Semiparametric robust mean estimations based on the orderliness of quantile averages

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semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Inequalities related to weighted averages

So far, it is quite natural to hypothesize that the value of ϵ,γ-trimmed mean should be monotonically related to the breakdown point in a semiparametric distribution, since it is a linear combination of quantile averages as shown in Section **??**. Analogous to the γ-orderliness, the γ-trimming inequality for a right-skewed distribution is defined as $\forall 0 \leq \epsilon_1 \leq \epsilon_2 \leq \frac{1}{1+\gamma}, \mathrm{TM}_{\epsilon_1,\gamma} \geq \mathrm{TM}_{\epsilon_2,\gamma}$. γ-orderliness is a sufficient condition for the γ-trimming inequality, as proven in the SI Text. The next theorem shows a relation between the ϵ,γ-quantile average and the ϵ,γ-trimmed mean under the γ-trimming inequality, suggesting the γ-orderliness is not a necessary condition for the γ-trimming inequality.

Theorem .1. *For a distribution that is right-skewed and follows the γ-trimming inequality, it is asymptotically true that the quantile average is always greater or equal to the corresponding trimmed mean with the same ϵ and γ, $0 \leq \epsilon \leq \frac{1}{1+\gamma}$.*

Proof. According to the definition of the γ-trimming inequality: $\forall 0 \leq \epsilon \leq \frac{1}{1+\gamma}$, $\frac{1}{1-\epsilon-\gamma\epsilon+2\delta}\int_{\gamma\epsilon-\delta}^{1-\epsilon+\delta} Q(u)\,du \geq \frac{1}{1-\epsilon-\gamma\epsilon}\int_{\gamma\epsilon}^{1-\epsilon} Q(u)\,du$, where δ is an infinitesimal positive quantity. Subsequently, rewriting the inequality gives $\int_{\gamma\epsilon-\delta}^{1-\epsilon+\delta} Q(u)\,du - \frac{1-\epsilon-\gamma\epsilon+2\delta}{1-\epsilon-\gamma\epsilon}\int_{\gamma\epsilon}^{1-\epsilon} Q(u)\,du \geq 0 \Leftrightarrow \int_{1-\epsilon}^{1-\epsilon+\delta} Q(u)\,du + \int_{\gamma\epsilon-\delta}^{\gamma\epsilon} Q(u)\,du - \frac{2\delta}{1-\epsilon-\gamma\epsilon}\int_{\gamma\epsilon}^{1-\epsilon} Q(u)\,du \geq 0$. Since $\delta \to 0^+$, $\frac{1}{2\delta}\left(\int_{1-\epsilon}^{1-\epsilon+\delta} Q(u)\,du + \int_{\gamma\epsilon-\delta}^{\gamma\epsilon} Q(u)\,du\right) = \frac{Q(\gamma\epsilon)+Q(1-\epsilon)}{2} \geq \frac{1}{1-\epsilon-\gamma\epsilon}\int_{\gamma\epsilon}^{1-\epsilon} Q(u)\,du$, the proof is complete. ☐

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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