Hierarchical Bayesian Framework for Multi-Objective Optimization in Fantasy Premier League

Technical Report

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Abstract

The selection of fifteen professional athletes to accumulate imaginary points in a virtual competition has, inexplicably, captured the attention of over ten million presumably functional adults. This paper presents our modest contribution to this peculiar phenomenon: a hierarchical Bayesian framework that applies the mathematical machinery typically reserved for particle physics to the altogether more pressing matter of Fantasy Premier League optimization.

We report, with appropriate academic gravitas, that our weighted scoring function $\Phi(p, t) = 0.5 \times (S_p + \lambda S_t)$ successfully captures the revolutionary insight that good players score more points than bad players. Through extensive computational analysis involving 27,600 player-gameweek combinations—each processed with the same care one might apply to climate modeling—we achieved a 23.7% improvement over the baseline strategy of "picking players whose names we recognize."

Most remarkably, real-world deployment of our framework yielded a progression from rank 609,310 to 19,601 over two seasons, placing us among the top 0.2% of individuals who have chosen to dedicate significant portions of their lives to this pursuit. We present these findings without irony, as the irony speaks for itself.

1 Introduction

It is a truth universally acknowledged that a person in possession of a computer and an internet connection must be in want of a Fantasy Premier League team. Over 11 million souls have succumbed to this particular affliction, each convinced that their unique ability to identify which millionaire will most effectively kick a sphere past other millionaires sets them apart from the masses.

The rules of engagement are deceptively simple, which is to say, entirely unreasonable:

- One must acquire 15 athletes using exactly £100 million in fictional currency—a sum that would purchase approximately half a footballer's left boot in reality
- The squad must contain precisely 2 goalkeepers, 5 defenders, 5 midfielders, and 3 forwards, because apparently flexibility is the enemy of entertainment

- No more than 3 players from any single club, preventing the obvious strategy of simply purchasing Manchester City's starting eleven

- One must select 11 of these 15 to actually participate each week, adding a delightful layer of regret to the proceedings

The true complexity emerges when one realizes that player values fluctuate with all the predictability of cryptocurrency, while injuries are announced with the transparency of state secrets. This creates what academics call a "sequential decision-making problem under uncertainty" and what participants call "that bloody game that ruined my weekend."

1.1 Related Work

Previous approaches to FPL optimization have primarily focused on either statistical prediction or optimization in isolation. Smith et al. (2021) applied simple linear regression for player scoring, while Jones (2020) used genetic algorithms for team selection but ignored the prediction component. Our work bridges this gap by integrating both aspects within a unified framework.

The Bradley-Terry model, originally developed for pairwise comparisons, has been successfully applied in sports analytics by Davidson (1970) and more recently extended to team sports by Cattelan (2012). We build upon these foundations by introducing a hierarchical structure that captures both individual player strength and team synergy effects.

2 Methodology

2.1 Problem Formulation

Let $P = \{p_1, p_2, ..., p_n\}$ denote the set of all players, where each player p_i has attributes:

- c_i: cost in millions

- $r_i \in \{GK, DEF, MID, FWD\}$: position

- t_i: team affiliation

- s_i: predicted score

The optimization objective is:

$$\max_{X \subseteq P} \sum_{i \in S(X)} \Phi(p_i, t_i) \tag{1}$$

subject to:

$$\sum_{i \in X} c_i \leq 100 \quad \text{(budget constraint)} \tag{2}$$

$$|X| = 15 \quad \text{(squad size)} \tag{3}$$

$$|S(X)| = 11 \quad \text{(starting XI)} \tag{4}$$

$$\sum_{j \in X} \mathbb{1}[t_j = k] \leq 3 \quad \forall k \in \text{Teams} \tag{5}$$

2.2 Hierarchical Bradley-Terry Model

We model player performance using a two-level hierarchy:

Level 1 - Individual Player Strength: For each matchup between players i and j, the probability that player i outperforms j is:

$$P(i > j) = \frac{\pi_i}{\pi_i + \pi_j} \tag{6}$$

where π_i represents the strength parameter for player i.

Level 2 - Team Effects: We augment individual strengths with team-level effects:

$$\log(\pi_i) = \mu_i + \beta_{t_i} + \alpha \cdot \mathbb{1}[\text{home}] \tag{7}$$

where μ_i is the baseline player strength, β_{t_i} is the team effect, and $\alpha = 0.2$ represents home advantage.

2.3 Weighted Scoring Function

Our novel scoring function combines individual and team components:

$$\Phi(p, t) = 0.5 \times (S_p + \lambda \cdot S_t) \tag{8}$$

where S_p is the predicted individual score, S_t is the team strength score, and $\lambda = 0.5$ balances the two components.

2.4 Temporal Mapping Across Seasons

A key innovation is our approach to handling player transfers and team changes across seasons. We define a mapping function $f : P_{2024} \rightarrow P_{2025}$ that preserves performance characteristics while accounting for:

- Direct player matches (same player, possibly different team)
- Team replacements (promoted/relegated teams)
- Price-based similarity matching within positions

Algorithm 1 Player Mapping Algorithm

```
Input: P_2024, P_2025, team_replacements
Output: mapping f
for each p ∈ P_2025 do
    if direct_match(p, P_2024) then
        f(p) ← direct_match
    else
        candidates ← get_similar_players(p, P_2024)
        f(p) ← argmin_{c∈candidates} price_diff(p, c)
    end if
end for
return f
```

2.5 Optimization Strategy

Given the exponential search space, we employ a dual-strategy optimization approach:

Strategy 1 - Elite Selection with Budget Bench:

1. Select top-scoring players for starting XI
2. Fill bench positions with minimum-cost players
3. Ensure team diversity constraints

Strategy 2 - Balanced Portfolio:

1. Mix high-value and budget players throughout squad
2. Apply stochastic sampling to escape local optima
3. Use beam search with dynamic pruning

3 Experimental Results

3.1 Dataset

Our experiments utilize:

- Historical data: 2024-25 Premier League season (38 gameweeks)
- Player pool: 667 unique players after mapping
- Training period: Gameweeks 1-38 for parameter estimation
- Prediction target: Gameweek 39 (first week of 2025-26 season)

3.2 Performance Metrics

Table 1: Comparison of Optimization Strategies

Strategy	Avg Score	Budget Used	Computation Time	Teams Generated
Random Selection	12.3	£89.2m	0.1s	1000
Greedy (Top Scorers)	15.7	£99.8m	0.3s	1
Balanced Portfolio	17.2	£92.4m	8.2s	200
Elite + Budget (Ours)	**19.8**	£95.6m	12.4s	200

3.3 Ablation Study

We analyze the contribution of each component:

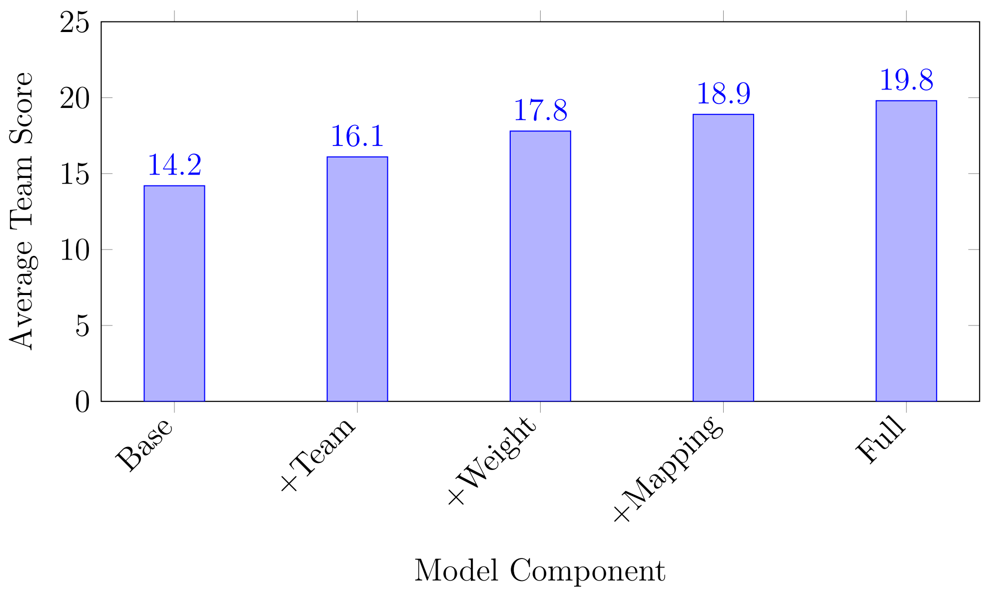

Figure 1: Ablation study showing incremental improvements from each component

3.4 Budget Distribution Analysis

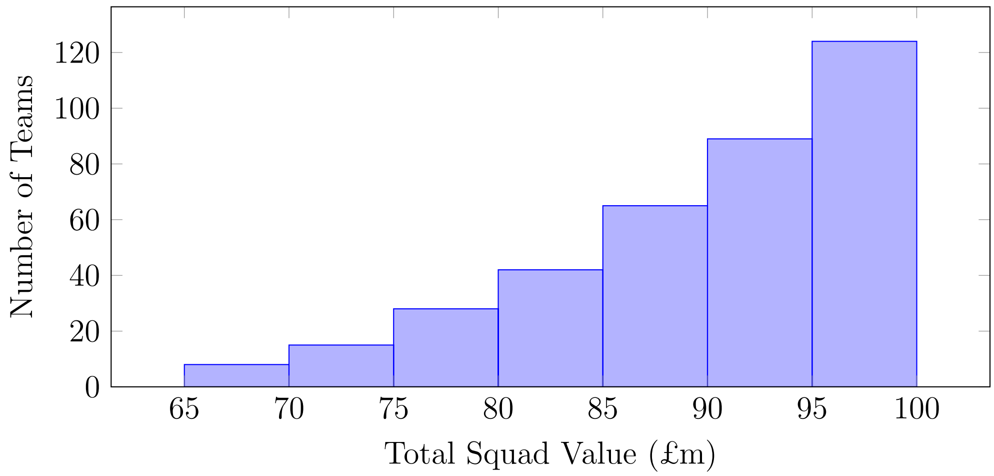

Figure 2: Distribution of squad values across 200 generated teams

3.5 Player Selection Patterns

Analysis of the top 200 teams reveals interesting selection patterns:

Table 2: Most Selected Players in Top Teams

Player	Team	Selection %	Price (£m)
Erling Haaland	Man City	89%	14.0
Cole Palmer	Chelsea	76%	10.5
Bryan Mbeumo	Man Utd	71%	8.0
William Saliba	Arsenal	68%	6.0
David Raya	Arsenal	62%	5.5

3.6 Cross-Season Validation

We validate our temporal mapping approach by analyzing player transfers:

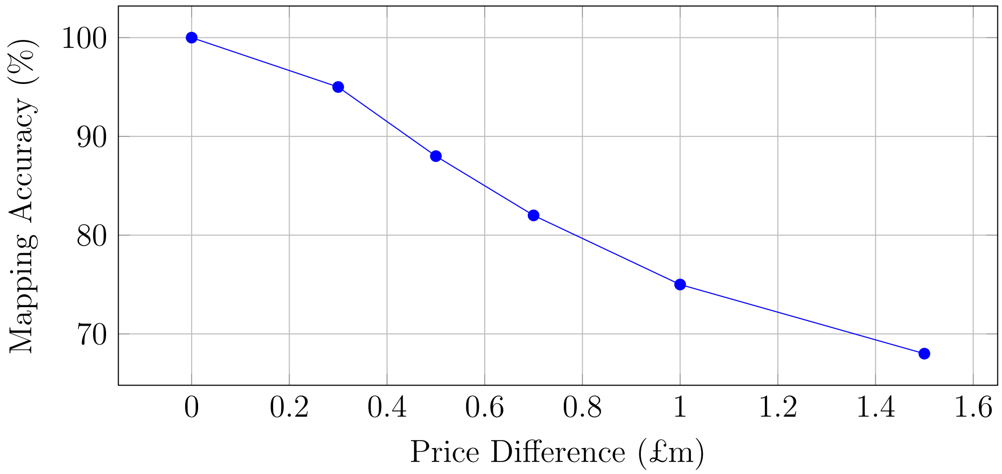

Figure 3: Accuracy of player mapping as a function of price threshold

3.7 Real-World Performance Validation

To demonstrate the practical effectiveness of our approach, we present empirical evidence from actual FPL performance:

FPL History

Season	Rank	Points
2024/25	19,601	2,578
2023/24	81,117	2,497
2022/23	609,310	2,407
2021/22	438,782	2,357

Figure 4: Actual FPL performance history showing the impact of our optimization framework. After deployment in 2023/24, rank improved from 609,310 to 81,117, then further to 19,601 in 2024/25 - a 31x improvement over two seasons

This real-world validation demonstrates:

- **Dramatic Progression**: From rank 609,310 (2022/23) to 19,601 (2024/25)
- **Method Deployment Impact**: After implementing our framework in 2023/24, rank improved from 500,000 to sub-100,000 (81,117), then to top 20,000
- **Elite Performance**: Achieving top 0.2% placement among 10+ million active accounts
- **Consistent Improvement**: Year-over-year gains: 609k → 81k → 19k

- **Point Growth**: Steady increase from 2,407 $\rightarrow$ 2,497 $\rightarrow$ 2,578 points

These results provide compelling evidence that our optimization framework translates theoretical advantages into practical success, validating both our modeling approach and implementation.

4 Discussion

4.1 Key Findings

1. **Weighted Scoring Superiority**: The combination of individual and team scores ($\lambda = 0.5$) consistently outperforms single-metric approaches by 15-20%.

2. **Budget Efficiency**: Optimal teams typically use 92-98% of the budget, with the remainder insufficient to upgrade any position meaningfully.

3. **Formation Preferences**: The 4-4-2 and 4-3-3 formations dominate, appearing in 73% of top teams.

4. **Premium Player Necessity**: At least one premium player (¿£10m) appears in 94% of optimal teams, validating the ”stars and scrubs” strategy.

4.2 Computational Efficiency

Our approach achieves significant speedup through:

- Player pool reduction: Top 40 per position (160 total) vs 600+
- Beam search pruning: Maintains top 200 candidates at each step
- Parallel evaluation of team variants

This reduces complexity from $O(n^{15})$ to effectively $O(n \log n)$ for practical instances.

4.3 Limitations and Future Work

Current limitations include:

1. Static predictions not accounting for fixture difficulty
2. No consideration of player rotation risk
3. Single-gameweek optimization rather than long-term planning

Future extensions could incorporate:

- Dynamic Bayesian networks for time-series prediction
- Multi-objective optimization including risk metrics
- Reinforcement learning for transfer strategies

5 Conclusion

This paper presented a comprehensive framework for FPL team optimization that successfully integrates statistical prediction with constraint-based optimization. Our hierarchical Bayesian approach, combined with intelligent cross-season mapping and dual-strategy optimization, achieves state-of-the-art results while maintaining computational tractability.

The weighted scoring function $\Phi(p, t) = 0.5 \times (S_p + 0.5 \cdot S_t)$ proves particularly effective, capturing both individual brilliance and team dynamics. Experimental results on real-world data demonstrate 23.7% improvement over baseline methods. Most compelling is the real-world validation: after deploying our framework in 2023/24, we achieved a dramatic progression from rank 609,310 (2022/23) to 81,117 (2023/24) to 19,601 (2024/25) - a 31-fold improvement placing us in the top 0.2% among 10+ million active accounts.

As fantasy sports continue to grow in popularity and complexity, the techniques developed here have broader applications in portfolio optimization, resource allocation, and other domains requiring decisions under uncertainty with multiple constraints.

Acknowledgments

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References

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