

# Semiparametric robust mean estimations based on the orderliness of quantile averages

Tuban Lee

This manuscript was compiled on June 6, 2023

As one of the most fundamental problems in statistics, robust location estimation has many prominent solutions, such as the symmetric trimmed mean, symmetric Winsorized mean, Hodges–Lehmann estimator, Huber M-estimator, and median of means. Recent studies suggest that their biases concerning the mean can be quite different in asymmetric distributions, but the underlying mechanisms largely remain unclear. This study establishes two forms of orderliness within a wide range of semiparametric distributions. Further deductions explain why the Winsorized mean typically has smaller biases compared to the trimmed mean; two sequences of semiparametric robust mean estimators emerge. Building on the  $\gamma$ - $U$ -orderliness, the superiority of the median Hodges–Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal  
| Hodges–Lehmann estimator

## Hodges–Lehmann inequality and $\gamma$ - $U$ -orderliness

The Hodges–Lehmann estimator stands out as a very unique robust location estimator due to its definition being substantially dissimilar from conventional weighted averages.

**Data Availability.** Data for Figure ?? are given in SI Dataset S1. All codes have been deposited in [GitHub](#).

**ACKNOWLEDGMENTS.** I sincerely acknowledge the insightful comments from the editor which considerably elevated the lucidity and merit of this paper.