Problem #1

Due: Wednesday, April 15, turn in to canvas, as described in syllabus.

The goal of this problem is to see if it is possible to stabilize the inverted pendulum by merely sinusoidally vibrating its support, as described by the following equation

$$\ddot{\theta} = -\frac{g}{\ell}\Big(1 + \alpha\cos(\omega t)\Big)\sin(\theta). \tag{1}$$

Another goal of this problem is for you to learn a little bit about Floquet theory. Yet another goal is to introduce a problem for which a discrete time map is a very natural object to consider in analysis of periodically-forced systems.

1. Choose a non-dimensionalization of time, and then rewrite the equation as a system of two first order equations, which should depend on two dimensionless parameters.
2. Linearize the equation about the inverted equilibrium position ($\theta = \pi$) to obtain a linear non-autonomous system to study:
$$\frac{d\mathbf{x}}{dt} = A(t)\mathbf{x},$$
where $A(t) = A(t+T)$ is a 2×2 T-periodic matrix, and $\mathbf{x} \in \mathbf{R}^2$ is your state space vector.
3. Summarize any key results you need from Floquet theory that will allow you to find potentially stable regions of your 2-parameter plane.
4. Estimate the boundary for this stable region numerically, exploiting part 3. This should appear as a plot in your two-parameter plane.
5. Explore how well this works by doing some numerical simulations of the original nonlinear system (1) with the vibration chosen appropriately, and for some suitable initial condition. This is an open ended question. Some things I'm curious about: What happens if you don't start near the inverted position as an initial condition? What happens if you start the pendulum near $\theta = 0$ with this vibration turned on? What might define an "optimal" choice for the control parameters α, ω?