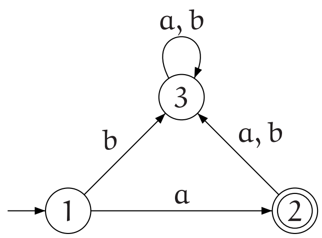
Tutorial 8

Exercise 1: For each of the following languages, construct a DFA accepting the given language. Represent the constructed automata by graphs and tables.

a) $L_1 = \{w \in \{a, b\}^* \mid w = a\}$

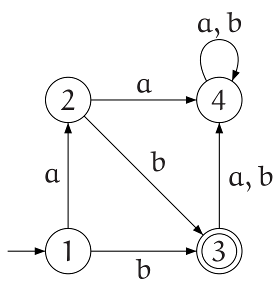
Solution:



	a	b
$\to$ 1	2	3
$\leftarrow$ 2	3	3
3	3	3

b) $L_2 = \{b, ab\}$

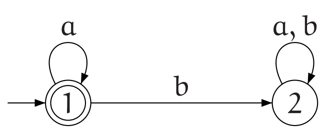
Solution:



	a	b
$\to$ 1	2	3
2	4	3
$\leftarrow$ 3	4	4
4	4	4

c) $L_3 = \{w \in \{a, b\}^* \mid \exists n \in \mathbb{N} : w = a^n\}$

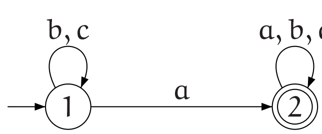
Solution:



	a	b
$\leftrightarrow$ 1	1	2
2	2	2

d) $L_4 = \{w \in \{a, b, c\}^* \mid |w|_a \geq 1\}$

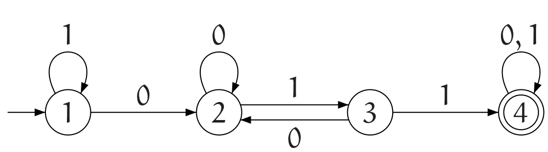
Solution:



	a	b	c
$\to$ 1	2	1	1
$\leftarrow$ 2	2	2	2

e) $L_5 = \{w \in \{0, 1\}^* \mid w$ contains subword $011\}$

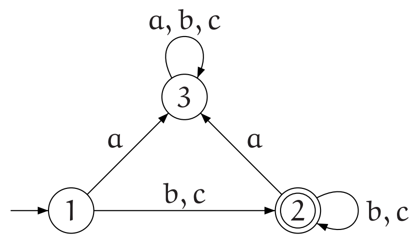
Solution:



	0	1
$\to$ 1	2	1
2	2	3
3	2	4
$\leftarrow$ 4	4	4

f) $L_6 = \{w \in \{a, b, c\}^* \mid |w| > 0 \ \wedge \ |w|_a = 0\}$

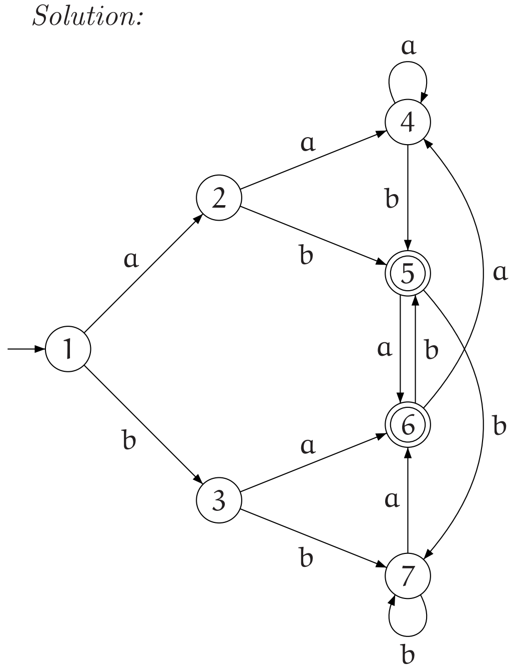
Solution:



	a	b	c
$\rightarrow 1$	3	2	2
$\leftarrow 2$	3	2	2
3	3	3	3

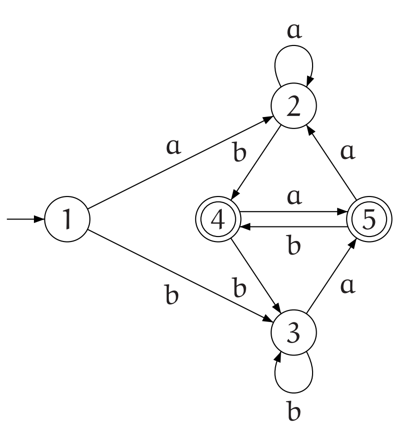
g) $L_7 = \{w \in \{a, b\}^* \mid |w| \geq 2$ and the last two symbols of w are not the same$\}$

Solution:



	a	b
$\rightarrow 1$	2	3
2	4	5
3	6	7
4	4	5
$\leftarrow 5$	6	7
$\leftarrow 6$	4	5
7	6	7

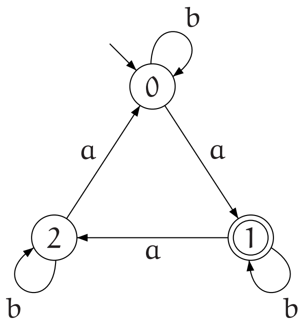
Alternative solution:



	a	b
$\rightarrow 1$	2	3
2	2	4
3	5	3
$\leftarrow 4$	5	3
$\leftarrow 5$	2	4

h) $L_8 = \{w \in \{a, b\}^* \mid |w|_a \bmod 3 = 1\}$

Solution:

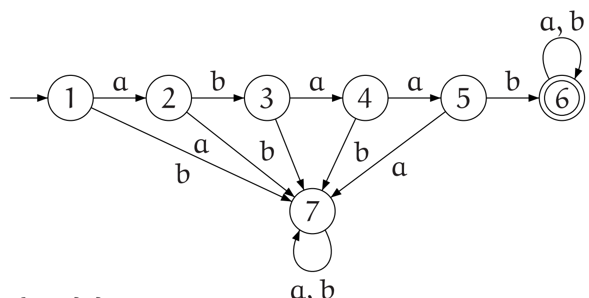


	$\mathtt{a}$	$\mathtt{b}$
$\rightarrow 0$	1	0
$\leftarrow 1$	2	1
2	0	2

Exercise 2: Construct DFA accepting words beginning with $\mathtt{abaab}$, ending with $\mathtt{abaab}$, and containing $\mathtt{abaab}$, i.e., construct deterministic finite automata accepting the following three languages:

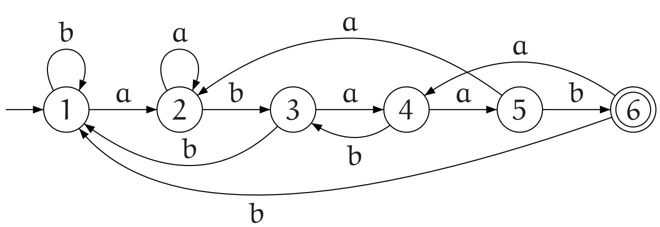
a) $L_1 = \{\mathtt{abaab}w \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$

Solution:



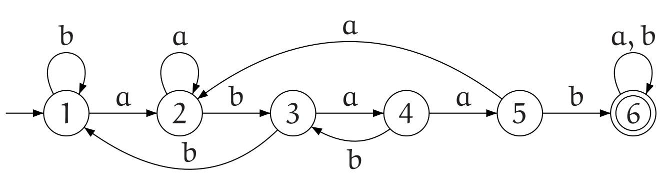
b) $L_2 = \{w\mathtt{abaab} \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$

Solution:



c) $L_3 = \{w_1\mathtt{abaab}w_2 \mid w_1, w_2 \in \{\mathtt{a}, \mathtt{b}\}^*\}$

Solution:



Exercise 3: Describe how to find out for a given DFA $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ if:

a) $\mathcal{L}(\mathcal{A}) = \emptyset$

b) $\mathcal{L}(\mathcal{A}) = \Sigma^*$

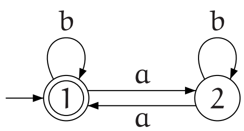
Solution: It is sufficient to compute the set of states that are reachable from q_0. We can use for example breadth-first search for this.

It holds that $\mathcal{L}(\mathcal{A}) = \emptyset$ iff none of reachable states is accepting, and $\mathcal{L}(\mathcal{A}) = \Sigma^*$ holds iff every reachable state is accepting.

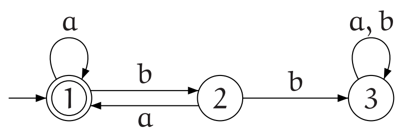
Exercise 4: Construct DFA $\mathcal{A}_1, \mathcal{A}_2$ such that:

$\mathcal{L}(\mathcal{A}_1) = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0\}$
$\mathcal{L}(\mathcal{A}_2) = \{w \in \{a, b\}^* \mid$ every occurence of symbol b in w is followed with symbol $a\}$

Solution: $\mathcal{A}_1$:



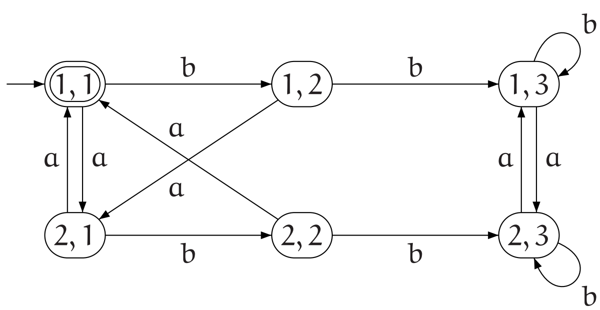
$\mathcal{A}_2$:



Using automata $\mathcal{A}_1, \mathcal{A}_2$, construct DFA accepting the following languages:

a) $L_1 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ and every occurence of symbol b in w is followed with symbol $a\}$

Solution:



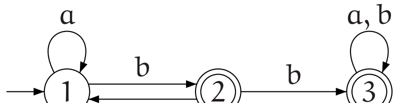
b) $L_2 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ or every occurrence of symbol b in w is followed with symbol $a\}$

Solution: The same automaton as in (a) but with the set of accepting states

$$F = \{(1, 1),\ (1, 2),\ (1, 3),\ (2, 1)\}$$

c) $L_3 = \{w \in \{a, b\}^* \mid$ some occurrence of symbol b in w is not followed with symbol $a\}$

Solution:



d) $L_4 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ and some occurrence of symbol b in w is not followed with symbol $a\}$

Solution: The same automaton as in (a) but with the set of accepting states

$$F = \{(1, 2),\ (1, 3)\}$$

e) $L_5 = \{w \in \{a, b\}^* \mid$ if $|w|_a \bmod 2 = 0$ then every occurrence of symbol b in w is followed with symbol $a\}$

Solution: The same automaton as in (a) but with the set of accepting states

$$F = \{(1, 1),\ (2, 1),\ (2, 2),\ (2, 3)\}$$

f) $L_6 = \{w \in \{a, b\}^* \mid |w|_a \bmod 2 = 0$ iff every occurrence of symbol b in w is followed with symbol $a\}$

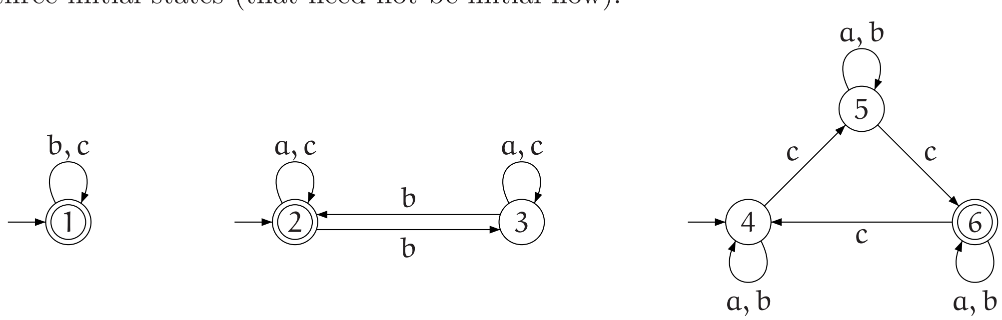
Solution: The same automaton as in (a) but with the set of accepting states

$$F = \{(1, 1),\ (2, 2),\ (2, 3)\}$$

Exercise 5: Construct NFA accepting the following languages:

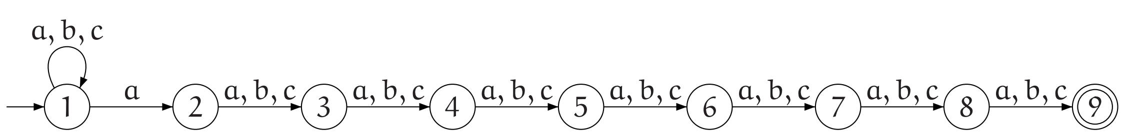
a) $L_1 = \{w \in \{a, b, c\}^* \mid |w|_a = 0 \ \vee\ |w|_b \bmod 2 = 0 \ \vee\ |w|_c \bmod 3 = 2\}$

Solution: The automaton could be easily constructed by combining three separate automata. Alternatively, we could add one new initial state with ε-transitions to the original three initial states (that need not be initial now).



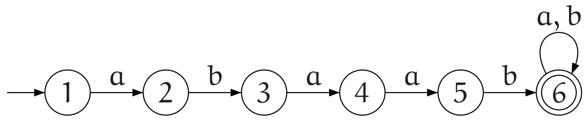
b) $L_2 = \{w \in \{a, b, c\}^* \mid |w| \geq 8$ and the eighth symbol from the end of word w is $a\}$

Solution:



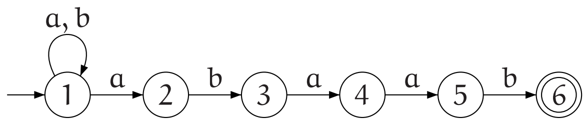
c) $L_3 = \{abaabw \mid w \in \{a, b\}^*\}$

Solution:



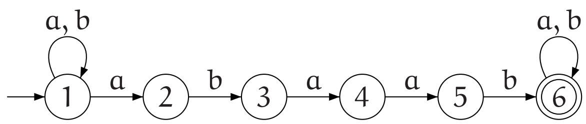
d) $L_4 = \{wabaab \mid w \in \{a, b\}^*\}$

Solution:



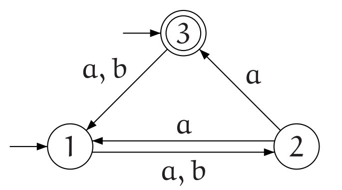
e) $L_5 = \{w_1 abaab w_2 \mid w_1, w_2 \in \{a, b\}^*\}$

Solution:



Exercise 6: Construct a DFA equivalent to the given NFA:

a)



Solution:

Original automaton:

	a	b
$\to 1$	2	2
2	1,3	–
$\leftrightarrow 3$	1	1

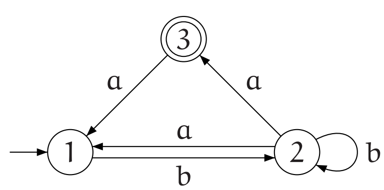
Resulting automaton:

	a	b
$\leftrightarrow \{1,3\}$	$\{1,2\}$	$\{1,2\}$
$\{1,2\}$	$\{1,2,3\}$	$\{2\}$
$\leftarrow \{1,2,3\}$	$\{1,2,3\}$	$\{1,2\}$
$\{2\}$	$\{1,3\}$	$\emptyset$
$\emptyset$	$\emptyset$	$\emptyset$

After renaming states:

	a	b
$\leftrightarrow 1$	2	2
2	3	4
$\leftarrow 3$	3	2
4	1	5
5	5	5

b)



Solution:

Original automaton:

	$\mathtt{a}$	$\mathtt{b}$
$\to 1$	–	2
2	1,3	2
$\leftrightarrow 3$	1	–

Resulting automaton:

	$\mathtt{a}$	$\mathtt{b}$
$\to \{1\}$	$\emptyset$	$\{2\}$
$\emptyset$	$\emptyset$	$\emptyset$
$\{2\}$	$\{1,3\}$	$\{2\}$
$\leftarrow \{1,3\}$	$\{1\}$	$\{2\}$

After renaming states:

	$\mathtt{a}$	$\mathtt{b}$
$\to 1$	2	3
2	2	2
3	4	3
$\leftarrow 4$	1	3