## Semiparametric robust mean estimations based on the orderliness of quantile averages

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- As one of the most fundamental problems in statistics, robust loca-
- 2 tion estimation has many prominent solutions, such as the symmetric
- 3 trimmed mean, symmetric Winsorized mean, Hodges-Lehmann es-
- 4 timator, Huber M-estimator, and median of means. Recent studies
- suggest that their biases concerning the mean can be quite different
- 6 in asymmetric distributions, but the underlying mechanisms largely
- remain unclear. This study establishes two forms of orderliness within
- $_{8}$  a wide range of semiparametric distributions. Further deductions ex-
- $_{\rm 9}$   $\,\,$  plain why the Winsorized mean typically has smaller biases compared
- to the trimmed mean; two sequences of semiparametric robust mean
- $_{\rm 11}$   $\,$  estimators emerge. Building on the  $\gamma\text{-}U\text{-}{\rm orderliness},$  the superiority

of the median Hodges-Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges—Lehmann estimator

## Hodges–Lehmann inequality and $\gamma$ -U-orderliness

- 2 The Hodges-Lehmann estimator is a very special robust loca-
- 3 tion estimator since its definition is dissimilar from conven-
- 4 tional L-estimators, R-estimators, and M-estimators.
- 5 Data Availability. Data for Figure ?? are given in SI Dataset
- 6 S1. All codes have been deposited in GitHub.
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