Semiparametric robust mean estimations based on the orderliness of quantile averages

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semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Inequalities related to weighted averages

So far, it seems plausible that the bias of a reasonable weighted average should be monotonically related to its degree of robustness in a semiparametric distribution, since it is a linear combination of quantile averages. Analogous to the γ-orderliness, the γ-trimming inequality for a right-skewed distribution is defined as $\forall 0 \leq \epsilon_1 \leq \epsilon_2 \leq \frac{1}{1+\gamma}, \gamma \geq 0, \mathrm{TM}_{\epsilon_1,\gamma} \geq \mathrm{TM}_{\epsilon_2,\gamma}$. While γ-orderliness is a sufficient condition for the γ-trimming inequality, as proven in the SI Text. The next theorem shows another relation between quantile average and trimmed mean.

Theorem .1. *For a distribution that is right-skewed and follows the γ-trimming inequality, it is asymptotically true that the quantile average is always greater or equal to the corresponding trimmed mean with the same ϵ and γ.*

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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