Near-consistent robust estimations of moments for unimodal distributions

Tuban Lee

This manuscript was compiled on May 24, 2023

- Descriptive statistics for parametric models currently heavily rely on
- the accuracy of distributional assumptions. Here, leveraging the in-
- variant structures of unimodal distributions, a series of sophisticated
- yet efficient estimators, robust to both gross errors and departures
- from parametric assumptions, are proposed for estimating mean and
- 6 central moments for common unimodal distributions. This article also
- 7 illuminates the understanding of the common nature of probability
- 8 distributions and the measures of them.

orderliness | invariant | unimodal | adaptive estimation | U-statistics

The potential inconsistencies between the sample mean (\bar{x}) and robust location estimators in distributions with finite moments have been noticed for more than two centuries (1), with numerous significant attempts made to address them. In the computation of a sample mean, extreme data points are incorporated; however, they are much less weighted in robust location estimators. The procedure of identifying and downweighting extreme values inherently necessitates the formulation of certain distributional assumptions.

- Data Availability. Data for Table ?? are given in SI Dataset S1.
 All codes have been deposited in GitHub.
- 12 **ACKNOWLEDGMENTS.** I gratefully acknowledge the constructive comments made by the editor which substantially improved the clarity and quality of this paper.
- CF Gauss, Theoria combinationis observationum erroribus minimis obnoxiae. (Henricus Dieterich), (1823).