Near-consistent robust estimations of moments for unimodal distributions

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A. Robust Estimations of the Central Moments. In 1976, Bickel and Lehmann (1), in their third paper of the landmark series *Descriptive Statistics for Nonparametric Models*, generalized nearly all robust scale estimators of that time as measures of the dispersion of a symmetric distribution around its center of symmetry. In 1979, the same series, they (2) proposed a class of estimators referred to as measures of spread, which consider the spread of a random variable, irrespective of its symmetry, throughout its distribution, rather than focusing on dispersion relative to a fixed point. Building on this, Rousseeuw and Croux proposed a popular efficient scale estimator (3) in 1993, but the importance of tackling the symmetry assumption has been greatly underestimated, as will be discussed later.

Theorem A.1.

Proof. □

1. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models. iii. dispersion in *Selected works of EL Lehmann*. (Springer), pp. 499–518 (2012).
2. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models iv. spread in *Selected Works of EL Lehmann*. (Springer), pp. 519–526 (2012).
3. PJ Rousseeuw, C Croux, Alternatives to the median absolute deviation. *J. Am. Stat. association* **88**, 1273–1283 (1993).