MATH 266 - NDSU
Fall 2025
Final Exam
December 17th, 2025
Time Limit: 120 Minutes

Name: Kassandra
ID Number: 1220452

This exam contains 10 pages (including this cover page) and 8 questions. Total of points is 100. Read all of the following information before starting the exam:

- Present your work clearly and in order, and justify your conclusions. We reserve the right to take off points if we cannot see how you arrived your answers or cannot read your handwriting clearly.
- **No calculators are allowed. You may use a double-sided full page note sheet.**
- Write your answers on the space provided. Feel free to use the other side of each page if more space is needed, but please indicate you have done so, otherwise the back side of each page will be regarded as scratch work and **will not be graded.**
- A Laplace transform table is provided separately.
- Good luck!

Grade Table

Question	Points	Score
1	10	3
2	15	8
3	20	12
4	10	5
5	10	4
6	10	6
7	10	10
8	15	5
Total:	100	53

volume

1. (10 points) A tank contains 10 gal of brine made by dissolving 6 lb of salt in water. Salt water containing 1 lb of salt per gallon runs in at the rate of 2 gal/min and the well-stirred mixture runs out at the rate of 3 gal/min. Find the amount of salt in the tank after t minutes.

$\frac{lb}{gal} \cdot \frac{gal}{min}$

$A'(t) = C_{in} r_{in} - C_{out} r_{out}$

$C_{in} = 1\,lb/gal$ $\quad A' = (2\,lb/min) - C_{out}(3\,gal/min)$

$r_{in} = 2\,gal/min$ $\quad A' = \frac{2\,lb}{min} - \frac{3A(t)}{10}$

$r_{out} = 3\,gal/min$

$C_{out} = A(t)/volume$ $\quad A'(t) - \frac{3A(t)}{10} = 2$

3/10

$A(t) = \frac{C_{out}}{volume}$

$M(t) = e^{\int -3/10\,dt}$

$M(t) = e^{-3/10t}$

$e^{-3/10t} y' + \frac{3}{10} e^{-3/10t} y = \frac{6}{10} e^{-3/10t}$

2 cont

$$As^2 + \underline{As} + \underline{A} + Bs^2 + \underline{Bs} + \underline{Cs} = e^{3s} \underline{-s}$$

$$As + Bs + Cs = -s$$

$$A + B + C = -1 \rightarrow C = -1$$

$$A\cancel{s^2} + B\cancel{s^2} = 0$$

$$A = -B$$

$$A = 0$$

$$\frac{+1}{(s+1)^2} = e^{-3s} - s(s+1)^2$$

$$e^{ax} \quad x^n$$

$$xe^x =$$

2. (15 points) Solve the initial value problem

$$y'' + 2y' + y = u(x-3),\ y(0) = 0,\ y'(0) = 1.$$

$\mathcal{L}\{y''+2y'+y\} = \mathcal{L}\{u(x-3)\} \to \frac{e^{-as}}{s} \to \frac{e^{-3s}}{s}$

$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + \mathcal{L}\{y\}$

$(s^2 Y(s) - s\cancel{y}^{0}(0) - \cancel{y'}^{1}(0)) + 2(sY(s) - \cancel{y}^{0}(0)) + Y(s)$

$(s^2 Y(s) - 1) + 2sY(s) + Y(s)$

$s^2 Y(s) + 2sY(s) + Y(s) - 1 = \frac{e^{-3s}}{s}$

$s^2 Y(s) + 2sY(s) + Y(s) = \frac{e^{-3s}}{s} - 1$

$Y(s)(s^2+2s+1) = \frac{e^{-3s} - s}{s}$

$Y(s)\ (s+1)^2 = \frac{e^{-3s} - s}{s}$

$Y(s) = \frac{e^{-3s} - s}{s(s+1)^2}$

8/15

$\frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} = \frac{e^{-3s} - s}{s(s+1)^2}$

$A(s+1)^2 + B(s)(s+1) + Cs = e^{-3s} - s$

$A(s^2+2s+1) + Bs^2 + Bs + Cs = e^{3s} - s$

3. (20 points) This problem consists of two parts.

   (a) (10 points) Find the general solution of the system of equations

$$\vec{x}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \vec{x}.$$

$\det(A - \lambda I) = 0$

$\begin{pmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{pmatrix} = 0$

$((-2-\lambda)(-2-\lambda) - 1) = 0$

$4 + 2\lambda + 2\lambda + \lambda^2 - 1 = 0$

$\lambda^2 + 4\lambda + 4 - 1 = 0$

$\lambda^2 + 4\lambda + 3 = 0$

$(\lambda+1)(\lambda+3) = 0$

$\lambda_1 = -1, \lambda_2 = -3$

$\lambda_1 = -1$

$(A - \lambda I)\vec{v}_1 = 0$

$\begin{pmatrix} -2+1 & 1 \\ 1 & -2+1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 = x_2$

$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda_2 = -3$

$(A - \lambda I)(\vec{v}_2) = 0$

$\begin{pmatrix} -2+3 & 1 \\ 1 & -2+3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 + x_2 = 0$

$x_1 = -x_2$

$\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

general soln

$\vec{x}(t) = C_1 e^{\lambda_1 t} \vec{v}_1 + C_2 e^{\lambda_2 t} \vec{v}_2$

$\boxed{\vec{x}(t) = C_1 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$

10

[1]This problem continues on next page.

(b) (10 points) Find a particular solution to the nonhomogeneous system of equations

$$\vec{x}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} 2e^{-t} \\ -2e^{-t} \end{pmatrix}.$$

Hint: You solved the homogeneous system in part (a).

$\vec{g}(t) = \begin{pmatrix} 2e^{-t} \\ -2e^{-t} \end{pmatrix}$

$\vec{X}' = A\vec{x} + \vec{g}(t)$

$\vec{X}(t) = \vec{x}_c(t) + \vec{x}_p(t)$

$\vec{x}_c = C_1 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\boxed{x(t) = C_1 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -12e^{-t} \\ 6e^{-t} \end{pmatrix}}$

$\vec{x}_p(t) = x(t)\vec{v}(t)$

$\vec{v}(t) = \int x^{-1}(t)\, \vec{g}(t)\, dt$

$\cancel{\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 2e^{-t} \\ -2e^{-t} \end{pmatrix} dt}$

$\vec{v}(t) \int \begin{pmatrix} -4e^{-t} & -2e^{-t} \\ 2e^{-t} & -2e^{-t} \end{pmatrix} dt$

2/10

$\vec{v} = \int \begin{matrix} -6e^{-t} \\ 0 \end{matrix}\, dt \quad \int -6e^{-t} \to 6e^{-t} \quad \to 0$

$\vec{v} = \begin{pmatrix} 6e^{-t} \\ 0 \end{pmatrix}$

$\vec{x}_p = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 6e^{-t} \\ 0 \end{pmatrix} \to \begin{pmatrix} -12e^{-t} \\ 6e^{-t} \end{pmatrix}$

4. (10 points) Use the method of **undetermined coefficients** to find the general solution of the equation

$$y'' - 2y' - 3y = -3e^{-x}.$$

$y = y_c + y_p$

$y_c \Rightarrow y'' - 2y' - 3y = 0$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$r_1 = 3, r_2 = -1$

$y_c = C_1 e^{3x} + C_2 e^{-1x}$

$F(x) = -3e^{-x}$

$ae^{rx} \rightarrow Ae^{rx}$

$-3e^{-1x}$

$y_p = -3ex^{-1x}$

$\boxed{y = C_1 e^{3x} + C_2 e^{-x} - 3ex^{-x}}$

5/10

5. (10 points) Solve the system of equations

$$\vec{x}' = \begin{pmatrix} 2 & -4 \\ 2 & -2 \end{pmatrix} \vec{x}.$$

$\det(A - \lambda I) = 0$

$\det\begin{pmatrix} 2-\lambda & -4 \\ 2 & -2-\lambda \end{pmatrix} = 0$

$(2-\lambda)(-2-\lambda) + 8 = 0$

$-4 - 2\lambda + 2\lambda + \lambda^2 + 8 = 0$

$\lambda^2 + 4 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\lambda = \frac{\pm\sqrt{-4(1)(4)}}{2}$

$\lambda = \frac{4i}{2} = \pm 2i$

$\lambda : \alpha \pm \beta i$

$\lambda = \pm 2i$

$(A - 2iI)\vec{v_1} = 0$

$\begin{pmatrix} 2-2i & -4 \\ 2 & -2-2i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$(2-2i)x_1 - 4x_2 = 0$

$x_1 = \frac{-4}{2-2i} x_2$

$x_1 = \frac{-2}{1-i} x_2$

$\vec{v_1} = \begin{pmatrix} -2/1-i \\ 1 \end{pmatrix} \to \begin{pmatrix} -2(1-i) \\ 1-i \end{pmatrix} \to \begin{pmatrix} -2-2i \\ 1-i \end{pmatrix} \to \vec{v_1} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + i\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\vec{v} = \vec{a} + \vec{b}$

$= \cancel{e^{\alpha t}}(\cos(\beta t)\vec{a} - \sin(\beta t)\vec{b})$

$\vec{x_1} = (\cos(2t)\begin{pmatrix} -2 \\ 1 \end{pmatrix} - \sin(2t)\begin{pmatrix} -2 \\ -1 \end{pmatrix})$

$\boxed{\vec{x_1}(t) = (\cos(2t)\begin{pmatrix} -2 \\ 1 \end{pmatrix} - \sin(2t)\begin{pmatrix} -2 \\ -1 \end{pmatrix})}$

$x_2(t) = \cancel{e^{\alpha t}}(\cos(\beta t)\vec{b} + \sin(\beta t)\vec{a})$

$\boxed{\vec{x_2}(t) = \cos(2t)\begin{pmatrix} -2 \\ -1 \end{pmatrix} + \sin(2t)\begin{pmatrix} -2 \\ 1 \end{pmatrix}}$

4/10

6. (10 points) Find the general solution of the equation

$$\left(2\sqrt{y}+\frac{1}{x^2}\right)\,dx+\left(\frac{x}{\sqrt{y}}+\frac{1}{1+y^2}\right)\,dy=0.$$

$$(2\sqrt{y}+1/x^2)\,dx = -\left(\frac{x}{\sqrt{y}} + \frac{1}{1+y^2}\,dy\right)$$

$$\frac{2(y)^{1/2}x^2+1}{x^2}\,dx = -\frac{x+1}{(y^{1/2})(1+y^2)}\,dy$$

$$\int 2y^{1/2}x^4 + x^2\,dx = \int \frac{-(x+1)}{y^{1/2}+y}\,dy$$

$$\int 2y^{1/2}x^4\,dx + \int x^2\,dx = -x+1\int\frac{1}{y^{1/2}+y}\,dy \qquad v = y^{1/2}+y$$

$$2y^{1/2}\int x^4\,dx + \int x^2\,dx =$$

$$(2y^{1/2})\left[\frac{1}{5}x^5 + \frac{1}{3}x^3\right] =$$

on scrap paper

answer. $y = \tan(1/x)$

6. $\left(2\sqrt{y} + 1/x^2\right)dx + \left(x/\sqrt{y} + 1/(1+y^2)\right)dy$

$\int 2\sqrt{y}\,dx + \int \frac{1}{x^2}dx = \int \frac{x}{\sqrt{y}}dy + \int \frac{1}{1+y^2}dy$ $\quad x^{-2}$

$2\sqrt{y}\,x + \frac{1}{x} = x\int y^{-1/2}dy + \tan^{-1}y$

$\cancel{2\sqrt{y}\,x} + \frac{1}{x} = \cancel{2x\sqrt{y}} + \tan^{-1}y$

$\tan^{-1}y = 1/x$

$\boxed{y = \tan 1/x}$ $= g(y)$

$\frac{1}{x^2}$

$\frac{1}{1}x^{-2+1}$

x^{-1}

$\frac{1}{x}$

$y^{-1/2+1}$

$2y^{1/2}$

6/10

7. (10 points) Use **any method learned in this class** to solve the initial value problem

$$y'' - 8y' + 16y = 0,\ y(0) = 5,\ y'(0) = 3.$$

$f.g = f'g + g'f$

$r^2 - 8r + 16 = 0$

$(r-4)^2 = 0,$

$r = 4$

$y = C_1 e^{rx} + C_2 x e^{rx}$

$y = C_1 e^{4x} + C_2 x e^{4x} \rightarrow 5 = C_1$

$y' = 4C_1 e^{4x} + C_2 e^{4x} + 4C_2 x e^{4x}$

$3 = 4C_1 \underbrace{e^{4(0)}}_{1} + C_2 \underbrace{e^{4(0)}}_{1} + \underbrace{\cancel{4(C_2)(0)e^{4(0)}}}_{0}$

$3 = 4C_1 + C_2$

$3 = 4(5) + C_2$

$3 = 20 + C_2$

$C_2 = -17$

$\boxed{y = 5e^{4x} - 17xe^{4y}}$

10

8. (15 points) Solve the system of equations

$$\vec{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 1 \end{pmatrix} \vec{x}.$$

$\det(A - \lambda I) = 0$

$\det\begin{pmatrix} 1-\lambda & 0 & 0 \\ 1 & 3-\lambda & 0 \\ 0 & 1 & 1-\lambda \end{pmatrix} =$

$(1-\lambda)(-1)^2 \det\begin{pmatrix} 3-\lambda & 0 \\ 1 & 1-\lambda \end{pmatrix} + 0 + 0 = 0$

$(1-\lambda)[(3-\lambda)(1-\lambda) - 1] = 0$

$(1-\lambda)[3 - 3\lambda - \lambda + \lambda^2 - 1] = 0$

$(1-\lambda)[\lambda^2 - 4\lambda + 2] = 0$

$(1-\lambda)(\lambda-2)^2 = 0$

$\lambda_1 = 1, \quad \lambda_{2,3} = 2$

$\lambda = 1$

$(A - \lambda I)(\vec{v_1}) = (0)$

$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x_1 + 2x_2 + 0x_3 = 0$

$x_1 = -2x_2$

$\therefore \vec{v_1} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$

$\lambda = 2$

$(A - \lambda I)(\vec{v_2}) = (0)$

$\begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{cases} x + x_2 = 0 \\ 0 \quad x_2 - x_3 = 0 \end{cases}$

$x_1 = -x_2$

$x_2 = +x_3$

$x_1 = -x_3$

$v_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

8 cont.

$\lambda_3 = 2$

$$(A - \lambda I)(\vec{w}) = (\vec{v}_2)$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -1 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \to$$

$$\vec{w} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{x}(t) = c_1 e^{\lambda t}\vec{r}_1 + c_2 e^{\lambda_2}\vec{v}_2 + c_3 e^{\lambda t}(t \cdot \vec{v}_2 + \vec{w})$$

$$\boxed{\vec{x}(t) = c_1 e^{t}\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + c_3 e^{2t}\begin{pmatrix} t-1 \\ -t \\ -t+1 \end{pmatrix}}$$

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MATH 266 FINAL FORMULAS

2θ: $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = 2\tan\theta / 1-\tan^2\theta$

$\frac{1}{2}\theta$: $\sin^2\theta = (1-\cos 2\theta)/2$

$\sin\theta/2 = \pm\sqrt{(1-\cos\theta)/2}$

$\cos^2\theta = (1+\cos 2\theta)/2$

$\cos\theta/2 = \pm\sqrt{1+\cos\theta/2}$

$\tan\theta/2 = \pm\sqrt{(1-\cos\theta)/(1+\cos\theta)}$

Int by parts: $\int u\,dv = uv - \int v\,du$

Sep. EQN: $dy/dx = f(x)g(y) \rightarrow \int \frac{1}{g(y)}dy = \int f(x)dx$

$g(y) \neq 0$; C: $y = y_0$, $g(y_0) = 0$

1st ord EQN: $P(x)y' + q(x)y = f(x)$; $P(x) \neq 0$

$\rightarrow y' + r(x)y = f(x)$

int fact: $M(x) = e^{\int r(x)dx}$

y' coef $= 1$; no $+C$

$e^{\ln x} = x, x>0$ & $a\ln x = \ln x^a$; $e^{a\ln x} = x^a$

gen soln: $M(x)y' + r(x)M(x)y = f(x)M(x)$

$\hookrightarrow d/dx\,(M(x)y) = f(x)Mx$

$\hookrightarrow$ int & $\div M(x)$

Exact EQN: $M = \Psi_x$ & $N = \Psi_y$

$M_y = \Psi_{xy}$ & $N_x = \Psi_{xy}$; $M_y = N_x$

$\Psi = \int M\,dx + g(y) = N_x$ to get $g'(y)$

int $g'(y)$ w/r to y sub into

Mixing: setup: $A'(t) = C_{in}r_{in} - C_{out}r_{out}$

$C_{OUT} = A(t)/\text{volume} \rightarrow$ find $A(0)$

2^{ND} ORDER Hom. Eqn: $ay'' + by' + cy = 0$; $y_1 \neq ky_2$

$W[y_1,y_2] = y_1y_2' - y_2y_1' = \det\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}$

y_1 & $y_2 \rightarrow$ LI if $W[\ ] \neq 0$

Char eqn: $ar^2 + br + c = 0$

gen soln:

2 dis: $y = C_1e^{r_1x} + C_2e^{r_2x}$

Rep: $y = C_1e^{rx} + C_2xe^{rx}$

Com: $\alpha \pm \beta i$: $y = e^{\alpha x}(C_1\cos(\beta x) + C_2\sin(\beta x))$

* IVP NEED 2 COND

Particular Soln Table	
$F(x)$	y_p
ae^{rx}	Ae^{rx}
$a\sin(rx)$ &/or $a\cos(rx)$	$A\sin(rx) + B(rx)$
$a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$	$Ax^n + A_{n-1}x^{n-1} + A_0$

Undet Co: $ay'' + by' + cy = F(x)$ $y = y_c + y_p$

$y_c \rightarrow = 0$; Table part soln; y_c can't overlap. if y_p by x if still by x^2

only for: poly, e^{rx}, and $\sin(rx)/\cos(rx)$

Vari of P.: $ay'' + by' + cy = F(x)$

$= 0 \rightarrow y_c = C_1y_1 + C_2y_2$ & $y_p = u_1y_1 + u_2y_2$

$u_1 = -\int \frac{y_2F}{a\cdot W[y_1,y_2]}dx$ $u_2 = \int \frac{y_1F}{a\,W[y_1,y_2]}dx$

Laplace Transform

def: $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)dt = Fs$

P.W.

$f(x) = \begin{cases} e^x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

def. $\mathcal{L}\{(x)\} = \int_0^\infty e^{-st} f(t)\,dt$

$\int_0^\pi e^{-st}e^t + \int_\pi^\infty 0e^{-st}dt$

$\int_0^\pi e^{-(s-1)t}dt$

to use table $\rightarrow \mathcal{L}\{f t\}$ & use prop. to sep individuals

simply w/ prop. co to front, correspond to table

Properties Laplace

1. $\mathcal{L}\{f(x) + g(x)\} = \mathcal{L}\{f(x)\} \pm \mathcal{L}\{g(x)\}$
2. $\mathcal{L}\{cf(x)\} = c\,\mathcal{L}\{f(x)\}$
3. $\mathcal{L}\{f(x)g(x)\} \neq \mathcal{L}\{f(x)\}\,\mathcal{L}\{g(x)\}$
4. $F(s)$ Laplace of $f(x) \rightarrow \mathcal{L}\{e^{ax}f(x)\} = F(s-a)$ shift dist a to right
5. $f(x)$ $[0,\infty)$ $f'(x)$ $[0,\infty)$ then $\mathcal{L}\{f'(x)\} = sF(s) - f(0)$ $F(s) = \mathcal{L}\{f(x)\}$
6. $\mathcal{L}(f')(s) = s\,\mathcal{L}(f)(s) - f(0)$
7. $\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'0$

Inverse Laplace Transform

look at dem of form match to table. correct numerator to correct num

$\mathcal{L}^{-1}\{f(s)\}$

$P(x)/Q(x)$, $\rightarrow$ poly & $\deg(P) < \deg(Q)$

PFD

$(x-a)(x-b) \rightarrow A/(x-a) + B/x-b$

$(x-a)^2 \rightarrow A/(x-a) + B/(x-a)^2$

$(x^2+c) \rightarrow Bx+c/x^2+c)$

get rid of dem, solve for $c(s)$

IVP w/ L T $y'' + 5y' + 6 = 0$ $y(0) = 0$ $y'(0) = 1$

$Y(s) = \mathcal{L}\{y\} \rightarrow$ get $y(t)$ by using $\mathcal{L}^{-1}\{Y(s)\}$

1. LT both sides $\rightarrow$ 2. prop & initial to obtain Alg. eqn for $Y(s)$. solve for. 3. $\mathcal{L}^{-1}\{Y(s)\}$

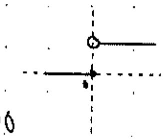
$y'' - y' - 2y \rightarrow s^2Y(s) - sy(0) - y'(0)$

$-(sY(s) - y(0)) - 2Y(s)$ c goes in front

Simp & solve w/ PFD

Unit Step Func

$u(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 & \text{if } x > 0 \end{cases}$



Notes:

L.S. → [A|0]

↳ RREF

↳ Leading 1 P. others free

$\mathcal{L}\{u(x-a)\}$, $\mathcal{L}\{f(x-a)\,u(x-a)\}$

if f(x) jump at x=a w/ height k

k is fun. or c. $k\,u(x-a)$

fun. $= x\,u(x-2)$, $y(0)=0$, $y'(0)=1$

$\mathcal{L}\{\ \} \to Y(s)$ simp.

LH need to be in form $f(x-a)\,u(x-a)$

find $f(x-2)=x$ $z=x-2 \to x=z+2$

$f(z)=z+2$ $f(x)=x+2$

$\mathcal{L}\{x\,u(x-2)\} = \sim$

$Y(s) =$

Solve for Y(s) → ILT

Homogen Linear Systems w/ constant co

↳ λ; $\det(A-\lambda I)=0$ → solve for λ

↳ $\vec{v}$; $A\vec{v}=\lambda\vec{v}$ or $(A-\lambda I)\vec{v}=0$ → solve for $\vec{v}$

↳ general soln $\vec{x}'=A\vec{x}$ (3 cases)

↳ distinct λ: $\vec{v_i}$ for λ_i, $\vec{x}(t)= C_1 e^{\lambda_1 t}\vec{v}_1 + \dots + C_n e^{\lambda_n t}\vec{v_n}$

↳ complex $\lambda: \alpha \pm \beta i$: if $\vec{v}=\vec{a}+\vec{b}i$ is eigenvect for $\alpha+\beta i$, then LI soln

↳ $\vec{x_1}(t) = e^{\alpha t}(\cos(\beta t)\vec{a} - \sin(\beta t)\vec{b})$, $\vec{x_2}(t) = e^{\alpha t}(\cos(\beta t)\vec{b} + \sin(\beta t)\vec{a})$

↳ rep: λ of multiplicity two

↳ if λ has 2 LI $\vec{v}_1$ & $\vec{v}_2$ then $\vec{x}(t) = C_1 e^{\lambda t}\vec{v}_1 + C_2 e^{\lambda t}\vec{v}_2$

↳ if λ has 1 $\vec{v}_1$, find $\vec{w}$ such that $(A-\lambda I)\vec{w}=\vec{v}$) & $\vec{x}(t) = C_1 e^{\lambda t}\vec{v} + C_2 e^{\lambda t}(t\vec{v}+\vec{w})$

Nonhomo

$\vec{x}' = A\vec{x} + \vec{g}(t)$

$\vec{x}(t) = \vec{x_c}(t) + \vec{x_p}(t)$ where $\vec{x_c}(t)$ solves $\vec{x}'=A\vec{x}$ & $\vec{x_p}(t) = X(t)\,\vec{v}(t)$ where $X(t)$ is a fundamental matrix

$\vec{v}(t) = \int X^{-1}(t)\,\vec{g}(t)\,dt$

$\frac{d}{dx}$ $e^x = x$

$\ln x = 1/x$

$n^x = n^x \ln x$ → $\int n^x\,dx = n^x/\ln n + C$

$\tan x = \sec^2 x$

$\cot x = -\csc^2 x$

$\sec x = \sec x \tan x$

$\csc x = -\csc x \cot$

$\arcsin x = 1/\sqrt{1-x^2}$

$\arccos x = -1/\sqrt{1-x^2}$

$\arctan x = 1/1+x^2$

$\text{arccot}\, x = -1/1+x^2$

$\text{arcsec}\, x = 1/x\sqrt{x^2-1}$

$\text{arccsc}\, x = -1/x\sqrt{x^2-1}$

$1+\tan^2(x) = \sec^2(x)$

$\tan = \frac{\sin}{\cos}$ $\sec = \frac{1}{\cos}$

Laplace Transform Table:

$f(x)$	$\mathcal{L}\{f(x)\} = F(s) = \int_0^\infty e^{-st} f(t)dt$
C	$\frac{C}{s}$
$x^n, n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{ax}	$\frac{1}{s-a}$
$\sin(ax)$	$\frac{a}{s^2+a^2}$
$\cos(ax)$	$\frac{s}{s^2+a^2}$
$e^{ax}x^n$	$\frac{n!}{(s-a)^{n+1}}$
$e^{ax}\sin(bx)$	$\frac{b}{(s-a)^2+b^2}$
$e^{ax}\cos(bx)$	$\frac{s-a}{(s-a)^2+b^2}$
$u(x-a)$	$\frac{e^{-as}}{s}$
$f'(x)$	$sF(s) - f(0)$
$f''(x)$	$s^2F(s) - sf(0) - f'(0)$
$f(x-a)u(x-a)$	$e^{-as}F(s)$
$e^{ax}f(x)$	$F(s-a)$
$-xf(x)$	$F'(s)$