1. <u>Calculations of “sliding RMS”:</u>

Let's say our sliding window is of size n and currently we are at $data[n+i] = x_{n+i}$.

Define R_i as the RMS of size n from x_{i+1} to our current value x_{n+i}. This means:

$$R_i^2 = \frac{1}{n}\sum_{k=i+1}^{n+i} x_k^2 = \frac{1}{n} \times [x_{i+1}^2 + x_{i+2}^2 + \cdots + x_{n+i-1}^2 + x_{n+i}^2]$$

And with the new datum coming in from the queue $\mathbf{Q}$ as $data[n+i+1] = x_{n+i+1}$, in which we perform *Enqueue* at the same time as calculating the new RMS value R_{i+1}, the square of which is:

$$R_{i+1}^2 = \frac{1}{n}\sum_{k=i+2}^{n+i+1} x_k^2 = \frac{1}{n} \times [x_{i+2}^2 + x_{i+3}^2 + \cdots + x_{n+i}^2 + x_{n+i+1}^2]$$

$$= \frac{1}{n} \times [x_{i+1}^2 + x_{i+2}^2 + x_{i+3}^2 + \cdots + x_{n+i}^2] + \frac{1}{n} \times [-x_{i+1}^2 + x_{n+i+1}^2]$$

$$= R_i^2 - \frac{x_{i+1}^2}{n} + \frac{x_{n+i+1}^2}{n}$$

Hence $R_{i+1} = \sqrt{R_i^2 - \frac{x_{i+1}^2}{n} + \frac{x_{n+i+1}^2}{n}}$

To generalize, as the new datum **Newest** comes in, we can *Enqueue* to $\mathbf{Q}$ as we calculate the new RMS value. The oldest data in the queue $\mathbf{Q}$ is x_{i+1}, which we can pop out as: $\mathbf{Oldest} = \mathbf{Q}.Dequeue$.

So we can calculate the current RMS value $\mathbf{RMS_{current}}$ from the previous $\mathbf{RMS_{previous}}$, and the scaled newest squared $\mathbf{NSq} = \frac{\mathbf{Newest}^2}{\boldsymbol{n}}$ and the scaled oldest squared $\mathbf{OSq} = \frac{\mathbf{Oldest}^2}{\boldsymbol{n}}$ as:

$$\mathbf{RMS_{current}} = \sqrt{\mathbf{RMS_{previous}^2} - \mathbf{OSq} + \mathbf{NSq}}$$

2. <u>Calculations of STFT and considerations:</u>

Let's say we want to calculate the Fourier Transform ($\mathbf{F}$)of an epoch size N as $X = \{x_i\}$ with $i = \overline{1, N}$.

Define taper operator as $\mathbf{W}$ with a given taper window w. Applying the taper window means:

$$A \overset{\mathbf{W}}{\Rightarrow} \mathbf{W}(A) = \{a_i \times w_i\} \text{ with any given signal array size } N\text{: } A = \{a_i\}, \qquad i = \overline{1, N}.$$

Define normalization operator as $\mathbf{Z}$ similar to calculation of the standard score:

$$A \overset{\mathbf{Z}}{\Rightarrow} \mathbf{Z}(A) = \left\{\frac{a_i - \mu_A}{\sigma_A}\right\} \text{ with } \mu_A \text{ and } \sigma_A \text{ is the mean and standard deviation of } A\text{, respectively.}$$

Calculating the STFT of the signal X can be done by applying the operator in this order: $\mathbf{S} \overset{\text{def}}{=} \mathbf{W} \to \mathbf{Z} \to \mathbf{F}$

$$X \overset{\mathbf{S}}{\Rightarrow} \mathbf{S}(X) = \mathbf{F}\{\mathbf{Z}[\mathbf{W}(X)]\}$$

In other words: apply the *taper*, then *normalize*, then calculate the *Fourier* Transform.

$Note$: I also did $\mathbf{Z} \rightarrow \mathbf{W} \rightarrow \mathbf{F}$ but the total power of some band was less stable than $\mathbf{W} \rightarrow \mathbf{Z} \rightarrow \mathbf{F}$

(per recommendation of $Wim\ van\ Drongelen$)

After STFT we obtain the frequency magnitude spectrum $S = S[f] = |\mathbf{S}(X)|$ with f as frequency.

Then we calculate the power spectral density (PSD) $P = P[f] = \alpha \times \{S[f]\}^2 = \alpha \times S^2$

with scaling (optional) factor $\alpha = \dfrac{1}{F_s \times \sum W^2} = \dfrac{1}{F_s \times \sum_{i=1}^{N} w_i^2}$ and F_s as sampling rate of X.

Then to calculate the power of a limited band (LBP) of a frequency band $\boldsymbol{b} = [f_l, f_h]$:

$$\mathbf{LBP}_b = \sum_{f=f_l}^{f=f_h} P[f]$$

3. <u>Evaluation of alarm:</u>

For a certain calculation $\mathbf{C}$ of the signal like RMS or LBP applied on signal epoch X to result in $\mathbf{c}$,

we predefine the alarm thresholds $\mathbf{w}_{\mathbf{th}}^{\mathbf{C}} = [w_L, w_U]$ and $\mathbf{d}_{\mathbf{th}}^{\mathbf{C}} = [d_L, d_U]$

for instantaneous update of alarm level $\mathbf{L} = \{Normal, Warning, Danger\}$

and the requirement of the threshold range is (strict order): $d_L < w_L < w_U < d_U$

Then the instantanous alarm level is:

$$\begin{cases} Danger & \text{if } \mathbf{c} < d_L \text{ or if } \mathbf{c} > d_U \\ Warning & \text{if } d_L < \mathbf{c} < w_L \text{ or if } w_U < \mathbf{c} < d_U \\ Normal & \text{otherwise } (w_L < \mathbf{c} < w_U) \end{cases}$$