Semiparametric robust mean estimations based on the orderliness of quantile averages

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As one of the most fundamental problems in statistics, robust location estimation has many prominent solutions, such as the symmetric trimmed mean, symmetric Winsorized mean, Hodges–Lehmann estimator, Huber M-estimator, and median of means. Recent studies suggest that their biases concerning the mean can be quite different in asymmetric distributions, but the underlying mechanisms largely remain unclear. This study establishes two forms of orderliness within a wide range of semiparametric distributions. Further deductions explain why the Winsorized mean typically has smaller biases compared to the trimmed mean; two sequences of semiparametric robust mean estimators emerge. Building on the γ-U-orderliness, the superiority of the median Hodges–Lehmann mean is discussed.

semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Classifying Distributions by the Signs of Derivatives

Let $\mathcal{P}_{\mathbb{R}}$ denote the set of all continuous distributions over $\mathbb{R}$ and $\mathcal{P}_{\mathbb{X}}$ denote the set of all discrete distributions over a countable set $\mathbb{X}$. While the focus of this article is primarily on the class of continuous distributions, $\mathcal{P}_{\mathbb{R}}$, most of the results and discussions presented can be extended to the discrete case, $\mathcal{P}_{\mathbb{X}}$, unless otherwise specified. Besides fully and smoothly parameterizing them by a Euclidean parameter or merely assuming regularity conditions, there exist additional methods for classifying distributions.

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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