Tutorial 4

Exercise 1: If $p \leftrightarrow q$ holds, what can be said about the truth value of formula $p \vee \neg q$?

Exercise 2: Let us assume that $\neg p \vee q$ is true. Which of the following formulas hold under this assumption (i.e., which of these formulas logically follow from this assumption)? Justify your answers (e.g., by the table method, by presenting a semantic contradiction, or by presenting a truth valuation where the assumption $\neg p \vee q$ is true but the conclusion is false).

a) p
b) $q \rightarrow p$
c) $p \rightarrow q$
d) $\neg q \rightarrow \neg p$
e) $\neg p \wedge q$

Exercise 3: Let us assume that $p \wedge q$ is true. Which of the following formulas hold under this assumption (i.e., which of the following formulas logically follow from this assumption)? Justify your answers.

a) p
b) q
c) $p \vee q$
d) $p \wedge \neg q$
e) $\neg p \vee q$
f) $\neg q \rightarrow p$
g) $p \leftrightarrow q$

Exercise 4: Consider the following formulas:

a) $\neg p$
b) $\neg q$
c) $\neg p \vee \neg q$
d) $\neg p \wedge \neg q$
e) $p \leftrightarrow \neg q$
f) $\neg(p \leftrightarrow q)$
g) $p \wedge \neg q$
h) $\neg p \wedge q$
i) $\neg(p \rightarrow q) \wedge \neg(q \rightarrow p)$

- From which of these formulas the conclusion $\neg(p \wedge q)$ logically follows?
- From which of these formulas the conclusion $\neg(p \vee q)$ logically follows?
- From which of these formulas the conclusion $\neg(p \rightarrow q)$ logically follows?

Justify your answers.

Exercise 5: Determine whether the given conclusion logically follows from the given assumptions. (Justify your answers.)

a) The conclusion p follows from the assumptions q and $p \rightarrow q$.
b) The conclusion $\neg q$ follows from the assumptions $\neg p$ and $p \rightarrow q$.

c) The conclusion $p \wedge q$ follows from the assumptions p and q.
d) The conclusion q follows from the assumptions p and $p \vee q$.
e) The conclusion p follows from the assumptions $\neg q$ and $p \vee q$.
f) The conclusion $\neg q$ follows from the assumptions $\neg p$ and $p \vee q$.
g) The conclusion $\neg p \wedge q$ follows from the assumption $\neg p \vee (q \rightarrow p)$.
h) The conclusion $q \vee \neg q$ follows from the assumption p.

Exercise 6: Formalize the following sentences as formulas of propositional logic. Then show that the given conclusion logically follows from the assumptions (by showing a semantic contradiction), or show that it does not follow from the assumptions by giving a truth valuation where the assumptions are true and the conclusion is false.

a) *Either logic is difficult, or students don't like it.*
If mathematics is easy, then logic is not difficult.

If students like logic, mathematics is not easy.

b) *If there is not enough experts in IT, they have high salaries.*
There is not enough experts in IT or many students want to study IT.
If many students want to study IT, there is not enough jobs for graduates.
There is enough jobs for graduates.

Experts in IT have high salaries.

c) *If company A did not enter into the contract with company B, or if company A complied with the conditions of the contract, then company B will not win the lawsuit.*
If company A failed to deliver the goods on the due date, then it did not complied with the conditions of the contract.
Company A entered into the contract with company B and failed to deliver the goods on the due date.

Company B will win the lawsuit.

Exercise 7: Find out whether the following assumptions are consistent or inconsistent. Justify your answers (in those cases where the assumptions are inconsistent, justify it by finding a semantic contradiction, and in those cases when they are consistent, give an example of a truth valuation where all these assumptions are true).

a) If Jones committed the murder then he was in the victim's apartment and he did not leave before eleven.
Jones was in the victim's apartment.
If he left before eleven then the doorman saw him.
It is not the case that the doorman saw him or that Jones committed the murder.

b) The contract is satisfied if and only if the building is completed by November 30.
The building is completed by November 30 if and only if the subcontractor completes his work by November 10.
The investor loses money if and only if the contract is not satisfied.
The subcontractor completes his work by November 10 if and only if the investor loses money.

Exercise 8: For problems from Exercise 6, determine by the resolution method whether the given conclusion logically follows from the given assumptions.

Exercise 9: For the problems from Exercise 7, determine by the resolution method whether the given assumptions are consistent or inconsistent.