

Discrete Math-2022 Fall-Quiz-2

Name:

Problem 1. (11 Points) Determine whether each of the following statements is correct or not.

1. $\{\emptyset\} \in \{\emptyset\}$ (true / false)
2. $\{\emptyset\} \in \{\{\emptyset\}\}$ (true / false)
3. $\{\emptyset\} \subset 2^\emptyset$ (true / false)
4. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ (true / false)
5. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ (true / false)
6. $\emptyset \in 2^\emptyset$ (true / false)
7. $\emptyset \subseteq \emptyset$ (true / false)
8. If $A \subseteq B$ and $B \in C$, then $A \in C$ (true / false)
9. If $A \in B$ and $B \not\subseteq C$, then $A \notin C$ (true / false)
10. If $A \notin B$ and $B \subseteq C$, then $A \notin C$ (true / false)
11. If $A \notin B$ and $B \not\subseteq C$, then $A \notin C$ (true / false)

Answer:

1-5. F,T,F,T,T

6-11. T,T,F,F,F,F.

Problem 2. (6 Points) Determine the cardinality for each of the following sets

1. $2^{2^\emptyset} - (\emptyset \times \{a, b\})$
2. $\emptyset \cup (2^{\{\emptyset\}} - \{\emptyset\})$
3. $2^{2^\emptyset} - 2^\emptyset$

Answer:

1. $\{\emptyset, \{\emptyset\}\}$, 2.
2. $\{\{\emptyset\}\}$, 1.
3. $\{\{\emptyset\}\}$, 1.

Problem 3. (10 Points) A relation $R \subseteq A \times A$ is called *circular* if

$$(\forall x)(\forall y)(\forall z)((x \in A \wedge y \in A \wedge z \in A \wedge xRy \wedge yRz) \rightarrow zRx)$$

1. Provide a relation that is circular but is not transitive
2. Provide a relation that is transitive but is not circular
3. Prove that, for any non-empty relation R , it is an equivalence relation if and only if it is both reflexive and circular.

Answer:

1. $a \xrightarrow{\quad} b \xrightarrow{\quad} c$
2. $a \xrightarrow{\quad} b \xrightarrow{\quad} c$

3. Let the binary relation be R on a non-empty set S . The relation R is called circular if aRb and bRc , then cRa . A relation is called equivalence if it is reflexive, symmetric, and transitive.

(\Leftarrow) Let the relation R be reflexive and circular with $(a, b) \in R$. By reflexivity, we have

$$(\forall b \in S)(\langle b, b \rangle \in R).$$

Then

$$\langle a, b \rangle, \langle b, b \rangle \in R.$$

For circular, we have $\langle b, a \rangle \in R$, so the relation is symmetric. Since R is symmetric and circular, R is transitive. So, the relation R is an equivalence relation.

(\Rightarrow) Since R is transitive and symmetric, we know that R is circular. Thus R is reflexive and circular.

Problem 4. (8 Points)

Suppose that $A \neq \emptyset$ is a non-empty set. Determine whether or not each of the following statements is correct. If correct, provide a short argument for why it is correct (do not need formal proof). Otherwise, provide a counter example.

1. If $R \subseteq A \times A$ is irreflexive, then R^2 is also irreflexive.
2. If $R \subseteq A \times A$ is anti-symmetric, then R^2 is also anti-symmetric.

Answer:

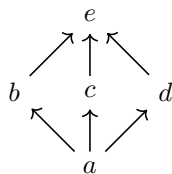
1. No, for instance $R = \{\langle 1, 2 \rangle, \langle 2, 1 \rangle\}$,
2. No, for instance $R = \{\langle 1, 3 \rangle, \langle 2, 4 \rangle, \langle 3, 2 \rangle, \langle 4, 1 \rangle\}$.

Problem 5. (15 Points) For poset $\langle A, R \rangle$, where

$$A = \{a, b, c, d, e\} \text{ and } R = \{\langle a, d \rangle, \langle a, c \rangle, \langle a, b \rangle, \langle a, e \rangle, \langle b, e \rangle, \langle c, e \rangle, \langle d, e \rangle\} \cup I_A,$$

1. Draw the Hasse diagram of $\langle A, R \rangle$
2. Find all minimal elements and all maximal elements of A , respectively
3. Find the greatest element and the least element exist of A , respectively
4. Find all upper bounds and the least upper bound of $\{d, b, c\}$.
5. Find all lower bounds and the greatest lower bound of $\{e\}$.

Answer:



- 1.
2. minimal elements: a, maximal elements: e.
3. greatest element: e; least element: a.
4. upper bounds: e; least upper bound: e.
5. upper bounds: a, b, c, d, e; greatest lower bound: e.