Near-consistent robust estimations of moments for unimodal distributions

Tuban Lee

This manuscript was compiled on June 8, 2023

A. Robust Estimations of the Central Moments. In 1976, Bickel and Lehmann (1), in their third paper of the landmark series *Descriptive Statistics for Nonparametric Models*, generalized nearly all robust scale estimators of that time as measures of the dispersion of a symmetric distribution around its center of symmetry. In 1979, the same series, they (2) proposed a class of estimators referred to as measures of spread, which consider the spread of a random variable, irrespective of its symmetry, throughout its distribution, rather than focusing on dispersion relative to a fixed point. Building on this, Oja (1981) (3) generalized measures of scatter, Rousseeuw and Croux proposed a popular efficient scale estimator (4) in 1993, but the importance of tackling the symmetry assumption has been greatly underestimated, as will be discussed later.

Theorem A.1.

Proof. □

1. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models. iii. dispersion in *Selected works of EL Lehmann.* (Springer), pp. 499–518 (2012).
2. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models iv. spread in *Selected Works of EL Lehmann.* (Springer), pp. 519–526 (2012).
3. H Oja, Descriptive statistics for multivariate distributions. *Stat. & Probab. Lett.* **1**, 327–332 (1983).
4. PJ Rousseeuw, C Croux, Alternatives to the median absolute deviation. *J. Am. Stat. association* **88**, 1273–1283 (1993).