Chapter 2

THE STANDARD MODEL OF PARTICLE PHYSICS AND SUPERSYMMETRY

The LHC was designed to explore Higgs physics and some beyond-Standard-Model physics accessible at the TeV scale. Three among the four currently known fundamental interactions[1], namely quantum electrodynamics (QED), the weak and the strong interactions (also known as quantum chromodynamics, or QCD), are expected to come into full play — indeed abundantly, providing physicists with ample opportunities to explore what had not been possible before. The theoretical foundation underlying these interactions has been worked out by physicists in the second half of the 20th century and laid out in the Standard Model of particle physics, a mathematical framework that makes possible quantitative predictions of particle interactions. This chapter touches on some aspects of the Standard Model, including the principle of symmetry, which is an important organizing principle of the Standard Model.

The Standard Model, on the other hand, has a number of problems, and some of them are reviewed in the present chapter. Supersymmetry, one among several attempts to go beyond the Standard Model, provides solutions to these problems; it has been and still is being actively pursued at the LHC. The theory of supersymmetry, including some key points on phenomenology, is discussed briefly at the end of the chapter.

2.1 The Standard Model of Particle Physics

Elementary particles and their fundamental interactions, excluding gravity, are fully represented in the Standard Model [1–3]. The concept of symmetry plays an essential role, as each component of the Standard Model — QED, the weak interaction, and the strong interaction — can be obtained by imposing an appropriate symmetry, a so-called local gauge symmetry; in addition, each component also respects spacetime symmetries [4]. Section 2.1.1 discusses briefly spacetime symmetries as well as the

[1]Gravitational interactions are negligible at the LHC energy scale. However, as will be discussed later in the chapter, they are no longer negligible at the Planck scale where fundamental questions not only about gravity but also about other aspects of the Standard Model arise.

local gauge symmetries that underline the Standard Model, while Section 2.1.2 discusses in more detail the particle contents of the Standard Model as well as several of its aspects.

2.1.1 Symmetries

A symmetry is mathematically represented by a group. Spacetime symmetries are represented by the Poincaré group, and local gauge symmetries local gauge groups, both of which are discussed in the following.

2.1.1.1 The Poincaré Group

Spacetime symmetries include the Lorentz symmetry, which refers to the equivalence between inertial observers with regard to physical laws. If K and K' are inertial frames, the relativity principle requires physical laws as observed in K to be also physical laws as observed in K'. Thus, let x, y, z and t be the coordinates of an event as measured in K, and x', y', z' and t' those of the same event as measured in K'. According to special relativity, these coordinates are related by a Lorentz transformation

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$$

The Lorentz transformations form a group, called the Lorentz group. It is made up of transformations that leave the four-dimensional distance

$$\eta_{\mu\nu} x^{\mu} x^{\nu} = (ct)^2 - x^2 - y^2 - z^2$$

invariant. Physically, Lorentz symmetry concerns rotations and boosts between inertial frames.

To the Lorenzt symmetry may be added possible displacements of the origins of the frames, including time and spatial displacements,

$$x'^{\mu} = x^{\mu} + a^{\mu}$$

In this way, we obtain the full spacetime symmetry, or Poincaré symmetry. The corresponding group is called the Poincaré group.

A physical theory is said to be constrained by spacetime symmetry if its Lagrangian is invariant with respect to the Poincaré group. This concept will be illustrated below when we discuss gauge groups.

2.1.1.2 Gauge Groups

The known fundamental interactions in the Standard Model, namely quantum electrodynamics, the weak force, and the strong force, have been discovered to follow the principle of symmetry, in the sense that each of them can be obtained when an appropriate symmetry is required. The corresponding symmetry groups can be classified into abelian groups and non-abelian groups. In this section we review some general considerations in the use of such groups.

The Abelian group $U(1)$ The global $U(1)$ group arises when we consider a transformation of the form

$$\psi \to e^{i\theta}\psi, \tag{2.1}$$

where θ is a real number. The gauge principle turns θ into a function of spacetime coordinates, $\theta(x)$, and the resulting transformation is called a local gauge transformation. The Lagrangian of the theory is then required to be invariant under the local gauge transformation.

We will consider as an example the Dirac Lagrangian

$$L = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi,$$

which conforms to spacetime symmetry, but which is not invariant under the local gauge transformation, since the transformed Lagrangian is

$$L' = L + \bar{\psi}\gamma_{\mu}\psi(\partial^{\mu}\theta).$$

To come up with the new Lagrangian that would be invariant, we introduce a new field, called a gauge field and denoted A_{μ}, together with the covariant derivative operation

$$D_{\mu} = \partial_{\mu} + ieA_{\mu},$$

and the requirement that under the local gauge transformation, the A_{μ} would have to transform according to

$$A_{\mu} \to A'_{\mu} = A_{\mu} + \frac{1}{e}\partial_{\mu}\theta.$$

The kinematics of A_{μ} will be taken into account in the new Lagrangian through the term

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where, by definition,

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}.$$

The new Lagrangian that would be invariant under the local gauge transform is

$$L = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

This Lagrangian is actually the Lagrangian of QED. The field A_{μ} represents the photon. If the covariant derivative is expanded we will see in the Lagrangian the term $e\bar{\psi}\gamma_{\mu}\psi A^{\mu}$ which involves not only the photon term A_{μ} but also the charged fermion terms $\bar{\psi}$ and ψ; it represents the elementary electromagnetic interaction and expresses the fact that at the most fundamental level currently known, electrodynamic interactions are to be understood in terms of one simple elementary interaction that always involves a photon and a pair of charged fermions. This knowledge is also usually expressed graphically in Figure 2.1 where the wiggly line represents the photon, and the two straight lines with arrows represent the charged particles. This single diagram encodes different possibilities, we may understand it as the annihilation of two charged particles in which a photon is seen at the end, or a process in which a charged particle radiates a photon and turns to an antiparticle, or a process where the photon radiates a pair of particle-antiparticle.[2]

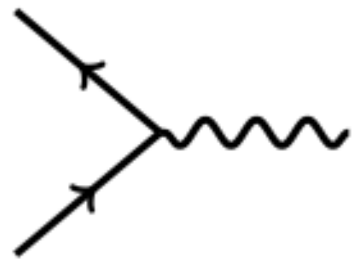

Figure 2.1: The Elementary QED vertex

Non-Abelian Groups $SU(n)$ $SU(n)$ may be understood as the group of all $n \times n$ unitary matrices whose determinants equal 1. Such a matrix, say U, may be written in the form

$$U = \exp\bigg(-i\frac{T^a}{2}\alpha^a \bigg)$$

The gauge principle turns the parameters α^a's into functions of spacetime coordinates $\alpha^a(x)$'s. The T^α are called the generators of the group and satisfy the relations

$$[T^a, T^b] = if_c^{ab}T^c$$

As in the case of QED discussed earlier, in order to write down a Lagrangian that would be invariant under the local gauge transformation $SU(n)$, we are forced to introduce new fields that represent bosons in the theory, the number of bosons correspond to the number of generators of the group, plus the covariant derivative operation and the kinematic terms involving these fields. The possible elementary interactions of the theory may then be read off by looking at the various terms in the Lagrangian. In QCD, for example, where the gauge group is $SU(3)$, there are eight generators that correspond to eight gluons in the theory. Here, in addition to a quark-gluon vertex, in Figure 2.2, of the type seen in QED, there are the three-point vertex and four-point vertex, in Figure 2.3, that correspond to gluon self-interactions; these additional self-interactions are characteristic of non-Abelian interactions.

[2]The last process, more particularly photon to electron-positron pair, is important to physics at the LHC.

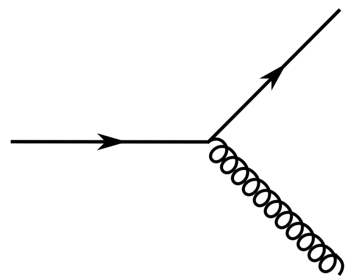

Figure 2.2: The QCD quark-gluon vertex

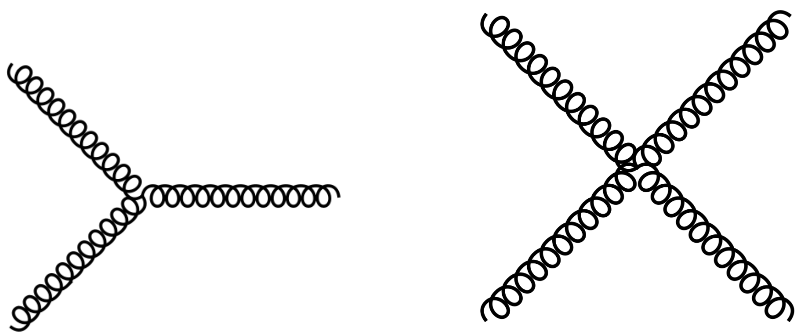

Figure 2.3: The QCD gluon self-interactions

2.1.2 The Standard Model Particles and Forces

The Standard Model is the quantitative implementation of the idea that physics is to be understood in terms of a small number elementary particles and their fundamental interactions. The elementary particles are classified into fermions and bosons. Fermions are further classfied into three families, each family is made up of a pair of leptons and two quarks; they are listed in table 2.1.

	Leptons				Quarks			
	Particle		Mass	Charge	Particle		Mass	Charge
I	electron	e	0.511 MeV	-1	Up	u	2.3 MeV	$+\frac{2}{3}$
	e neutrino	ν_e	< 2 eV	0	Down	d	4.8 MeV	$-\frac{1}{3}$
II	muon	μ	105.658 MeV	-1	Charm	c	1.275 GeV	$+\frac{2}{3}$
	μ neutrino	ν_μ	< 2 eV	0	Strange	s	95 MeV	$-\frac{1}{3}$
III	tau	e	1776.82 MeV	-1	Top	t	173.07 GeV	$+\frac{2}{3}$
	τ neutrino	ν_τ	< 2 eV	0	Bottom	b	4.18 MeV	$-\frac{1}{3}$

Table 2.1: The Standard Model fermions. All are spin 1/2 particles

The only difference between the families is the masses of the particles, and it is still an open question why three families in fact exist.

The bosons, on the other hand, mediate the forces between the fermions. They carry integer spins, and are listed in table 2.2.

Particle		Mass	Charge	Spin
Photon	γ	–	0	1
$W^{\pm}$		80.385 GeV	± 1	1
Z		91.1876 GeV	0	1
Gluon	g	–	0	1
Higgs	h	125.9 GeV	0	0

Table 2.2: The Standard Model bosons. All have integer spins

The fundamental interactions are QED, QCD, and weak interactions, mediated by the photon, the gluons, and the weak gauge bosons respectively. QED and the weak theory have been unified into a single electroweak theory. The relevant gauge groups are $SU(2)\otimes U(1)$ for electroweak and $SU(3)$ for QCD. It is a general property of gauge theories that the gauge bosons are massless. The photon and the gluons are massless, but the weak bosons are not. This discrepancy was resolved with a mechanism known as spontaneous symmetry breaking [5–7]. The result is the introduction of a new scalar field, the Higgs field, whose interactions with elementary particles would give them masses. Electroweak symmetry is said to be broken into QED symmetry around the energy 246 GeV. The Higgs particle was discovered in 2012 [8, 9], its mass has been measured to be $\sim$ 125 GeV, giving a confirmation of electroweak unification and, at the same time, motivates the hope for the unification of all three Standard Model interactions.

2.2 Beyond the Standard Model

The Standard Model has been tested very extensively in terms of its quantitative predictions of elementary particle interactions, and has hitherto withstood all the tests. It, however, is not a physics theory where many of our important questions about the physical world can be understood satisfactorily. This section discusses some problems that cannot be answered within the framework of the Standard Model; it also discusses supersymmetry, an attempt to go beyond the Standard Model to address some of the questions that we still have.

2.2.1 Problems with the Standard Model

Gravity Gravitational interactions were first proposed by Newton. A mass M was postulated to exert an attractive force on another mass m, given by

$$\mathbf{F} = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector pointing from M to m, and G the gravitational constant. Einstein proposed a fundamental change to gravitational interactions where forces are completely eliminated. In general relativity there is a direct link between the distribution of matter and energy in a spacetime region to the geometry of spacetime, the link being given according to the equation

$$G_{\mu\nu} = -\kappa T_{\mu\nu}$$

where $\kappa = 8\pi G/c^4$, and

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

is the Einstein tensor, which has been written in terms of the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R, both of which are functions of the metric tensor $g_{\mu\nu}$ which characterizes a geometry. In this new scheme the paths of objects follow the geodesics of a spacetime geometry.

At present, gravitational interactions are not accounted for in the Standard Model. Thus, as long as we are still searching for a single, all-encompassing theoretical framework to address all of our questions about the physical world, the Standard Model is not a complete theory.

The hierachy problem [17, 18] At the energy scale of the LHC, gravitational interactions are completely negligible. A rough comparison between the gravitational force between two equal masses M separated by a distance r, which is GM^2/r^2, with the electrostatic force between two charges $|e|$ separated by a distance r, which is e^2/r^2, will indicate the relative weakness of gravity. Indeed, taking as the unit of mass $Mc^2 = 1$ GeV, the electromagnetic coupling

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{136.036}$$

is to be compared with

$$\frac{GM^2}{4\pi\hbar c} = 5.3 \times 10^{-40}.$$

It follows that gravity is negligible at the GeV or TeV scale. In fact, gravitational interactions are negligible up to the Planck scale $(hc/G)^{1/2} \sim 10^{19}$ GeV. Given that there are only four forces currently known that span from the scale of a few hundreds GeV to the Planck scale, it might seem reasonable to assume that the Standard Model physics is valid up to Planck scale, i.e. there is no new physics up to Planck scale. However, it has been pointed out that this assumption leads to the following issue.

In quantum field theory the physical mass of an elementary particle is a sum of its bare mass plus corrections due to interactions. The Higgs is self-interacting and due to its mass receives a major correction from self-interaction; in addition it receives a major correction from its interaction with the top quark, the heaviest Standard Model particle. If μ denotes the Higgs mass, μ_B its bare mass, then the corrections have been determined to take the form

$$\mu^2 \simeq \mu_B^2 + \frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \dots$$

where Λ is the momentum scale up to which corrections are applied, λ is the Higgs coupling strength, and y_t is the coupling strengh between the top and the Higgs. If Λ is taken to be the Planck scale, the corrections have to be extremely precise to fit the physical Higgs mass $\mu \sim 100$ GeV. This has been judged to be very unnatural; in addition, the fact that the corrections are so much bigger than the Higgs mass itself has also been considered unsatisfactory.

Dark matter Astronomical and cosmological measurements accumulated over the years [10–13] have argued overwhelmingly for the inadequacy of ordinary matter to account for the total matter in the universe. Indeed, it is currently estimated that Standard Model particles account for about only 5% of all matter in the universe, while dark matter and dark energy account for the rest, about 27% and 68% respectively [14, 15]. At present, the nature of dark matter is still unknown, even though there have been many indirect cosmological evidences that point to its existence. Thus, for instance, theoretically we expect to see smaller rotational velocity of objects that are increasingly distant from the galaxy to which they belong, shown by the dashed line in Figure 2.4. Actual measurements, however, have shown that the rotational curve is rather flat, as indicated by the solid line in the same figure. It is thus concluded that there is invisible mass that not only cannot be seen but is also distributed differently from ordinary matter.

There are many other examples as well, among which gravitational lensing furnishes another convincing evidence that indicates the existence of dark matter. The amount of deflection of light from distant galaxies may be used to estimate the amount of matter in the galaxy clusters between the Earth and the distant galaxies, and has led to the conclusion that the galaxy clusters are in co-existence with an enormous amount of dark matter [10–15].

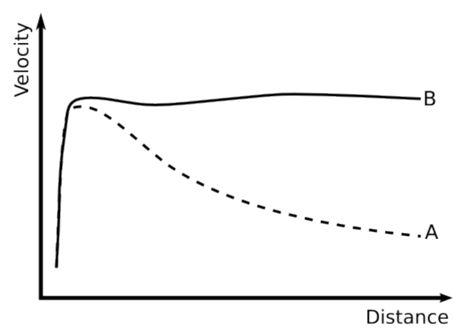

Figure 2.4: The rotational velocity of spiral galaxy with distance [16].

The majority of dark matter is expected to be cold dark matter made up of unrelativistic particles. Most Standard Model particles are not dark matter candidates except neutrinos, which are both stable and weakly interacting. However, neutrinos are relativistic particles and might only account for the so-called hot dark matter, which is only a fraction of the total amount of dark matter.

2.2.2 Supersymmetry

Supersymmetry [91–96] is an extension of the Standard Model that offers potential solutions to many currently unsolved problems [19–23]. It started with the question whether or not spacetime symmetry, the Poincaré group, can be extended in a non-trivial way. This is to be contrasted with gauge symmetries, which are trivial extensions of spacetime symmetries, in the sense that the generators of the gauge groups commute with the generators of the Poincaré group.

This section gives a brief discussion of supersymmetry. For a full reference, see [24].

The Poincaré Algebra and Supersymmetry The Poincaré group, as discussed in Section 2.1.1.1, is made up of the Lorentz group and the group of spacetime translations. The elements of the groups are functions of ten real continuous parameters, six coming from rotations and boosts in the Lorentz group, and four from the translation group. Mathematically the Poincaré group is associated with a set of ten generators, six associated with the Lorentz group and usually denoted $J^{\mu\nu}$, and four associated with the translation group, which will be denoted P^{μ}. Among these generators there exist commutation relations

$$[P^{\mu}, J^{\rho\sigma}], \quad [P^{\mu}, P^{\nu}], \quad [J^{\mu\nu}, J^{\rho\sigma}]$$

whose expressions involve only the generators $J^{\mu\nu}$ and P^{μ}. The generators and the commutation relations are said to form the Poincaré algebra. The question of the extension of the Poincaré group becomes the question of whether or not new generators could be added to the existing set of generators, such that the new commutation relations that arise are not all trivial, and that they are expressions that involve only the old and the new generators.

The Poincaré algebra was found to be extensible, but on the condition that, when adding the new generators, we have to consider not only the commutation relations between the old and the new generators, but also the anticommutation relations among the new generators themselves. The result is a set of generators and commutation and anticommutation relations among them that form a system called the super-Poincaré algebra. One of the consequences that follows is that the new generators map bosons into fermions and vice versa. Theoretical considerations then require supersymmetric theory to contain only two possible multiplets, the chiral supermultiplet that consists of two scalar and two spinor fields, or the vector supermultiplet that consists of two spinor and two vector fields. It was found necessary, also on theoretical ground, to introduce one or more new particles for every Standard Model particle that differs by spin 1/2, called its superparners. The particles in a multiplet otherwise have the same mass and other quantum numbers.

Supersymmetry may be classified depending to the number of new generators that are added to the Poincaré generator. The case where there is only one new generator added is called $N = 1$ supersymmetry. The Minimal Supersymmetric Standard Model (MSSM) is $N = 1$ supersymmetry [24, 25], it is the extension of the Standard Model with the least possible number of new particles that need to be introduced. The particle contents are listed in Table 2.3.

Boson		Fermions		$SU(3), SU(2), U(1)$
Gluons	g	Gluinos	$\tilde{g}$	$(8, 1, 0)$
Gauge bosons	$W^{\pm}, W^0$	Gauginos	$\tilde{W}^{\pm}, \tilde{W}^0$	$(1, 3, 0)$
B boson	B	Bino	$\tilde{B}$	$(1, 1, 0)$
Sleptons	$\tilde{\nu}_L, \tilde{e}_L$	Leptons	ν_L, e_L	$(1, 2, -1)$
	$\tilde{\bar{e}}_L$		$\bar{e}_L$	$(1, 1, -2)$
Squarks	$\tilde{u}_L, \tilde{d}_L$	Quarks	u_l, d_L	$(3, 2, \frac{1}{3})$
	$\tilde{u}_R$		u_R	$(3, 1, \frac{4}{3})$
	$\tilde{d}_R$		d_R	$(3, 1, \frac{-2}{3})$
Higgs	H_d^0, H_d^-	Higgsinos	$\tilde{H}_d^0, \tilde{H}_d^-$	$(1, 2, -1)$
	H_d^+, H_u^0		$\tilde{H}_u^+, \tilde{H}_u^0$	$(1, 2, 1)$

Table 2.3: Particles in the MSSM

If supersymmetry exists, it has to be broken, for otherwise supersymmetric particles would have been detected alongside Standard Model particles. Supersymmetry is thought be broken spontaneously, in the same way the Standard Model electroweak theory is broken spontaneously. A more detail discussion on supersymmetry breaking is provided in [24].

Supersymmetry Phenomenology at the LHC At the LHC, a class of MSSM models known as R-parity conserving models figure predominantly. R-parity [24, 25] is a multiplicative quantum number defined by

$$P_R = (-1)^{3(B-L)+2S}$$

where B is the baryon number, L the lepton number, and S the spin. Then each Standard Model is assigned the value $+1$ while each supersymmetric particle the value -1. Phenomenologically, R-parity conservation implies that

- The lightest supersymmetric particle (LSP) is stable;
- All other supersymmetric particles decay into a state that has an odd number of LSPs;
- Supersymmetric particles are produced in pairs.

The LSP does not participate in known interactions and manifests as missing transverse energy, it is a good candidate for dark matter. At the LHC, both strong and electroweak interactions are expected to be sources of supersymmetric particles. Many searches for supersymmetric particles have been carried out since the start of the LHC, one of which will be discussed in Chapter 6 of this thesis.