$$r(A^TA) \le r(A^TA, A^Tb)$$

$$= r(A^T(A, b))$$

$$\le \min\{r(A^T), r(A, b)\}$$

$$\le r(A^T)$$

$$r(A^TA) = r(AA^T) = r(A) = r(A^T)$$

$$\therefore r(A^TA) = r(A^TA, A^Tb).$$

$Ax=b$ 无解.

寻找近似解.

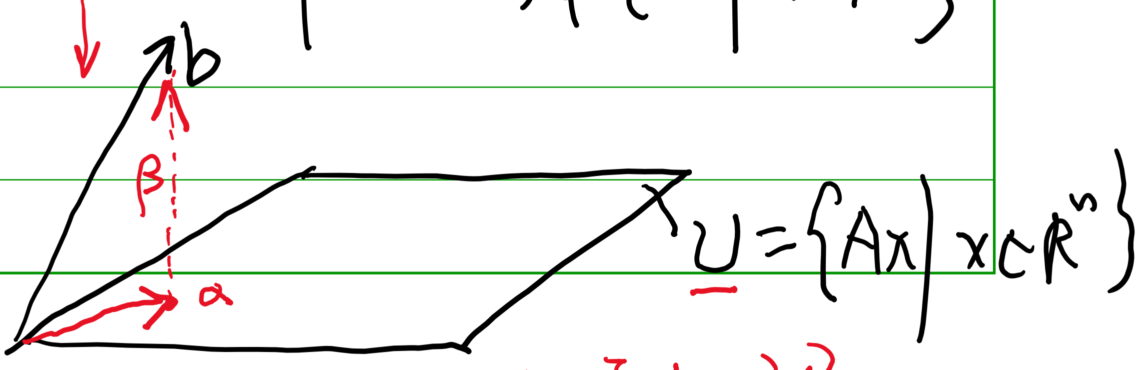
$\because Ax=b$ 无解,

$\therefore b$ 不能由 A 的列向量表示.

$$\{Ax \mid x\in R^n\} = L(\alpha_1, \alpha_2, \cdots, \alpha_n)$$

α_i 为 A 的第 i 列

$\because Ax=b$ 无解, $\therefore b \notin \{Ax \mid x\in R^n\}$



α 叫做 $Ax=b$ 的近似解

$\beta = b - \alpha$

$\beta \perp \{Ax \mid x \in R^n\}$ 的充要条件

$\Leftrightarrow \beta \perp A$ 的列向量

$(\beta, \alpha_i) = 0 \Rightarrow \alpha_i^T \beta = 0$

$$\begin{cases} \alpha_1^T \beta = 0 \\ \alpha_2^T \beta = 0 \\ \alpha_n^T \beta = 0 \end{cases} \qquad \begin{pmatrix} \alpha_1^T \\ \alpha_2^T \\ \vdots \\ \alpha_n^T \end{pmatrix} \beta = 0$$

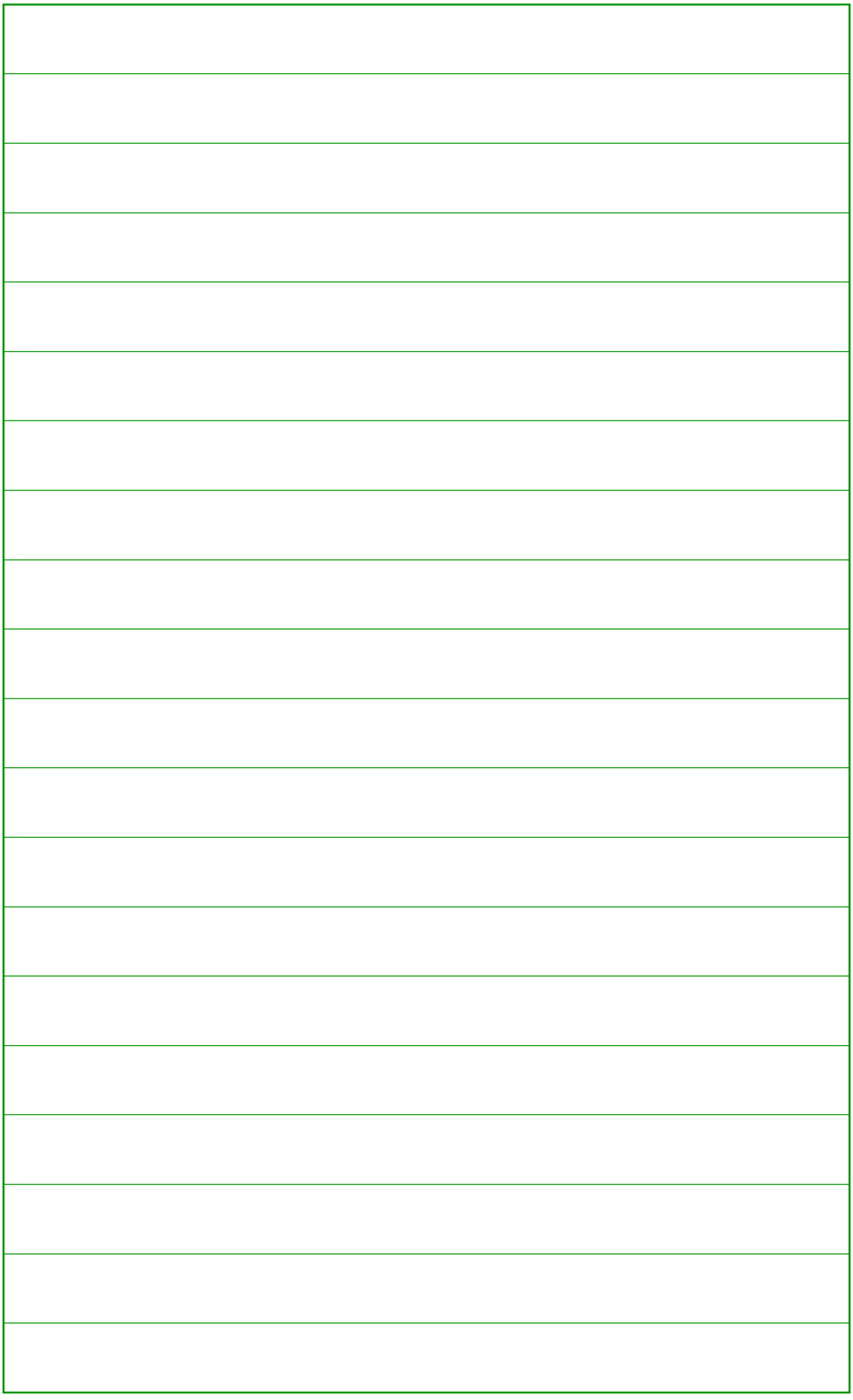
$\therefore A^T \beta = 0 \quad \therefore A^T (b - \alpha) = 0$

$\because \alpha \in U, \therefore$ 存在 $\tilde{x}$, 使 $A\tilde{x} = \alpha$

$\therefore A^T (b - A\tilde{x}) = 0$

$\therefore A^T A\tilde{x} = A^T b$

解的形式: $A^T A x = A^T b$.



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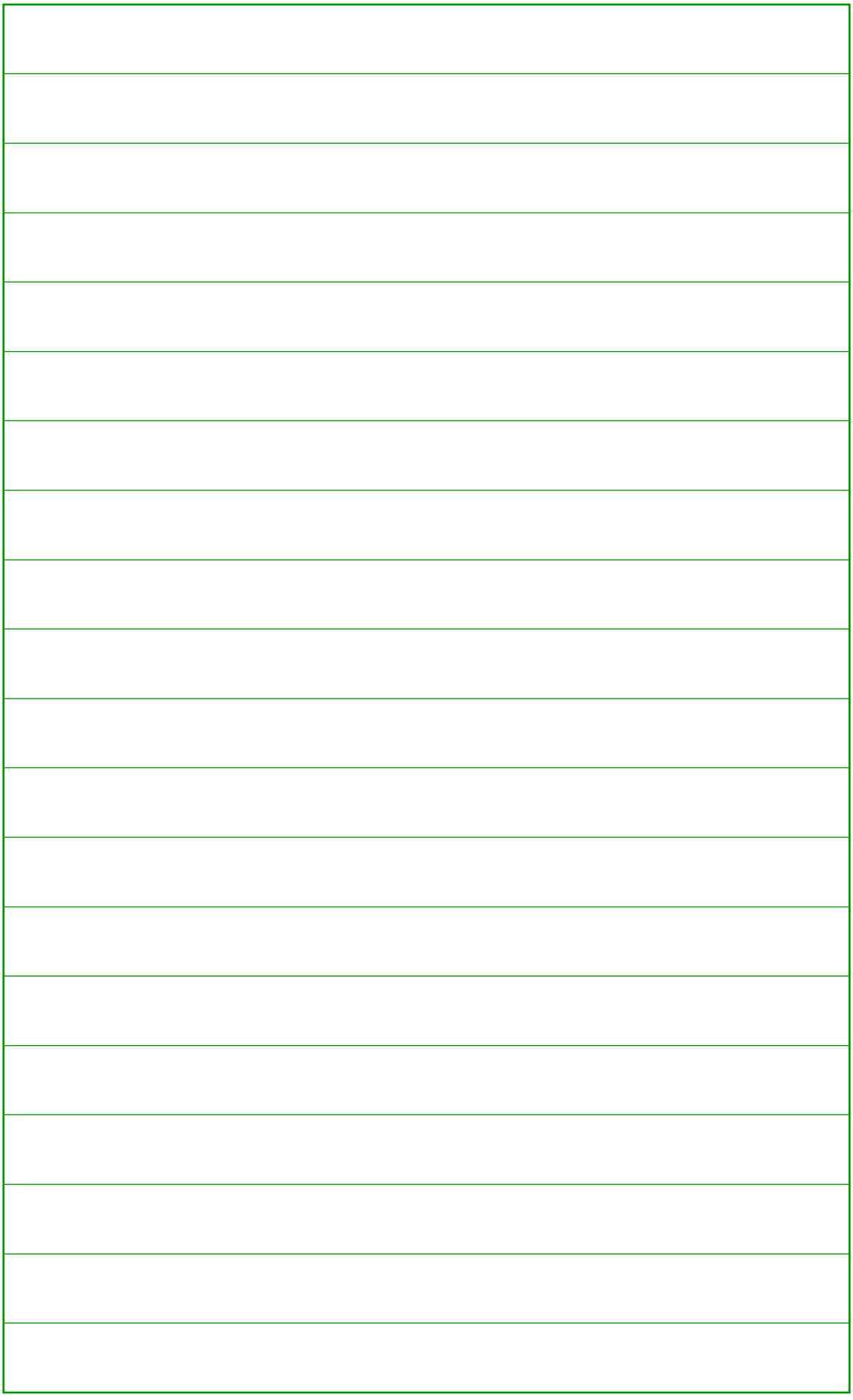

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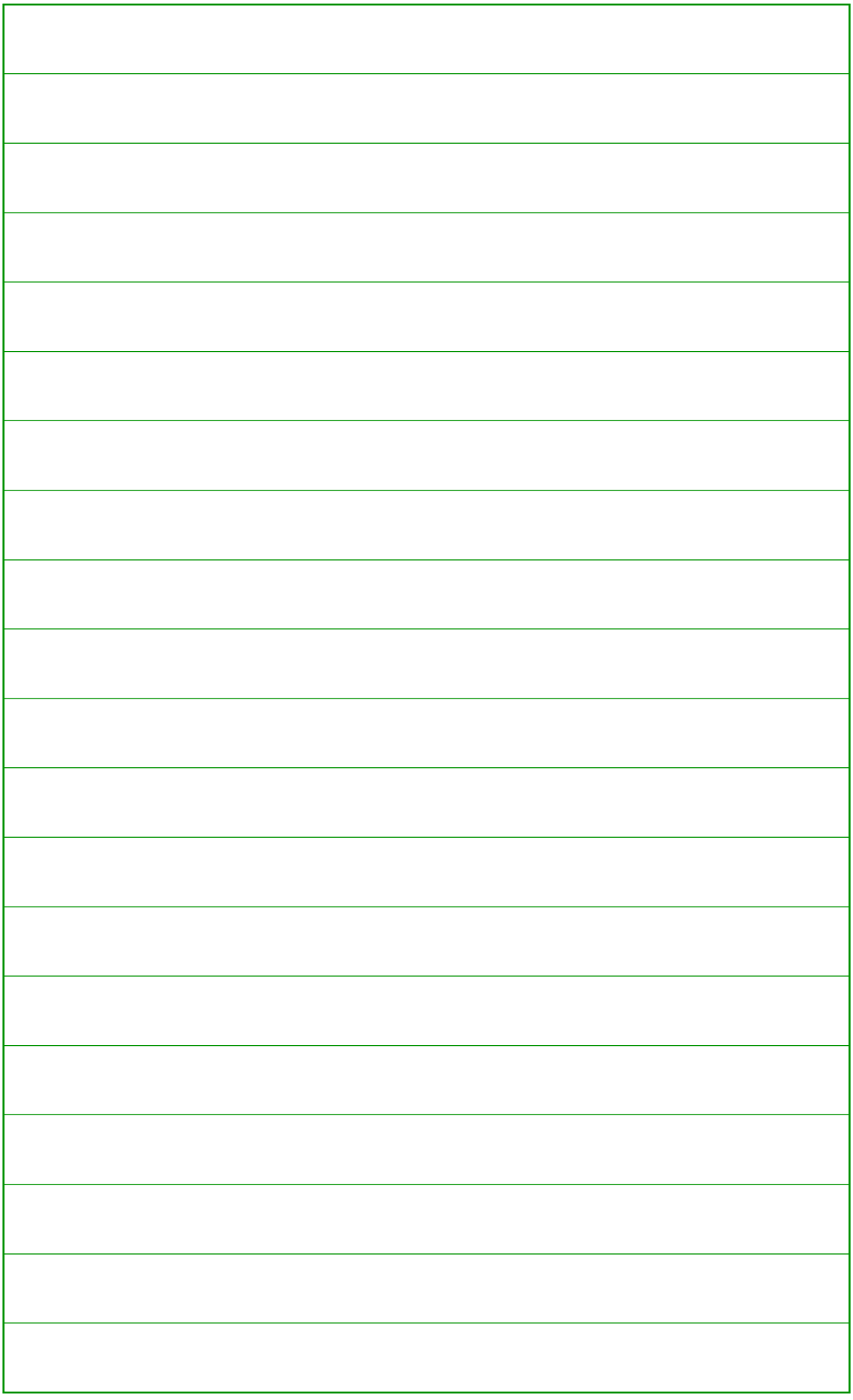

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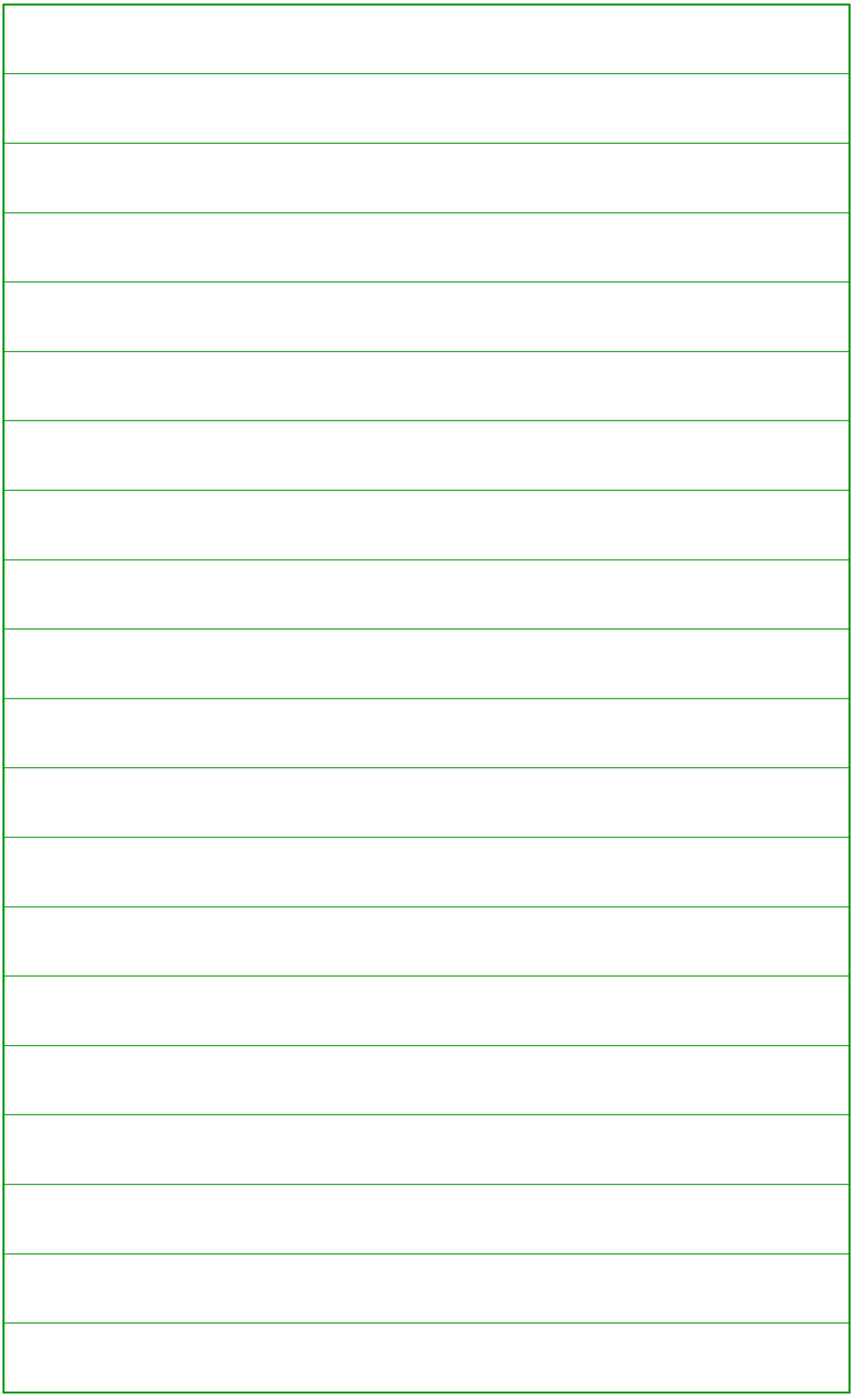

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