

# Semiparametric robust mean estimations based on the orderliness of quantile averages

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1 As one of the most fundamental problems in statistics, robust loca-  
2 tion estimation has many prominent solutions, such as the symmetric  
3 trimmed mean, symmetric Winsorized mean, Hodges–Lehmann es-  
4 timator, Huber M-estimator, and median of means. Recent studies  
5 suggest that their biases concerning the mean can be quite different  
6 in asymmetric distributions, but the underlying mechanisms remain  
7 largely unclear. This study establishes two forms of orderliness  
8 within a wide range of semiparametric distributions. From this, a  
9 sequence of advanced robust mean estimators emerges, which also  
10 explains why the Winsorized mean and median of means typically  
11 have smaller biases compared to the trimmed mean. Building on  
12 the  $\gamma$ - $U$ -orderliness, the superiority of the median Hodges–Lehmann  
13 mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal  
| Hodges–Lehmann estimator

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