Semiparametric robust mean estimations based on the orderliness of quantile averages

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As one of the most fundamental problem in statistics, robust location estimation has many prominent solutions, such as the symmetric trimmed mean, symmetric Winsorized mean, Hodges–Lehmann estimator, Huber M-estimator, and median of means. Recent studies suggest that their maximum biases concerning the mean can be quite different in asymmetric distributions, but the underlying mechanisms and average performance remain largely unclear. This study establishes several forms of orderliness among quantile averages, similar to the mean-median-mode inequality, within a wide range of semiparametric distributions. From this, a sequence of advanced robust mean estimators emerges, which also explains why the Winsorized mean typically have smaller biases compared to the trimmed mean. Building on the νth orderliness, the binomial mean is proposed as the bias-optimal semiparametric L-estimator.

semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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