Near-consistent robust estimations of moments for unimodal distributions

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This manuscript was compiled on May 24, 2023

Descriptive statistics for parametric models currently heavily rely on the accuracy of distributional assumptions. Here, leveraging the invariant structures of unimodal distributions, a series of sophisticated yet efficient estimators, robust to both gross errors and departures from parametric assumptions, are proposed for estimating mean and central moments for common unimodal distributions. This article also illuminates the understanding of the common nature of probability distributions and the measures of them.

orderliness | invariant | unimodal | adaptive estimation | U-statistics

The potential inconsistencies between the sample mean ($\bar{x}$) and robust location estimators in distributions with finite moments have been noticed for more than two centuries (1), with numerous significant attempts made to address them. Robustness in both L-statistics and R-statistics is attained through the trimming of extreme values. However, trimming removes some information about the original distribution, it is impossible to estimate the values of the removed parts without a parametric assumption.

Data Availability. Data for Table **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

ACKNOWLEDGMENTS. I gratefully acknowledge the constructive comments made by the editor which substantially improved the clarity and quality of this paper.

1. CF Gauss, *Theoria combinationis observationum erroribus minimis obnoxiae.* (Henricus Dieterich), (1823).