# Semiparametric robust mean estimations based on the orderliness of quantile averages

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## Hodges–Lehmann inequality and $\gamma$-$U$-orderliness

The Hodges–Lehmann estimator stands out as a unique robust location estimator due to its definition being substantially dissimilar from conventional $L$-estimators, $R$-estimators, and $M$-estimators. In their landmark paper, *Estimates of location based on rank tests*, Hodges and Lehmann (1) proposed two methods for computing the H-L estimator: the Wilcoxon score $R$-estimator and the median of pairwise means. The Wilcoxon score $R$-estimator is a location estimator based on signed-rank test, or $R$-estimator, (1) and was later independently discovered by Sen (1963) (2, 3). However, the median of pairwise means is a generalized $L$-statistic and a trimmed $U$-statistic, as classified by Serfling in his novel conceptualized study in 1984 (4). Serfling further advanced the understanding by generalizing the H-L kernel as $hl_k(x_1, \ldots, x_k) = \frac{1}{k}\sum_{i=1}^{k} x_i$, where $k \in \mathbb{N}$ (4). Here, the weighted H-L kernel is defined as $whl_k(x_1, \ldots, x_k) = \frac{\sum_{i=1}^{k} x_i \mathbf{w}_i}{\sum_{i=1}^{k} \mathbf{w}_i}$, where $\mathbf{w}_i$s are the weights applied to each element.

**Data Availability.** Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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