

# Semiparametric robust mean estimations based on the orderliness of quantile averages

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1 As one of the most fundamental problems in statistics, robust loca-  
2 tion estimation has many prominent solutions, such as the symmetric  
3 trimmed mean, symmetric Winsorized mean, Hodges–Lehmann es-  
4 timator, Huber M-estimator, and median of means. Recent studies  
5 suggest that their maximum biases concerning the mean can be quite  
6 different in asymmetric distributions, but the underlying mechanisms  
7 and average performance remain largely unclear. In this article, simi-  
8 lar to the mean-median-mode inequality, it is proven that within the  
9 context of nearly all common unimodal distributions, there is an order-  
10 liness of symmetric quantile averages with varying breakdown points.  
11 Further deductions explain why the Winsorized mean and median of  
12 means typically have smaller biases compared to the trimmed mean.  
13 Building on the  $\nu$ th orderliness, the binomial mean is proposed as  
14 the bias optimal semiparametric  $L$ -estimator.

semiparametric | mean-median-mode inequality | asymptotic | unimodal  
| Hodges–Lehmann estimator

1 **Data Availability.** Data for Figure ?? are given in SI Dataset  
2 S1. All codes have been deposited in [GitHub](#).

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