Supplementary Data of ”Modeling and Bifurcation Analysis of an Intraguild Predation System”

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Abstract

This is a supplement data for author’s paper titled *Modeling and Bifurcation Analysis of an Intraguild Predation System* [1].

APPENDIX A
DETAIL EXPRESSION OF EQUILIBRIUM ELEMENT

The detail expression for variable $\mathcal{X}$ in the elements of equilibrium E_{11} discussed in [1, Section 3.1] is given as follows.

$$\mathcal{X} = \frac{1}{3}(1-2k_0) + \mathcal{X}_0 - \frac{\mathcal{X}_n}{\mathcal{X}_d} \tag{1}$$

where $\mathcal{X}_0$, $\mathcal{X}_n$, and $\mathcal{X}_d$ are are of the following forms.

$$\begin{aligned}
\mathcal{X}_0 =& \Bigg\{ \Bigg[\left(\frac{(\alpha_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha 20 d_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta)}{(3\alpha_{13}\zeta)} - \frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^2}{(9\alpha_{13}^2\zeta^2)} \right)^3 \\
&+ \left(\frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^3}{(27\alpha_{13}^3\zeta^3)} + \frac{(\alpha_{13}k_0^2\zeta + \alpha_{13}\alpha_{20}\delta_1 k_0 - \alpha_{10}\gamma_3 k_0 k_3\zeta)}{(2\alpha_{13}\zeta)} \right. \\
&\left. - \frac{((\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)(\alpha_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha_{20}\delta_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta))}{(6\alpha_{13}^2\zeta^2)} \right)^2 \Bigg]^{1/2} \\
&+ \frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^3}{(27\alpha_{13}^3\zeta^3)} + \frac{(\alpha_{13}k_0^2\zeta + \alpha_{13}\alpha_{20}\delta_1 k_0 - \alpha_{10}\gamma_3 k_0 k_3\zeta)}{(2\alpha_{13}\zeta)} \\
&- \frac{((\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)(\alpha_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha_{20}\delta_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta))}{(6\alpha_{13}^2\zeta^2)} \Bigg\}^{1/3} \\
\mathcal{X}_n =& \left[\frac{(\alpha_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha_{20}\delta_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta)}{(3\alpha_{13}\zeta)} - \frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^2}{(9\alpha_{13}^2\zeta^2)} \right] \\
\mathcal{X}_d =& \Bigg\{ \Bigg[\left(\frac{(\alpha_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha_{20}\delta_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta)}{(3\alpha_{13}\zeta)} - \frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^2}{(9\alpha_{13}^2\zeta^2)} \right)^3 \\
&+ \left(\frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^3}{(27\alpha_{13}^3\zeta^3)} + \frac{(\alpha_{13}k_0^2\zeta + \alpha_{13}\alpha_{20}\delta_1 k_0 - \alpha_{10}\gamma_3 k_0 k_3\zeta)}{(2\alpha_{13}\zeta)} \right. \\
&\left. - \frac{((\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)(\alpha_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha_{20}\delta_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta))}{(6\alpha_{13}^2\zeta^2)} \right)^2 \Bigg]^{(1/2)} \\
&+ \frac{(\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)^3}{(27\alpha_{13}^3\zeta^3)} + \frac{(\alpha_{13}k_0^2\zeta + \alpha_{13}\alpha_{20}\delta_1 k_0 - \alpha_{10}\gamma_3 k_0 k_3\zeta)}{(2\alpha_{13}\zeta)} \\
&- \frac{((\alpha_{13}\zeta - 2\alpha_{13}k_0\zeta)(a_{13}\zeta k_0^2 - 2\alpha_{13}\zeta k_0 - \alpha_{13}\alpha_{20}\delta_1 + \alpha_{13}\alpha_{20}\varepsilon_1 + \alpha_{10}\gamma_3 k_3\zeta))}{(6\alpha_{13}^2\zeta^2)} \Bigg\}^{(1/3)}
\end{aligned} \tag{2}$$

in which ζ is given below.

$$\zeta = \frac{c\gamma_3 k_3}{\alpha_{13}\left(q^2 + \frac{\gamma_3^2 k_3^2}{\alpha_{13}^2}\right)} \tag{3}$$

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Appendix B
Elements of Linearized System Model

The elements of matrix A in the linearized system model of the IGP system as discussed in [1, Section 3.2] are as follows.

$$
\begin{aligned}
&A_{1,1}=\frac{\alpha_{10}x_0x_1}{(k_0+x_0)^2}-\frac{\alpha_{10}x_1}{(k_0+x_0)}-\frac{\alpha_{20}x_2}{(k_0+x_0)}+\frac{\alpha_{20}x_0x_2}{(k_0+x_0)^2}-2x_0+1,\\
&A_{1,2}=-\frac{\alpha_{10}x_0}{(k_0+x_0)},\qquad A_{1,3}=-\frac{\alpha_{20}x_0}{(k_0+x_0)},\qquad A_{1,4}=A_{1,5}=0,\\
&A_{2,1}=\frac{\varepsilon_1x_1}{(k_0+x_0)}-\frac{\varepsilon_1x_0x_1}{(k_0+x_0)^2},\qquad A_{2,2}=\frac{\varepsilon_1x_0}{(k_0+x_0)}+\frac{\varepsilon_3x_3}{(k_3+x_3)}+\frac{2cx_1^3x_2}{(q^2+x_1^2)^2}-\frac{2cx_1x_2}{q^2+x_1^2}-\delta_1,\\
&A_{2,3}=\frac{-cx_1^2}{q^2+x_1^2},\qquad A_{2,4}=\frac{\varepsilon_3x_1}{k_3+x_3}-\frac{\varepsilon_3x_1x_3}{(k_3+x_3)^2},\qquad A_{2,5}=0,\\
&A_{3,1}=\frac{\varepsilon_1x_2}{(k_0+x_0)}-\frac{\varepsilon_1x_0x_2}{(k_0+x_0)^2},\qquad A_{3,2}=\frac{-2cx_1^3x_2}{(q^2+x_1^2)^2}+\frac{2cx_1x_2}{(q^2+x_1^2)},\\
&A_{3,3}=\frac{\varepsilon_1x_0}{(k_0+x_0)}+\frac{\varepsilon_4x_4}{k_4+x_4}+\frac{cx_1^2}{q^2+x_1^2}-\delta_2,\qquad A_{3,4}=0,\qquad A_{3,5}=\frac{\varepsilon_4x_2}{k_4+x_4}-\frac{\varepsilon_4x_2x_4}{(k_4+x_4)^2},\\
&A_{4,1}=A_{4,3}=A_{4,5}=0,\qquad A_{4,2}=\frac{-\alpha_{13}x_3}{k_3+x_3},\qquad A_{4,4}=\frac{\alpha_{13}x_1x_3}{(k_3+x_3)^2}-\frac{\alpha_{13}x_1}{(k_3+x_3)}-\gamma_3(2x_3-1),\\
&A_{5,1}=A_{5,2}=A_{5,4}=0,\qquad A_{5,3}=\frac{-\alpha_{24}x_4}{k_4+x_4},\qquad A_{5,5}=\frac{\alpha_{24}x_2x_4}{(k_4+x_4)^2}-\frac{\alpha_{24}x_2}{k_4+x_4}-\gamma_4(2x_4-1).
\end{aligned}
\tag{4}
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References

[1] T. A. Tamba, ”Modeling and bifurcation analysis of an intraguild predation system,” *International Journal of Advances in Applied Sciences (IJAAS)*, 2023.