

PS Bootstrap Binary Standardization

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2/9/2022

The goal of this document is to combine a lot of the functions that Hun has made and made them into functions that can easily be called for the main loop of the simulations function.

Function to find the intercept to attain the desired treatment rate

```
treatment_rate_coef <- function(desired_prop, seq_try, df, logit_formula ) {  
  # setting up data setting variables  
  seq_select = tibble(seq_try, mean_A = rep(0, length(seq_try)))  
  
  # for loop that goes through all intercept options.  
  for(i in c(1:length(seq_try))) {  
    beta_0_treat = seq_try[i]  
  
    # this will be where the beta is updated. Must have beta_0_treat in formula  
    A_logit <- eval(parse(text=logit_formula ))  
    p_A      <- exp(A_logit)/(1+ exp(A_logit))  
    A        <- rbinom(N, 1, p_A) # this decided treat and and non treated  
  
    # getting the mean treated  
    seq_select$mean_A[i] = mean(A)  
  }  
  
  # # plotting visualizing check  
  # seq_select %>%  
  #   ggplot(aes(x=seq_try, y=mean_A))+  
  #   geom_point()  
  
  # selecting coeff that gives desired treated untreated proportion  
  if( sum(seq_select$mean_A == desired_prop) > 0) {  
    beta_0_treat = seq_select %>% filter(mean_A == desired_prop ) %>% pull(seq_try)  
  } else {  
    beta_0_treat = seq_select %>%  
      filter(mean_A < desired_prop + 0.01) %>%  
      filter(mean_A > desired_prop - 0.01) %>%  
      summarise(  
        avg = mean(seq_try)  
      ) %>% pull(avg)  
  }  
  
  # returning the beta value  
  return(mean(beta_0_treat))  
}
```

```
}
```

Finding the right beta_effect estimate for binary data

```
binary_outcome_rate <- function(desired_diff, seq_try, df, logit_formula ) {  
  seq_select = tibble(seq_try, prop_save = rep(0, length(seq_try)))  
  for(i in c(1:length(seq_try))) {  
    beta_effect = seq_try[i]  
  
    # binary outcome  
    y_logit <-eval(parse(text=logit_formula ))  
    p_outcome <- exp(y_logit)/(1+ exp(y_logit))  
    y_binary <- rbinom(N, 1, p_outcome)  
  
    # finding difference between groups  
    prop = tibble(y_binary, A = df$A) %>%  
      group_by(A) %>%  
      summarise( mean_out = mean(y_binary)) %>%  
      pivot_wider(  
        names_from = A,  
        values_from = mean_out  
      ) %>%  
      mutate (diff = `1`-`0`) %>%  
      pull(diff)  
  
    # having the difference  
    seq_select$prop_save[i] = prop  
  }  
  
  # graphing  
  # seq_select %>%  
  #   ggplot(aes(x=seq_try, y=prop_save))+  
  #   geom_point()  
  
  # saving the best beta.  
  beta_effect = seq_select %>%  
    filter(prop_save < desired_diff + 0.01) %>%  
    filter(prop_save > desired_diff - 0.01) %>%  
    summarise(  
      avg = mean(seq_try)  
    ) %>% pull(avg)  
  return( beta_effect)  
}  
  
# logit_formula <- "beta_effect*df$A + 0.5*df$L2 + 0.3*df$L3"  
# seq_try = seq(-4,4, 0.01)  
# desired_diff = 0.4  
# beta_effect = binary_outcome_rate(desired_diff, seq_try, df, logit_formula)
```

Generating data

```
data_gen <- function(N, desired_prop) {  
  
  # Setting the Observed Variables  
  L1 <- rnorm(N, 0, 1)  
  L2 <- rnorm(N, 0, 1)  
  L3 <- rnorm(N, 0, 1)  
  
  # adding the covaraites to dataset  
  df <- tibble(L1, L2, L3)  
  
  desired_prop = 0.2  
  seq_try      = seq(-2,2, 0.01)  
  logit_formula <- "beta_0_treat+ 0.5*df$L1 + 0.4*df$L2"  
  
  # get the desited treatment rate  
  beta_0_treat = treatment_rate_coef(desired_prop, seq_try, df, logit_formula)  
  
  # Creating the treatment assigment  
  A_logit <- beta_0_treat+ 0.5*df$L1 + 0.4*df$L2  
  p_A      <- exp(A_logit)/(1+ exp(A_logit))  
  A        <- rbinom(N, 1, p_A)  
  
  # adding the treated untreated data to the dataset  
  df$A = A  
  
  # continuous outcome  
  y_continuous <- 1*df$A + 0.5*df$L2 + 0.3*df$L3 + rnorm(N, 0,1)  
  
  df$y_continuous = y_continuous  
  
  # binary outcome  
  logit_formula <- "beta_effect*df$A + 0.5*df$L2 + 0.3*df$L3"  
  seq_try = seq(-4,4, 0.01)  
  desired_diff = 0.4  
  beta_effect = binary_outcome_rate(desired_diff, seq_try, df, logit_formula)  
  
  y_logit <-eval(parse(text=logit_formula ))  
  p_outcome <- exp(y_logit)/(1+ exp(y_logit))  
  y_binary <- rbinom(N, 1, p_outcome)  
  
  df$y_binary = y_binary  
  
  return(df)  
}  
N = 100  
desired_prop = 0.2  
data_gen(N, desired_prop)
```

```
## # A tibble: 100 x 6  
##       L1      L2      L3      A y_continuous y_binary  
##   <dbl> <dbl> <dbl> <int>      <dbl>      <int>
```

```
## 1 -0.969 -1.92 -0.0297 0 -1.20 0
## 2 -1.54 0.375 -0.300 0 -0.896 0
## 3 0.362 0.240 0.699 0 1.13 1
## 4 1.25 -0.781 -1.06 0 0.282 0
## 5 -0.266 -0.517 0.103 0 -0.990 0
## 6 -0.132 -1.09 -1.87 0 -1.53 0
## 7 0.882 0.733 -1.06 0 -0.175 1
## 8 -0.154 -0.802 -1.73 0 0.454 0
## 9 0.821 -0.792 0.836 1 2.60 1
## 10 0.374 -0.698 0.594 0 0.168 1
## # ... with 90 more rows

pre_data <- defData(varname = "L1", formula = "0", variance = 1, dist = "normal")
pre_data <- defData(pre_data, varname = "L2", formula = "0", variance = 1,
  dist = "normal")
pre_data <- defData(pre_data, varname = "L3", formula = "0", variance = 1,
  dist = "normal")
pre_data <- defData(pre_data, varname = "A", formula = "0.5*L1 + 0.27*L2 -0.17*L3",
  dist = "binary", link = "logit")
pre_data <- defData(pre_data, varname = "Y", formula = "0.5*A + 0.8*L2 + -0.1*L3",
  dist = "binary", link = "logit")

set.seed(7777)
df <- genData(1000, pre_data)
expit <- function(beta) {
  return(exp(beta)/(1 + exp(beta)))
}
ATE <- expit(sum(0.5 + 0.8*df$L2 - 0.1*df$L3)) - expit(sum(0.8*df$L2 - 0.1*df$L3))
# this is not true ATE
# True log odds ratio: 0.5
```

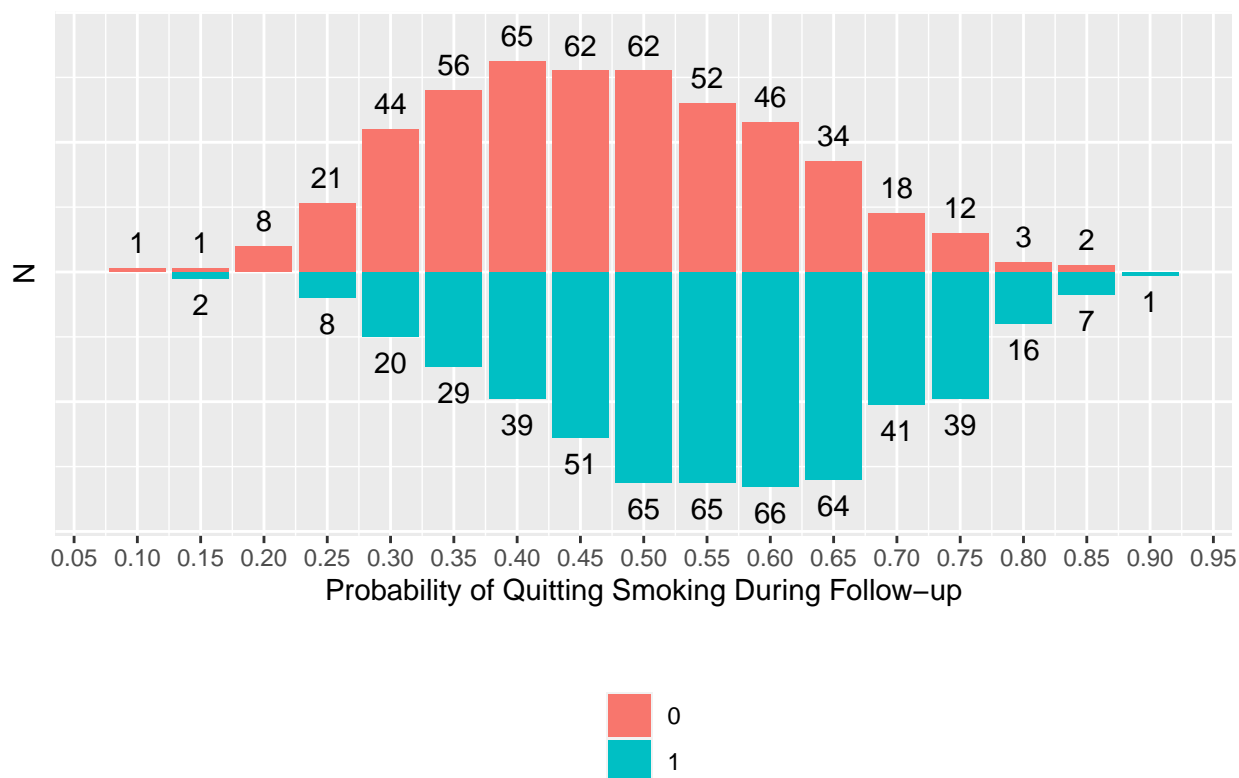
Propensity Score Model

500 pairs Propensity Score distribution

```
df %>%
  mutate(ps.grp = round(ps/0.05) * 0.05) %>%
  group_by(A, ps.grp) %>%
  summarize(n = n()) %>%
  ungroup() %>%
  mutate(n2 = ifelse(A == 0, yes = n, no = -1*n)) %>%
  ggplot(aes(x = ps.grp, y = n2, fill = as.factor(A))) +
  geom_bar(stat = 'identity', position = 'identity') +
  geom_text(aes(label = n, x = ps.grp, y = n2 + ifelse(A == 0, 8, -8))) +
  xlab('Probability of Quitting Smoking During Follow-up') +
  ylab('N') +
  ggtitle('Propensity Score Distribution by Treatment Group') +
  scale_fill_discrete('') +
  scale_x_continuous(breaks = seq(0, 1, 0.05)) +
  theme(legend.position = 'bottom', legend.direction = 'vertical',
    axis.ticks.y = element_blank(),
    axis.text.y = element_blank())

## `summarise()` regrouping output by 'A' (override with `.groups` argument)
```

Propensity Score Distribution by Treatment Group



Nearest neighbor propensity score matching

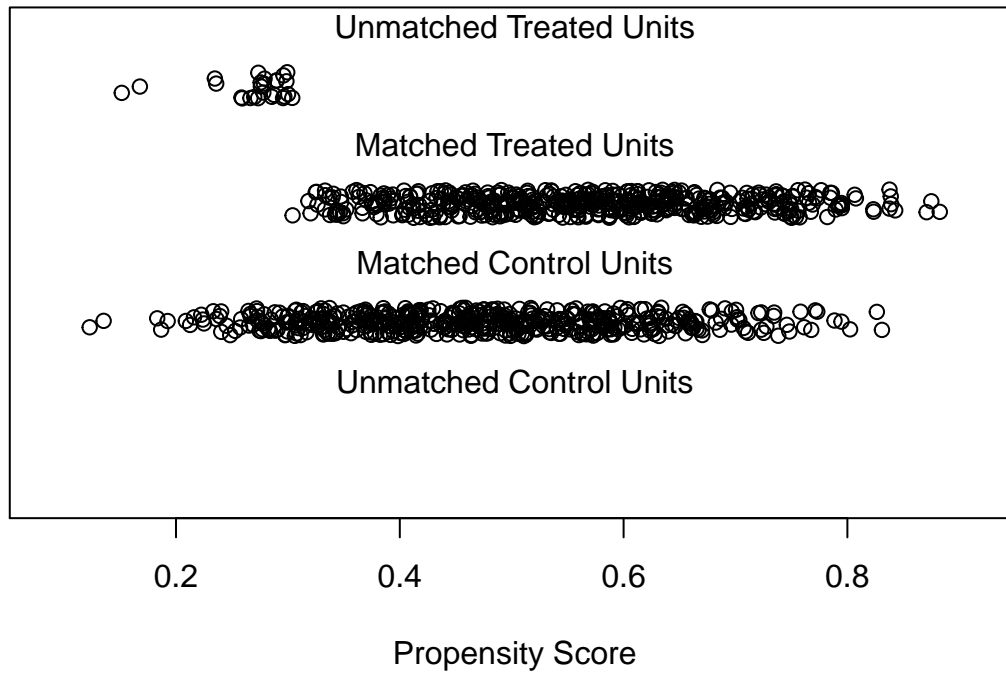
```
matched <- matchit(A ~ L1 + L2 + L3, data = df,
  distance = "glm", link = "logit",
  method = "nearest", ratio = 1)
```

```
summary(matched)[2]
```

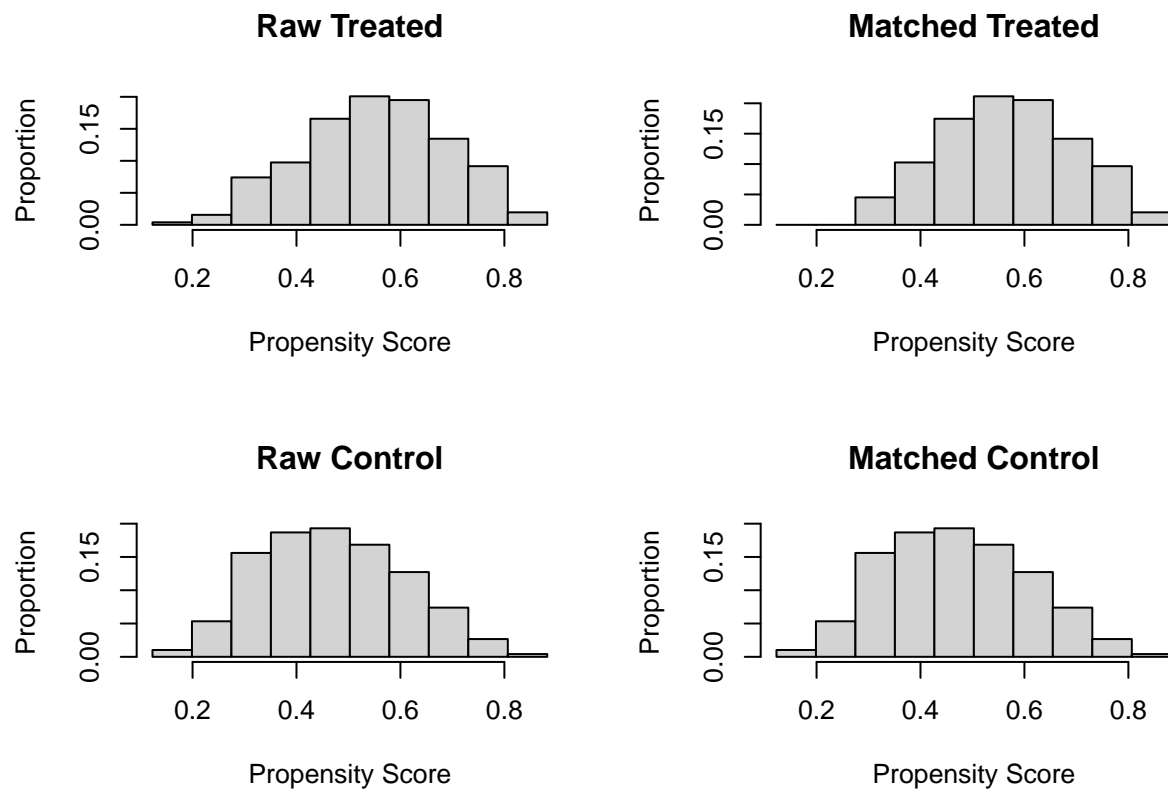
```
## $nn
##           Control Treated
## All (ESS)      487      513
## All           487      513
## Matched (ESS)  487      487
## Matched       487      487
## Unmatched      0       26
## Discarded      0        0
```

```
plot(matched, type = "jitter", interactive = FALSE)
```

Distribution of Propensity Scores



```
plot(matched, type = "histogram")
```



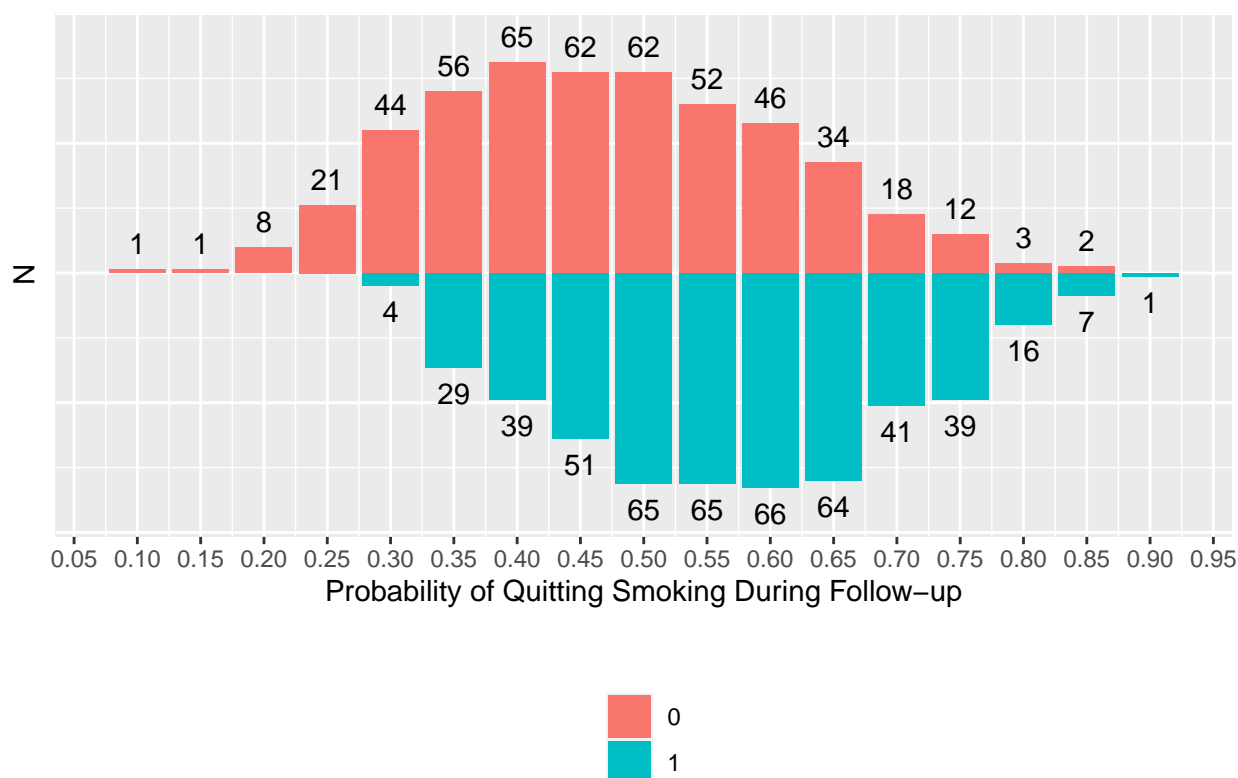
```
matched_df <-  
  match.data(matched)
```

495 pairs propensity score distribution

```
matched_df %>%
  mutate(ps.grp = round(ps/0.05) * 0.05) %>%
  group_by(A, ps.grp) %>%
  summarize(n = n()) %>%
  ungroup() %>%
  mutate(n2 = ifelse(A == 0, yes = n, no = -1*n)) %>%
  ggplot(aes(x = ps.grp, y = n2, fill = as.factor(A))) +
  geom_bar(stat = 'identity', position = 'identity') +
  geom_text(aes(label = n, x = ps.grp, y = n2 + ifelse(A == 0, 8, -8))) +
  xlab('Probability of Quitting Smoking During Follow-up') +
  ylab('N') +
  ggtitle('Propensity Score Distribution by Treatment Group') +
  scale_fill_discrete('') +
  scale_x_continuous(breaks = seq(0, 1, 0.05)) +
  theme(legend.position = 'bottom', legend.direction = 'vertical',
        axis.ticks.y = element_blank(),
        axis.text.y = element_blank())
```

`summarise()` regrouping output by 'A' (override with `.groups` argument)

Propensity Score Distribution by Treatment Group



simple bootstrap

```
nboot <- 100
# set up a matrix to store results
```

```

boots <- data.frame(i = 1:nboot,
  se_ATE = NA,
  se_OR = NA,
  log_OR = NA,
  mean1 = NA,
  mean0 = NA,
  difference = NA
)
# loop to perform the bootstrapping
for (i in 1:nboot) {
  # sample with replacement
  sampl <- matched_df %>% filter(subclass %in% sample(levels(subclass),500, replace = TRUE))

  bootmod <- glm(Y ~ A + ps, data = sampl,
    weights = weights, family = binomial)

  # create new data sets
  sampl.treated <- sampl %>%
    mutate(A = 1)

  sampl.untreated <- sampl %>%
    mutate(A = 0)

  # predict values
  sampl.treated$pred.y <-
    predict(bootmod, sampl.treated, type = "response")

  sampl.untreated$pred.y <-
    predict(bootmod, sampl.untreated, type = "response")

  # output results

  boots[i, "log_OR"] <- summary(bootmod)$coeff[2,1]

  boots[i, "se_OR"] <- summary(bootmod)$coeff[2,2]

  boots[i, "se_ATE"] <-
    sqrt((summary(bootmod)$coeff[2,2]*mean(sampl.treated$pred.y) *
      (1 - mean(sampl.treated$pred.y)))^2 +
      (summary(bootmod)$coeff[2,2]*mean(sampl.untreated$pred.y) *
      (1 - mean(sampl.untreated$pred.y)))^2)

  boots[i, "mean1"] <- mean(sampl.treated$pred.y)
  boots[i, "mean0"] <- mean(sampl.untreated$pred.y)
  boots[i, "difference"] <- boots[i, "mean1"] - boots[i, "mean0"]

  mean_log_OR <- mean(boots$log_OR)

  Empirical_se_ATE <- sd(boots$difference)

  mean_se_ATE <- mean(boots$se_ATE)

```



```

Empirical_se_log_OR <- sd(boots$log_OR)

mean_se_log_OR <- mean(boots$se_OR)

ATE <- mean(boots$difference)

# once loop is done, print the results
if (i == nboot) {
  cat("ATE:")
  cat(ATE)
  cat("\n")
  cat("\n")
  cat("Empirical_se_ATE:")
  cat(Empirical_se_ATE)
  cat("\n")
  cat("\n")
  cat("mean_se_ATE:")
  cat(mean_se_ATE)
  cat("\n")
  cat("\n")
  cat("95% CI for ATE:")
  cat(ATE - 1.96*Empirical_se_ATE,
      ", ",
      ATE + 1.96*Empirical_se_ATE)
  cat("\n")
  cat("\n")
  cat("mean_log_OR:")
  cat(mean_log_OR)
  cat("\n")
  cat("\n")
  cat("Empirical_se_log_OR:")
  cat(Empirical_se_log_OR)
  cat("\n")
  cat("\n")
  cat("mean_se_log_OR:")
  cat(mean_se_log_OR)
  cat("\n")
  cat("\n")
  cat("95% CI for log odds ratio:")
  cat(mean_log_OR - 1.96*mean_se_log_OR,
      ", ",
      mean_log_OR + 1.96*mean_se_log_OR)
}
}

```

```

## ATE:0.1068614
##
## Empirical_se_ATE:0.025629
##
## mean_se_ATE:0.0601082
##
## 95% CI for ATE:0.05662853 , 0.1570942
##

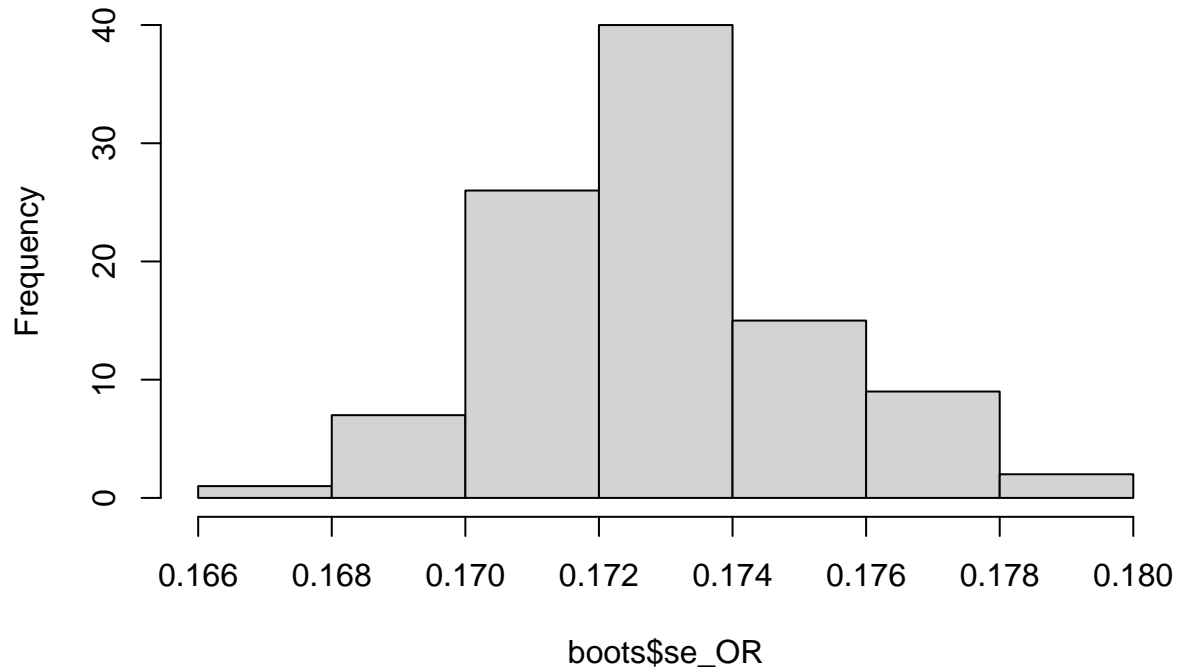
```

```
## mean_log_OR:0.4353995
##
## Empirical_se_log_OR:0.1046255
##
## mean_se_log_OR:0.172854
##
## 95% CI for log odds ratio:0.09660568 , 0.7741934
hist(boots$log_OR)
```



```
hist(boots$se_OR)
```

Histogram of boots\$se_OR



```
a <- glm(Y ~ A + ps, data = sampl,
          weights = weights, family = binomial)
```

```
summary(a)
```

```
##
## Call:
## glm(formula = Y ~ A + ps, family = binomial, data = sampl, weights = weights)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.5387  -1.1270   0.8968   1.0713   1.4361
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -0.8868     0.3264  -2.717  0.00660 **
## A              0.5997     0.1728   3.470  0.00052 ***
## ps            1.3180     0.6373   2.068  0.03863 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 868.96  on 627  degrees of freedom
## Residual deviance: 844.52  on 625  degrees of freedom
## AIC: 850.52
##
## Number of Fisher Scoring iterations: 4
```