# Near-consistent robust estimations of moments for unimodal distributions

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**Descriptive statistics for parametric models currently heavily rely on the accuracy of distributional assumptions. Here, leveraging the invariant structures of unimodal distributions, a series of sophisticated yet efficient estimators, robust to both gross errors and departures from parametric assumptions, are proposed for estimating mean and central moments for common unimodal distributions. This article also illuminates the understanding of the common nature of probability distributions and the measures of them.**

orderliness | invariant | unimodal | adaptive estimation | $U$-statistics

The potential inconsistencies between the sample mean ($\bar{x}$) and robust location estimators in distributions with finite moments have been noticed for more than two centuries (1), with numerous significant attempts made to address them. In calculating the sample mean, extreme values are incorporated; however, they are much less weighted in robust location estimators. The procedure of identifying and downweighting extreme values inherently necessitates the formulation of certain distributional assumptions. Inconsistencies may arise when these assumptions, parametric or semiparametric, are violated. Due to the presence of infinite dimensional nuisance shape parameters, the semiparametric approach struggles to adequately address distributions with more intricate shapes. Newcomb (1886) provided the first modern approach to robust parametric estimation by developing a class of estimators that gives "less weight to the more discordant observations" (2).

**Data Availability.** Data for Table **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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1. CF Gauss, *Theoria combinationis observationum erroribus minimis obnoxiae*. (Henricus Dieterich), (1823).
2. S Newcomb, A generalized theory of the combination of observations so as to obtain the best result. *Am. journal Math.* **8**, 343–366 (1886).