Tutorial 2

Exercise 1: Consider the following atomic propositions:

p — *„the sun is shining“*
q — *„it is raining“*
r — *„we can see a rainbow“*
s — *„it is snowing“*

Describe in a natural language what the propositions represented by the following formulas of propositional logic say:

a) $(p \wedge q) \rightarrow r$
b) $p \rightarrow (\neg q \wedge \neg s)$
c) $\neg\neg r$
d) $(p \vee q) \vee s$
e) $q \rightarrow q$
f) $\neg s \rightarrow q$
g) $(\neg r \wedge q) \leftrightarrow \neg s$
h) $\neg(\neg p \rightarrow \neg q)$

Exercise 2: Represent the following propositions by formulas of propositional logic (for each formula, specify precisely what are the atomic propositions):

a) If it is not Monday today then it won't be Wednesday after tomorrow.
b) If is Monday or Wednesday today and it not Friday after tomorrow, then it is Monday today.
c) It is not Monday nor Thursday today.

Determine the days in a week, on which these propositions are true, and on which are false.

Exercise 3: Write the following propositions as formulas of propositional logic (for each formula specify what are the atomic propositions):

a) If barometric pressure drops, then it will be raining or snowing.
b) If a packet with a request comes, this request will be processed and a packet with an acknoledgement will be sent, or a packet with information about error will be sent.
c) If new oilfields are not found and there is a crisis in the Middle East, then oil prices will increase.
d) If Mr. Smith has bought a new car and has not sold the old one, then he has already payed off his mortage or he has got a new loan.
e) Sister has a blue coat and a white coat.
f) If John testifies and tells the truth, he will be found guilty; and if he does not testify, he will be found guilty.
g) A sufficient condition for a number x to be odd is that x is a prime and it is greater than 2.
h) Necessary condition for a sequence to converge is that it is bounded from above and from below.
i) This amount will be paid if and only if the goods will be delivered.
j) If x is positive, then x^2 is positive.

k) If triangle ABC is not isosceles then it is not equilateral.
l) Graph G is planar if and only if it does not contain as a subgraph a subdivision of graph K_5 nor a subdivision of graph $K_{3,3}$.
m) It is not true that if this candidate won't be elected for president then the economical situation does not get worse.
n) If the culprit forged this document, bribed the taxi-driver, and haven't cleared the fingerprints, then an evidence will be found against him.

Exercise 4: Consider the following propositions:

p — *„Prague is larger than Liberec"*
q — *„Carlsbad is situated in western Bohemia"*
r — *„the Elbe flows through České Budějovice"*

(So propositions p and q are true, and proposition r is false.)
Which of the following propositions are true, and which are false? (Formulate these propositions also in a natural language.)

a) $p \vee r$
b) $p \wedge r$
c) $\neg p \wedge \neg r$
d) $p \leftrightarrow (\neg q \vee r)$
e) $(q \vee \neg r) \rightarrow p$
f) $(q \vee p) \rightarrow (q \rightarrow \neg r)$
g) $(q \leftrightarrow \neg p) \leftrightarrow (p \leftrightarrow r)$
h) $(q \rightarrow p) \rightarrow ((p \rightarrow \neg r) \rightarrow (\neg r \rightarrow q))$

Exercise 5: For each of the following sequences of symbols, do the following:

a) Decide if it is a well-formed formula of propositional logic (according to the formal definition).
b) Decide if it is a well-formed formula of propositional logic when the conventions for omitting parentheses can be used.
c) If it is a well-formed formula (either according to (a) or (b)):
   - Write this formula according to formal definition (i.e., without omitting parentheses).
   - Write this formula with as much parentheses omitted as possible.
   - Draw a corresponding abstract syntax tree.

(Justify your answers in points (a) and (b).)

1. $)(p)\neg\wedge\wedge$
2. $\forall x : q(x) \wedge r(x, x)$
3. p
4. $(\neg(\neg q))$
5. $(\neg(\neg q()))$
6. $(\neg(\neg)q)$
7. $(p\neg q)$
8. $\wedge pq$
9. $p \wedge q$
10. $(p \wedge q)$
11. $((p \wedge q))$
12. $((p \wedge q) \vee r)$
13. $((\neg p) \vee (q \leftrightarrow (\neg r)))$
14. $r \vee (\neg q \vee s)$

15. $((\neg r \vee \neg p) \vee s) \wedge (\neg q \vee s)$

16. $(\neg((\neg p) \rightarrow (\neg(\neg r))))$

Exercise 6: Using the table method, determine all models of the following formulas and decide, which of these formulas are tautologies, which are satisfiable, and which are contradictions:

a) $p \vee q$
b) $p \vee \neg p$
c) $p \vee q \rightarrow q \vee p$
d) $p \rightarrow (p \vee q) \vee r$
e) $p \rightarrow (\neg p \rightarrow q)$
f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$
g) $((p \rightarrow q) \leftrightarrow q) \rightarrow p$
h) $p \rightarrow (q \rightarrow (q \rightarrow p))$
i) $p \wedge \neg(q \rightarrow p)$
j) $p \wedge q \rightarrow p \vee r$
k) $(p \vee (\neg p \wedge q)) \vee (\neg p \wedge \neg q)$
l) $p \wedge q \rightarrow (p \leftrightarrow q \vee r)$
m) $(p \wedge q \rightarrow (p \wedge \neg p \rightarrow q \vee \neg q)) \wedge (q \rightarrow q)$
n) $p \leftrightarrow q$
o) $p \leftrightarrow p \vee p$
p) $p \vee q \leftrightarrow q \vee p$
q) $(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
r) $(p \leftrightarrow p) \leftrightarrow p$