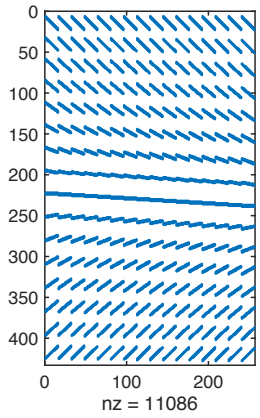
DAY 7

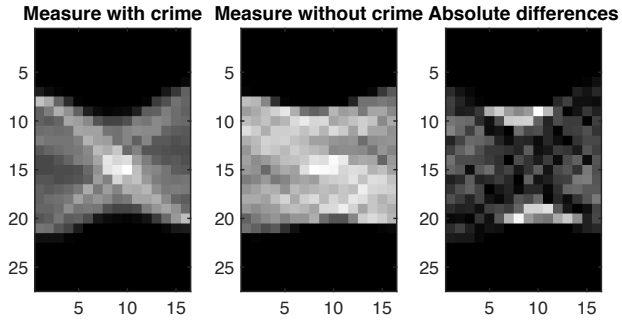
1 Exercise 2

1.1 Naïve reconstruction

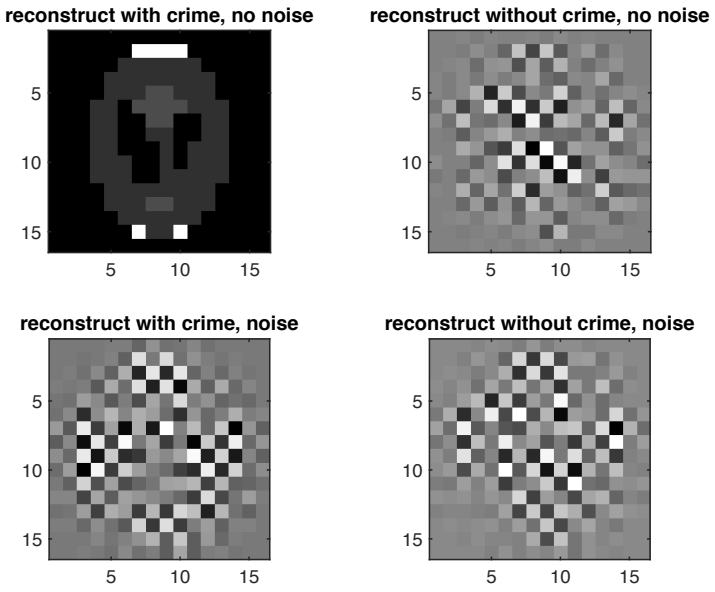
Run "XRMA_matrix_comp.m" with $N = 16$. Matrix A is sparse matrix:



Data with crime and without crime are shown in the below figure. The absolute differences are shown in the right figure.

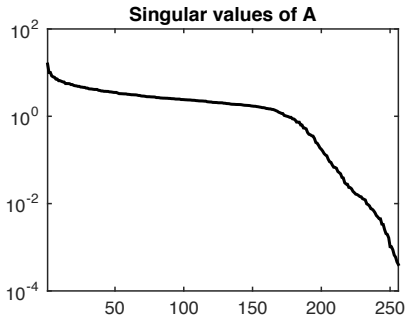


Naïve reconstruction of data with inverse-crime and without inverse-crime is shown in the below figure. We can see that the results are useless in case the measurement has noise.

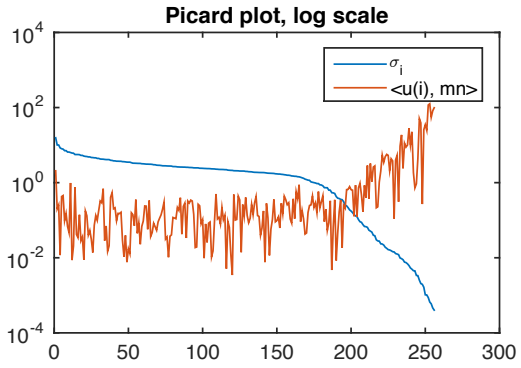


1.2 TSVD

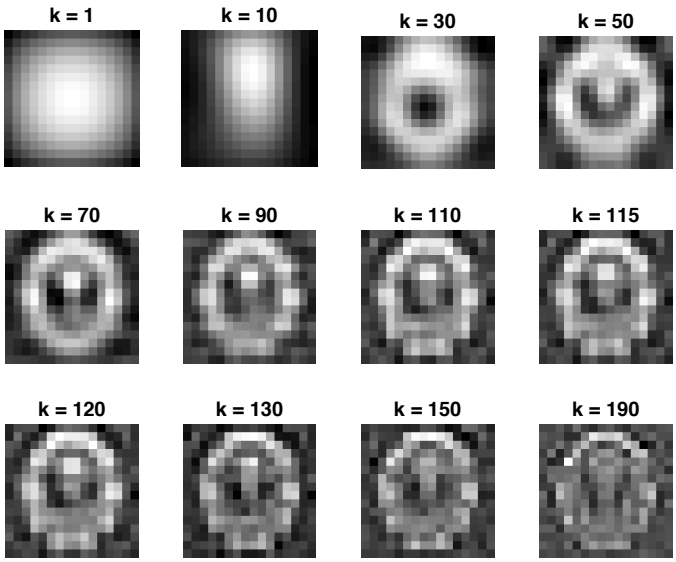
Decompose A to [U S V]. The decay of singular values is shown in below figure. The singular values decrease dramatically from the element 160.



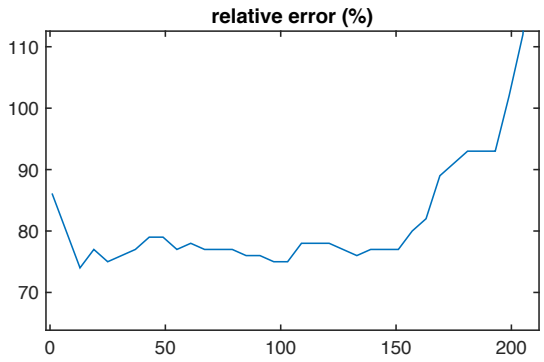
Picard plot: The decay speed of singular value σ_i and $|\langle u_i, b\rangle|$. We can see that the Picard condition break after around $k = 150$, when $|\langle u_i, b\rangle|$ starts having increase trend.



Reconstructed image in relation to number of truncated value

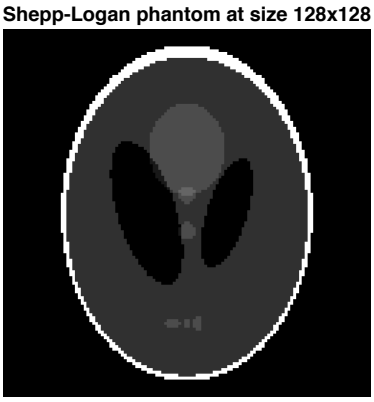


From observation, a good result is reconstructed with truncate value $k\sim130$. The relative error graph shows that the errors are quite same with $k = 30{:}\,150$. This may be the results of low resolution of test problem and high noise.

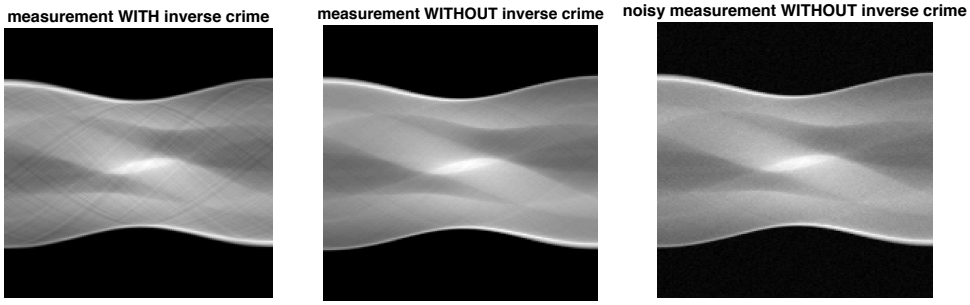


2 Exercise 3

2.1 Simulate tomographic data

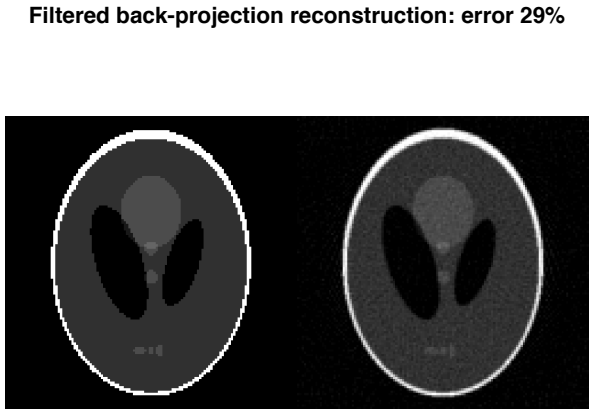


Dummy data



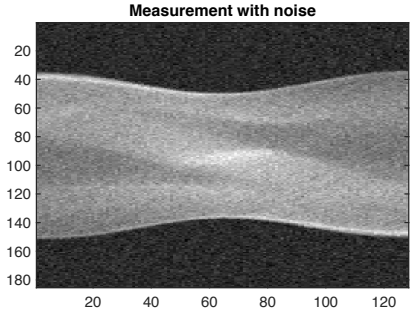
Dummy measurement

2.2 Reconstruction using back-projection

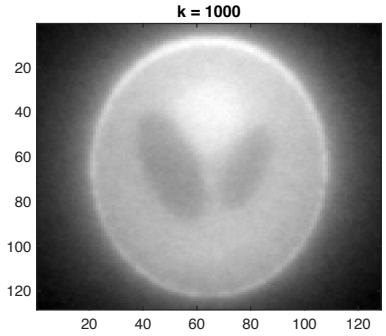


2.3 CGLS

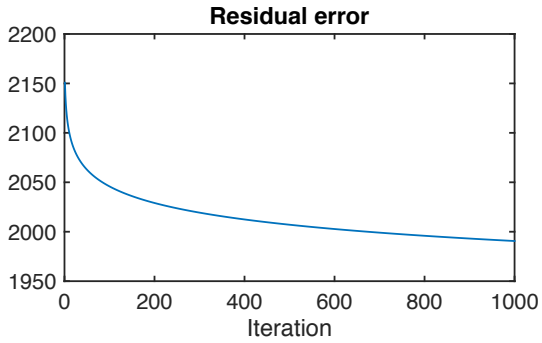
Modify CGLS by replacing: Ax by $radon(x, theta)$ and $A'b$ by $iradon(b, theta)$.



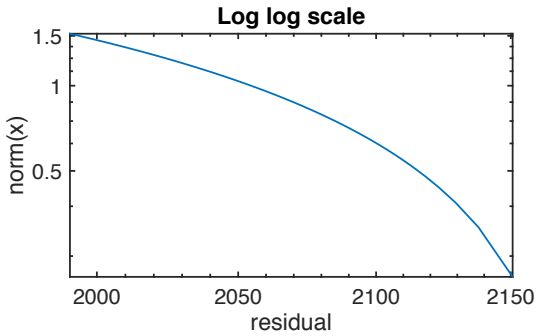
Constructed by CGLS



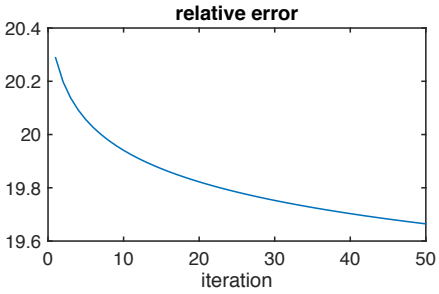
Residual error continues to decrease, but stay quite large



The L-curve is not formed



Relative error continues decreasing but still stay quite large.



2.4 Discrepancy principle

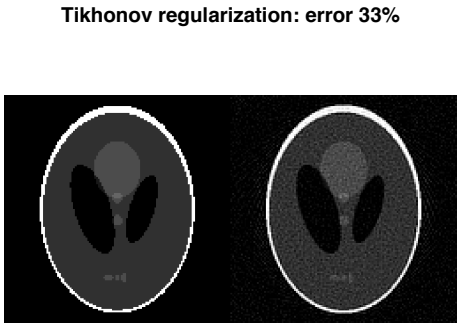
Pick k that's

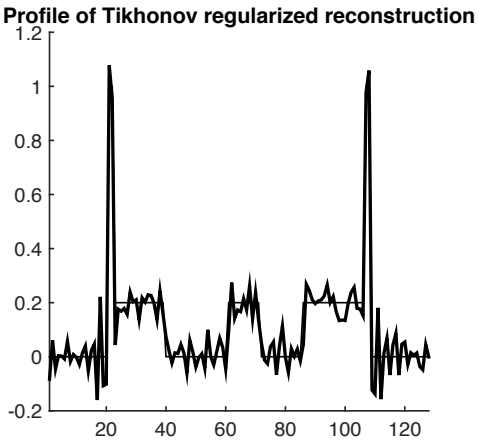
$$\|Ax_k - b\|_2 > \nu_{dp}\|e\|_2 > \|Ax_{k+1} - b\|_2$$

Here noise level is 1%: $\|e\|_2 = 0.01\|b\|_2 = 0.33$

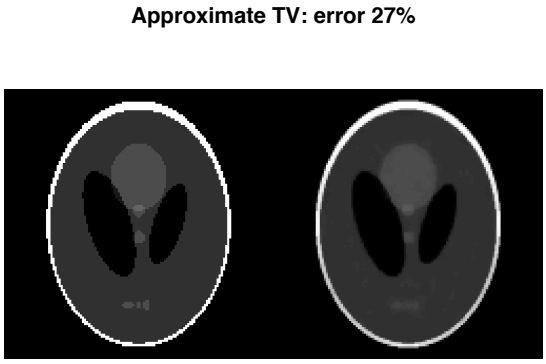
However, there is an error in implementation of CGLS that keep residual error high. I have not fix this.

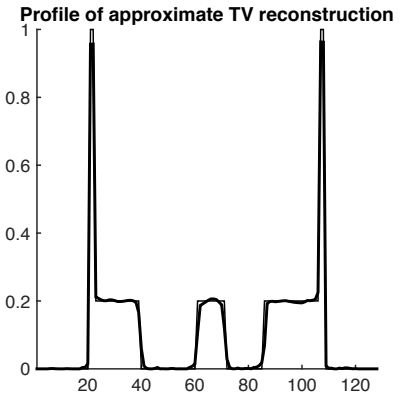
2.5 Matrix-free X-ray tomography

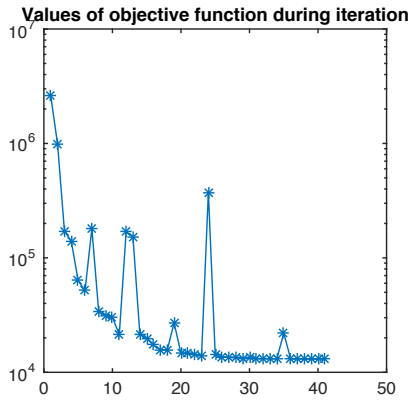




2.6 Total variation regularization







There is some jumps in the objective function.

2.7 Summary

Total variation gives the best solution because the target is piecewise constant.