Exercises

1 Introduction to Inverse Problems

- **Read:** Chapter 1. Chapter 2.1, 2.2.
- **Exercises:** (Some necessary codes can be found in `http://wiki.helsinki.fi/display/mathstatHenkilokunta/Inverse+Problems+Book+Page`)

1. In the following applications, which ones are inverse problems?
   (a) The monitor of the volcano activities (based on the magnetic field above the volcano).
   (b) Data fitting.
   (c) Calculation of areas and perimeters on geometric figures.
   (d) Restoration of cracks in old photographs.

   In addition, try to find 2 or 3 examples on inverse problems close to your dayly life.
2. Consider the following linear inverse problems in the form of $Ax = b$. Here, A is the forward model, b is the data (the effect), and $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is the state (the cause). Which requirement for being a well-posed inverse problem is violated? Any idea to overcome it and modify the problem as well-posed?
   (a) $A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 2.2 \end{pmatrix}$
   (b) $A = \begin{pmatrix} 1 & 1 \end{pmatrix}$ and $b = 1$
   (c) $A = \begin{pmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{pmatrix}$ and $b = A \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0.01 \\ -0.03 \\ 0.02 \end{pmatrix} = \begin{pmatrix} 0.27 \\ 0.25 \\ 3.33 \end{pmatrix}$
3. [MS] P14, Exercise 2.1.2.
4. [MS] P14, Exercise 2.1.3. Further, try with different choice of a in ψ_0 and different noise level, then observe how the naive reconstruction changes.
5. [MS] P21, Exercise 2.2.1. Write on paper