

# Near-consistent robust estimations of moments for unimodal distributions

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**Descriptive statistics for parametric models currently heavily rely on the accuracy of distributional assumptions. Here, leveraging the invariant structures of unimodal distributions, a series of sophisticated yet efficient estimators, robust to both gross errors and departures from parametric assumptions, are proposed for estimating mean and central moments for common unimodal distributions. This article also illuminates the understanding of the common nature of probability distributions and the measures of them.**

orderliness | invariant | unimodal | adaptive estimation |  $U$ -statistics

The potential inconsistencies between the sample mean ( $\bar{x}$ ) and robust location estimators with non-zero asymptotic breakdown points in distributions with finite moments have been noticed for more than two centuries (1), yet remain unsolved. Strictly speaking, it is unsolvable as by trimming, some information about the original distribution is removed, making it impossible to estimate the values of the removed parts without distributional assumptions.

**Data Availability.** Data for Table ?? are given in SI Dataset S1. All codes have been deposited in [GitHub](#).

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1. CF Gauss, *Theoria combinationis observationum erroribus minimis obnoxiae*. (Henricus Dieterich), (1823).