Near-consistent robust estimations of moments for unimodal distributions

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Descriptive statistics for parametric models currently heavily rely on the accuracy of distributional assumptions. Here, leveraging the structures of parametric distributions and their central moment kernel distributions, a series of sophisticated yet efficient estimators, robust to both gross errors and departures from parametric assumptions, are proposed for estimating mean and central moments for common unimodal distributions. This article also illuminates the understanding of the common nature of probability distributions and the measures of them.

orderliness | invariant | unimodal | adaptive estimation | U-statistics

The potential biases of robust location estimators in estimating the population mean have been noticed for more than two centuries (1), with numerous significant attempts made to address them. In calculating a robust estimator, the procedure of identifying and downweighting extreme values inherently necessitates the formulation of distributional assumptions. Biases naturally arise when these assumptions, parametric and/or semiparametric, are violated. Previously, it was demonstrated that, due to the presence of infinite-dimensional nuisance shape parameters, the semiparametric approach struggles to consistently address distributions with shapes more intricate than γ-symmetry.

Theorem .1.

Proof. $\square$

Data Availability. Data for Table **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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1. CF Gauss, *Theoria combinationis observationum erroribus minimis obnoxiae.* (Henricus Dieterich), (1823).