Robust estimations of moments for unimodal distributions

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A. Robust Estimations of the Central Moments. In 1976, Bickel and Lehmann (1), in their third paper of the landmark series *Descriptive Statistics for Nonparametric Models*, generalized nearly all robust scale estimators of that time as measures of the dispersion of a symmetric distribution around its center of symmetry. In 1979, the same series, they (2) proposed a class of estimators referred to as measures of spread, which consider the pairwise differences of a random variable, irrespective of its symmetry, throughout its distribution, rather than focusing on dispersion relative to a fixed point. While they had already considered one version of the trimmed standard deviation, which is essentially a trimmed second raw moment, in the third paper of that series (1); in the final section of the fourth paper (2), they explored another two versions of the trimmed standard deviation based on symmetric differences and pairwise differences, the latter is modified here for comparison,

$$\left[\binom{n}{2}(1-\epsilon-\gamma\epsilon)\right]^{-\frac{1}{2}}\left[\sum_{i=\binom{n}{2}\gamma\epsilon}^{\binom{n}{2}(1-\epsilon)}\left(X-X'\right)_i^2\right]^{\frac{1}{2}}, \quad [1]$$

where $(X-X')_1 \leq \ldots \leq (X-X')_{\binom{n}{2}}$ are the order statistics of the pairwise differences, $X_{\mathbf{i}}-X_{\mathbf{j}}$, $\mathbf{i}<\mathbf{j}$, provided that $\binom{n}{2}\gamma\epsilon \in \mathbb{N}$ and $\binom{n}{2}(1-\epsilon) \in \mathbb{N}$. They showed that, when $\epsilon=0$, the result from [1] is equal to $\sqrt{2}$ times the sample standard deviation of the distribution.

Theorem A.1.

Proof. □

1. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models. iii. dispersion in *Selected works of EL Lehmann.* (Springer), pp. 499–518 (2012).
2. PJ Bickel, EL Lehmann, Descriptive statistics for nonparametric models iv. spread in *Selected Works of EL Lehmann.* (Springer), pp. 519–526 (2012).