Robust estimations of moments for unimodal distributions

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A. Robust Estimations of the Central Moments. In 1976, Bickel and Lehmann (1), in their third paper of the landmark series *Descriptive Statistics for Nonparametric Models*, generalized nearly all robust scale estimators of that time as measures of the dispersion of a symmetric distribution around its center of symmetry. In 1979, the same series, they (2) proposed a class of estimators referred to as measures of spread, which consider the pairwise differences of a random variable, irrespective of its symmetry, throughout its distribution, rather than focusing on dispersion relative to a fixed point. Oja (1981, 1983) (3, 4) provided a more comprehensive and generalized examination of these concepts, integrated the measures of location, dispersion, and spread as proposed by Bickel and Lehmann (1, 2, 5), along with van Zwet's convex transformation order of skewness and kurtosis (1964) (6) for univariate and multivariate distributions, resulting a greater degree of generality and a broader perspective on these statistical constructs.

Theorem A.1.

Proof. $\square$

1. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models. iii. dispersion in *Selected works of EL Lehmann.* (Springer), pp. 499–518 (2012).
2. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models iv. spread in *Selected Works of EL Lehmann.* (Springer), pp. 519–526 (2012).
3. H Oja, On location, scale, skewness and kurtosis of univariate distributions. *Scand. J. statistics* pp. 154–168 (1981).
4. H Oja, Descriptive statistics for multivariate distributions. *Stat. & Probab. Lett.* **1**, 327–332 (1983).
5. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models ii. location in *selected works of EL Lehmann.* (Springer), pp. 473–497 (2012).
6. W van Zwet, Convex transformations: A new approach to skewness and kurtosis in *Selected Works of Willem van Zwet.* (Springer), pp. 3–11 (2012).