# Tomography with Explicit Mesh

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## ABSTRACT

We reconstruct object from X-ray tomography data using explicit mesh (or Lagrangian mesh). In compare to normal methods, which use uniform grid points (or Eulerian mesh), Lagrangian mesh offers some advantages:

- Explicit information can be extracted easier
- Represent object with lower resolution of the mesh

Even though they are different in representation, the problem is still linear. This report applies regularization methods (TSVD and total variation) to explicit mesh model. We also evaluate the method with sparse data.

## EXPLICIT MODEL

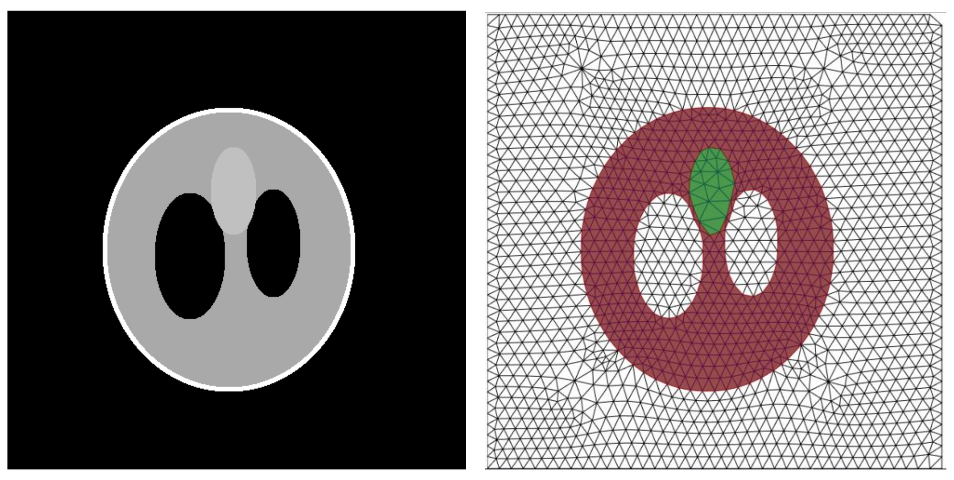
### *Explicit mesh representation*

We discretize the 2D domain to triangular mesh. The attenuation inside each triangle is constant.

There are two jobs

- Find attenuation in each triangle
- Refine the mesh to reserve edge

My work focuses on finding the attenuations. We assume the mesh already tracks the object.



Phantom object — Mesh representation

### *Forward model*

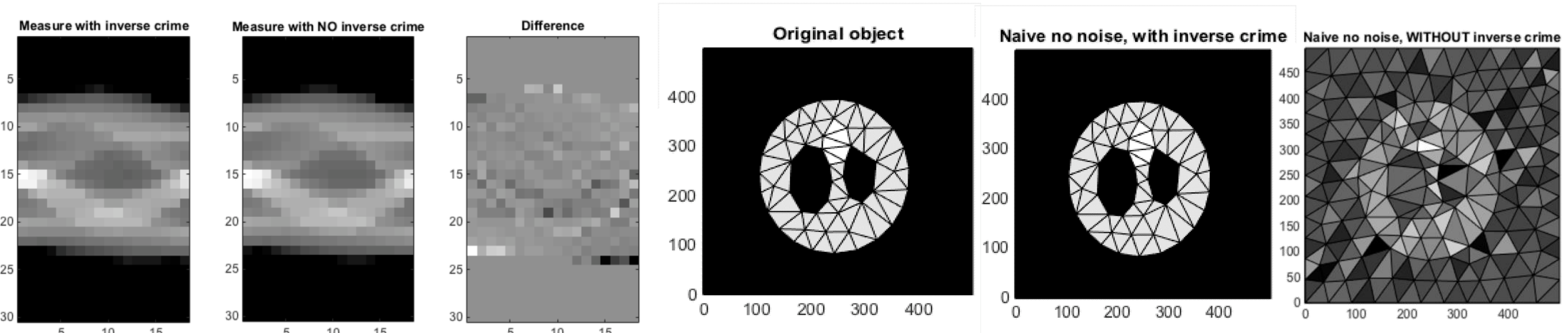
We compute each measurement base on intersection between ray and triangles. The relation between them are linear

$$m_i = \sum_{j=1}^{all\ intersections} l_j f_j \qquad \boldsymbol{Af = m}$$

The inverse problem can be solved with discrete inverse methods.

## AVOID INVERSE CRIME

Inverse crime: To avoid inverse crime, we construct forward model with higher resolution phantom.




## TOTAL VARIATION METHOD

Regularization term is total variation of attenuation

$$T(\alpha) = \min \|Af - m\|^2 + \alpha |\nabla f|$$

For triangle mesh, the total variation is

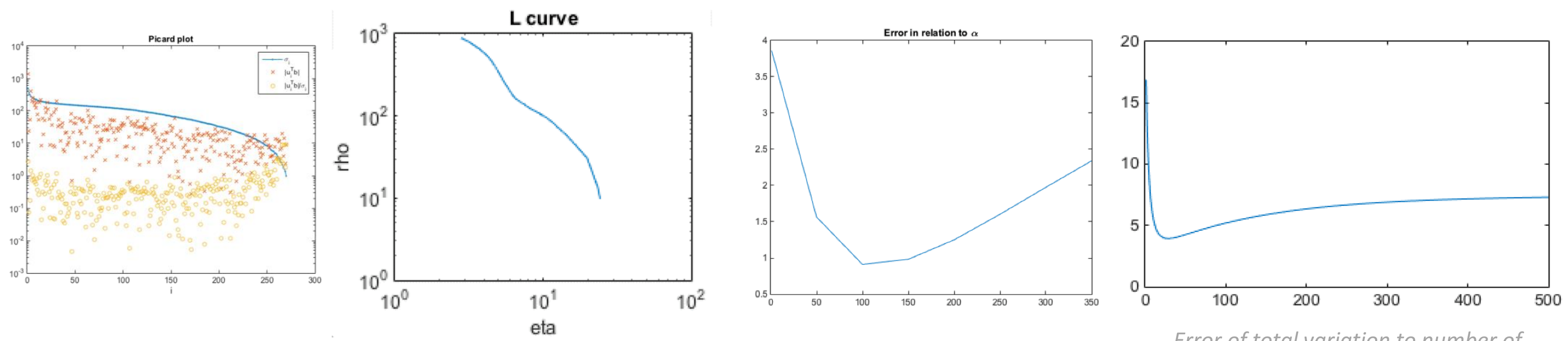
$$|\nabla f| = \sum_{i=1}^{no.\ of\ edges} length_{edge_i} |f_1 - f_2|$$

Apply conjugate gradient method with approximation $|t|_\beta = \sqrt{t^2 + \beta}$

$$\frac{\partial}{\partial f_i} = 2A^T A f - 2A^T m + \sum_{e=1}^{3} \frac{f_i - f_e}{(f_i - f_e)^2 + \beta}$$

## PARAMETERS CHOICE

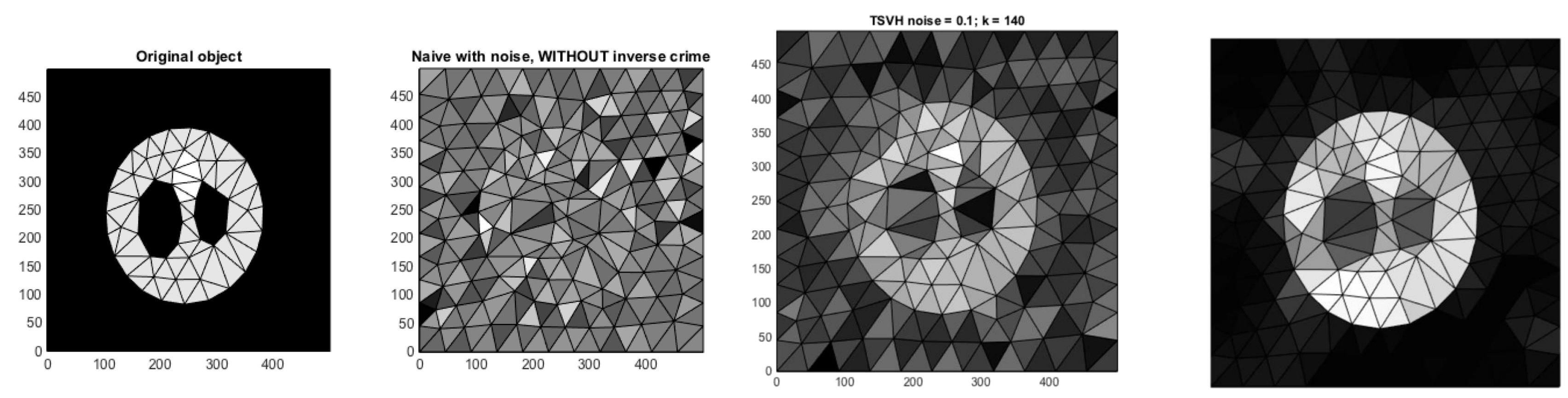
L-curve can not be applied. We use error to real solution to find optimal parameters




Picard plot, TSVD — L curve, TSVD — Error and coefficient α, total variation — Error of total variation to number of iterations, total variation

## COMPARE REGULARIZATION METHODS

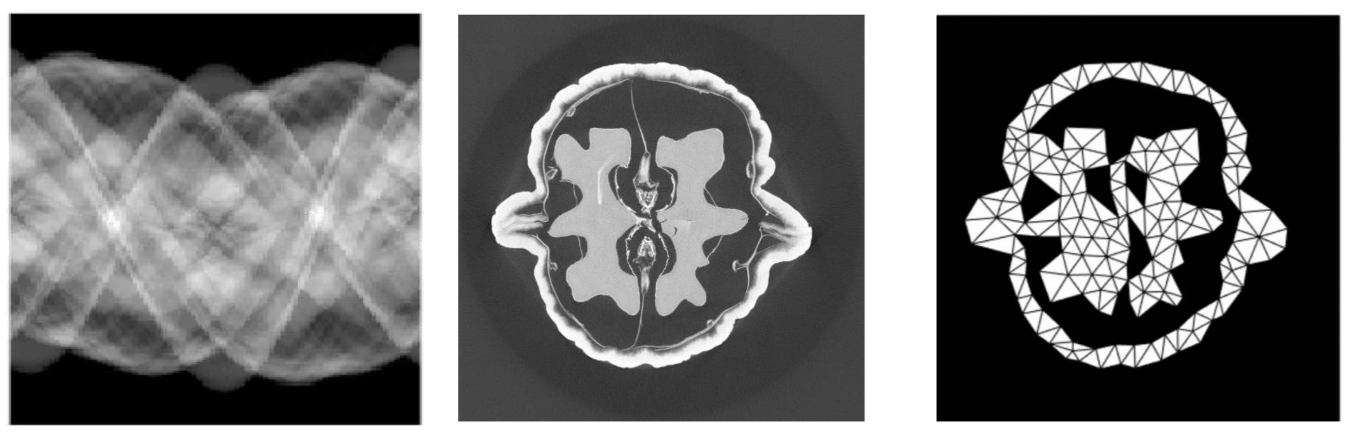
10x18 measurements; 284 triangle; 10% noise




Original object — Naive solution — TSVD solution — Total variation

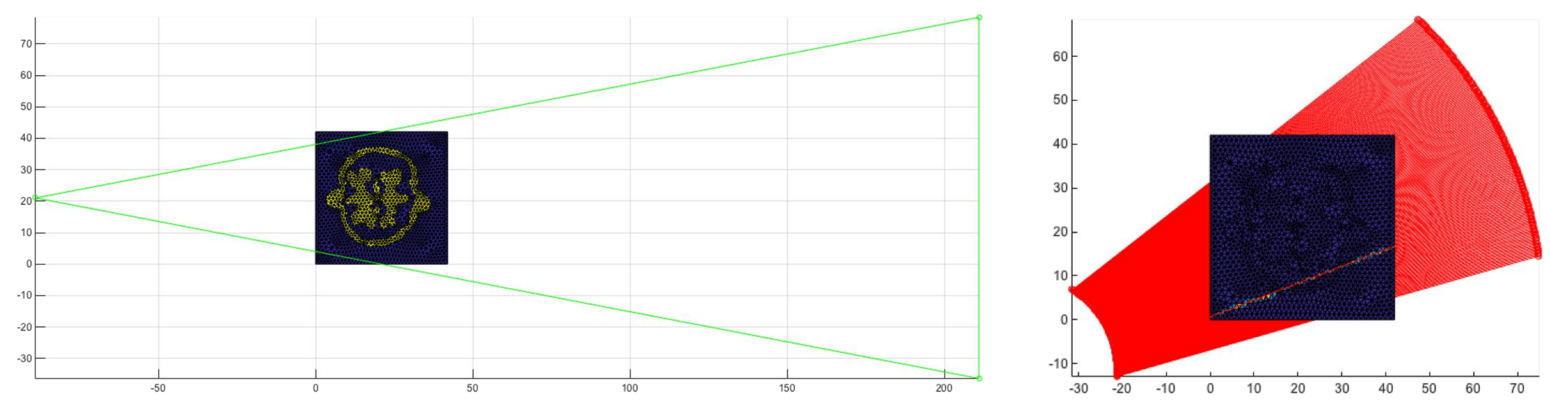
## TOMOGRAPHY WITH WALNUT SCAN

Tomography walnut model with $164 \times 120$ measurements.



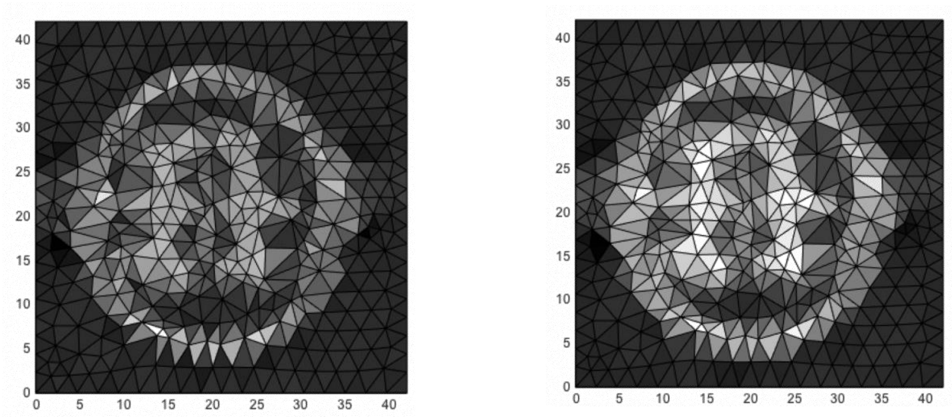
Sinogram with 120 projections — Construction from high resolution — Initialized mesh

Build matrix A



Cone beams set up — Find intersections to build A

Naïve solution and total variation



Naive solution — Total variation

## CONCLUSIONS AND DISCUSSIONS

Explicit mesh in compare to grid point

- Lower resolution, faster in solving $Af = m$
- May represent piecewise constant function better
- Have to refine the mesh, which is neglected in this report

Regularization for explicit mesh: Total variation shows better results than TSVD and naïve. Choosing parameters is difficult.

Future works:

- Optimize and test with higher resolution mesh
- Derive force model to deform the mesh