Semiparametric robust mean estimations based on the orderliness of quantile averages

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semiparametric | mean-median-mode inequality | asymptotic | unimodal | Hodges–Lehmann estimator

Classifying Distributions by the Signs of Derivatives

Let $\mathcal{P}_{\mathbb{R}}$ denote the set of all continuous distributions over $\mathbb{R}$ and $\mathcal{P}_{\mathbb{X}}$ denote the set of all discrete distributions over a countable set $\mathbb{X}$. The primary focus of this article will be on the class of continuous distributions, $\mathcal{P}_{\mathbb{R}}$. However, it's worth noting that most discussions and results can be extended to encompass the discrete case, $\mathcal{P}_{\mathbb{X}}$, unless explicitly specified otherwise. Besides fully and smoothly parameterizing them by a Euclidean parameter or merely assuming regularity conditions, there exist additional methods for classifying distributions based on their characteristics, such as their skewness, peakedness, modality, and supported interval. In 1956, Stein initiated the study of estimating parameters in the presence of an infinite-dimensional nuisance shape parameter (1) and proposed a necessary condition for this type of problem, a contribution later explicitly recognized as initiating the field of semiparametric statistics (2). In 1982, Bickel simplified Stein's general heuristic necessary condition (1), derived sufficient conditions, and used them in formulating adaptive estimates (2). A notable example discussed in these groundbreaking works was the adaptive estimation of the center of symmetry for an unknown symmetric distribution, which is a semiparametric model.

Data Availability. Data for Figure **??** are given in SI Dataset S1. All codes have been deposited in GitHub.

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1. CM Stein, Efficient nonparametric testing and estimation in *Proceedings of the third Berkeley symposium on mathematical statistics and probability.* Vol. 1, pp. 187–195 (1956).
2. PJ Bickel, On adaptive estimation. *The Annals Stat.* **10**, 647–671 (1982).