

Robust estimations of moments for unimodal distributions

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A. Robust Estimations of the Central Moments. In 1976, Bickel and Lehmann (1), in their third paper of the landmark series *Descriptive Statistics for Nonparametric Models*, generalized nearly all robust scale estimators of that time as measures of the dispersion of a symmetric distribution around its center of symmetry. In 1979, the same series, they (2) proposed a class of estimators referred to as measures of spread, which consider the pairwise differences of a random variable, irrespective of its symmetry, throughout its distribution, rather than focusing on dispersion relative to a fixed point. Two examples of the impacts of this series are as follows. Oja (1981, 1983) (3, 4) provided a more comprehensive and generalized examination of these concepts, and integrated the measures of location, dispersion, and spread as proposed by Bickel and Lehmann (1, 2, 5), along with van Zwet's convex transformation order of skewness and kurtosis (1964) (6) for univariate and multivariate distributions, resulting a greater degree of generality and a broader perspective on these statistical constructs. Rousseeuw and Croux proposed a popular efficient scale estimator (7) in 1993. However the importance of tackling the symmetry assumption has been greatly underestimated, as will be discussed later.

Theorem A.1.

Proof. \square

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3. H. Oja, On location, scale, skewness and kurtosis of univariate distributions. *Scand. J. statistics* pp. 154–168 (1981).
4. H. Oja, Descriptive statistics for multivariate distributions. *Stat. & Probab. Lett.* 1, 327–332 (1983).
5. P.J. Bickel, E.L. Lehmann, Descriptive statistics for nonparametric models ii. location in *selected works of E.L. Lehmann*. (Springer), pp. 473–497 (2012).
6. W. van Zwet, Convex transformations: A new approach to skewness and kurtosis in *Selected Works of Willem van Zwet*. (Springer), pp. 3–11 (2012).
7. P.J. Rousseeuw, C. Croux, Alternatives to the median absolute deviation. *J. Am. Stat. association* 88, 1273–1283 (1993).