Semiparametric robust mean estimations based on the orderliness of quantile averages

Tuban Lee

This manuscript was compiled on June 7, 2023

- As one of the most fundamental problems in statistics, robust loca-
- 2 tion estimation has many prominent solutions, such as the symmetric
- 3 trimmed mean, symmetric Winsorized mean, Hodges-Lehmann es-
- 4 timator, Huber M-estimator, and median of means. Recent studies
- suggest that their biases concerning the mean can be quite different
- 6 in asymmetric distributions, but the underlying mechanisms largely
- 7 remain unclear. This study establishes two forms of orderliness within
- a wide range of semiparametric distributions. Further deductions ex-
- $_{\rm 9}$ $\,\,$ plain why the Winsorized mean typically has smaller biases compared
- to the trimmed mean; two sequences of semiparametric robust mean
- $_{\rm 11}$ $\,$ estimators emerge. Building on the $\gamma\text{-}U\text{-}{\rm orderliness},$ the superiority
- of the median Hodges-Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges—Lehmann estimator

Hodges–Lehmann inequality and γ -U-orderliness

- ² The Hodges–Lehmann estimator is a very unique robust loca-
- 3 tion estimator since its definition is substantially dissimilar
- from conventional L-estimators and M-estimators.
- 5 **Data Availability.** Data for Figure ?? are given in SI Dataset
- 6 S1. All codes have been deposited in GitHub.
- 7 ACKNOWLEDGMENTS. I sincerely acknowledge the insightful
- 8 comments from the editor which considerably elevated the lucidity
- 9 and merit of this paper.