

Semiparametric robust mean estimations based on the orderliness of quantile averages

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This manuscript was compiled on May 24, 2023

1 As one of the most fundamental problems in statistics, robust loca-
2 tion estimation has many prominent solutions, such as the symmetric
3 trimmed mean, symmetric Winsorized mean, Hodges–Lehmann es-
4 timator, Huber M-estimator, and median of means. Recent studies
5 suggest that their maximum biases concerning the mean can be quite
6 different in asymmetric distributions, but the underlying mechanisms
7 and average performance remain largely unclear. In this article, simi-
8 lar to the mean-median-mode inequality, it is proven that within the
9 context of nearly all common unimodal distributions, there is an or-
10 derliness of symmetric quantile averages with varying breakdown
11 points. Further deductions explain why the Winsorized mean and me-
12 dian of means typically have smaller biases compared to the trimmed
13 mean. Building on the U -orderliness, the superiority of the median
14 Hodges–Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal
| Hodges–Lehmann estimator

1 **Data Availability.** Data for Figure ?? are given in SI Dataset
2 S1. All codes have been deposited in [GitHub](#).

3 **ACKNOWLEDGMENTS.** I sincerely acknowledge the insightful
4 comments from the editor which considerably elevated the lucidity
5 and merit of this paper.