

# Semiparametric robust mean estimations based on the orderliness of quantile averages

Tuban Lee

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As one of the most fundamental problems in statistics, robust location estimation has many prominent solutions, such as the symmetric trimmed mean, symmetric Winsorized mean, Hodges–Lehmann estimator, Huber M-estimator, and median of means. Recent studies suggest that their maximum biases concerning the mean can be quite different in asymmetric distributions, but the underlying mechanisms and average performance remain largely unclear. In this article, similar to the mean-median-mode inequality, it is proven that within the context of nearly all common unimodal distributions, there is an orderliness of symmetric quantile averages with varying breakdown points. Further deductions explain why the Winsorized mean and median of means typically have smaller biases compared to the trimmed mean. Building upon the  $\nu$ th orderliness, the binomial mean is proposed to be the bias-optimal semiparametric  $L$ -estimator.

semiparametric | mean-median-mode inequality | asymptotic | unimodal  
| Hodges–Lehmann estimator

**Data Availability.** Data for Figure ?? are given in SI Dataset S1. All codes have been deposited in [GitHub](#).

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