Applied dynamical systems – *project proposal*

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Chaos in random recurrent neural networks

Project description

This topic could be categorized under the section of *Dynamical systems and neural networks*. This is not really related to machine learning as there would be no tasks to be solved or targets to be optimized. The goal of my project is to quantitatively describe the stability of random neural networks.

More specifically, much of this project would be to partially replicate and/or build from [1]. Essentially, the subject of study is random recurrent networks of nonlinear units, which can be interpreted as artificial neurons due to nonlinearity properties. The general objective is to characterize stability and chaos regimes of the system based on certain properties of the networks using mean field theory, for example the gain and variability of the weights like in the paper.

Why does chaos matter in neural networks at all? There are examples that the brain sits at "edges of chaos". However, a systematic experimental and computational review of such a topic requires much work that I don't think I have time to invest in. But as an example implication of chaos, recurrent neural networks initialized near the edge of chaos or at highly chaotic regimes [2] can affect the network's classification performance during training, as the dimensions of the network's activation can be expanded or reduced depending on the input dimensions.

Project objectives

Since I have always struggled to read this paper [1], I'm using this opportunity to re-read this paper but also trying to understand the math and numerics behind the stability dependency on the network properties. So the first objective is to be able to grasp the mathematical gist of the paper, then run a few simulations to examine/confirm the dependency of the network stability on the nonlinearity gain and weights variability. Additionally, although I won't be implementing estimation of Lyapunov exponents, I am aiming to read through and summarize frequently-mentioned QR-method for computing them [3].

Depending on further examination of feasibility, the following objective is a tentative addition. This involves examining the additional dependency on the biases to be added to the nonlinearity. The original nonlinearity usually does not consider the existence of a bias in any of the unit, only the gain parameter. While the gain could be interpreted as the input resistance of a neuron, the nonlinearity lacks a bias term to represent the threshold of a biological neuron. Hence I want to see how the variability in the biases might affect the stability of the network, in addition to the aforementioned properties. [1]

References

[1] H. Sompolinsky, A. Crisanti, and H. J. Sommers. Chaos in random neural networks. *Phys. Rev. Lett.*, 61:259–262, Jul 1988.

[2] Matthew Farrell, Stefano Recanatesi, Timothy Moore, Guillaume Lajoie, and Eric Shea-Brown. Recurrent neural networks learn robust representations by dynamically balancing compression and expansion. *bioRxiv*, 2019.

[3] Hubertus F. [von Bremen], Firdaus E. Udwadia, and Wlodek Proskurowski. An efficient qr based method for the computation of lyapunov exponents. *Physica D: Nonlinear Phenomena*, 101(1):1 – 16, 1997.

[1] Disclaimer: My research is on neural intrinsic excitability, so this objective is quite related to my research that I have not gotten the time to fully invest in.