In [1]:
```
# In this project, I tried to back-test a trading strategy using the Euro-Dollar 3 months future index.
# The long-short signals is based on the moving average price and the volatility regime is predicted by GARCH model.
# The strategy can obtain a 0.97 Sharpe ratio for optimize paramettersand 0.77 for out-of-sample Walk Forward Analysis.
# I also test this strategy with S&P 500 e-Mini future index and it obtains 0.71 for Sharpe ratio
# This strategy is based on paper 'Quantitative Investment Strategies Research: Investing in Central Bank Momentum'
# of Morgan Stanley Research
```

```
C:\Users\18183432\OneDrive - LA TROBE UNIVERSITY\Python\trading strategy\aws\Walk_forward_testing
```

In [ ]:
```
import os
# os.chdir('C:\\Users\\18183432\\OneDrive - LA TROBE UNIVERSITY\\Python\\trading strategy\\aws\\Walk_forward_testing')
cwd = os.getcwd()
print(cwd)
import pandas as pd
import numpy as np
import scipy.optimize as sco

#from itertools import product
#import functools

# matplotlib for plotting
import matplotlib.pyplot as plt

# GARCH model for estimation
from arch import arch_model
```

In [113]:
```
def drawdown (final):
    '''
    function to calculate drawdown of security/portfolio

    Input:
        final: return serires of security/portfolio
    Output:
        drawdown of portfolio
    '''
    cum_rets = (1 + final).cumprod()
    drawdown =  -(1-cum_rets.div(cum_rets.cummax()))

    return drawdown

def future_analyzing_contract(price,window = 252*3, MA1 = 66, MA2=262, forecast_horizon = 1, agg ='mean', update_regime =
    '''
    This function is to calculate the return of a future constract with a trading strategy based of volatility regime and
    moving average price of 66 days (for low volatility regime) and 252 days (for high volatility regime)

    Input
    -----
        Price: a data frame with a Time column and Price column
        window: number of days to fit GARCH(1,1)
        forecast_horizon: horizon to forecast volatility regime
        update_regime: the frequency of regime update

    Output
    ------
        price_minus_window: a dataframe with 10 columns including Time,price, log_ret,low66, high252, forecasted _varianc
        annualized_returns
        annualized_sharpe
        annualized_standard_deviation
        proportion_long_over_short
        max_draw_down

    '''
    price.columns =['Time', 'price'] # Reset the name of colums
    price['Time'] = pd.to_datetime(price['Time'], format=('%m/%d/%Y')) # Convert the time column to time object
    price.dropna(axis = 0, inplace = True) # Eliminate NA values
    #price.shape
```

```
    ### 1. Calculate the return of contract
    price['log_ret'] =  price['price'].astype(float).pct_change()*100
    price = price.reset_index().drop(columns =['index'])
    ### 2. Create the moving averange price with window = 66 days for low volatility regime and window = 252 days for hig
    price['low66'] = price['price'].rolling(MA1).mean() # 66
    price['high252'] = price['price'].rolling(MA2).mean() #252
    ### 3.Calculate GARCH volatility with 3 years window (=3*252 days)
    ret = np.array(price['log_ret'].dropna())
    predictions =[]
    realized_vars =[]
    price_minus_window = price.iloc[window:,:].reset_index().drop(columns = ['index'])

    for i in range(len(ret)-window+1): # i = 1
        ret1 = np.array(ret[i:i+window])# len(ret1)
        # Record realized variance for comparison with forecast values
        realized_var = np.var(ret1)
        realized_vars.append(realized_var)
        # set up GARCH model
        model = arch_model(ret1, mean='Zero', vol='GARCH', p=1, q=1)#
        # fit model
        model_fit = model.fit()

        # Record forecast values of Variance
        prediction = model_fit.forecast(horizon=forecast_horizon).variance.values[-1]
        predictions.append(np.mean(prediction))
# ==============================================================================
#         if agg == 'median':
#             predictions.append(np.median(prediction))
#         elif agg == 'min':
#             predictions.append(np.min(prediction))
#         elif agg == 'max':
#             predictions.append(np.max(prediction))
#         else:
#             predictions.append(np.mean(prediction))
# ==============================================================================

    # put the variance and relized variance into data frame
    price_minus_window ['forecasted_variance'] = np.array(predictions, float)
    price_minus_window ['realized_vars'] = realized_vars
    # calculate the median of rolling-3-year window variance
    price_minus_window['median_forecast_variance'] = price_minus_window['forecasted_variance'].rolling(window).median()
    # eleminate NA values
```

```python
    price_minus_window = price_minus_window.dropna().reset_index().drop(columns = ['index'])
    # plot the actual variance and forecast variance for comparison
    var = price_minus_window ['realized_vars']
    forecast_variance = price_minus_window ['forecasted_variance']
# =============================================================================
#     plt.plot(var,'r-', label = 'realized')
#     plt.plot(forecast_variance,'b-',label = 'forecast')
#     plt.legend()
#     plt.title('Realized volatility and Forecast volatility from GARCH(1,1)')
#     plt.show()
# =============================================================================
    price_minus_window1 = price_minus_window.copy()
    price_minus_window =   price_minus_window1.copy()
    price_minus_window1.head()
    ### 4.Create a column to indicate volatility regime
    regime = []
    #update_regime = 5 # as the paper said that they only update regime 1 time a week, then update_regime is set by 5 bus
    for i in range(update_regime, price_minus_window.shape[0], update_regime): # i = 5, i = 5 +5
        #print(i)
        if price_minus_window['forecasted_variance'][i] > price_minus_window['median_forecast_variance'][i-1]:
            regime += ['low']* update_regime
        else:
            regime += ['high']*update_regime

    # Handle the remainder of (number of days / update-regime)
    if price_minus_window['forecasted_variance'].iloc[-1] > price_minus_window['median_forecast_variance'].iloc[-2]:
        if (price_minus_window.shape[0] % update_regime) == 0:
            regime += ['low']* update_regime
        else:
            regime += ['low']* (price_minus_window.shape[0] % update_regime)
    else:
        if (price_minus_window.shape[0] % update_regime) == 0:
            regime += ['high']* update_regime
        else:
            regime += ['high']* (price_minus_window.shape[0] % update_regime)
   # Put regime back into dataframe
    price_minus_window['regime'] = regime


    ### 5. Calculate return based on the volatility levels (regime) and moving average value of stock prices
    price_minus_window['price'] = np.array(price_minus_window['price'], float)
    price_minus_window['low66'] = np.array(price_minus_window['low66'], float)
```

```
    price_minus_window['high252'] = np.array(price_minus_window['high252'], float)
    price_minus_window ['position'] = np.where(price_minus_window['regime'] =='low', np.where(price_minus_window['price']
    price_minus_window ['ret'] = price_minus_window ['position'].shift(1)* price_minus_window ['log_ret']


    #### 6.Statistic information of future contract

    # return
    price_minus_window['Cum_ret'] = (1+price_minus_window['ret']).cumprod() -1
    avg_annualized_return = (1+price_minus_window['Cum_ret'].iloc[-1])**(252/price_minus_window.shape[0])-1

    # standard deviation
    annualized_standard_deviation = price_minus_window['ret'].std()*np.sqrt(252)
    # sharpe ratio
    annualized_sharpe = avg_annualized_return/annualized_standard_deviation

    # % of time long/% of time short
    if price_minus_window.loc[price_minus_window['position'] == -1].shape[0] ==0:
        proportion_long_over_short = 'Full Short'
    else:
        proportion_long_over_short = price_minus_window.loc[price_minus_window['position'] == 1].shape[0]/price_minus_win
    # print('Proportion of Long days over Short day is',proportion_long_over_short)

    # Maximum draw down
    max_draw_down = max(-drawdown(price_minus_window['ret'].dropna()))
    #print('Maximum draw down of contract is', max_draw_down)

    # create a new subplot for each source
    # plt.hist(price_minus_window['ret'], bins='auto')
    # plt.title("Histogram of contract" + str(contract))
    # plt.xlabel("Return")
    # plt.ylabel('Frequency')
    # plt.savefig('histogram of contract'+str(contract)+'.png')

    return price_minus_window, avg_annualized_return, annualized_sharpe, annualized_standard_deviation,proportion_long_ov

def future_analyzing_port_baseline(data, MA1 = 5, MA2=210, window = 3,  forecast_horizon = 5, update_regime = 5):
    '''
    This function will will create a trading strategy based of volatility regime and moving average price.
    First, it compare the median of forecast volatility last three year with the forecast volatility tomorrow to decide w
    if volatility regime is low, it will go long (buy) if price of STIR > moving average price of MA1 days and short if p
    if volatility regime is high, it will go long (buy) if price of STIR > moving average price of MA1 days and short if
```

```
    Input
    -----
        data: a csv file containing a set of columns. Each column provides price over time of 1 future contract
        window: number of years to fit GARCH(1,1), can be from 1-4, otherwise we can not have enough observations for str
        weight: for calcualting the portfolio return, can be 'max_sharpe', 'min_vol' or equal
        MA1: moving average window for low volatility regime
        MA2: moving average window for high volatility regime
        forecast_horizon: horizon to forecast volatility regime
        agg: the method to choose the calculate the volatility forecast if  forecast_horizon > 1, the method can be 'min'
        update_regime: the frequency of regime update

    Output
    ------
        port_ret: a dataframe which includes Time and return of each contract
        summary_port: a data frame containing avg_annualized_return, annualized_standard_deviation and annualized_Sharpe

    Time
    ------
        It took around 20 minutes to run this function for 'STIR_futures_sample.csv' in Phong's Laptop
    '''
    # Read data
    df = data   # df = valid_set[2]
    # create the name for columns (the name of column is the number of investigated contract)
    col =['Time']
    for i in range(1, df.shape[1],1):
        col.append(str(i))
    df.columns = col # set the name for columns
    df['Time'] = pd.to_datetime(df['Time'], format=('%m/%d/%Y')) # Convert 'Time' column to Time object

    # Eliminate NA values
    valid_contracts=[]
    n= 2# n= 2 if we use the median of forecast values from GARCH(1,1) as a bench macrh to choose volatility regime, and
    for i in range(1,df.shape[1],1):
        col[i]= df[['Time',str(i)]].dropna() # Create dataframe for different contracts
        if col[i].shape[0]>(window+3)*252:
            valid_contracts.append(str(i))
    valid_contracts

    ### Calculate return for each contract and put the dataframe results into a list called individual_ret
    # We will have 19 dataframes in this list in the case n = 2. I also calclute the annualized_ret, ann_sd, annualized_s
    individual_ret = []
```

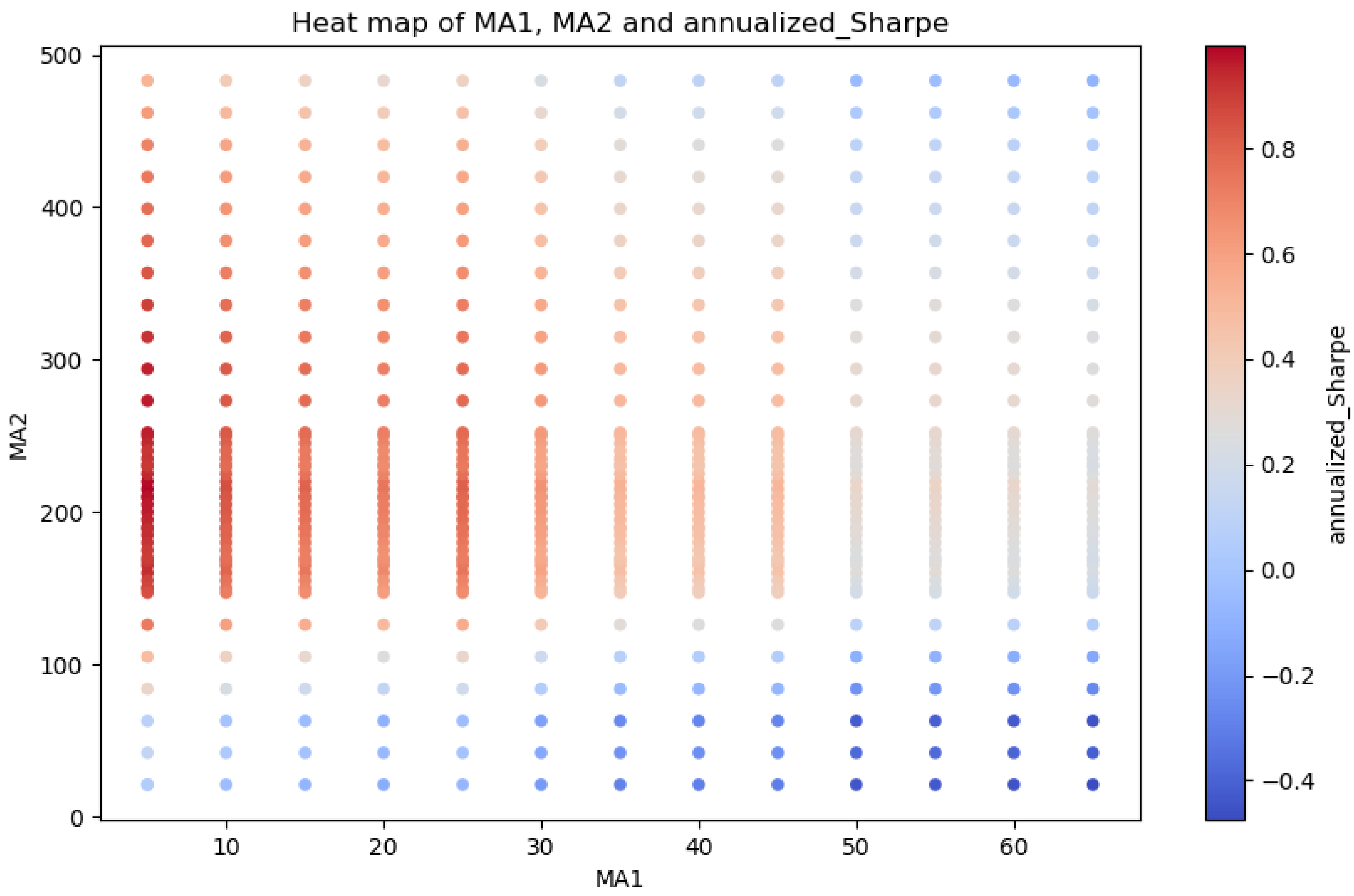
```
    annualized_ret=[]
    annualized_sharpe =[]
    standard_deviations =[]
    proportion_long_over_shorts=[]
    max_draw_downs=[]
    long_short =[]
    for contract in valid_contracts: # contract = 43
        price = col[int(contract)].reset_index().drop(columns =['index'])
        price_minus_window, annualized_returns, a_sharpe, standard_deviation, proportion_long_over_short, max_draw_down =

        individual_ret.append(price_minus_window[['Time','ret']])


    ### Collect the returns columns of each contract and merger them tegether based on 'Time' columns
    port_ret =pd.DataFrame(individual_ret[0])
    for i in range(1,len(individual_ret),1): # i = 1
        port_ret = port_ret.merge(pd.DataFrame(individual_ret[i]),on='Time', how='outer')

    ### Create the name for columns in port_ret dataframe
    column1 = valid_contracts.copy()
    column1.insert(0,'Time')
    port_ret.columns = column1
    port_ret.fillna(0, inplace=True)
    port_ret1 = port_ret.set_index('Time')
    noa =len(valid_contracts)
    weight_port = np.array(noa*[1./noa,]) # Equal weights

# =============================================================================
#     # Calculate weight for portfolio
#     def portfolio_ret(weights,port_ret = port_ret1):
#         return np.sum(port_ret.mean()*weights)*252
#     def portfolio_vol(weights,port_ret = port_ret1):
#         return np.sqrt(np.dot(weights.T,np.dot(port_ret.cov()*252,weights)))
#     def min_func_sharpe(weights): # function to be optimized
#         return -portfolio_ret(weights)/portfolio_vol(weights)
#
#     cons = ({'type': 'eq', 'fun': lambda x: np.sum(x)-1}) # equality constrain sum(weights) =1
#     bnds = tuple((0,1) for x in range(noa)) # bound for individual weight
#
#     ## Equal weight
#
#     ## Optimal weight for maximizing Sharpe ratio
```

```
#     if weight == 'max_sharpe':
#         opts =sco.minimize(min_func_sharpe,eweights, method ='SLSQP', bounds = bnds, constraints=cons)
#         weight_port = opts['x']
#     ## Optimal weight for minimizing the variance of the portfolio
#     if weight == 'min_vol':
#         optv = sco.minimize(portfolio_vol,eweights, method ='SLSQP', bounds = bnds, constraints=cons)
#         weight_port = optv['x']
# ==============================================================================
    # Calculate Total return for each contract and cummulative return

    port_ret['Total Ret'] = port_ret.drop(columns = 'Time').mul(weight_port,axis=1).sum(axis=1)
    port_ret1['Total Ret'] = np.array(port_ret['Total Ret'])
    port_ret['Cummulative return'] = (1+port_ret['Total Ret']).cumprod() -1 #.cumsum)()
    port_ret1['Cummulative return']= np.array(port_ret['Cummulative return'])
    winning_percentage = len(port_ret[port_ret['Total Ret']>0])/len(port_ret)

    # Plot the return of porfolio over time
# ==============================================================================
#     import matplotlib.pyplot as plt
#     plt.style.use('fivethirtyeight')
#
#     port_ret1['Cummulative return'].plot(figsize=(10,8))
#     plt.title('Cummulative return of portfolio '+ str(valid_contracts))
#     plt.show()
# ==============================================================================

    avg_annualized_return_port = (1+port_ret['Cummulative return'].iloc[-1])**(252/port_ret1['Cummulative return'].shape[
    #print('Average annualized daily return of porfolio is', round(avg_annualized_return_port*100,2),'%')
    # standard deviation
    annualized_standard_deviation = port_ret1['Total Ret'].std()*np.sqrt(252)
    #print('Annualized standard deviation return of portfolio is', round(annualized_standard_deviation,2)*100, '%')
    # sharpe ratio
    annualized_Sharpe = avg_annualized_return_port/annualized_standard_deviation

    max_draw_down_port = max(-drawdown(port_ret['Total Ret']))
    summary_port = pd.DataFrame({'Start':port_ret['Time'].iloc[0] ,'Finish':port_ret['Time'].iloc[-1],'MA1': MA1,'MA2': M
                                 'annualized_Sharpe': annualized_Sharpe,'max_draw_down': max_draw_down_port, 'winning_per


    #weights = pd.DataFrame({'contract': valid_contracts, 'weights':weight_port})
    return  summary_port # port_ret #
```

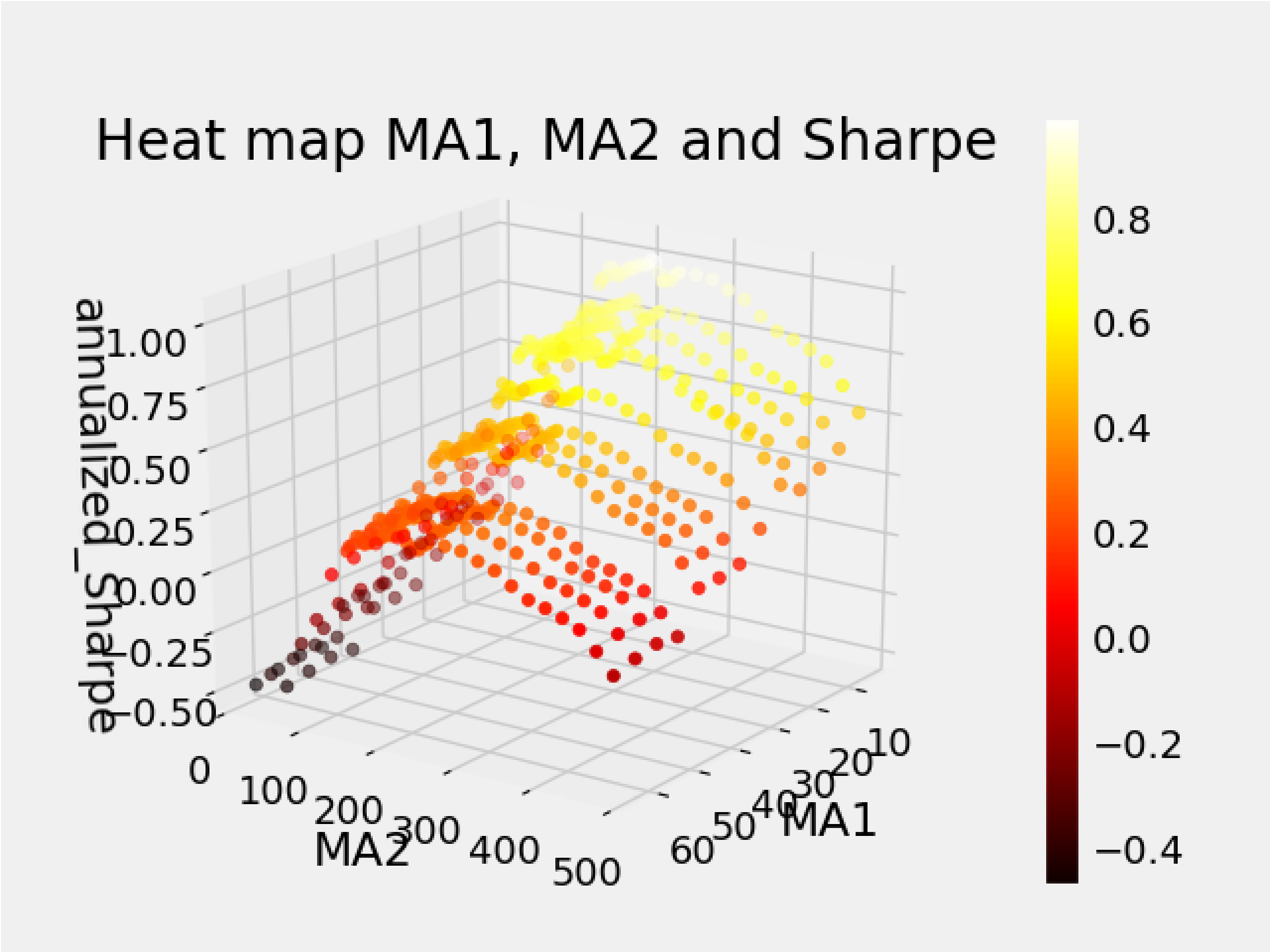
In [ ]:

```
# Run Grid Search to find How MA1 and MA2 affect the Sharpe value
import itertools
ma1 = range(5, 70, 5) # ma1 =[5]
ma2 = range(21,252*2, 21) # ma2 =range(150,252, 5)
results = pd.DataFrame()
for MA1, MA2 in product(ma1,ma2): # MA1 = 5, MA2 = 63, wi =3
    port_ret, port_ret1, summary_port,summary_contracts, annualized_Sharpe, weights, max_draw_down_port = future_analyzin
        data = 'STIR_futures_sample.csv',window = 3,weight = 'equal', MA1 = MA1, MA2=MA2, forecast_horizon = 1, update_re
    results = results.append(summary_port)
results.sort_values('annualized_Sharpe', ascending = False).to_csv('grid_search.csv', index = False)
```

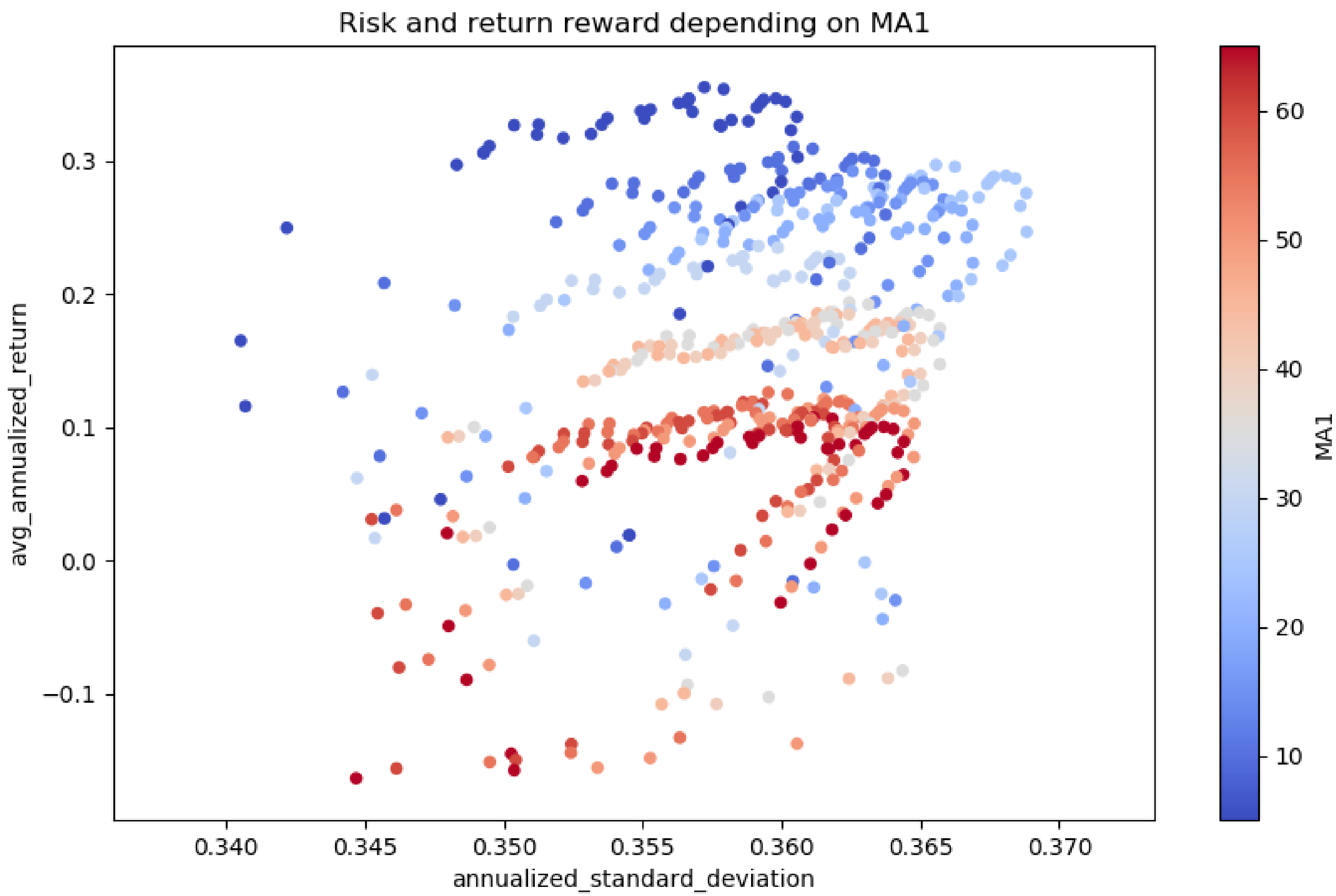
In [9]:
```
# Plot heat map of Sharpe ratio depending on MA1 and MA2
%matplotlib notebook
b= pd.read_csv('grid_search.csv')
b.plot.scatter(x='MA1', y ='MA2', c='annualized_Sharpe',cmap ='coolwarm', figsize = (10,6), colorbar = True)
plt.title('Heat map of MA1, MA2 and annualized_Sharpe')
# plt.scatter(x=b['MA1'], y =b['MA2'], s=b['annualized_Sharpe']*100)
plt.show()
```



In [66]:
```
########## plot 3D HEat Map MA1 and MA2
from mpl_toolkits import mplot3d
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.set_xlabel('MA1')
ax.set_ylabel('MA2')
ax.set_zlabel('annualized_Sharpe')
ax.set_title('Heat map MA1, MA2 and Sharpe')
ax.view_init(20, 35)
z = b['annualized_Sharpe']
c = b['annualized_Sharpe']
x = b['MA1']
y = b['MA2']


img = ax.scatter(x, y, z, c=c, cmap=plt.hot())
fig.colorbar(img)
plt.show()
```

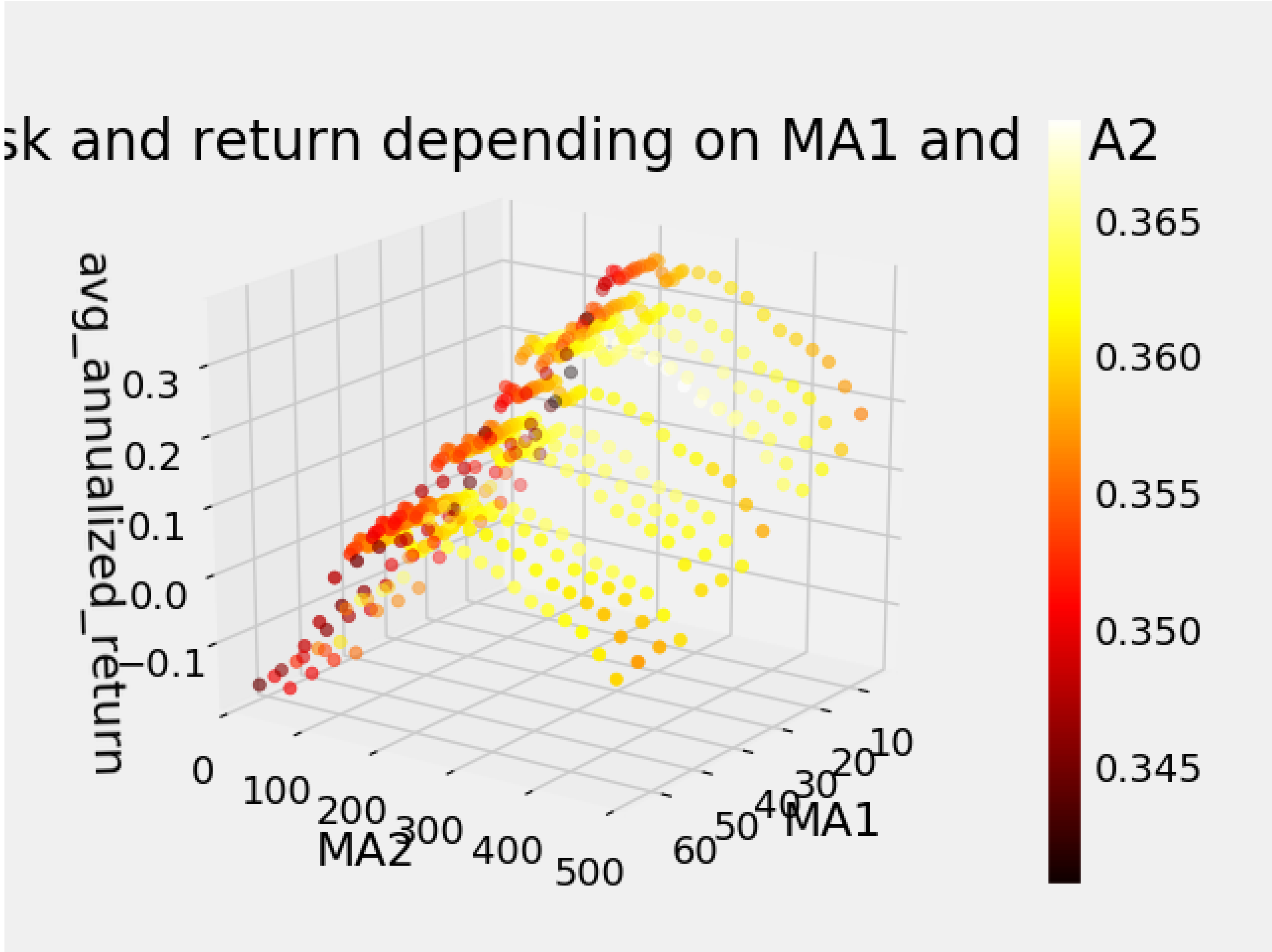

Heat map MA1, MA2 and Sharpe
annualized_Sharpe
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
0
100
200
300
400
500
MA2
10
20
30
40
50
60
MA1
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4

In [12]:
```
# How changes in MA1 and MA2 affect risk and return

b.plot.scatter(y='avg_annualized_return', x ='annualized_standard_deviation', c='MA1',cmap ='coolwarm', figsize = (10,6),
plt.title('Risk and return reward depending on MA1')
# plt.scatter(x=b['MA1'], y =b['MA2'], s=b['annualized_Sharpe']*100)
plt.show()
```



In [13]:
```
b.plot.scatter(y='avg_annualized_return', x ='annualized_standard_deviation', c='MA2',cmap ='coolwarm', figsize = (10,6),
plt.title('Risk and return reward depending on MA2')
# plt.scatter(x=b['MA1'], y =b['MA2'], s=b['annualized_Sharpe']*100)
plt.show()
```

In [67]:
```
########## plot 3D Risk and return reward depending on MA1 and MA2
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.set_xlabel('MA1')
ax.set_ylabel('MA2')
ax.set_zlabel('avg_annualized_return')
ax.set_title('Risk and return depending on MA1 and MA2')
ax.view_init(20, 35)
z = b['avg_annualized_return']
c = b['annualized_standard_deviation']
x = b['MA1']
y = b['MA2']


img = ax.scatter(x, y, z, c=c, cmap=plt.hot())
fig.colorbar(img)
plt.show()
```



In [ ]:
```
# For overfitting check, I run Walk Forward Analysis to see how the  strategies work out_of_sample
```

In [ ]:
```
# First I run a for loop to find an optimal training window and testing out-of-sample window

############################## FoR Loop to find optimal train and valid period ##############################
#1. Rolling window for the training set
from itertools import product
from joblib import Parallel, delayed
import multiprocessing
ma1 = range(5, 70, 5) # ma1 =[5] ma2 = [210]
ma2 = range(21,21*15, 21) # ma2 = range(21,21*15, 21)
inputs = list(product(ma1,ma2))
num_cores = multiprocessing.cpu_count()
def grid_search (data1): # ma2 =range(150,252, 5)

    results = Parallel(n_jobs=num_cores)(delayed(future_analyzing_port_baseline)(data1,MA1,MA2) for MA1, MA2 in inputs)
    for i in range(len(results)):
        if i ==0:
            summary = results[i]
        summary = pd.concat([summary,results[i]], axis =0)
    return summary.sort_values('annualized_Sharpe', ascending = False)

df = pd.read_csv('STIR_futures_sample.csv')
df = df.drop(df.index[0:7]).reset_index().drop(columns =['index'])
from datetime import timedelta
col =['Time']
for i in range(1, df.shape[1],1):
    col.append(str(i))
df.columns = col # set the name for columns
df['Time'] = pd.to_datetime(df['Time'], format=('%m/%d/%Y'))

train_valid_set = df.iloc[0:df.shape[0]-252,].reset_index().drop(columns=['index']) #
# train_valid_set = df.iloc[int(252*2.75):df.shape[0],] # test test set
test_set = df.copy()


train_valid_results = pd.DataFrame()

import time
start = time.time()
tp = [6.375]#[7, 7.5,8] # TP =7.5 VP =1
vp = [0.125]#,[0.25,0.5, 0.75,1]
for TP, VP in product(tp,vp):
```

```
    train_period = TP
    valid_period = VP
    #if (TP == 7.5) and (VP ==0.5):
    #    continue
    number_cross_validate = ((train_valid_set['Time'][train_valid_set.shape[0]-1]-train_valid_set['Time'][0]).days-train_

    train_set =[0]*int(number_cross_validate+1)
    valid_set = [0]*int(number_cross_validate+1)

    for i in range(1,int(number_cross_validate+1),1): # i=2
        print(i)
        if i ==1:
            train_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]) & (train_valid_set['T
        else:
            train_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]+timedelta(int(valid_pe

        valid_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]+timedelta(int((valid_perio


    training_results = []
    valid_results = []
    annualized_Sharpes = []
    for i in range(1, len(train_set)): # i = 1, MAX_EVALS = 3
        random_results = grid_search( data1= train_set[i])
        training_results.append(random_results)
        best_para = random_results.sort_values('annualized_Sharpe', ascending = False).head(1)
        summary_port  = future_analyzing_port_baseline(data = valid_set[i], MA1 = best_para['MA1'].iloc[0], MA2=best_para
        valid_results.append (summary_port)
        annualized_Sharpes.append(summary_port['annualized_Sharpe'].iloc[0])
        print(i)

    for i in range(0, len(training_results)):# i =3

       # training_results[i].to_csv('traning_results_walk_forward_'+str(i)+'_'+str(train_period)+'_'+str(valid_period)+'.
        if i ==0:
            valid_result = valid_results[0]
        else:
            valid_result = pd.concat([valid_result,valid_results[i]], axis=0)

    pd.DataFrame(valid_result).to_csv('valid_results_walk_forward_'+str(i)+'_'+str(train_period)+'_'+str(valid_period)+'.
    train_valid_summary = pd.DataFrame({'train_period': train_period,'valid_period': valid_period,'mean_valid_sharpe': np
    train_valid_results = train_valid_results.append(train_valid_summary)
```

```
train_valid_results
end = time.time()
(end - start)/3600
```

In [ ]:
```
# We got the results like this:
       train_period  valid_period  mean_valid_sharpe
 0          6.375         0.125          3.331654
 0            7           0.125          0.112202
 0            7           0.25           1.228202
 0           7.5          0.25           0.277203
 0           8.0          0.25          -0.103875
```

In [ ]:
```
############# CASE 1 ########################################################
# From the above results I choose 1606 days (6.375*252) for training period from that we got optimal MA1 and MA2
# and then apply these MA1, MA2 into testing period of 31 days (0.125*252). The results for each period are as follows:
df = pd.read_csv('STIR_futures_sample.csv')
df = df.drop(df.index[0:7]).reset_index().drop(columns =['index'])
from datetime import timedelta
col =['Time']
for i in range(1, df.shape[1],1):
    col.append(str(i))
df.columns = col # set the name for columns
df['Time'] = pd.to_datetime(df['Time'], format=('%m/%d/%Y'))

train_valid_set = df
# train_valid_set = df.iloc[int(252*2.75):df.shape[0],] # test test set
test_set = df.copy()


train_valid_results = pd.DataFrame()

import time
start = time.time()
tp = [6.375]#[7, 7.5,8] # TP =7.5 VP =1
vp = [0.125]#,[0.25,0.5, 0.75,1]
for TP, VP in product(tp,vp):
    train_period = TP
    valid_period = VP
    #if (TP == 7.5) and (VP ==0.5):
    #    continue
    number_cross_validate = ((train_valid_set['Time'][train_valid_set.shape[0]-1]-train_valid_set['Time'][0]).days-train_

    train_set =[0]*int(number_cross_validate+1)
    valid_set = [0]*int(number_cross_validate+1)

    for i in range(1,int(number_cross_validate+1),1): # i=2

        if i ==1:
            train_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]) & (train_valid_set['T
        else:
            train_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]+timedelta(int(valid_pe

        valid_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]+timedelta(int((valid_perio
```

```
from itertools import product
from joblib import Parallel, delayed
import multiprocessing
num_cores = multiprocessing.cpu_count()
valid_results = []
annualized_Sharpes = []
i_set = range(1,int(number_cross_validate+1),1)
#ma1 = list(valid_result['MA1'])
#ma2 = list(valid_result['MA2'])
ma1 = [5,25,60,60,20, 30, 30, 5,15,25,30,65,65,35, 20, 60, 30, 25, 60,15, 65,50,5,35,35,35,5,5,5]#6.375 - 0.125 updat
ma2 = [273,210,210,252,210,42,273,294,252,252,252,295,294, 147, 63, 105,84,84,42,105,105,105,21,147,147,147,294,126,1
#ma1 = [5, 10, 25, 25,50,5,35,25,25,25,65,65, 45, 55, 65, 65, 60, 25, 55, 45, 40, 20,5,5,5,35,35,35,5,35,35] # 6.375
#ma2 = [294, 252, 210, 294, 252, 231,252,42,294,21,294,294, 231, 273, 294, 84, 105, 105, 105, 105, 105, 105,21,21,21,
# ma1 = [5,30,45,35,25,50,30,45,5,5,30,15,65,5,35] # 6.75 - 0.25
# ma2 = [252,294,210,273,294,273,147,147,126,126,105, 105,84,294,147]
#ma1 = [5,30,45,35,25,35,30,5,5,5,5,5,5]  # 7 -0.25 - big grid search
#ma2 =[252,210,294,273,273,210,147,126,126,126,126,126,294]
#ma1 = [5,30,45,35,25,35,30,5,30,25,35,50,5]  # 7 -0.25 - big grid search
#ma2 =[252,210,294,273,273,210,147,126,105,105,42,105,294] #
# 7.5 - 0.25 - small grid
# ma1 = [5,30,45,35,25,35,45,45,5,65,65] #
# ma2 =[260,280,260,220,220,220,160,160,150,290,290]
inputs = zip(i_set, ma1, ma2)
results = Parallel(n_jobs=num_cores)(delayed(future_analyzing_port_baseline)(valid_set[i],MA1,MA2) for i, MA1, MA2 in

# CHeck performance of WFA period by period
for i in range(len(results)):# i =3

   # training_results[i].to_csv('traning_results_walk_forward_'+str(i)+'_'+str(train_period)+'_'+str(valid_period)+'.
    if i ==0:
        valid_result = results[0]
    else:
        valid_result = pd.concat([valid_result, results[i]], axis=0)
```

In [96]:
```
# Add index into valid_result to know how many times we have iterated and show it
cols = valid_result .columns.tolist()
cols = cols[-1:] + cols[:-1]
valid_result  = valid_result [cols]
valid_result
```

Out[96]:

	index	Start	Finish	MA1	MA2	annualized_return	annualized_sd	annualized_Sharpe	max_draw_down	winning_percentage
0	1	2015-05-12	2015-06-29	5	273	1.205105	0.496489	2.427253	0.098972	0.588235
0	2	2015-06-26	2015-08-14	25	210	-0.517570	0.686469	-0.753960	0.193396	0.428571
0	3	2015-08-11	2015-09-28	60	210	6.778134	0.697462	9.718281	0.132557	0.500000
0	4	2015-09-25	2015-11-13	60	252	-0.190740	0.658533	-0.289643	0.259746	0.500000
0	5	2015-11-10	2015-12-29	20	210	-0.473486	0.511561	-0.925570	0.184890	0.441176
0	6	2015-12-28	2016-02-12	30	42	33.056746	0.655367	50.440041	0.097649	0.636364
0	7	2016-02-09	2016-03-29	30	273	-0.653102	0.712658	-0.916431	0.325741	0.470588
0	8	2016-03-29	2016-05-13	5	294	0.745190	0.506713	1.470635	0.195182	0.470588
0	9	2016-05-11	2016-06-28	15	252	1.432937	0.704742	2.033278	0.198417	0.500000
0	10	2016-06-28	2016-08-12	25	252	-0.789390	0.573439	-1.376590	0.246269	0.424242
0	11	2016-08-11	2016-09-27	30	252	-0.066693	0.441926	-0.150914	0.118516	0.454545
0	12	2016-09-27	2016-11-11	65	295	-0.772266	0.421546	-1.831983	0.225865	0.470588
0	13	2016-11-10	2016-12-28	65	294	10.567245	0.505084	20.921773	0.059112	0.666667
0	14	2016-12-28	2017-02-10	35	147	-0.536519	0.523899	-1.024090	0.143666	0.354839
0	15	2017-02-10	2017-03-29	20	63	-0.518708	0.400004	-1.296759	0.173177	0.363636
0	16	2017-03-30	2017-05-12	60	105	-0.209228	0.424462	-0.492925	0.137665	0.419355
0	17	2017-05-12	2017-06-28	30	84	0.205265	0.364754	0.562749	0.079983	0.484848
0	18	2017-06-29	2017-08-11	25	84	-0.020898	0.288320	-0.072480	0.091859	0.451613
0	19	2017-08-14	2017-09-27	60	42	0.754365	0.362501	2.081001	0.115567	0.593750
0	20	2017-09-28	2017-11-10	15	105	1.578413	0.264159	5.975250	0.044146	0.625000
0	21	2017-11-13	2017-12-28	65	105	1.915169	0.288565	6.636867	0.043076	0.656250

	index	Start	Finish	MA1	MA2	annualized_return	annualized_sd	annualized_Sharpe	max_draw_down	winning_percentage
0	22	2017-12-28	2018-02-09	50	105	4.656074	0.345322	13.483280	0.062257	0.766667
0	23	2018-02-12	2018-03-29	5	21	0.544175	0.481380	1.130450	0.125270	0.545455
0	24	2018-03-29	2018-05-11	35	147	2.011034	0.299715	6.709824	0.036736	0.548387
0	25	2018-05-15	2018-06-28	35	147	-0.490205	0.235682	-2.079939	0.106642	0.406250
0	26	2018-06-29	2018-08-13	35	147	-0.219582	0.249365	-0.880565	0.100350	0.451613
0	27	2018-08-15	2018-09-27	5	294	1.278299	0.254070	5.031276	0.038469	0.516129
0	28	2018-09-28	2018-11-12	5	126	0.713387	0.281664	2.532759	0.076817	0.531250
0	29	2018-11-14	2018-12-28	5	147	0.577714	0.292356	1.976063	0.087119	0.516129

In [ ]:
```
######  Calculate return for the whole test period
def future_analyzing_port_baseline(data, MA1 = 5, MA2=210, window = 3,  forecast_horizon = 5, update_regime = 5):
    '''
    This function will will create a trading strategy based of volatility regime and moving average price.
    First, it compare the median of forecast volatility last three year with the forecast volatility tomorrow to decide w
    if volatility regime is low, it will go long (buy) if price of STIR > moving average price of MA1 days and short if p
    if volatility regime is high, it will go long (buy) if price of STIR > moving average price of MA1 days and short if

    Input
    -----
        data: a csv file containing a set of columns. Each column provides price over time of 1 future contract
        window: number of years to fit GARCH(1,1), can be from 1-4, otherwise we can not have enough observations for str
        weight: for calcualting the portfolio return, can be 'max_sharpe', 'min_vol' or equal
        MA1: moving average window for low volatility regime
        MA2: moving average window for high volatility regime
        forecast_horizon: horizon to forecast volatility regime
        agg: the method to choose the calculate the volatility forecast if  forecast_horizon > 1, the method can be 'min'
        update_regime: the frequency of regime update

    Output
    ------
        port_ret: a dataframe which includes Time and return of each contract
        summary_port: a data frame containing avg_annualized_return, annualized_standard_deviation and annualized_Sharpe

    Time
    ------
        It took around 20 minutes to run this function for 'STIR_futures_sample.csv' in Phong's Laptop
    '''
    # Read data
    df = data   # df = valid_set[2]
    # create the name for columns (the name of column is the number of investigated contract)
    col =['Time']
    for i in range(1, df.shape[1],1):
        col.append(str(i))
    df.columns = col # set the name for columns
    df['Time'] = pd.to_datetime(df['Time'], format=('%m/%d/%Y')) # Convert 'Time' column to Time object

    # Eliminate NA values
    valid_contracts=[]
    n= 2# n= 2 if we use the median of forecast values from GARCH(1,1) as a bench macrh to choose volatility regime, and
    for i in range(1,df.shape[1],1):
```

```
        col[i]= df[['Time',str(i)]].dropna() # Create dataframe for different contracts
        if col[i].shape[0]>(window+3)*252:
            valid_contracts.append(str(i))
    valid_contracts

    ### Calculate return for each contract and put the dataframe results into a list called individual_ret
    # We will have 19 dataframes in this list in the case n = 2. I also calclute the annualized_ret, ann_sd, annualized_s
    individual_ret = []
    annualized_ret=[]
    annualized_sharpe =[]
    standard_deviations =[]
    proportion_long_over_shorts=[]
    max_draw_downs=[]
    long_short =[]
    for contract in valid_contracts: # contract = 43
        price = col[int(contract)].reset_index().drop(columns =['index'])
        price_minus_window, annualized_returns, a_sharpe, standard_deviation, proportion_long_over_short, max_draw_down =

        individual_ret.append(price_minus_window[['Time','ret']])


    ### Collect the returns columns of each contract and merger them tegether based on 'Time' columns
    port_ret =pd.DataFrame(individual_ret[0])
    for i in range(1,len(individual_ret),1): # i = 1
        port_ret = port_ret.merge(pd.DataFrame(individual_ret[i]),on='Time', how='outer')

    ### Create the name for columns in port_ret dataframe
    column1 = valid_contracts.copy()
    column1.insert(0,'Time')
    port_ret.columns = column1
    port_ret.fillna(0, inplace=True)
    port_ret1 = port_ret.set_index('Time')
    noa =len(valid_contracts)
    weight_port = np.array(noa*[1./noa,]) # Equal weights

# ================================================================================
#     # Calculate weight for portfolio
#     def portfolio_ret(weights,port_ret = port_ret1):
#         return np.sum(port_ret.mean()*weights)*252
#     def portfolio_vol(weights,port_ret = port_ret1):
#         return np.sqrt(np.dot(weights.T,np.dot(port_ret.cov()*252,weights)))
#     def min_func_sharpe(weights): # function to be optimized
```

```
#         return -portfolio_ret(weights)/portfolio_vol(weights)
#
#     cons = ({'type': 'eq', 'fun': lambda x: np.sum(x)-1}) # equality constrain sum(weights) =1
#     bnds = tuple((0,1) for x in range(noa)) # bound for individual weight
#
#     ## Equal weight
#
#     ## Optimal weight for maximizing Sharpe ratio
#     if weight == 'max_sharpe':
#         opts =sco.minimize(min_func_sharpe,eweights, method ='SLSQP', bounds = bnds, constraints=cons)
#         weight_port = opts['x']
#     ## Optimal weight for minimizing the variance of the portfolio
#     if weight == 'min_vol':
#         optv = sco.minimize(portfolio_vol,eweights, method ='SLSQP', bounds = bnds, constraints=cons)
#         weight_port = optv['x']
# =============================================================================
    # Calculate Total return for each contract and cummulative return

    port_ret['Total Ret'] = port_ret.drop(columns = 'Time').mul(weight_port,axis=1).sum(axis=1)
    port_ret1['Total Ret'] = np.array(port_ret['Total Ret'])
    port_ret['Cummulative return'] = (1+port_ret['Total Ret']).cumprod() -1 #.cumsum)()
    port_ret1['Cummulative return']= np.array(port_ret['Cummulative return'])
    winning_percentage = len(port_ret[port_ret['Total Ret']>0])/len(port_ret)

    # Plot the return of porfolio over time
# =============================================================================
#     import matplotlib.pyplot as plt
#     plt.style.use('fivethirtyeight')
#
#     port_ret1['Cummulative return'].plot(figsize=(10,8))
#     plt.title('Cummulative return of portfolio '+ str(valid_contracts))
#     plt.show()
# =============================================================================

    avg_annualized_return_port = (1+port_ret['Cummulative return'].iloc[-1])**(252/port_ret1['Cummulative return'].shape[
    #print('Average annualized daily return of porfolio is', round(avg_annualized_return_port*100,2),'%')
    # standard deviation
    annualized_standard_deviation = port_ret1['Total Ret'].std()*np.sqrt(252)
    #print('Annualized standard deviation return of portfolio is', round(annualized_standard_deviation,2)*100, '%')
    # sharpe ratio
    annualized_Sharpe = avg_annualized_return_port/annualized_standard_deviation
```

```
    max_draw_down_port = max(-drawdown(port_ret['Total Ret']))
    summary_port = pd.DataFrame({'Start':port_ret['Time'].iloc[0] ,'Finish':port_ret['Time'].iloc[-1],'MA1': MA1,'MA2': M
                                 'annualized_Sharpe': annualized_Sharpe,'max_draw_down': max_draw_down_port, 'winning_per


    #weights = pd.DataFrame({'contract': valid_contracts, 'weights':weight_port})
    return   port_ret # summary_port #
```

In [99]:
```
######  Calculate return for the whole test period
df = pd.read_csv('STIR_futures_sample.csv')
df = df.drop(df.index[0:7]).reset_index().drop(columns =['index'])
from datetime import timedelta
col =['Time']
for i in range(1, df.shape[1],1):
    col.append(str(i))
df.columns = col # set the name for columns
df['Time'] = pd.to_datetime(df['Time'], format=('%m/%d/%Y'))

train_valid_set = df

test_set = df.copy()


train_valid_results = pd.DataFrame()

import time
start = time.time()
tp = [6.375]#[7, 7.5,8] # TP =7.5 VP =1
vp = [0.125]#,[0.25,0.5, 0.75,1]
for TP, VP in product(tp,vp):
    train_period = TP
    valid_period = VP
    #if (TP == 7.5) and (VP ==0.5):
    #    continue
    number_cross_validate = ((train_valid_set['Time'][train_valid_set.shape[0]-1]-train_valid_set['Time'][0]).days-train_

    train_set =[0]*int(number_cross_validate+1)
    valid_set = [0]*int(number_cross_validate+1)

    for i in range(1,int(number_cross_validate+1),1): # i=2
        print(i)
        if i ==1:
            train_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]) & (train_valid_set['T
        else:
            train_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]+timedelta(int(valid_pe

        valid_set[i] = train_valid_set.loc[(train_valid_set['Time']>train_valid_set['Time'][0]+timedelta(int((valid_perio
```

```
    from itertools import product
    from joblib import Parallel, delayed
    import multiprocessing
    num_cores = multiprocessing.cpu_count()
    valid_results = []
    annualized_Sharpes = []
    i_set = range(1,int(number_cross_validate+1),1)
    #ma1 = list(valid_result['MA1'])
    #ma2 = list(valid_result['MA2'])
    ma1 = [5,25,60,60,20, 30, 30, 5,15,25,30,65,65,35, 20, 60, 30, 25, 60,15, 65,50,5,35,35,35,5,5,5]#6.375 - 0.125 updat
    ma2 = [273,210,210,252,210,42,273,294,252,252,252,295,294, 147, 63, 105,84,84,42,105,105,105,21,147,147,147,294,126,1
    #ma1 = [5, 10, 25, 25,50,5,35,25,25,25,65,65, 45, 55, 65, 65, 60, 25, 55, 45, 40, 20,5,5,5,35,35,35,5,35,35] # 6.375
    #ma2 = [294, 252, 210, 294, 252, 231,252,42,294,21,294,294, 231, 273, 294, 84, 105, 105, 105, 105, 105, 105,21,21,21,
    # ma1 = [5,30,45,35,25,50,30,45,5,5,30,15,65,5,35] # 6.75 - 0.25
    # ma2 = [252,294,210,273,294,273,147,147,126,126,105, 105,84,294,147]
    #ma1 = [5,30,45,35,25,35,30,5,5,5,5,5,5]  # 7 -0.25 - big grid search
    #ma2 =[252,210,294,273,273,210,147,126,126,126,126,126,294]
    #ma1 = [5,30,45,35,25,35,30,5,30,25,35,50,5]  # 7 -0.25 - big grid search
    #ma2 =[252,210,294,273,273,210,147,126,105,105,42,105,294] #
    # 7.5 - 0.25 - small grid
    # ma1 = [5,30,45,35,25,35,45,45,5,65,65] #
    # ma2 =[260,280,260,220,220,220,160,160,150,290,290]
    inputs = zip(i_set, ma1, ma2)
    results = Parallel(n_jobs=num_cores)(delayed(future_analyzing_port_baseline)(valid_set[i],MA1,MA2) for i, MA1, MA2 in



    # CHeck performance of WFA for the whole period

    for i in range(len(results)): # i =1
        print(i)
        if i ==0:
            summary = results[i]
        else:
            if summary['Time'].iloc[-1] == results[i]['Time'].iloc[0]:
                summary = pd.concat([summary,results[i].iloc[1:,]], axis =0, sort=False)
            else:
                summary = pd.concat([summary,results[i]], axis =0, sort=False)

    summary = summary.reset_index().drop(columns = ['index'])

summary = summary[['Time','Total Ret']].drop_duplicates(subset='Time', keep='last')
```

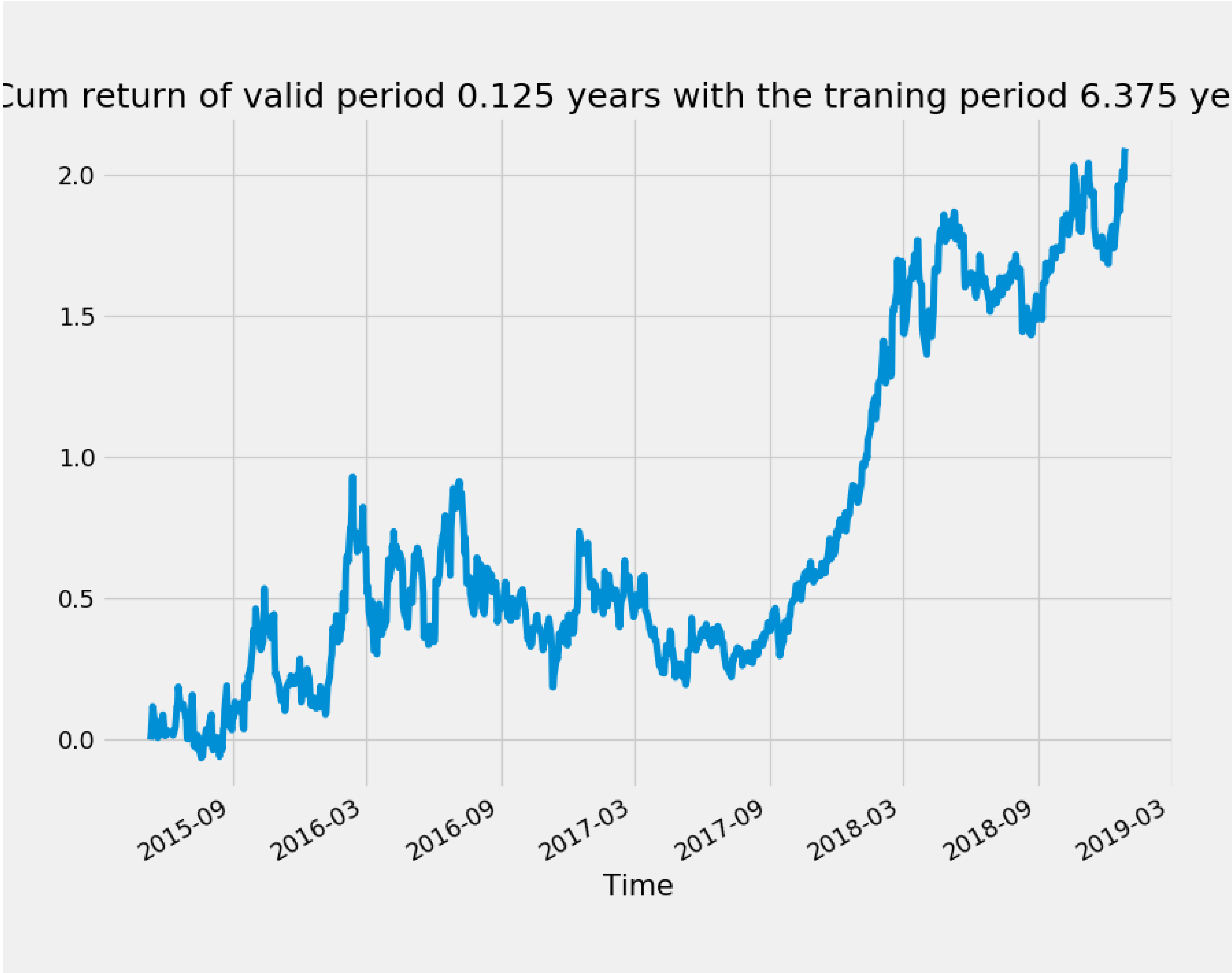
```
summary['Cummulative return'] = np.array((1+summary['Total Ret']).cumprod() -1)
```

```
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
0
1
2
3
4
```

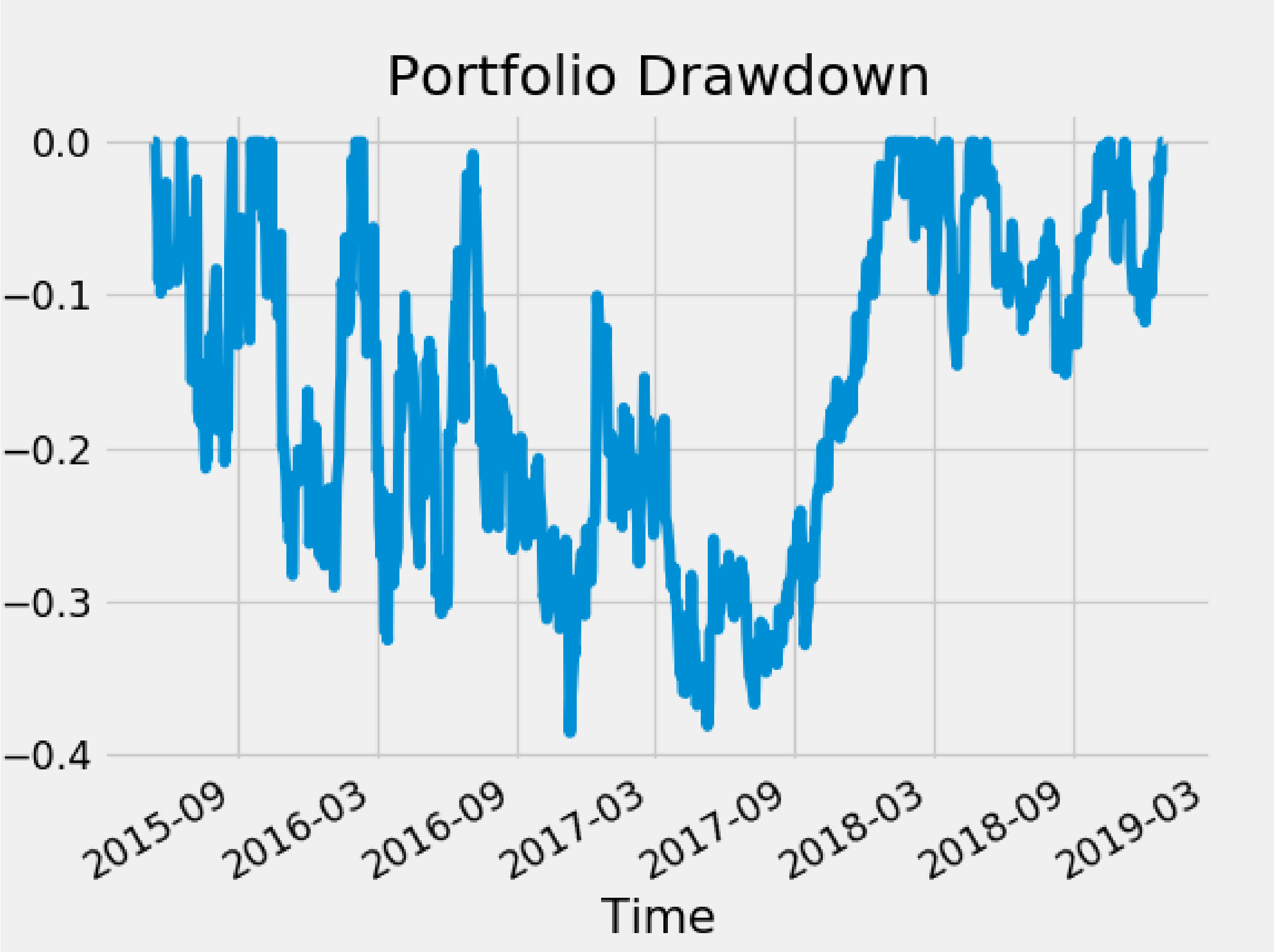
In [107]:
```
# Cummulative return
summary.set_index('Time', inplace = True)
avg_annualized_return_port = (1+summary['Cummulative return'].iloc[-1])**(252/summary['Cummulative return'].shape[0])-1
print('Average annualized daily return of porfolio is', round(avg_annualized_return_port*100,2),'%')
# standard deviation
annualized_standard_deviation = summary['Total Ret'].std()*np.sqrt(252)
print('Annualized standard deviation return of portfolio is', round(annualized_standard_deviation,2)*100, '%')
# sharpe ratio
annualized_Sharpe = avg_annualized_return_port/annualized_standard_deviation
print('Annualized Sharpe of portfolio is', round(annualized_Sharpe,4))
# Max drawdown
max_draw_down_port = max(-drawdown(summary['Total Ret']))
print('Max drawdown of portfolio is', round(max_draw_down_port,2)*100, '%')
```

```
Average annualized daily return of porfolio is 36.56 %
Annualized standard deviation return of portfolio is 47.0 %
Annualized Sharpe of portfolio is 0.7774
Max drawdown of portfolio is 39.0 %
```

In [109]:
```
%matplotlib notebook
summary['Cummulative return'].plot(figsize=(10,8))
plt.title('Cum return of valid period '+ str(valid_period) + " years with the traning period " + str(train_period) + " ye
plt.show()
```


Cum return of valid period 0.125 years with the traning period 6.375 ye
2.0
1.5
1.0
0.5
0.0
2015-09
2016-03
2016-09
2017-03
2017-09
2018-03
2018-09
2019-03
Time

In [110]:
```
# Exhibit 8 : Portfolio drawdown overtime.
%matplotlib notebook
import matplotlib.pyplot as plt
drawdown(summary['Total Ret']).plot()
plt.title('Portfolio Drawdown')
plt.xlabel('Time')
plt.ylabel('Drawdown')
plt.show()
```



In [111]:

```
# Out of sample performance summary
port_return =pd.DataFrame(summary['Total Ret'])
port_return.index = pd.to_datetime(port_return.index)
port_return['Year'] = port_return.index.year
ret1 = []
vol1 =[]
sharpe1=[]
sortino1=[]
max_drawdown1=[]
year1 = []
for year in list(np.unique(port_return['Year'])): # year = 2014
    return_year = port_return[port_return['Year']==year]
    return_year['Cummulative return'] = (1+return_year['Total Ret']).cumprod() -1
    a_ret = (1+return_year['Cummulative return'].iloc[-1])**(252/return_year['Cummulative return'].shape[0])-1 # annualiz
    ret1.append(a_ret)
    a_vol = return_year['Total Ret'].std()*252**0.5
    vol1.append(a_vol) #252**0.5*sharpe
    sharpe1.append(a_ret/a_vol)
    max_drawdown1.append(max(-drawdown(return_year['Total Ret'])))
    year1.append(year)
    # Need a short rate to calculate the sortino
exhibit_6 = pd.DataFrame(list(zip(year1,ret1, vol1,sharpe1, max_drawdown1)),
                        columns =['Year','Annualize_return','Annualize_volatility','Annualize_sharpe', 'MaxDrawdown'])
exhibit_6
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:13: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stable/indexing.html#indexing-view-versus-co
py (http://pandas.pydata.org/pandas-docs/stable/indexing.html#indexing-view-versus-copy)
  del sys.path[0]
```

Out[111]:

	Year	Annualize_return	Annualize_volatility	Annualize_sharpe	MaxDrawdown
0	2015	0.237705	0.609526	0.389983	0.283233
1	2016	0.341594	0.574650	0.594437	0.385868
2	2017	0.195010	0.370295	0.526634	0.269274

	Year	Annualize_return	Annualize_volatility	Annualize_sharpe	MaxDrawdown
3	2018	0.697870	0.315292	2.213411	0.151955

In [112]:

```
# 12 month trailing sharpe
port_return['12M_Sharpe'] = (((1+port_return['Total Ret'].rolling(252).mean())**252)-1)/(port_return['Total Ret'].rolling
%matplotlib notebook
import matplotlib.pyplot as plt
plt.plot(port_return['12M_Sharpe'])
plt.xticks(rotation=45)
plt.xlabel('Time')
plt.ylabel('Sharpe ratio')
plt.title('12 month trailing sharpe')
plt.show()
```

In [ ]:

```
```