# Near-consistent robust estimations of moments for unimodal distributions

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**A. Congruent distribution.** In the realm of nonparametric statistics, the relative differences, or orders, of robust estimators are of primary importance. A key implication of this principle is that when there is a shift in the parameters of the underlying distribution, all nonparametric estimates should asymptotically change in the same direction, if they are estimating the same attribute of the distribution. If, on the other hand, the mean suggests an increase in the location of the distribution while the median indicates a decrease, a contradiction arises. It is worth noting that such contradiction is not possible for any $LL$-statistics in a location-scale distribution, as explained in the previous article on semiparametric robust mean. However, it is possible to construct counterexamples to the aforementioned implication in a shape-scale distribution. In the case of the Weibull distribution, its quantile function is $Q_{Wei}(p) = \lambda(-\ln(1-p))^{1/\alpha}$, where $0 \leq p \leq 1$, $\alpha > 0$, $\lambda > 0$, $\lambda$ is a scale parameter, $\alpha$ is a shape parameter, ln is the natural logarithm function. Then, $m = \lambda \sqrt[\alpha]{\ln(2)}$, $\mu = \lambda\Gamma\left(1+\frac{1}{\alpha}\right)$, where $\Gamma$ is the gamma function. When $\alpha = 1$, $m = \lambda \ln(2) \approx 0.693\lambda$, $\mu = \lambda$, when $\alpha = \frac{1}{2}$, $m = \lambda \ln^2(2) \approx 0.480\lambda$, $\mu = 2\lambda$, the mean increases as $\alpha$ changes from 1 to $\frac{1}{2}$, but the median decreases.

**Data Availability.** Data for Table **??** are given in SI Dataset S1. All codes are attached.

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