Discrete Math-2022 Fall-Quiz-1

Name:

Problem 1. *(10 Points)* Determine if each of the following propositional formulas is tautology, contradiction or satisfiable.

1. $\neg(P \leftrightarrow Q) \to ((P \wedge \neg Q) \vee (\neg P \wedge Q))$ (tautology / contradiction / satisfiable)
2. $(\neg P \to \neg Q) \to ((\neg P \to Q) \to P)$ (tautology / contradiction / satisfiable)
3. $\neg(P \to (Q \to P))$ (tautology / contradiction / satisfiable)
4. $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ (tautology / contradiction / satisfiable)
5. $\neg(Q \to R) \wedge R$ (tautology / contradiction / satisfiable)

Answer: 1. tautology, 2. tautology, 3. contradiction, 4. tautology , 5. contradiction.

Problem 2. *(10 Points)* Write down formula α in both CNF and DNF based on the following truth table.

P	Q	R	α
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

CNF

$(P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)$

CNF

$(P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee Q)$

CNF

$(P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee R)$

CNF

$(\neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

DNF

$(\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge Q \wedge R)$

DNF

$(\neg P \wedge \neg Q) \vee (Q \wedge R)$

Problem 3. *(10 Points)* Write the following formula in CNF

$$P \to ((Q \to R) \wedge (P \vee \neg R))$$

Answer: $\neg P \vee \neg Q \vee R$

Problem 4. *(10 Points)* Prove the following inference **by resolution**

$$(\exists x)(P(x) \to Q(x)) \Rightarrow (\forall x)P(x) \to (\exists x)Q(x)$$

Answer: Write the Skolem normal form of $\alpha \wedge \neg\beta$

$$\begin{aligned}
&(\exists x)(P(x) \to Q(x)) \wedge \neg((\forall x)P(x) \to (\exists x)Q(x)) \\
=&(\exists x)(\neg P(x) \vee Q(x)) \wedge \neg(\neg(\forall x)P(x) \vee (\exists x)Q(x)) \\
=&(\exists x)(\neg P(x) \vee Q(x)) \wedge ((\forall x)P(x) \wedge \neg(\exists x)Q(x)) \\
=&(\exists x)(\forall y)(\forall z)((\neg P(x) \vee Q(x)) \wedge P(y) \wedge \neg Q(z))
\end{aligned}$$

The clause set is $S = \{\neg P(a) \vee Q(a), P(y), \neg Q(z)\}$

$$\left.\begin{array}{r}\neg P(a) \vee Q(a) \\ P(y)\end{array}\right\} \xrightarrow{\sigma=\{y/a\}} Q(a), \qquad \neg Q(z) \left.\right\} \xrightarrow{\sigma=\{z/a\}} \neg Q(a)$$

This gives us a contradiction $Q(a) \wedge \neg Q(a)$

Note: You will lose half of score if you answer this question by inference rules.

Problem 5. *(15 Points)*

Let $S(x)$ be "*x is a student*", $G(x)$ be "*x is a game*", $L(x, y)$ be "*x likes y*" and $E(x, y)$ be "$x = y$". Formalize each of the following sentences by predicate formula.

1. 有些学生喜欢所有游戏.
   right: $(\exists x)(S(x) \wedge (\forall y)(G(y) \to L(x, y)))$
   wrong: $(\exists x)(\forall y)((S(x) \wedge G(y)) \to L(x, y))$

2. 每个学生都有不喜欢的游戏.
   right: $(\forall x)(S(x) \to (\exists y)(G(y) \wedge \neg L(x, y)))$
   wrong: $(\forall x)(\exists y)((S(x) \wedge G(y)) \to \neg L(x, y))$

3. 有些游戏只有一个学生喜欢.
   right: $(\exists x)(G(x) \wedge (\exists y)(S(y) \wedge L(y, x) \wedge (\forall z)((S(z) \wedge L(z, x)) \to E(y, z))))$
   wrong: $(\exists x)(\exists y)((S(x) \wedge G(y)) \to L(y, x)) \wedge (\forall z)((S(z) \wedge L(z, x)) \to E(y, z))))$

4. 每个学生最多喜欢一种游戏.
   right: $(\forall x)(S(x) \to (\forall y)(\forall z)(G(y) \wedge L(x, y) \wedge G(z) \wedge L(x, z) \to E(y, z)))$

5. 有些学生恰好喜欢两种游戏.
   right: $(\exists x)(\exists y)(\exists z)(S(x) \wedge G(y) \wedge G(z) \wedge L(x, y) \wedge L(x, z) \wedge \neg E(y, z) \wedge (\forall w)((G(w) \wedge L(x, w)) \to (E(y, w) \vee E(z, w))))$

Note: If you make mistake like wrong answers above, you will lose all score. These mistake is like examples 1,2 in page 4 of handout of Predicate Logic. For example, for first question, the wrong answer is wrong because if there exist a in domain D which is not student, i.e., $S(a) = \mathbf{F}$, we have $(\exists x)(\forall y)((S(x) \wedge G(y)) \to L(x, y)) = \mathbf{T}$.

If you make other types of mistakes, you may lose part of score.