Semiparametric robust mean estimations based on the orderliness of quantile averages

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This manuscript was compiled on May 17, 2023

As one of the most fundamental problems in statistics, robust location estimation has many prominent solutions, such as the symmetric trimmed mean, symmetric Winsorized mean, Hodges–Lehmann estimator, Huber M-estimator, and median of means. Recent studies suggest that their maximum biases concerning the mean can be quite different in asymmetric distributions, but the underlying mechanisms and average performance remain largely unclear. In this article, similar to the mean-median-mode inequality, it is proven that in the context of nearly all common unimodal distributions, there exists an orderliness of symmetric quantile averages with different breakdown points. Further deductions explain why the Winsorized mean and median of means typically have smaller biases compared to the trimmed mean. Building on the U-orderliness, the superiority of the median Hodges–Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

ACKNOWLEDGMENTS. I sincerely acknowledge the insightful comments from the editor which considerably elevated the lucidity and merit of this paper.