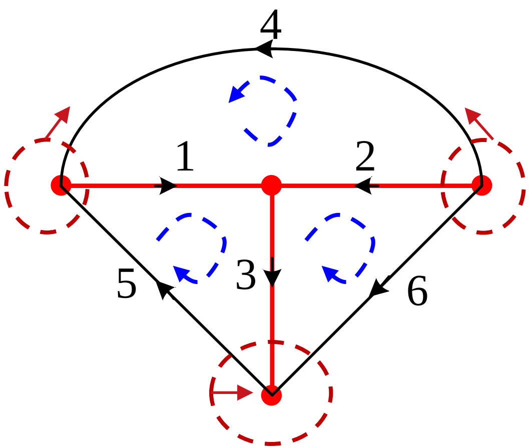
基本结论

一个具有*n*个节点，*b*条支路的连通图G及G的一个树T



1、树支数：$n_t = n\text{-}1$

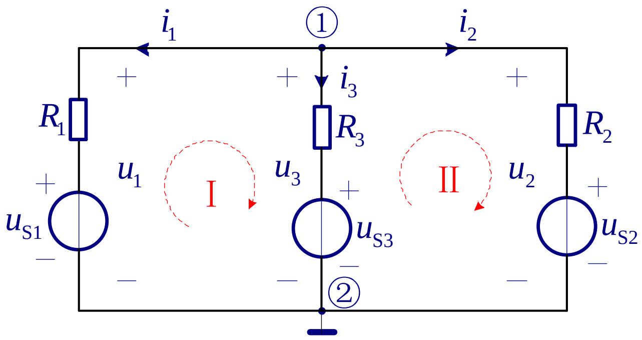
2、连支数：$n_l = b\text{-}n_t = b\text{-}n+1$

3、单连支回路（基本回路），$b\text{-}n+1$个 KVL

4、单树支割集（基本割集），$n\text{-}1$个 KCL

支路的端口特性，b个 VCR

2b



$$\begin{cases} i_1 + i_2 + i_3 = 0 \\ -u_1 + u_3 = 0 \\ -u_3 + u_2 = 0 \\ u_1 = R_1 i_1 + u_{s1} \\ u_2 = R_2 i_2 + u_{s2} \\ u_3 = R_3 i_3 + u_{s3} \end{cases}$$

2b法

结点2个，支路3条，可列:
2-1=1个独立的KCL方程
3-2+1=2个独立的KVL方程

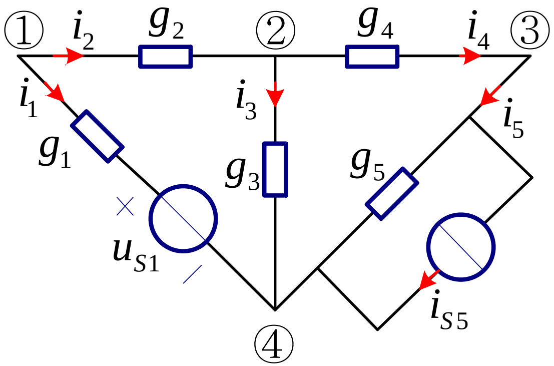
$$\begin{cases} i_1 + i_2 + i_3 = 0 \\ R_1 i_1 + u_{s1} = R_3 i_3 + u_{s3} \\ R_2 i_2 + u_{s2} = R_3 i_3 + u_{s3} \end{cases}$$

1b法

支路电流法

$$\begin{cases} \dfrac{u_1 - u_{s1}}{R_1} + \dfrac{u_2 - u_{s2}}{R_2} + \dfrac{u_3 - u_{s3}}{R_3} = 0 \\ -u_1 + u_3 = 0 \\ -u_3 + u_2 = 0 \end{cases}$$

支路电压法



$$1\cdot i_1 + 1\cdot i_2 + 0\cdot i_3 + 0\cdot i_4 + 0\cdot i_5 = 0$$
$$0\cdot i_1 - 1\cdot i_2 + 1\cdot i_3 + 1\cdot i_4 + 0\cdot i_5 = 0$$
$$0\cdot i_1 + 0\cdot i_2 + 0\cdot i_3 - 1\cdot i_4 + 1\cdot i_5 = 0$$
$$-1\cdot i_1 + 0\cdot i_2 - 1\cdot i_3 + 0\cdot i_4 - 1\cdot i_5 = 0$$

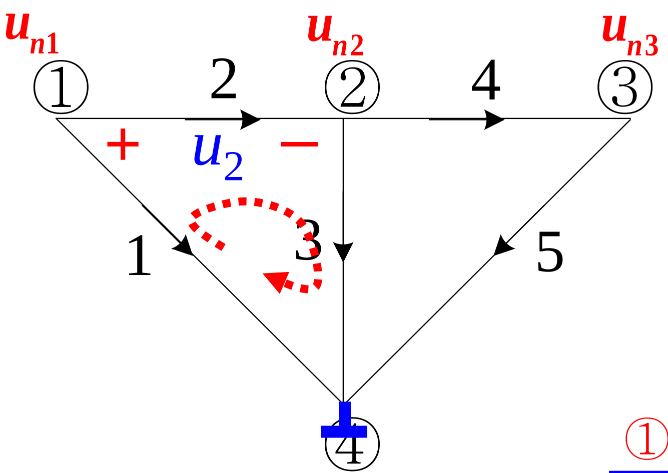
$$i_1 + i_2 = 0$$
$$-i_2 + i_3 + i_4 = 0$$
$$-i_4 + i_5 = 0$$
$$-i_1 - i_3 - i_5 = 0$$

线性无关的降阶关联矩阵A

$$\begin{array}{c} ① \\ ② \\ ③ \\ ④ \end{array}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ & & & & \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

关联矩阵

$$A_a I_b = 0$$



设u_1、u_2、u_3、u_4、u_5为支路电压，则：

$$
\begin{array}{c}
\\ ① \\ ② \\ ③
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}
\end{array}
$$

$$u_1 = u_{n1}$$

$$u_2 = u_{n1} - u_{n2}$$

$$u_3 = u_{n2}$$

$$u_4 = u_{n2} - u_{n3}$$

$$u_5 = u_{n3}$$

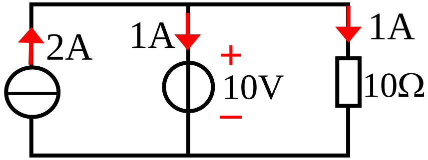
$$
\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{pmatrix}
=
\begin{array}{c|c}
 & ① \quad ② \quad ③ \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}
\end{array}
\begin{pmatrix} u_{n1} \\ u_{n2} \\ u_{n3} \end{pmatrix}
$$

$$\boldsymbol{U}_{\mathrm{b}} = \mathrm{A}^{\mathrm{T}} \boldsymbol{U}_{n}$$

探究：以下几种矩阵的构成与用途

支路 结点　A	支路 网孔　M
支路 基本回路　B_f	支路 基本割集　Q_f

（先连支后树支）



$$\because \mathrm{A}\boldsymbol{I}_{\mathrm{b}} = 0,\ \boldsymbol{U}_{\mathrm{b}} = \mathrm{A}^{\mathrm{T}}\boldsymbol{U}_{n}$$

$$\therefore \boldsymbol{U}_{\mathrm{b}}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{b}} = \boldsymbol{U}_{n}^{\mathrm{T}}\mathrm{A}\boldsymbol{I}_{\mathrm{b}} = 0$$

题图中电压源发出的功率为 –10 W，即吸收的功率为 10 W

题图中电流源发出的功率为 20 W，即吸收的功率为 –20 W

题图中电阻吸收的功率为 10 W

电路中所有元件吸收的功率之和为：10–20+10=0W

特勒根定理

具有n个结点、b条支路的网络（集中参数电路），支路电压和支路电流取一致参考方向，支路电压向量$U_b=(u_1,u_2,\ldots,u_b)^T$，支路电流向量$I_b=(i_1,i_2,\ldots,i_b)^T$，则：

- 若U_b和I_b是同一网络同一时刻的值

$$U_b^T I_b = 0 \quad 或 \quad \sum_{k=1}^{b} u_k i_k = 0$$

特勒根第一定理（功率定理）

- 若U_b和I_b是同一网络不同时刻的值

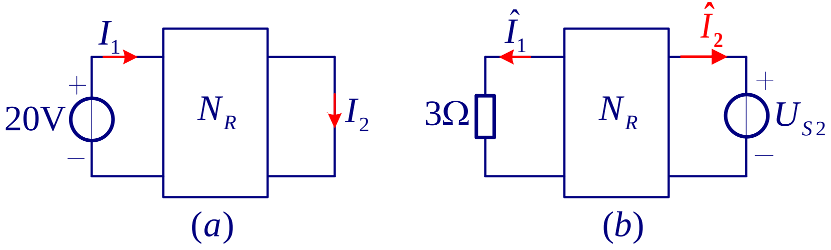
$$U_b^T(t_1) I_b(t_2) = 0 \qquad \sum_{k=1}^{b} u_k(t_1) i_k(t_2) = 0$$

- 若U_b和I_b分别是有向图相同的不同网络的值

$$U_b^T \hat{I}_b = 0 \qquad \sum_{k=1}^{b} u_k \hat{i}_k = 0$$

特勒根第二定理（似功率定理）

例： N_R 为线性无源电阻网络，已知 $I_1=10\text{A}$，$I_2=2\text{A}$，$\hat{I}_1=4\text{A}$ ，试求 U_{S2}



$$20\times\hat{I}_1+0\times\hat{I}_2=3\hat{I}_1\times(-I_1)+U_{S2}I_2$$

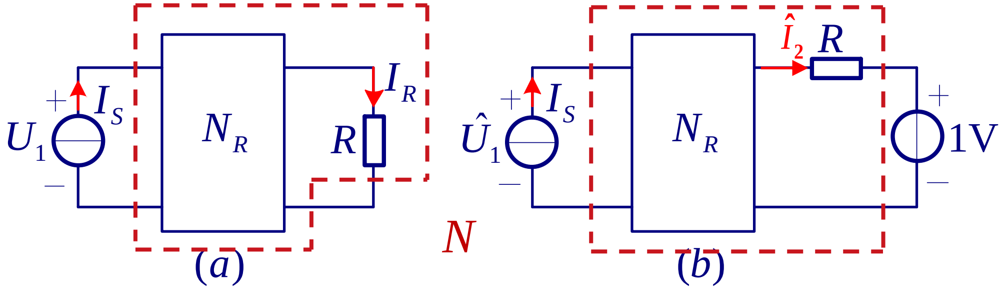
$$U_{S2}=100\text{V}$$

由特勒根定理有： $\sum_{k=1}^{b}u_k\hat{i}_k=0$ $\quad\sum_{k=1}^{b}\hat{u}_k i_k=0$

$$u_1\hat{i}_1+u_2\hat{i}_2+\sum_{k=3}^{b}R_k i_k\hat{i}_k=0 \qquad \hat{u}_1 i_1+\hat{u}_2 i_2+\sum_{k=3}^{b}R_k\hat{i}_k i_k=0$$

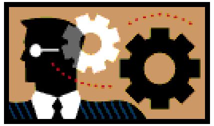
$$u_1\hat{i}_1+u_2\hat{i}_2=\hat{u}_1 i_1+\hat{u}_2 i_2$$

练习：N_R为线性无源电阻网络，已知 $I_S=2\text{A}$，$U_1=1\text{V}$，$I_R=2\text{A}$ ，试求 $\hat{U}_1$



方式一：对电阻网络N_R有： $U_1(-I_S)+I_R R\hat{I}_2=\hat{U}_1\times(-I_S)+(1+R\hat{I}_2)\times I_R$ $\therefore \hat{U}_1=2\text{V}$

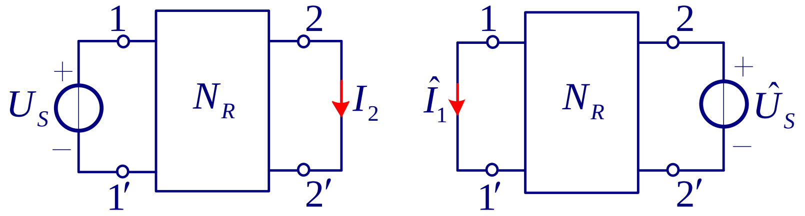
方式二：对电阻网络N有： $U_1(-I_S)=\hat{U}_1\times(-I_S)+1\times I_R$ $\therefore \hat{U}_1=U_1+\dfrac{I_R}{I_S}=2\text{V}$



思考：

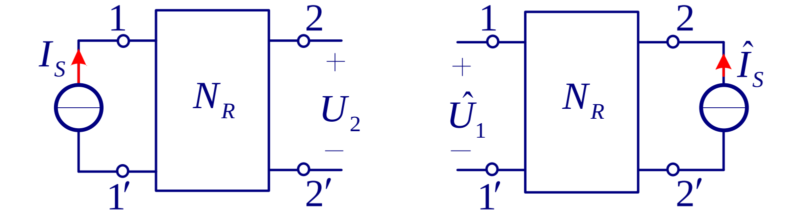
1、参考方向变化

2、激励与响应类型

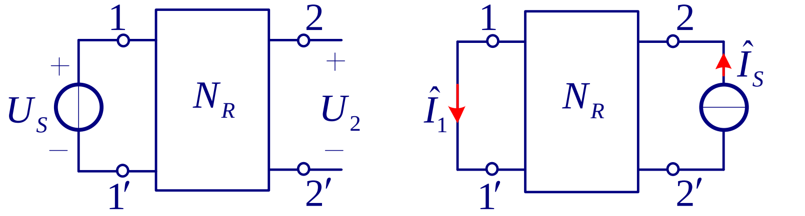


$$\because U_S\hat{I}_1 + 0\times\hat{I}_2 = 0\times I_1 + \hat{U}_S I_2$$

$$\therefore \frac{I_2}{U_S} = \frac{\hat{I}_1}{\hat{U}_S}$$



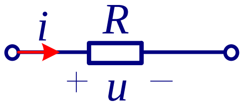
$$\frac{U_2}{I_S} = \frac{\hat{U}_1}{\hat{I}_S}$$



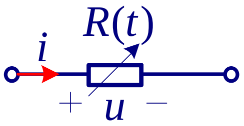
$$\frac{U_2}{U_S} = \frac{\hat{I}_1}{\hat{I}_S}$$

电阻电路元件

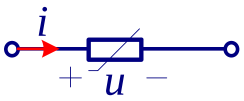
1、电阻元件



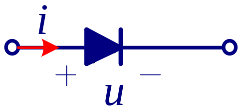
$u = Ri$ 或 $i = Gu$



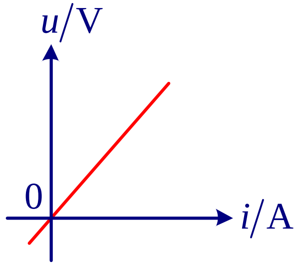
$u(t) = R(t)i(t)$ 或 $i(t) = G(t)u(t)$

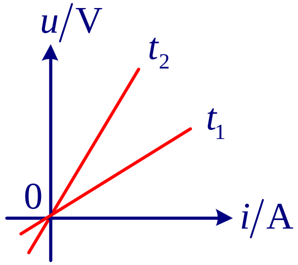


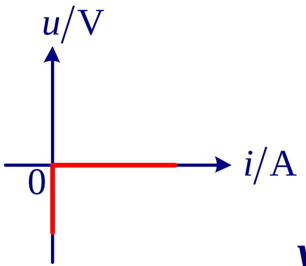
$f(u,i) = 0$ 或 $f(u,i,t) = 0$



$$\begin{cases} i = 0 & u < 0 \\ u = 0 & i > 0 \end{cases}$$







$$\boldsymbol{p(t) = u(t)i(t)}$$

$$\boldsymbol{w(t_0, t) = \int_{t_0}^{t} p(\tau)d\tau}$$

$\boldsymbol{w(-\infty, t) \geq 0}$ 无源元件