# Near-consistent robust estimations of moments for unimodal distributions

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**Descriptive statistics for parametric models currently heavily rely on the accuracy of distributional assumptions. Here, leveraging the invariant structures of unimodal distributions, a series of sophisticated yet efficient estimators, robust to both gross errors and departures from parametric assumptions, are proposed for estimating mean and central moments for common unimodal distributions. This article also illuminates the understanding of the common nature of probability distributions and the measures of them.**

orderliness | invariant | unimodal | adaptive estimation | $U$-statistics

The potential inconsistencies between the sample mean ($\bar{x}$) and robust location estimators in distributions with finite moments have been noticed for more than two centuries (1), with numerous significant attempts (2–8) made to address them. A contradiction arises where the computation of the sample mean includes extreme values, which are much less weighted in robust location estimators, making it impossible to accurately infer the values of the downweighted portions without a parametric assumption.

**Data Availability.** Data for Table **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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