Name: NIM: Class:

Assignment 2: Combinatorics
Discrete Mathematics A (MSH2A3)
Second Term 2018-2019

Instructions:

1. This assignment is due **Tuesday March 5 at 5:00 p.m.**. Please submit your work at School of Computing academic roster (*roster akademik Fakultas Informatika*), room A203A (building A room A203A). Do not forget to write your identity on the space provided. You may submit this assignment as of **Monday March 4 at 8:00 a.m.**.
2. In order to prevent any academic misconduct, you also need to submit a readable scan or photograph of this assignment to the provided submission slot in CeLoE. Please submit in a **.pdf file**. Please contact your class instructor for more detailed information. The due date of this online submission is the same as the hardcopy. **Please make sure that your file size do not exceed the maximum file size allowed**.
3. Please upload your assignment to the CeLoE under the file name: `A2-<student ID>.pdf`, for example: `A2-1301189999.pdf`.
4. To save paper, you may print and reproduce this assignment double-sided.
5. Your answers must be handwritten. You may use: HB or 2B pencil, or pen with blue or black ink.
6. All problems in this assignment are adapted from the textbooks. **The problems are written in English**. If you are a student in a regular class, you may answer the problems in Bahasa. However, if you are a student in international class, your answers must be written in English—otherwise your assignment will not be graded. You may ask your class instructor or teaching assistant for helping you understanding the problem, but you should not ask them to give the solution of any problem.
7. Write your solutions on the space provided. If you need more space, you may use additional A4 papers and attach them to your assignment.
8. Be neat and write legibly. You will be graded not only on the correctness of your answers, but also on the clarity with which you express them.
9. This assignment consists of **10 problems** and each problem is worth **10 points**.
10. Please retain yourself from copying answers from elsewhere without understanding the steps. This assignment is an individual evaluation.
11. **Important:** late submission without reasonable explanation will not be graded.

Problem 1 A palindromic number is a natural number whose reversal is identical to itself. For instance, the numbers: 1, 99, 101, 2112, 44444 are respectively palindromic numbers of 1, 2, 3, 4, and 5 digits. How many palindromic numbers between 1000 and $1000\,000$ are there?

ANSWER:

Problem 2 A ternary string is a string whose characters based on the set $\{0, 1, 2\}$. How many ternary strings of length 10 start with three 1s or end with two 1s but not both? (Examples of these strings are **111**2100210 and 1102122**11**, but the string **111**2102**11** is not included because it starts with three 1s and ends with two 1s.)

ANSWER:

Problem 3 How many students are enrolled in a course in Calculus, Discrete Mathematics, Data Structures, or Basic Programming at a school if:

1. 507 students are enrolled in Calculus,
2. 292 students are enrolled in Discrete Mathematics,
3. 312 students are enrolled in Data Structures,
4. 344 students are enrolled in Basic Programming,
5. 14 students are enrolled in both Calculus and Data Structures,
6. 213 students are enrolled in both Calculus and Basic Programming,
7. 211 students are enrolled in both Discrete Mathematics and Data Structures,
8. 43 students are enrolled in both Discrete Mathematics and Basic Programming,
9. No student take Calculus and Discrete Mathematics concurrently,
10. No student take Data Structures and Basic Programming concurrently.

(Hint: determine the cardinality of the union of four sets using inclusion-exclusion principle.)

ANSWER:

Problem 4 Personal identification numbers (PINs) of a bank account consist of six decimal digits. However, because the software engineer hired by the bank was so clumsy, the valid PINs cannot start with the digit 0. For examples, the sequences 123321, 170845, and 999999 are valid PINs, whereas the sequences 070317, 001100, and 012345 are not valid because their first digit are 0.

(a). **[5 points]** How many possible valid PINs are there? Explain your reasoning and express your answer as an integer!

(b). **[5 points]** Gyro Gearloose, an unexperienced hacker, wants to hack three different bank accounts. However, he expected that all of these accounts use the same PINs. What is the minimum number of different accounts he need to ensure that at least three of them have the same PINs? Explain your reasoning and express your answer as an integer!

ANSWER:

Problem 5 Bojongsoang University has four faculties, i.e.: Faculty of Engineering, Faculty of Arts and Design, Faculty of Economics, and Faculty of Law. Faculty of Engineering and Faculty of Economics consist of **seven** majors each, while Faculty of Arts and Design and Faculty of Law have **four** majors each. In the admission year of 2018, each major in the Faculty of Engineering and Faculty of Arts and Design admitted 50 students, while every major under the Faculty of Economics and Faculty of Law admitted 75 students.

(a). **[5 points]** Determine the total number of students registered to Bojongsoang University in the admission year of 2018. Express your answer as an integer!

(b). **[5 points]** What is the **minimum** number of students required to ensure that three of them enrolled in the **same major**? Express your answer as an integer!

ANSWER:

Problem 6 Six students, Alice, Bob, Carlos, David, Emma, and Fiona are going to the cinema. There are ten seats remaining: B1, B2, B3, . . . , B10.

(a). **[5 points]** Suppose Alice insists to sit on B1 while Bob insists to sit on B10. How many different seating arrangements are there?

(b). **[5 points]** Suppose Fiona must not be seated on B9. How many different seatings arrangements are there? (Hint: firstly, consider the case when Fiona must be seated on B9.)

ANSWER:

Problem 7 The competitive programming club in a university has **twenty-five** members, **five** of them are the former medalists of the national informatics olympiad. How many ways are there to make **a team of four** students if:

(a). **[5 points]** the team consists of exactly two former medalists?

(b). **[5 points]** the team consists of at least one former medalist?

ANSWER:

Problem 8 How many different strings can be made from the word **MILLIMICRON** when all letters are used and such strings do not contain the substring **RON**? (examples: **MILLIMICORN**, **MILLIMICONR**, **RILLIMICMON**, etc.)

ANSWER:

Problem 9 An ice cream parlor sell ice creams with three different flavors: blueberry, chocolate, and vanilla.

(a). **[4 points]** How many ways are there to choose a dozen ice creams (one dozen consists of twelve ice creams)? (Some examples are: 4 blueberries, 4 chocolates, and 4 vanillas; 6 blueberry, 6 chocolates, and 0 vanilla; etc..)

(b). **[3 points]** How many ways are there to choose a dozen ice creams with at least seven vanilla ice creams? (Some examples are: 2 blueberries, 3 chocolates, and 7 vanilla; 0 blueberry, 2 chocolates, and 10 vanillas; etc..)

(c). **[3 points]** How many ways are there to choose a dozen ice creams with at most six vanilla ice creams? (Some examples are: 4 blueberries, 2 chocolates, and 6 vanilla; 6 blueberry, 6 chocolates, and 0 vanilla; etc..)

ANSWER:

Problem 10 Peter Rabbit, his cousin Benjamin, and his triplet sisters Flopsy, Mopsy, and Cottontail, spend most of their days stealing vegetables from Mr. McGregor garden. One day, Peter stole **twenty-five carrots** and wanted to distribute them to his cousin and his sisters.

(a). **[5 points]** How many ways are there to divide ~~twenty~~ twenty-five carrots among five rabbits (i.e., Peter, Benjamin, and his sisters) if **each of them gets at least two carrots**? Some admissible distribution are:
Peter: 10 carrots, Benjamin: 9 carrots, Flopsy: 2 carrots, Mopsy: 2 carrots, Cottontail: 2 carrots;
Peter: 6 carrots, Benjamin: 6 carrots, Flopsy: 6 carrots, Mopsy: 4 carrots, Cottontail: 3 carrots;
Peter: 7 carrots, Benjamin: 6 carrots, Flopsy: 5 carrots, Mopsy: 4 carrots, Cottontail: 3 carrots;
Peter: 3 carrots, Benjamin: 4 carrots, Flopsy: 5 carrots, Mopsy: 6 carrots, Cottontail: 7 carrots;
Peter: 5 carrots, Benjamin: 5 carrots, Flopsy: 5 carrots, Mopsy: 5 carrots, Cottontail: 5 carrots.

ANSWER:

(b). **[5 points]** How many ways are there to divide twenty-five carrots among five rabbits if **each of them gets at least two carrots but Benjamin gets at most five carrots**? Some admissible distribution are: Peter: 3 carrots, Benjamin: 4 carrots, Flopsy: 5 carrots, Mopsy: 6 carrots, Cottontail: 7 carrots; Peter: 5 carrots, Benjamin: 5 carrots, Flopsy: 5 carrots, Mopsy: 5 carrots, Cottontail: 5 carrots. (The first, second,and the third examples in part (a) cannot be used because Benjamin gets more than five carrots.)

ANSWER: