Discrete Math (Honor) 2022-Fall Homework-11

Instructor: Xiang YIN
Due: 2022.12.30 Friday

Problem 1. *(10 Points)*
Determine whether each of the following statements is true or false.

1. For any simple graph $G = \langle V, E\rangle$, if $\forall v, v' \in V : d(v) + d(v') \geq |V|$, then G has a Hamilton path. (True / False)
2. Any complete graph contains a Hamilton circuit. (True / False)
3. For any simple graph $G = \langle V, E\rangle$, if $\forall v \in V : d(v) \geq \frac{n}{2}$, then G has a Hamilton circuit. (True / False)
4. For any undirected graph $G = \langle V, E\rangle$, we have $\sum_{v\in V} d(v) = 2|E|$. (True / False)
5. Connected graph G is a tree if and only if each edge of G is a bridge. (True / False)

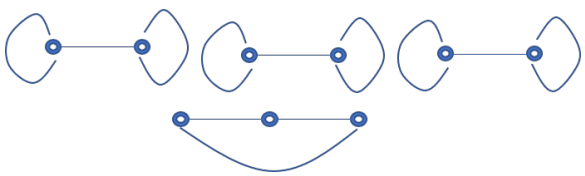
Answer: 1-5. T,T,T,T,T
Note: 1 and 3 is right from theorem in page 8 of lecture 6. For complete graph $K_n = (V, E)$, every $v \in V$ satisfies that $d(v) = n - 1$. Therefore, for $K \geq 2$ we have $d(v) + d(v') = 2n - 2 \geq n$. For $K = 1$, since K_1 contains only one vertex v. The path v is a Hamilton circuit. Note that for graph $K_2 = (V = \{v_1, v_2\}, E = \{e = (v_1, v_2)\})$, we have $v_1 e v_2 e v_1$ which is a Hamilton circuit.

Problem 2. *(10 Points)*

1. If G has 7 vertices, where 6 vertices have degree 3 and one vertex has degree 6, then G has _____edges.
2. If G has n vertices and m edges, where the degree of each vertex is either k or $k + 1$, then G contains_____vertices whose degrees are k.
3. If undirected graph G has 12 edges, 6 vertices with degree 3, and the degrees of the rest of the vertices are smaller than 3, then G contains at least _____ vertices.

Answer:

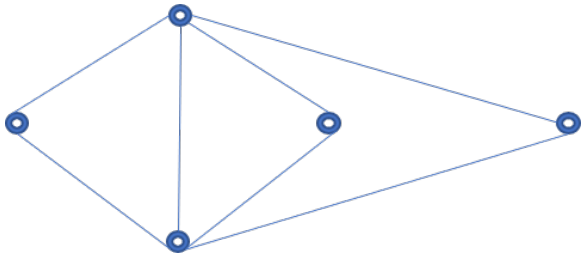
1. 12;
2. $(k + 1)n - 2m$;
3. 9.



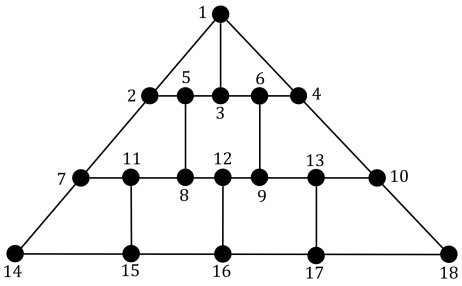
Problem 3. *(10 Points)*
1. Provide a simple graph with at least 4 vertices such that it has an Euler circuit but has no Hamilton circuit.

2. Provide a simple graph with at least 5 vertices such that it has a Hamilton circuit but has no Euler circuit.

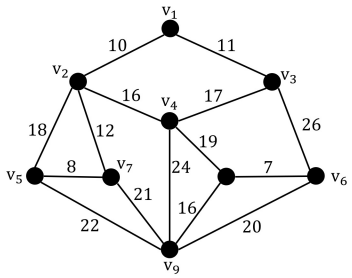
Answer: 1.



2.The example in homework 10 satisfies the condition.



Problem 4. *(10 Points)*
Construct the minimum spanning tree of the following graph using both the Kruskal's Algorithm and the Prim's Algorithm (specific which edge is added at each step).



Answer: Kruskal's Algorithm:
(v_6, v_8), (v_5, v_7), (v_1, v_2), (v_1, v_3), (v_2, v_7), (v_2, v_4), (v_8, v_9), (v_4, v_8).

Prim's Algorithm:
(v_5, v_7), (v_7, v_2), (v_1, v_2), (v_1, v_3), (v_2, v_4), (v_4, v_8), (v_6, v_8), (v_8, v_9).