# Discrete Math-2022 Fall-Quiz-2

Name:

**Problem 1.** *(11 Points)* Determine whether each of the following statements is correct or not.

1. $\{\emptyset\} \in \{\emptyset\}$ ( true / false)
2. $\{\emptyset\} \in \{\{\emptyset\}\}$ ( true / false)
3. $\{\emptyset\} \subset 2^{\emptyset}$ ( true / false)
4. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ ( true / false)
5. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ ( true / false)
6. $\emptyset \in 2^{\emptyset}$ ( true / false)
7. $\emptyset \subseteq \emptyset$ ( true / false)
8. If $A \subseteq B$ and $B \in C$, then $A \in C$ ( true / false)
9. If $A \in B$ and $B \not\subseteq C$, then $A \notin C$ ( true / false)
10. If $A \notin B$ and $B \subseteq C$, then $A \notin C$ ( true / false)
11. If $A \notin B$ and $B \not\subseteq C$, then $A \notin C$ ( true / false)

**Answer:**

1-5. F,T,F,T,T

6-11. T,T,F,F,F,F.

**Problem 2.** *(6 Points)* Determine the cardinality for each of the following sets

1. $2^{2^{\emptyset}} - (\emptyset \times \{a, b\})$
2. $\emptyset \cup (2^{\{\emptyset\}} - \{\emptyset\})$
3. $2^{2^{\emptyset}} - 2^{\emptyset}$

**Answer:**

1. $\{\emptyset, \{\emptyset\}\}$, 2.
2. $\{\{\emptyset\}\}$, 1.
3. $\{\{\emptyset\}\}$, 1.

**Problem 3.** *(10 Points)* A relation $R \subseteq A \times A$ is called *circular* if

$$(\forall x)(\forall y)(\forall z)((x \in A \wedge y \in A \wedge z \in A \wedge xRy \wedge yRz) \rightarrow zRx)$$

1. Provide a relation that is circular but is not transitive
2. Provide a relation that is transitive but is not circular
3. Prove that, for any non-empty relation $R$, it is an equivalence relation if and only if it is both reflexive and circular.

**Answer:**

1. $$\begin{tikzcd} a \arrow[r] & b \arrow[r] & c \arrow[ll, bend right] \end{tikzcd}$$

2. $$\begin{tikzcd} a \arrow[r] \arrow[rr, bend left] & b \arrow[r] & c \end{tikzcd}$$

3. Let the binary relation be $R$ on a non-empty set $S$. The relation $R$ is called circular if $aRb$ and $bRc$, then $cRa$. A relation is called equivalence if it is reflexive, symmetric, and transitive.

($\Longleftarrow$)Let the relation $R$ be reflexive and circular with $(a, b) \in R$. By reflexivity, we have

$$(\forall b \in S)(\langle b, b\rangle \in R).$$

Then

$$\langle a, b\rangle, \langle b, b\rangle \in R.$$

For circular, we have $\langle b, a\rangle \in R$, so the relation is symmetric. Since $R$ is symmetric and circular, $R$ is transitive. So, the relation $R$ is an equivalence relation.

($\Longrightarrow$) Since $R$ is transitive and symmetric, we know that $R$ is circular. Thus $R$ is reflexive and circular.

**Problem 4.** *(8 Points)*
Suppose that $A \neq \emptyset$ is a non-empty set. Determine whether or not each of the following statements is correct. If correct, provide a short argument for why it is correct (do not need formal proof). Otherwise, provide a counter example.

1. If $R \subseteq A \times A$ is inreflexive, then $R^2$ is also inreflexive.
2. If $R \subseteq A \times A$ is anti-symmetric, then $R^2$ is also anti-symmetric.

**Answer:**

1. No, for instance $R = \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$,
2. No, for instance $R = \{\langle 1, 3\rangle, \langle 2, 4\rangle, \langle 3, 2\rangle, \langle 4, 1\rangle\}$.

**Problem 5.** *(15 Points)* For poset $\langle A, R\rangle$, where

$$A = \{a, b, c, d, e\} \text{ and } R = \{\langle a, d\rangle, \langle a, c\rangle, \langle a, b\rangle, \langle a, e\rangle, \langle b, e\rangle, \langle c, e\rangle, \langle d, e\rangle\} \cup I_A,$$

1. Draw the Hasse diagram of $\langle A, R\rangle$
2. Final all minimal elements and all maximal elements of $A$, respectively
3. Find the greatest element and the least element exist of $A$, respectively
4. Find all upper bounds and the least upper bound of $\{d, b, c\}$.
5. Find all lower bounds and the greatest lower bound of $\{e\}$.

**Answer:**

1. $$\begin{tikzcd} & e & \\ b \arrow[ur] & c \arrow[u] & d \arrow[ul] \\ & a \arrow[ul] \arrow[u] \arrow[ur] & \end{tikzcd}$$

2. minimal elements: a, maximal elements: e.

3. greatest element: e; least element: a.

4. upper bounds: e; least upper bound: e.

5. upper bounds: a,b,c,d,e; greatest lower bound: e.