

# Near-consistent robust estimations of moments for unimodal distributions

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**A. Congruent distribution.** In the realm of nonparametric statistics, the relative differences, or orders, of robust estimators are of primary importance. A conclusion from this principle is that when there is a shift in the parameters of the underlying distribution, all nonparametric estimates should asymptotically change in the same direction, if they are estimating the same attribute of the distribution. Otherwise, if the mean indicates an increase in the location of the distribution, but the median indicates a decrease, a contradiction arises. However, while such contradiction is impossible for any  $LL$ -statistics in a location-scale distribution, as discussed in the previous semiparametric robust mean article, counterexamples of the above conclusion can be constructed in a shape-scale distribution.

**Data Availability.** Data for Table ?? are given in SI Dataset S1. All codes are attached.

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