CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)

$a_n = -2^{n+1}$
$a_{n-1} = -2^{(n-1)+1}$
$a_{n-1} = -2^n$
Now, we can write the expression of a_{n-1} to check that $a_n = -2^{n+1}$ is a solution of our recurrence relation or not.
$a_n = 3a_{n-1} + 2^n$
$a_n = 3(-2^n) + 2^n$
$a_n = -2.2^n$
$a_n = -2^{n+1}$
Then we can say that, $a_n = -2^{n+1}$ is a solution of the $a_n = 3a_{n-1} + 2^n$

(b) Find the solution with $a_0 = 1$.
(Solution)
It's a non-homogeneous recurrence relation, so we can define it like,
$a_n^{(g)} = a_n^{(h)} + a_n^{(p)}$
Firstly we should find the homogeneous part to write the characteristic equation.
$a_{n-1} = 1$
$a_n = r$
$a_n = 3a_{n-1}$(homogeneous part of the given recurrence relation)

r = 3 (characteristic root)
$a_n^{(h)} = c_1.3^n$
to find the particular part of the given recurrence relation,
$a_n^{(p)} = A.2^n$
$a_{n-1}^{(p)} = A.2^{n-1}$
$a_n = 3a_{n-1} + 2^n$ (The given recurrence relation)
$A.2^n = 3.A.2^{n-1} + 2^n$
$= ((3.A)/2).2^n + 2^n$
$A.2^n = 2^n.(((3.A)/2) + 1)$
$(((3.A)/2) + 1) =$ A
2.A - 2 = 3.A
A = -2
$a_n^{(p)} = A.2^n = -2.2^n = -2^{n+1}$
$a_n^{(g)} = a_n^{(h)} + a_n^{(p)}$
$a_n^{(g)} = c_1.3^n - 2^{n+1}$
if $a_0 = 1$ then $a_0 = c_1 - 2 = 1$
$c_1 = 3$ so,
$a_n = 3^{n+1} - 2^{n+1}$

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)
linear nonhomogeneous recurrence with constant coefficient
$f_n^{(g)} = f_n^{(h)} + f_n^{(p)}$
$f_n^{(h)}$Associated homogeneous recurrence relation is an = 4f(n1)- 4f(n2).
Characteristic equation: r 2 4r + 4 = 0 Factor. r 2 4r + 4 = 0
(r 2)(r 2)= 0 r = 2
$f_n^{(h)} = c1(2)^n + c2(2)^n.n$
$f_n^{(h)} = 2^n(c1 + c2.n)$
$f_n^{(p)} = n^2 + 8n + 20$
$f_n^{(g)} = f_n^{(h)} + f_n^{(p)}$
$f_n = 2^n(c1 + c2.n) + n^2 + 8n + 20$
$f(0) = 2^0(c1 + c2.0) + 0^2 + 8.0 + 20 = 2$
c1+20 = 2
c1 = -18
$f(1) = 2^1(c1 + c2.1) + 1^2 + 8.1 + 20 = 5$
$f(1) = 2(c1 + c2) + 29 = 5$
$f(1) = 2(-18 + c2) + 29 = 5$
c2 = 6
$f_n = 2^n(6.n - 18) + n^2 + 8n + 20$

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation a_n = $2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)
It's homogeneous , so we can define it like this,
$a_n = a_n^{(h)}$
To find the characteristic equation.
$a_{n-2} = 1$
$a_{n-1} =$ r
$a_n = r^2$
a_n = $2a_{n-1}$ - $2a_{n-2}$ (The given recurrence relation)
r^2 = 2r-2
r^2-2r+2 = 0
We will use discriminant to find the characteristic roots.
$\Delta = b^2$ - 4.a.c
$\Delta = -2^2$ - 4.(1).(2) = 4 - 8 = -4
$\Delta < 0$ (It means that there is no root in real numbers)
r_1 = -b+$\sqrt{\Delta}$/(2.a) = -(-2)+$\sqrt{-}$4/(2.1) = 2+$\sqrt{-}$4/2 = (2 + 2i)/2 = 1 + i
r_2 = -b-$\sqrt{\Delta}$/(2.a) = -(-2)-$\sqrt{-}$4/(2.1) = 2-$\sqrt{-}$4/2 = (2 - 2i)/2 = 1 - i
The characteristic roots are 1 + i and 1 - i

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)
We should write its general equation like this,
$a_n^{(h)} = c_1.r_1^n + c_2.r_2^n$
$a_n = c_1.(1+i)^n + c_2.(1-i)^n$
Now, we can use the $a_0 = 1$ and $a_1 = 2$ to find the values of c_1 and c_2.
Let's start with $a_0 = 1$ If n = 0, then $c_1 + c_2 = 1$.
for $a_1 = 2$
$a_n = c_1.(1+i)^n + c_2.(1-i)^n$
$a_1 = c_1 + i.c_1 + c_2 - i.c_2 = 2$
we already know that,
$c_1 + c_2 = 1$
1 + i($c_1 - c_2$) = 2
$c_1 - c_2$ = 1/i = i^{-1}
$c_1 + c_2 = 1$
$c_1 - c_2 = i^{-1}$
$2c_1 = 1 + i^{-1}$ so $c_1 = (1 + i^{-1})/2$
$c_2 = 1 - ((1 + i^{-1})/2)$ so $c_2 = (1 - i^{-1})/2$
As a conclusion, the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$ is,
a_n=$((1 + i^{-1})/2).(1 + i)^n$ + $((1 - i^{-1})/2).(1 - i)^n$