initialize weights

- Normally, weights are initialized with random small values
- If the weights have large initial values
  - Sigmoid functions will reach saturation soon
  - The system will deadlock at a saddle / stationary points

# BP algorithm: Learning rate

- Important effect on the efficiency and convergence of BP algorithm
  - A large value of $\eta$ can accelerate the convergence of the learning process, but can cause the system to ignore the global optimal point or focus on bad points (saddle points).
  - A small $\eta$ value can make the learning process take a long time
- Often select it empirically
- Good values of learning rate at the beginning (learning process) may not be good at a later time
  - Using an adaptive (dynamic) learning rate?

# BP algorithm: Momentum

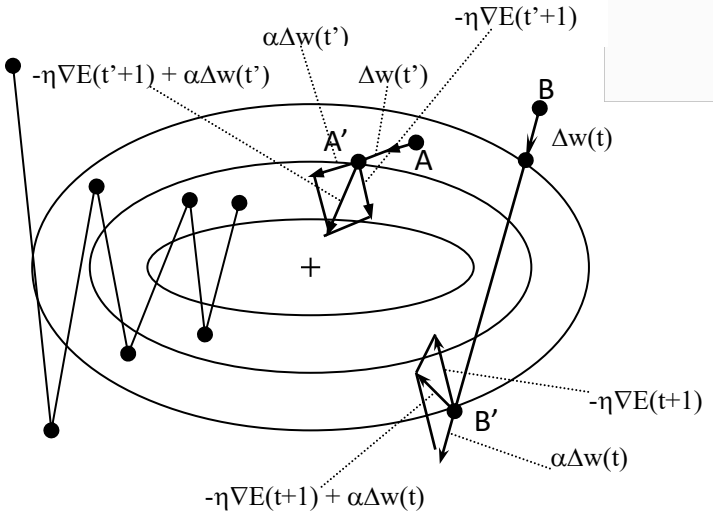
- The gradient descent method can be very slow if $\eta$ is small and can fluctuate greatly if $\eta$ is too large
- To reduce the level of fluctuations, it is necessary to add a momentum component

$$\Delta w^{(t)} = -\eta \nabla E^{(t)} + \alpha \Delta w^{(t-1)}$$

- where $\alpha$ ($\in[0,1]$) is a momentum parameter (usually assign = 0.9)
- We should choose reasonable values for learning rate and satisfying momentum

$$(\eta + \alpha) \gtrsim 1$$

where $\alpha > \eta$ to avoid fluctuations




Gradient descent for a simple square error function.

The left trajectory does not use momentum.

The right trajectory uses momentum.

# BP algorithm: Number of neurons

- The size (number of neurons) of the hidden layer is an important question for the application of multi-layer neural network to solve practical problems
- In fact, it is difficult to identify the exact number of neurons needed to achieve the desired system accuracy
- The size of the hidden layer is usually determined through experiments (experiment/trial and test)

# ANN: Learning limit

- Boolean functions
  - Any binary function can be learnt (approximately well) by an ANN using one hidden layer
- Continuous functions
  - Any bounded continuous function can be learnt (approximately) by an ANN using one hidden layer
    [Cybenko, 1989; Hornik et al., 1991]

# ANN: advantages, disadvantages

- Advantages
  - Nature (structure) supports high-level parallel computation
  - Obtain high accuracy in many problems (photos, video, audio, text)
  - Very flexible in network architecture
- Disadvantages
  - There are no general rules for determining the network architecture and optimal parameters for a given problem
  - There is no general method for assessing ANN's inner workings (thus, the ANN system is viewed as a "black box").
  - It is difficult (impossible) to give explanations to the user
  - Fundamental theories are few, to help explain the real successes

# ANN: When?

- The form of the function does not predetermined
- It is not necessary (or unimportant) to provide an explanation to the user about the results
- Accept long time for the training process
- Can collect a large number of labels for data
- Domains related to image, video, speech, text

# Open library

Keras

Caffe2

TensorFlow

PYTORCH

# References


- Cybenko, G. (1989) "Approximations by superpositions of sigmoidal functions", Mathematics of Control, Signals, and Systems, 2 (4), 303-314
- Kurt Hornik (1991) "Approximation Capabilities of Multilayer Feedforward Networks", Neural Networks, 4(2), 251–257