Coursework ENGG406

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1 Advanced Monte Carlo Method

In the performance-based design and operation of modern engineered systems, the accurate assessment of reliability is of paramount importance, particularly for civil, nuclear, aerospace, and chemical systems and plants which are safety-critical and must be designed and operated within a risk-informed approach. Monte Carlo simulation (MCS) offers a powerful means for evaluating the reliability of a system, due to the modelling flexibility that it offers indifferently of the type and dimension of the problem. The method is based on the repeated sampling of realisations of system configurations, which, however, are seldom failures so that a large number of realisations must be simulated in order to achieve an acceptable accuracy in the estimated failure probability, with costly, large computing times. For this reason, techniques of efficient sampling of system failure realisations are of interest (e.g. line sampling, importance sampling, subset simulation), in order to reduce the computational effort.

1.1 Problem I

The system considered in this problem is given by the following equation:

$$f(X) = X_1 - \frac{32}{\pi X_2^3} \times \sqrt{\frac{X_3^2 X_4^2}{16} + X_5^2} \tag{1}$$

Variable	Distribution	θ_1	θ_2	Mean
X_1	Uniform	70	80	75
X_2	Normal	39	0.1	39
X_3	Gumbel	1342	272.9	1500
X_4	Normal	400	0.1	400
X_5	Normal	250000	35000	250000

Table 1: Random variables and their parameters

1.1.1 Task

- Your task in this exercise is to perform a reliability analysis (i.e., to evaluate $P(f(X) \leq 0)$ using a simulation approach (i.e. compute the failure probability of the system).
- In your analysis you need to compare the two advanced Monte Carlo methods (subset simulation and line sampling) and the reason why you chose one particular method over the other method.

2 Artificial Neural Network as a Surrogate Model

Highly reliable systems are built to last a long time. Hence, the probability of failure of these systems are usually low (e.g. 10^{-4}). Monte Carlo methods could be used to estimate the probability of failure p_F of these systems. However, this comes at a huge computational cost. Computing p_F of a highly reliable system requires a repeated number of model runs. This might be infeasible when dealing with high-fidelity models of complex systems (i.e., models with many parameters). Artificial neural networks (ANN) can be used as substitute for these expensive models to reduce the computational cost. ANN learn to perform tasks by considering examples, generally without being programmed with task-specific rules.

2.1 Problem II

The numerical problem considered in this task is given by the following equation:

$$f(X) = 15.59\times 10^4 - \frac{X_1 \times X_2^2}{2 \times X_2^3} \times \frac{X_4^2 - 4 \times X_5 \times X_6 \times X_7^2 + X_4 \times (X_6 + 4 \times X_5 + 2 \times X_6 \times X_7)}{X_4 \times X_5 \times (X_4 + X_6 + 2 \times X_6 \times X_7)} \tag{2}$$

Variable	Distribution	Mean	Std
X_1	Normal	350	35
X_2	Normal	50.8	5.08
X_3	Normal	3.81	0.381
X_4	Normal	173	17.3
X_5	Normal	9.38	0.938
X_6	Normal	33.1	3.31
X_7	Normal	0.036	0.0036

Table 2: Random variables and their parameters

2.1.1 Task

- The task here is to train a neural network $\hat{f}(X)$ that mimics an expensive high fidelity model, then use $\hat{f}(X)$ to estimate the probability of failure p_F of the system using standard Monte Carlo method.

- You should explore different ANN architecture and identify the best architecture for the problem.
- Investigate how the number of samples affects p_F.

3 Probability Boxes

A probability box (or p-box) is a characterisation of an uncertain number consisting of both aleatory and epistemic uncertainties that is often used to model uncertainty. P-boxes are specified by left and right bounds on the cumulative probability distribution function.

3.1 Problem III

The example considered here is two-branched function:

$$\begin{aligned} f_1(X) &= X_1 + X_2 + \sqrt{33} \\ f_2(X) &= -X_3 + 10 \end{aligned} \tag{3}$$

$$f(x) = \min \begin{cases} f_1(X) \\ f_2(X) \end{cases} \tag{4}$$

If X_1, X_2 and X_3 are probability boxes with parameters given in the table

Variable	Distribution	Mean	Std
X_1	Normal	[-1, 1]	[1, 1.5]
X_2	Lognormal	[1, 2]	[1, 1.5]
X_3	Normal	[5, 6]	[1, 2]

Table 3: Imprecise random variables and their parameters

3.1.1 Task

- The task here is to estimate p_F of $f(x)$ using classical Monte Carol method.

4 Global Sensitivity Analysis

Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model $f(x)$ can be divided and allocated to different sources of uncertainty in its inputs. Variance-based sensitivity analysis (often referred to as Sobol indices S_i and T_i) is a form of global sensitivity analysis.

4.1 Problem IV

Using $f(X)$ given in question 2 to solve this problem.

4.1.1 Task

- Compute the first-order sensitivity indices.
- Compute the total-effect sensitivity indices.
- Show how the number of samples affect the sensitivity indices computed.