Near-consistent robust estimations of moments for unimodal distributions

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Descriptive statistics for parametric models currently rely heavily on the accuracy of distributional assumptions. Here, leveraging the structures of parametric distributions and their central moment kernel distributions, a class of estimators, consistent simultanously for both a semiparametric distribution and a distinct parametric distribution, is proposed. These efficient estimators are robust to both gross errors and departures from parametric assumptions, making them ideal for estimating the mean and central moments of common unimodal distributions. This article also illuminates the understanding of the common nature of probability distributions and the measures of them.

A. Invariant Moments. Most robust location estimators commonly used are symmetric owing to the prevalence of symmetric distributions. A γ-weighted Hodges-Lehmann mean ($\mathrm{WHLM}_{k,\epsilon,\gamma,n}$) can achieve consistency for any γ-symmetric distribution. However, it falls considerably short of effectively handling a broad spectrum of other common distributions. Shifting from semiparametrics to parametrics, consider an estimator with a non-sample-dependent breakdown point (defined in Subsection **??**) that is consistent simultanously for both a semiparametric class of distributions and a distinct parametric distribution, such a robust estimator is named with the prefix 'invariant' followed by the population parameter it is consistent with. Here, the recombined I-statistic is defined as

$$\mathrm{RI}_{d,\mathbf{k}_1,\mathbf{k}_2,k_1,k_2,\epsilon_1,\epsilon_2,\gamma_1,\gamma_2,n,LU_1,LU_2} \coloneqq \lim_{c\to\infty}\left(\frac{\left(LU_{1\mathbf{k}_1,k_1,\epsilon_1,\gamma_1,n}+c\right)^{d+1}}{\left(LU_{2\mathbf{k}_2,k_2,\epsilon_2,\gamma_2,n}+c\right)^{d}}-c\right),$$