Near-consistent robust estimations of moments for unimodal distributions

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This manuscript was compiled on June 8, 2023

orderliness | invariant | unimodal | adaptive estimation | U-statistics

A. Congruent distribution. In the realm of nonparametric statistics, the relative differences, or orders, of robust estimators are of primary importance. Deducting from this principle, it is natural to assume that when there is a shift in the parameters of the underlying distribution, all nonparametric estimates should asymptotically change in the same direction, if they are estimating the same attribute of the distribution. Otherwise, if the mean indicates an increase in the location of the distribution, but the median indicates a decrease, a contradiction arises. While such contradiction is impossible for a location-scale distribution, as discussed in the previous article, it is possible for a shape-scale distribution. For example, in the case of the Weibull distribution, $m = \lambda \sqrt[\alpha]{\ln(2)}$, $\mu = \lambda\Gamma\left(1 + \frac{1}{\alpha}\right)$, then, when $\alpha = 1$, $m = \lambda \ln(2) \approx 0.693\lambda$, $\mu = \lambda$, but when $\alpha = \frac{1}{2}$, $m = \lambda \ln^2(2) \approx 0.480\lambda$, $\mu = 2\lambda$, the mean increases, but the median decreases. To avoid such scenarios, let the quantile average function of a parametric distribution be denoted as $\mathrm{QA}\left(\epsilon, \gamma, \alpha_1, \cdots, \alpha_i, \cdots, \alpha_k\right)$, where α_i represent the parameters of the distribution

Data Availability. Data for Table **??** are given in SI Dataset S1. All codes are attached.

ACKNOWLEDGMENTS. I gratefully acknowledge the constructive comments made by the editor which substantially improved the clarity and quality of this paper.