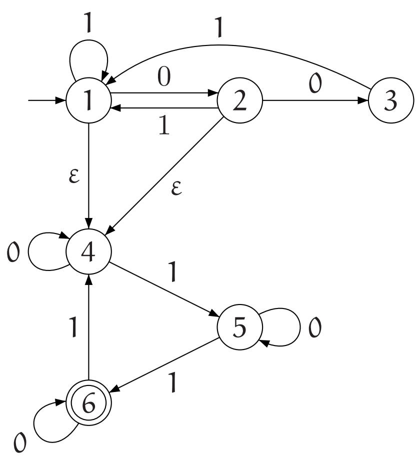
# Tutorial 9

**Exercise 1:** Construct GNFA accepting languages $L_1$ and $L_4$:

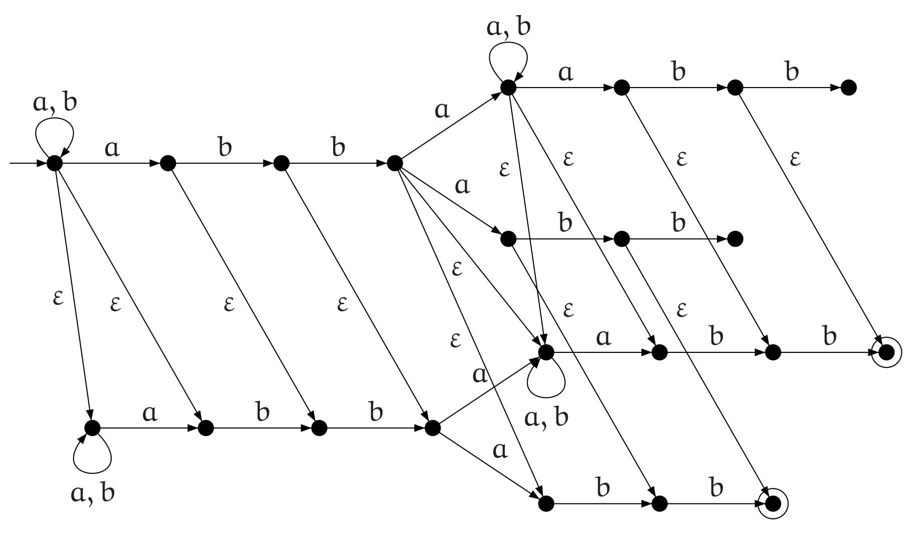
a) $L_1 = L_2 \cdot L_3$, where
$L_2 = \{w \in \{0,1\}^* \mid \text{every occurrence of } 00 \text{ in } w \text{ is immediately followed by } 1\}$
$L_3 = \{w \in \{0,1\}^* \mid |w|_1 \bmod 3 = 2\}$

*Solution:*

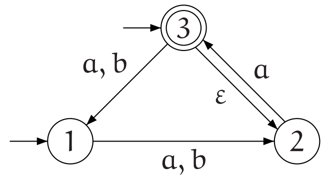



b) $L_4 = \{w \in \{a,b\}^* \mid w \text{ is obtained from some word } w' \in L_5 \text{ by ommiting of one symbol}\}$, where $L_5$ is the language consisting of those words over alphabet $\{a,b\}$ that contain subword $abba$ and end with suffix $abb$.

*Solution:*




**Exercise 2:** Construct equivalent DFA for the given GNFA:




a)

*Solution:*

Original automaton:

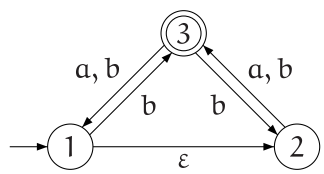
| | a | b | ε |
|---|---|---|---|
| → 1 | 2 | 2 | – |
| 2 | 3 | – | – |
| ↔ 3 | 1 | 1 | 2 |

Resulting automaton:

| | a | b |
|---|---|---|
| ↔ {1, 2, 3} | {1, 2, 3} | {1, 2} |
| {1, 2} | {2, 3} | {2} |
| ← {2, 3} | {1, 2, 3} | {1} |
| {2} | {2, 3} | ∅ |
| {1} | {2} | {2} |
| ∅ | ∅ | ∅ |

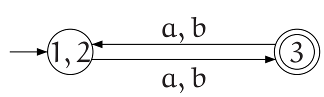
After renaming states:

| | a | b |
|---|---|---|
| ↔ 1 | 1 | 2 |
| 2 | 3 | 4 |
| ← 3 | 1 | 5 |
| 4 | 3 | 6 |
| 5 | 4 | 4 |
| 6 | 6 | 6 |




b)

*Solution:*




Original automaton:

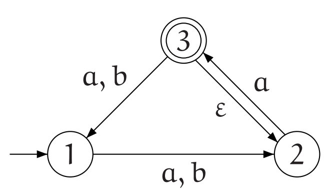
| | a | b | ε |
|---|---|---|---|
| → 1 | – | 3 | 2 |
| 2 | 3 | 3 | – |
| ← 3 | 1 | 1,2 | – |

Resulting automaton:

| | a | b |
|---|---|---|
| → {1, 2} | {3} | {3} |
| ← {3} | {1, 2} | {1, 2} |

After renaming states:

| | a | b |
|---|---|---|
| → 1 | 2 | 2 |
| ← 2 | 1 | 1 |




c)

*Solution:*

Original automaton:

| | $a$ | $b$ | $\varepsilon$ |
|---|---|---|---|
| $\to 1$ | 2 | 2 | – |
| 2 | 3 | – | – |
| $\leftarrow 3$ | 1 | 1 | 2 |

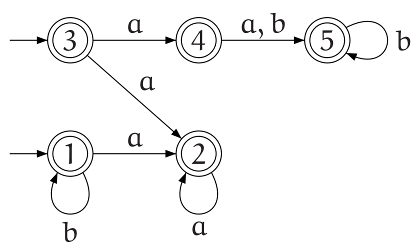
Resulting automaton:

| | $a$ | $b$ |
|---|---|---|
| $\to \{1\}$ | $\{2\}$ | $\{2\}$ |
| $\{2\}$ | $\{2,3\}$ | $\emptyset$ |
| $\leftarrow \{2,3\}$ | $\{1,2,3\}$ | $\{1\}$ |
| $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $\leftarrow \{1,2,3\}$ | $\{1,2,3\}$ | $\{1,2\}$ |
| $\{1,2\}$ | $\{2,3\}$ | $\{2\}$ |

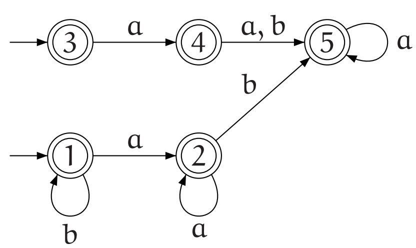
After renaming states:

| | $a$ | $b$ |
|---|---|---|
| $\to 1$ | 2 | 2 |
| 2 | 3 | 4 |
| $\leftarrow 3$ | 5 | 1 |
| 4 | 4 | 4 |
| $\leftarrow 5$ | 5 | 6 |
| 6 | 3 | 2 |

**Exercise 3:** For each of the following automata find at least one word over alphabet $\{a, b\}$, which is not accepted by the given automaton.

a) 


*Solution:* For example $bab$.

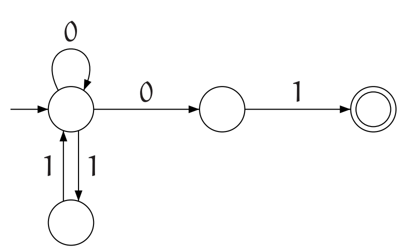
b) 


*Solution:* For example $abb$.

**Exercise 4:** For each of the following regular expressions, construct an equivalent finite automaton (it can be a GNFA):

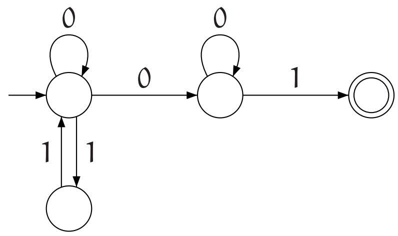
a) $(0 + 11)^*01$

*Solution:*



b) $(0+11)^*00^*1$

*Solution:*




c) $(a+bab)^* + a^*(ba+\varepsilon)$

**Exercise 5:** Describe an algorithm that for a given NFA $\mathcal{A} = (Q, \Sigma, \delta, I, F)$ decides if:

a) $\mathcal{L}(\mathcal{A}) = \emptyset$

b) $\mathcal{L}(\mathcal{A}) = \Sigma^*$

**Exercise 6:** Describe an algorithm that for given NFA $\mathcal{A}_1 = (Q_1, \Sigma, \delta_1, I_1, F_1)$ and $\mathcal{A}_2 = (Q_2, \Sigma, \delta_2, I_2, F_2)$ decides if $\mathcal{L}(\mathcal{A}_1) = \mathcal{L}(\mathcal{A}_2)$.

**Exercise 7:** Describe an algorithm that for given GNFA $\mathcal{A}$ constructs an equivalent NFA $\mathcal{A}'$ such that the sets of states of automata $\mathcal{A}$ and $\mathcal{A}'$ are the same.

***Exercise 8:** Consider an arbitrary alphabet $\Sigma$.
The ***Hamming distance*** $h(u,v)$ of a pair of words $u, v \in \Sigma^*$, such that $|u| = |v|$, is the number of positions in the words $u, v$ where these two words differ. Formally, $h(u,v)$ can be defined as follows: $h(\varepsilon,\varepsilon) = 0$, and for all symbols $a, b \in \Sigma$ and words $u, v \in \Sigma^*$, such that $|u| = |v|$, we have

$$h(au, bv) = \begin{cases} h(u,v) & \text{if } a = b \\ 1 + h(u,v) & \text{if } a \neq b \end{cases}$$

For a language $L \subseteq \Sigma^*$ and each $k \geq 0$ we define the language $H_k(L)$ as

$$H_k(L) = \{w \in \Sigma^* \mid \exists w' \in L : |w| = |w'| \wedge h(w,w') \leq k\}.$$

Show that for each $k \geq 0$ holds that if a language $L$ is regular then also language $H_k(L)$ is regular.