Semiparametric robust mean estimations based on the orderliness of quantile averages

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semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Hodges–Lehmann inequality and γ-U-orderliness

The Hodges–Lehmann estimator stands out as a unique robust location estimator due to its definition being substantially dissimilar from conventional L-estimators, R-estimators, and M-estimators. In their landmark paper, *Estimates of location based on rank tests*, Hodges and Lehmann (1) proposed two methods for computing the H-L estimator: the Wilcoxon score R-estimator and the median of pairwise means. The Wilcoxon score R-estimator is a location estimator based on signed-rank test, or R-estimator, (1) and was later independently discovered by Sen (1963) (2, 3). However, the median of pairwise means is a generalized L-statistic and a trimmed U-statistic, as classified by Serfling in his novel conceptualized study in 1984 (4). Serfling further advanced the understanding by generalizing the H-L kernel function as $hl_k\left(x_1,\ldots,x_k\right)=\frac{1}{k}\sum_{i=1}^{k}x_i$, where $k\in\mathbb{N}$ (4). Here, the weighted H-L kernel function is defined as $whl_k\left(x_1,\ldots,x_k\right)=\frac{\sum_{i=1}^{k}x_i\mathbf{w}_i}{\sum_{i=1}^{k}\mathbf{w}_i}$, where $\mathbf{w}_i$s are the weights applied to each element.

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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