

# Semiparametric robust mean estimations based on the orderliness of quantile averages

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As one of the most fundamental problems in statistics, robust location estimation has many prominent solutions, such as the symmetric trimmed mean, symmetric Winsorized mean, Hodges–Lehmann estimator, Huber  $M$ -estimator, and median of means. Recent studies suggest that their biases concerning the mean can be quite different in asymmetric distributions, but the underlying mechanisms largely remain unclear. This study establishes two forms of orderliness within a wide range of semiparametric distributions. Further deductions explain why the Winsorized mean typically has smaller biases compared to the trimmed mean; two sequences of semiparametric robust mean estimators emerge. Building on the  $\gamma$ - $U$ -orderliness, the superiority of the median Hodges–Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal  
| Hodges–Lehmann estimator

## Hodges–Lehmann inequality and $\gamma$ - $U$ -orderliness

The Hodges–Lehmann estimator and the median of means are very special robust location estimators since their definitions are substantially dissimilar from conventional  $L$ -estimators and  $M$ -estimators.

**Data Availability.** Data for Figure ?? are given in SI Dataset S1. All codes have been deposited in [GitHub](#).

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