Computer Vision I

Categorization – 12.06.2013

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vi visual inference

Bag-of-Words Model (BoW)

Object → **Bag of 'words'**

Bag of Words

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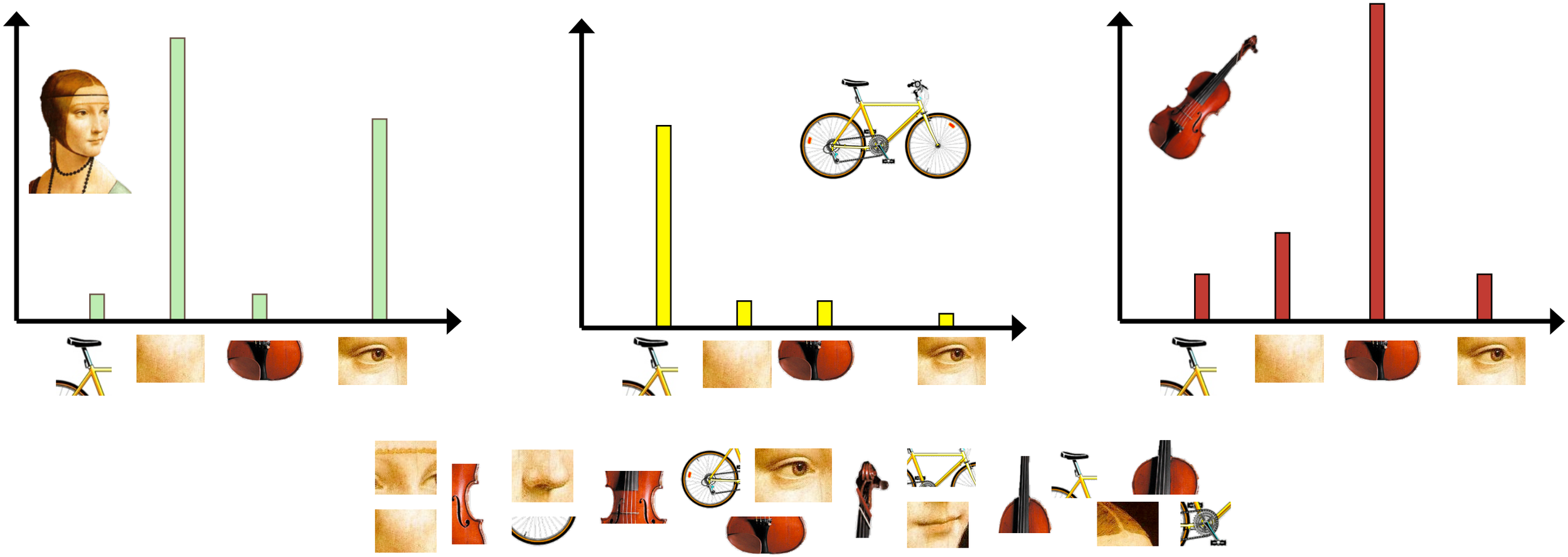
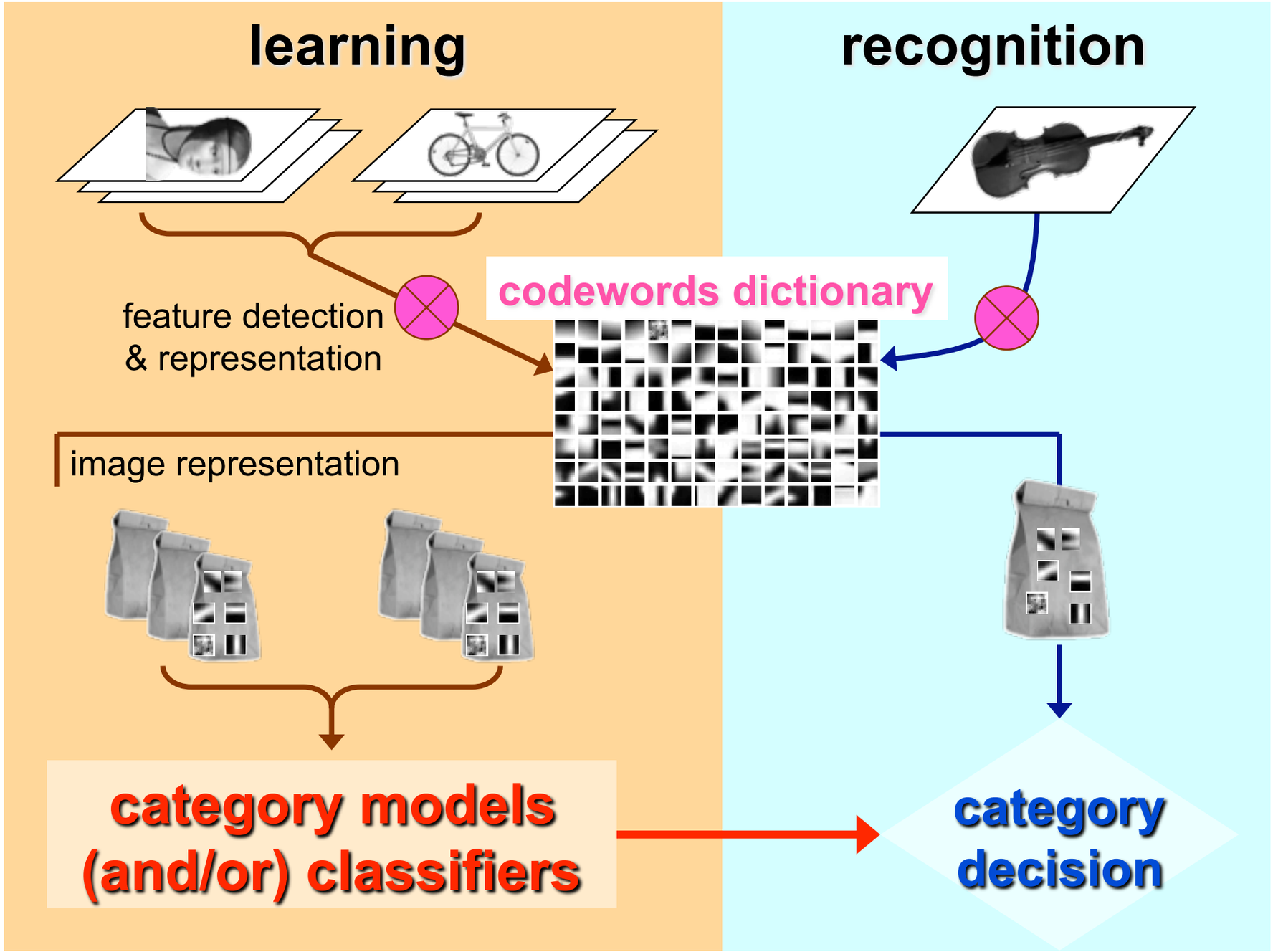
- Independent features
- Histogram representation



Bag-of-Words Model: Overview



Bag-of-Words Model: Object Representation & Learning

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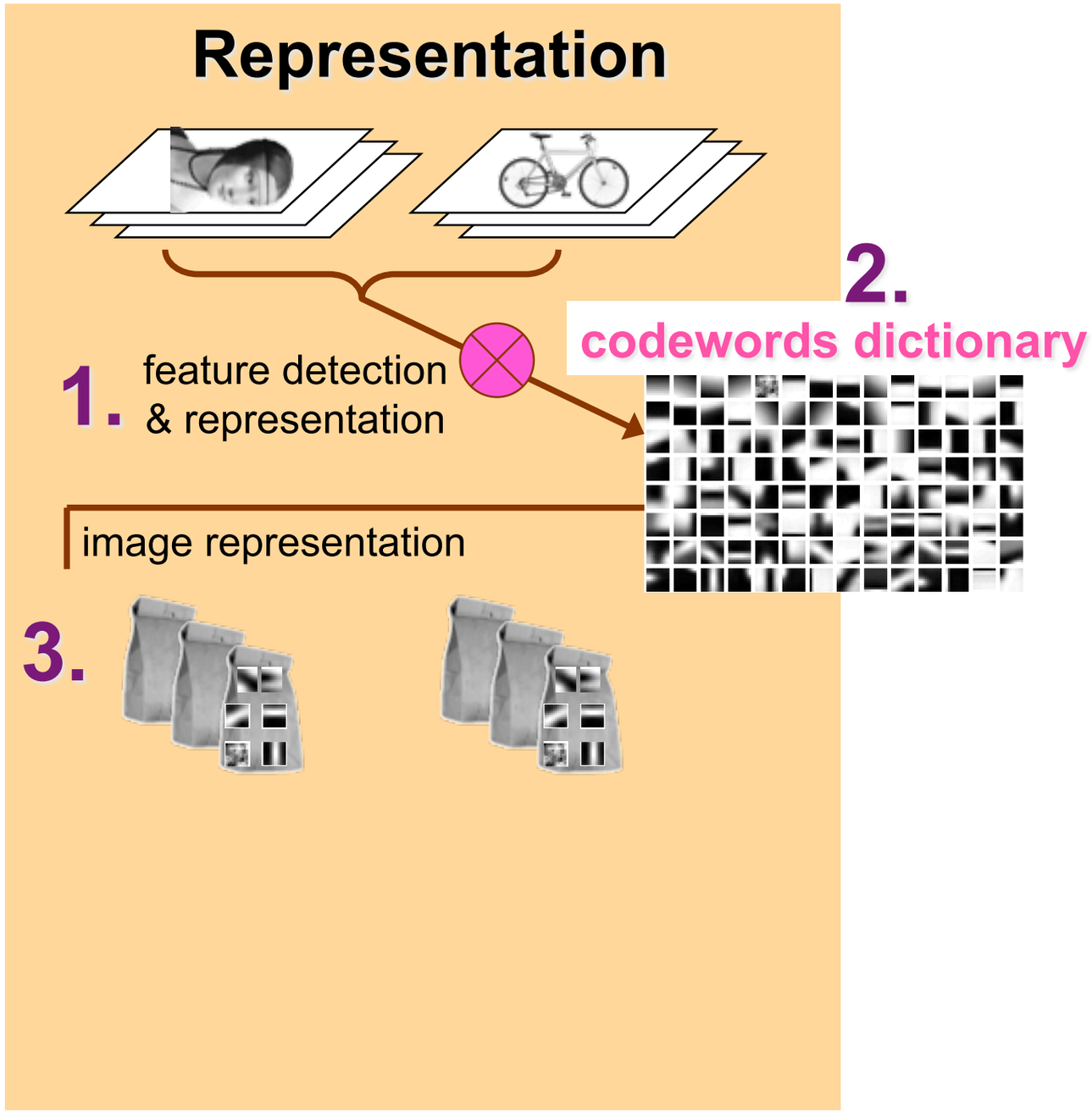

BoW-1. Feature detection and representation

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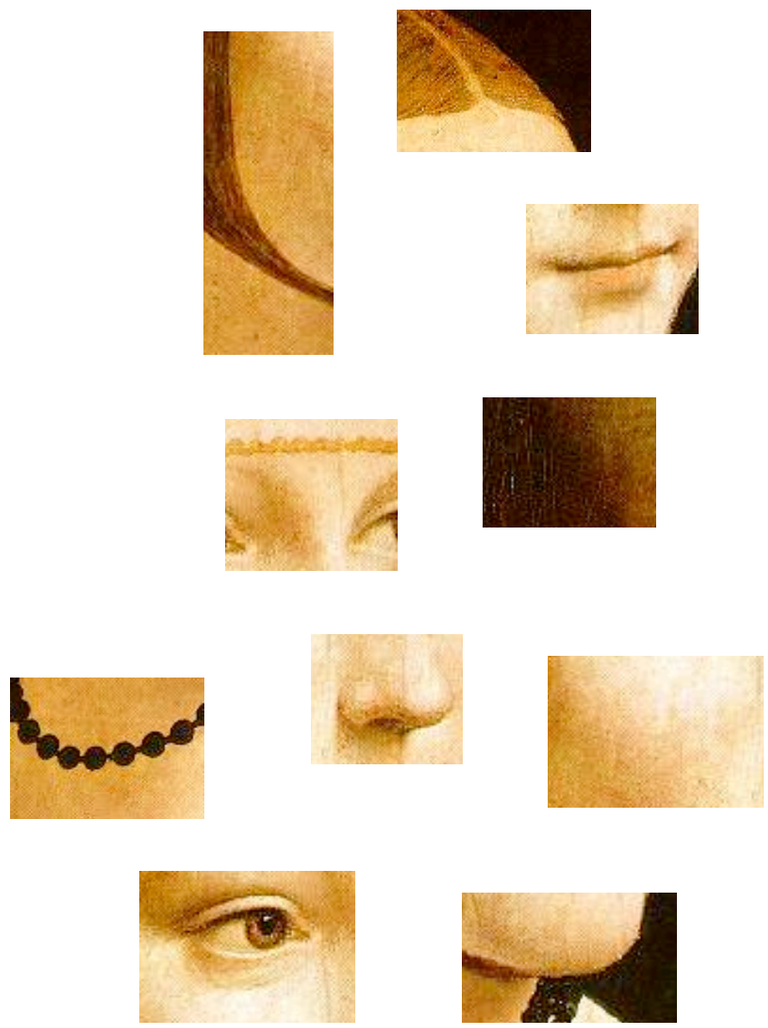
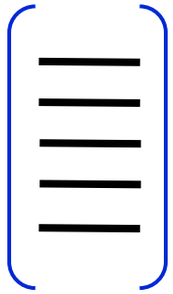
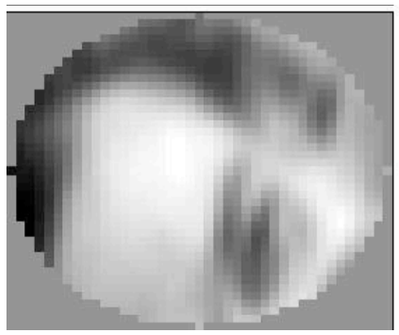


BoW-1. Feature detection and representation

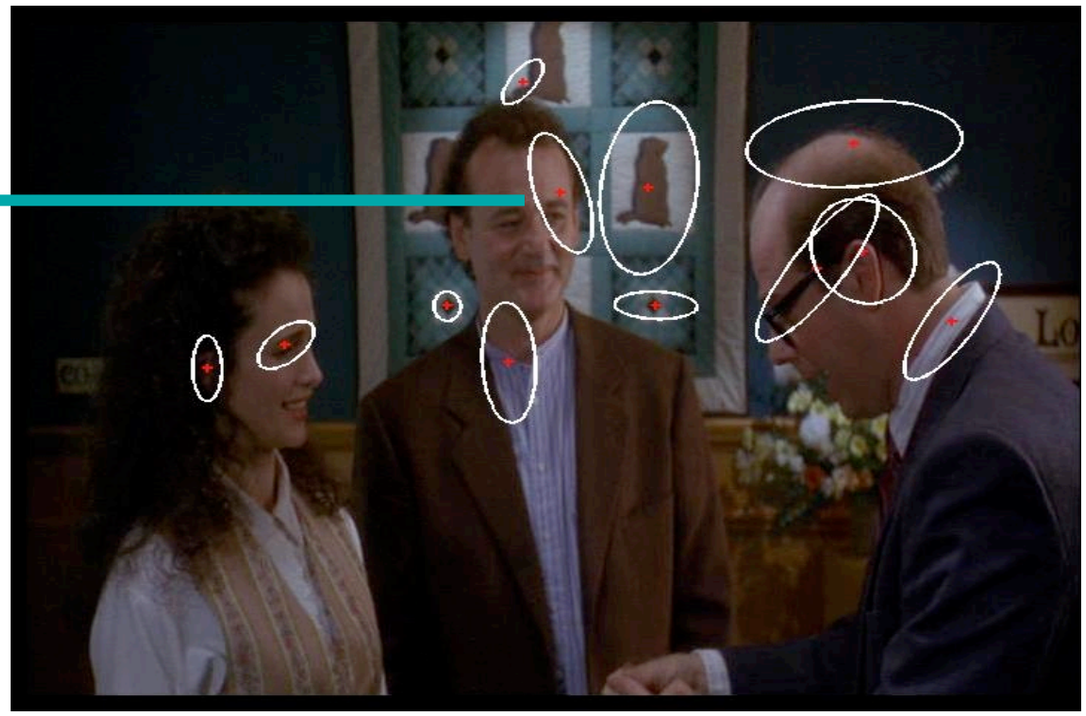


Compute descriptor

e.g. SIFT, shape context, etc.



Normalize patch



Detect patches

Local interest operator
(e.g. Harris-Laplace)
or regular grid

BoW-1. Feature detection and representation

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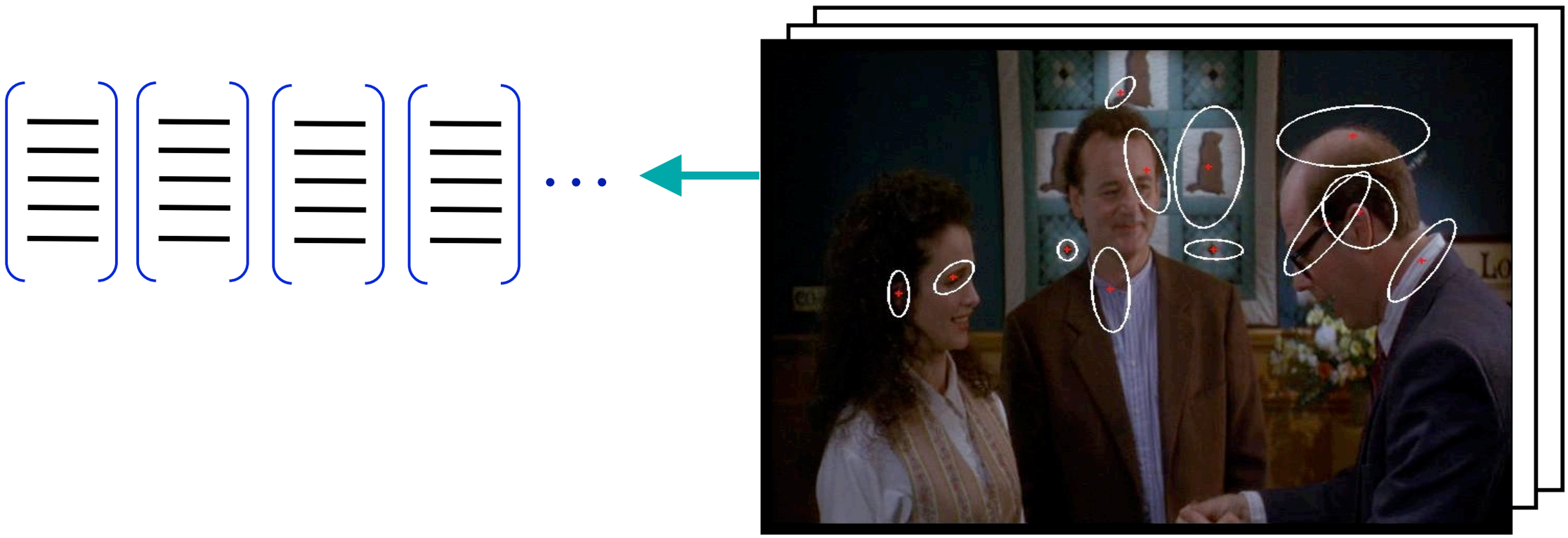

BoW-2. Codeword dictionary formation

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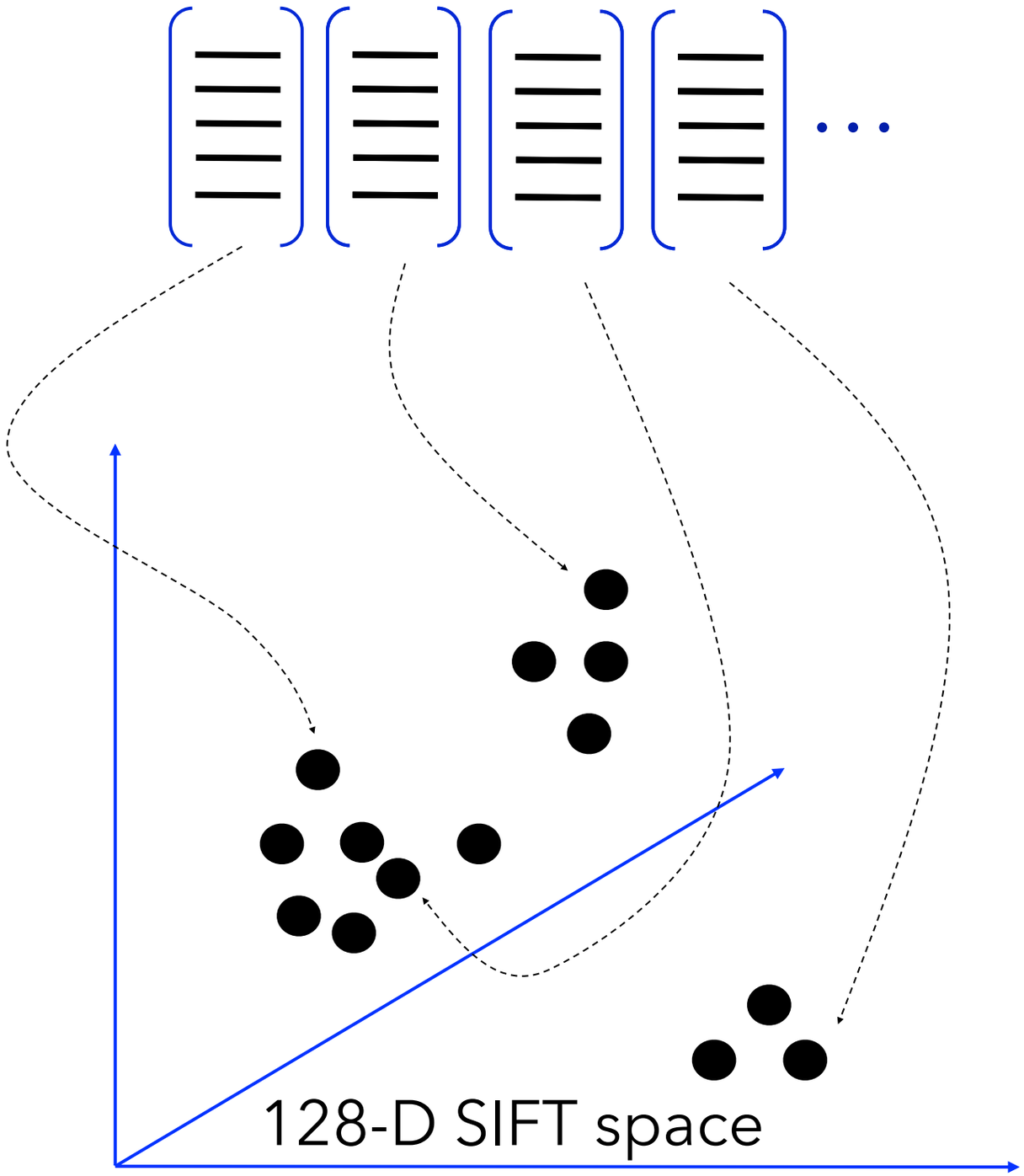

BoW-2. Codeword dictionary formation

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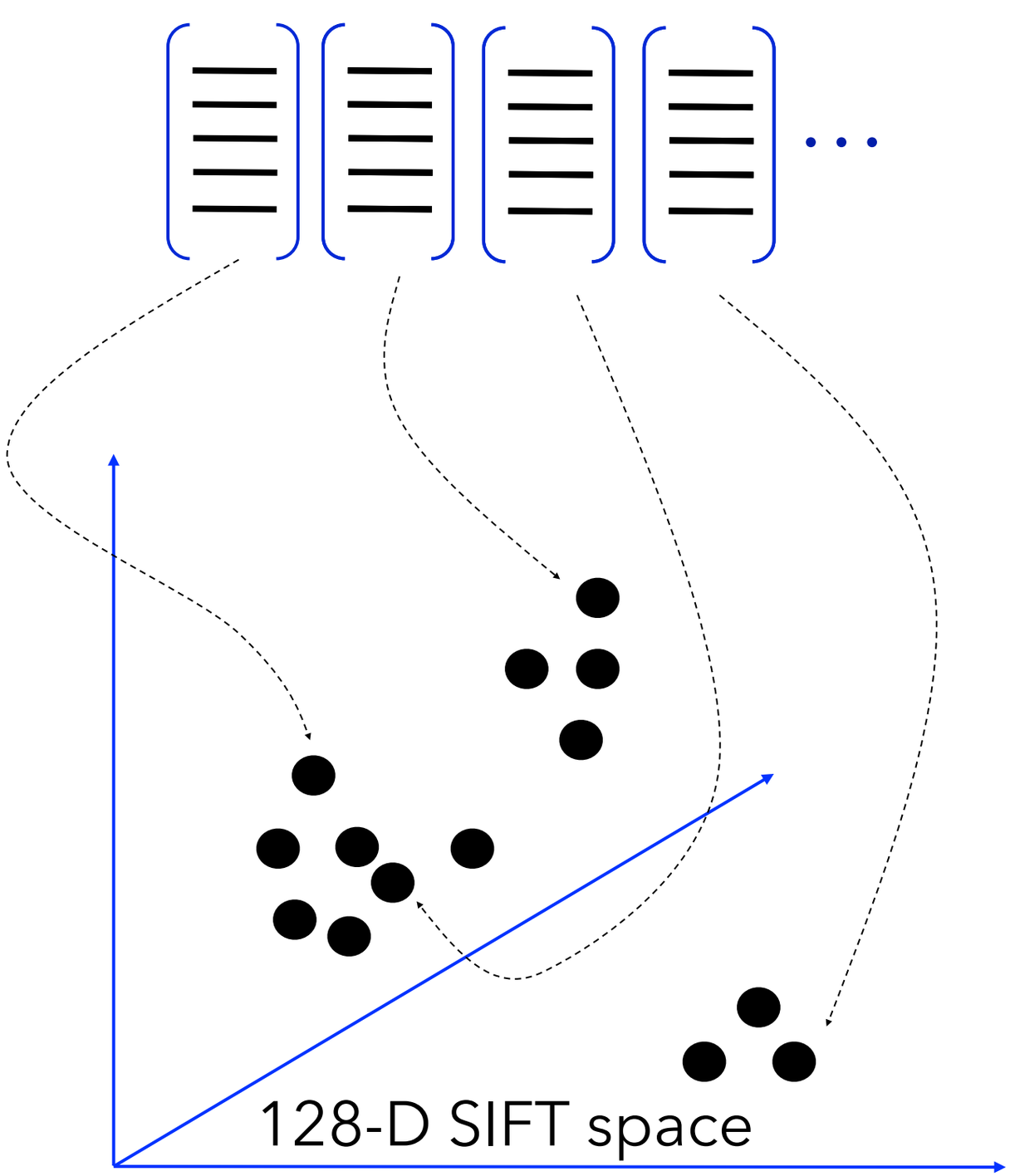
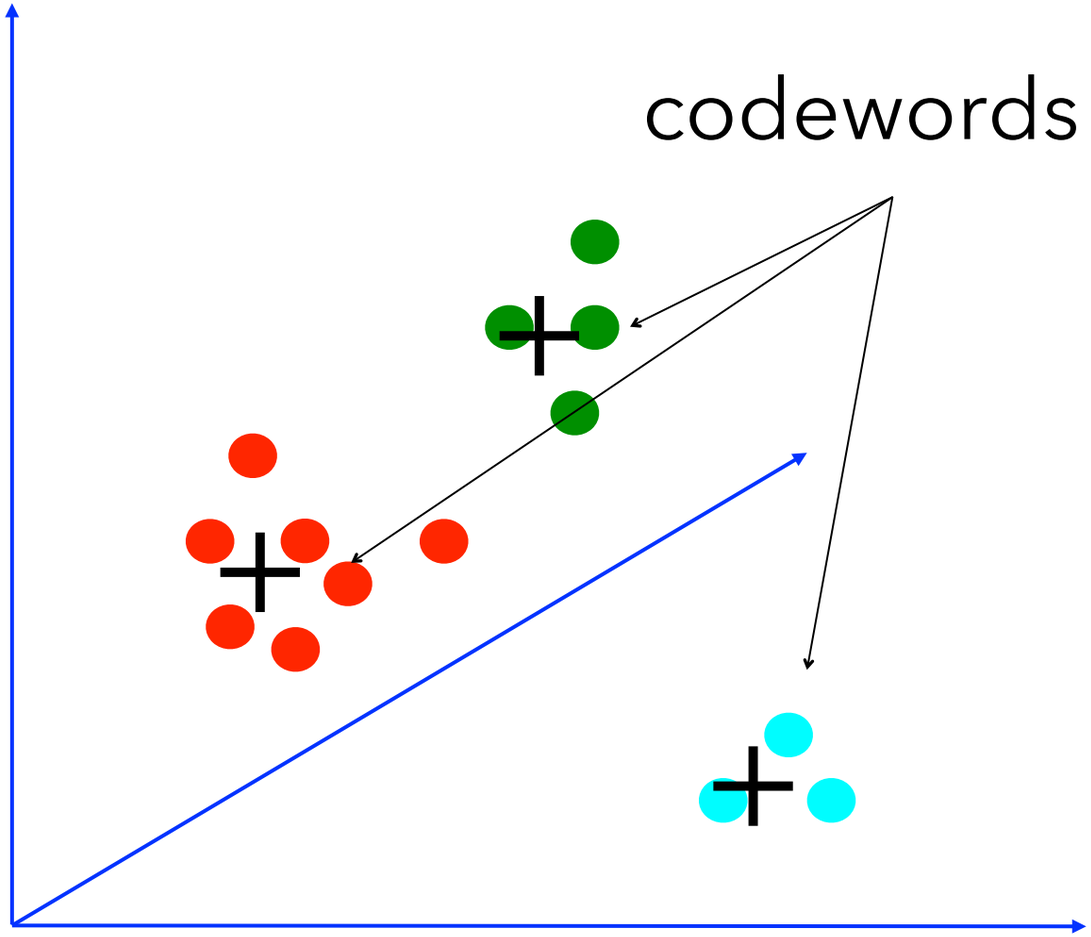
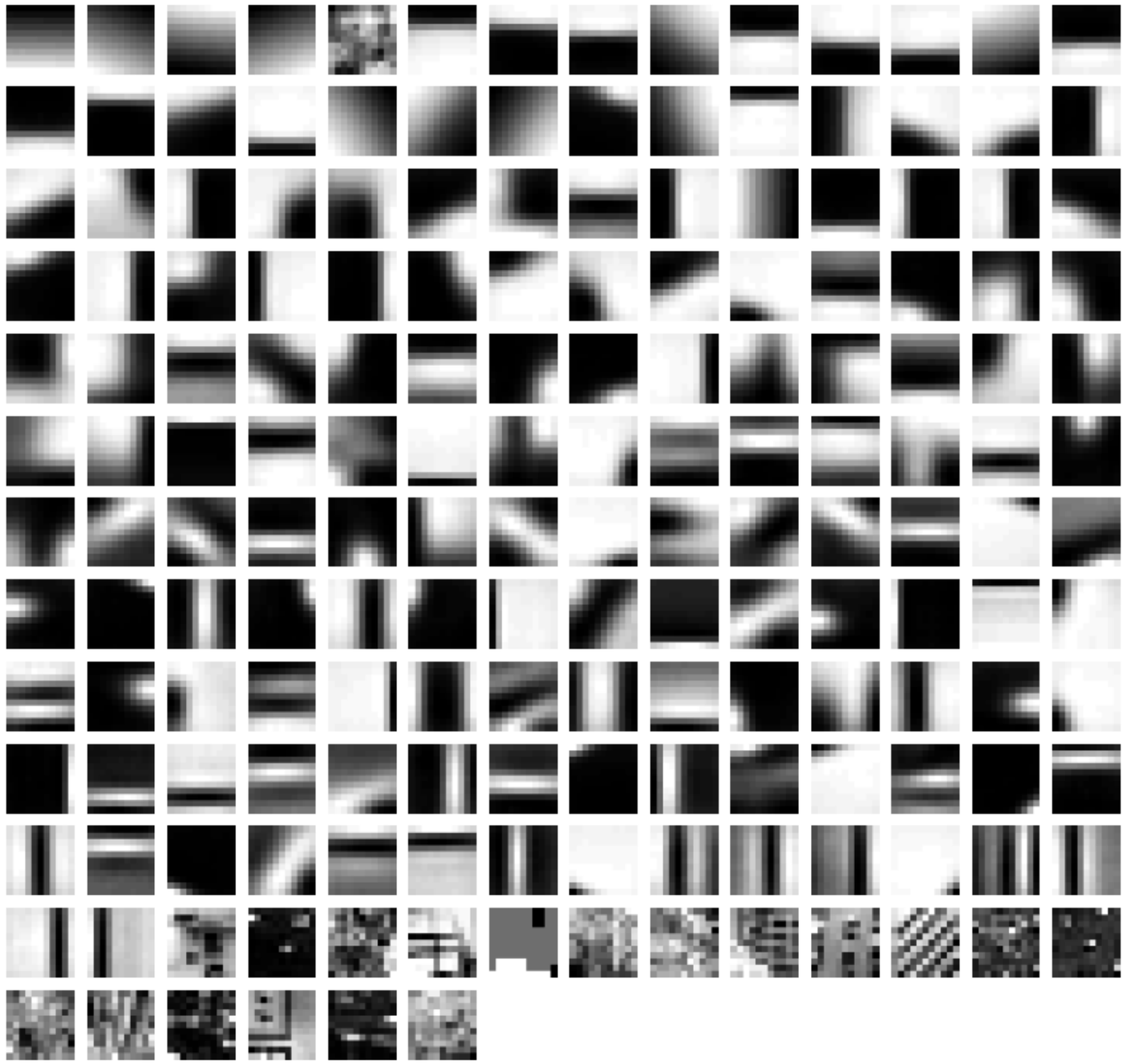




Vector quantization

e.g. k-means

BoW-2. Codeword dictionary formation



[Fei-Fei]

BoW-3. Image representation

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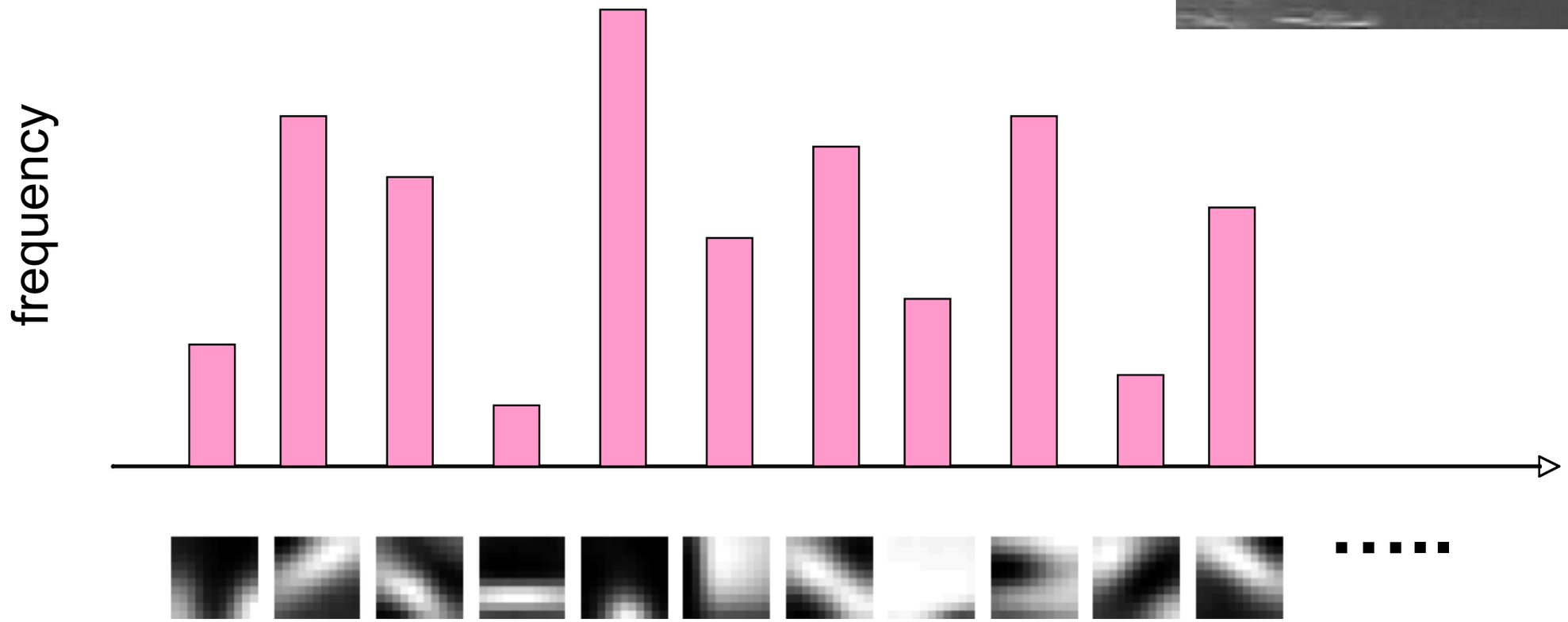
- Histogram of features assigned to each cluster



Next: Actual Recognition

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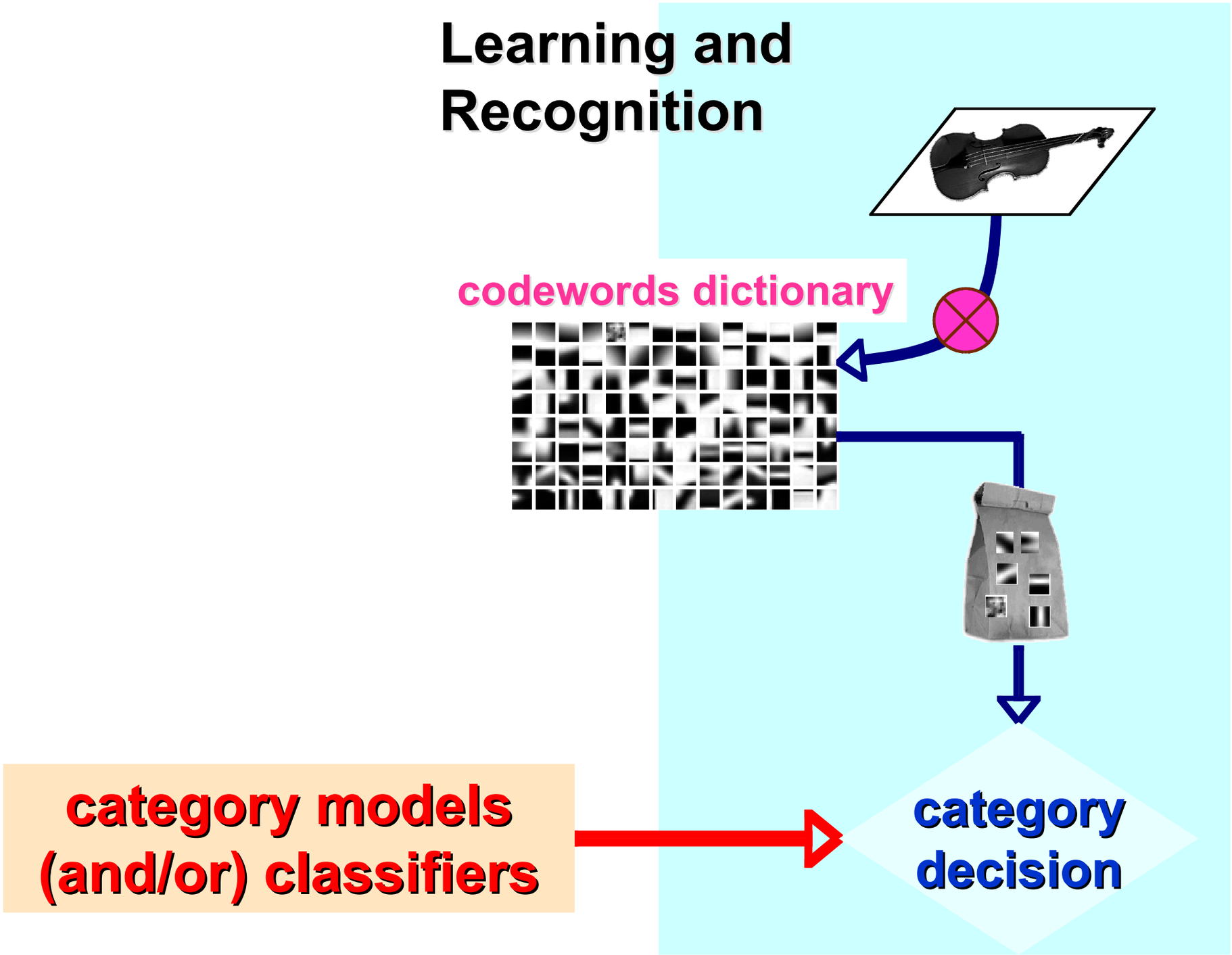

Learning and Recognition

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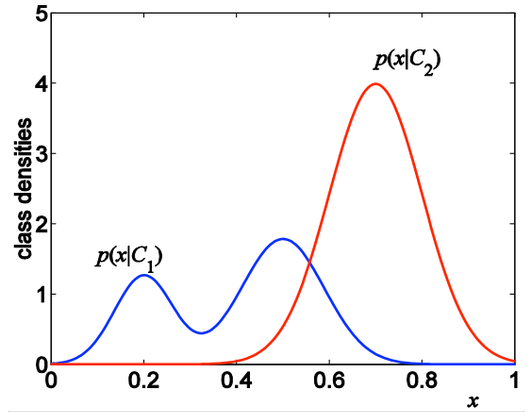
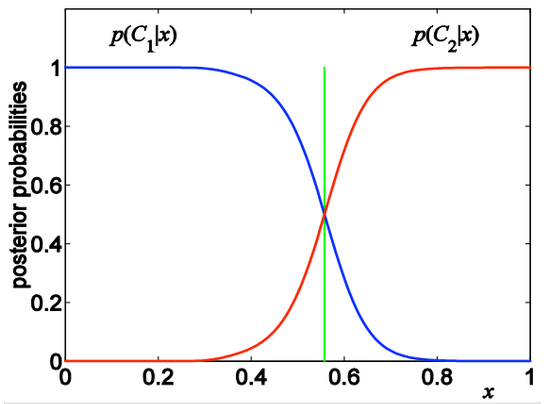
- **Generative approach:**
  - graphical models



- Discriminative approach:
  - Support Vector Machine (SVM)



category models (and/or) classifiers

Bayesian Decision Theory

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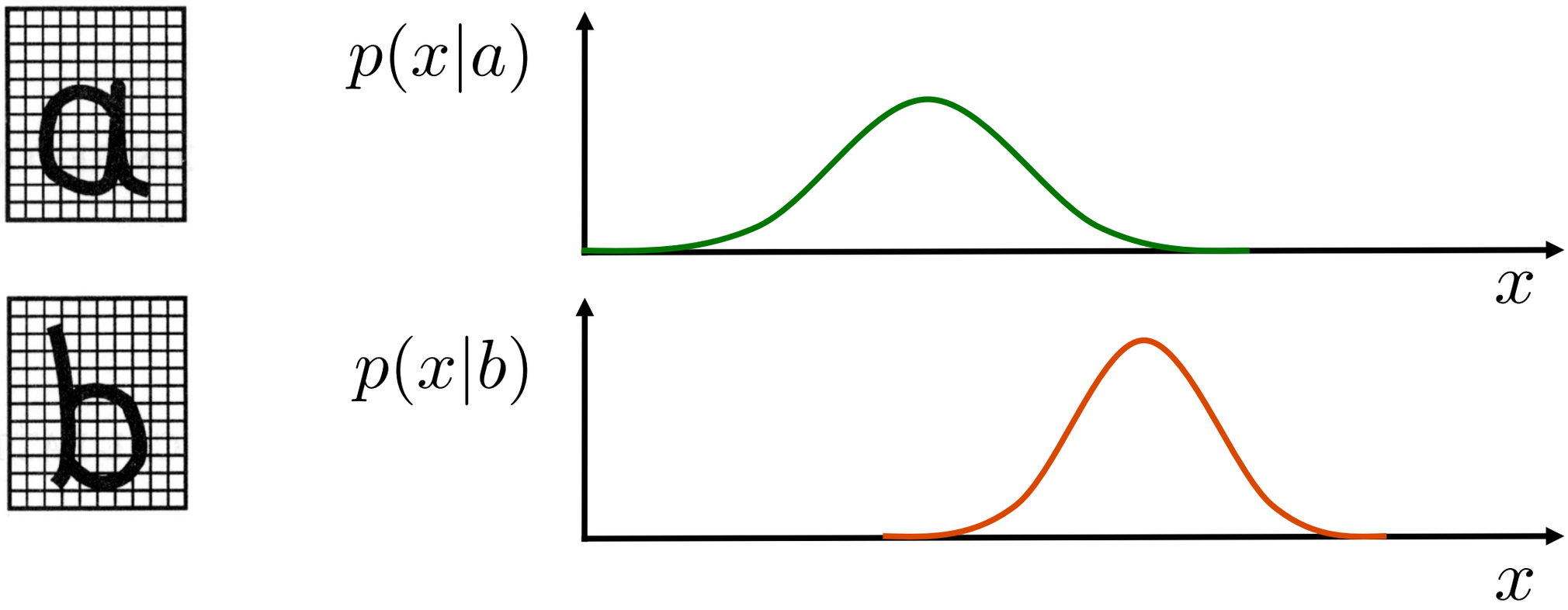
- 1st concept: Class conditional probabilities

$$p(x|C_k)$$

  - Probability of making an observation x knowing that it comes from some class C_k.
  - Here x is a feature (vector).
  - x measures / describes properties of the data.



Bayesian Decision Theory

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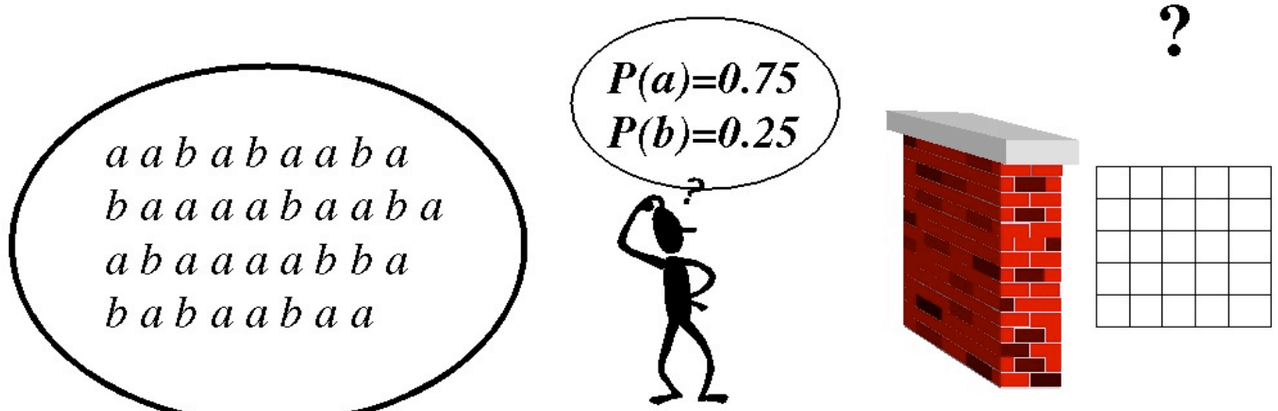
- 2nd concept: **Class priors** $p(C_k)$

(a priori probability of a data point belonging to a particular class)

- Example:



$$C_1 = a \qquad p(C_1) = 0.75$$

$$C_2 = b \qquad p(C_2) = 0.25$$

- Generally: $\sum_k p(C_k) = 1$

Bayesian Decision Theory

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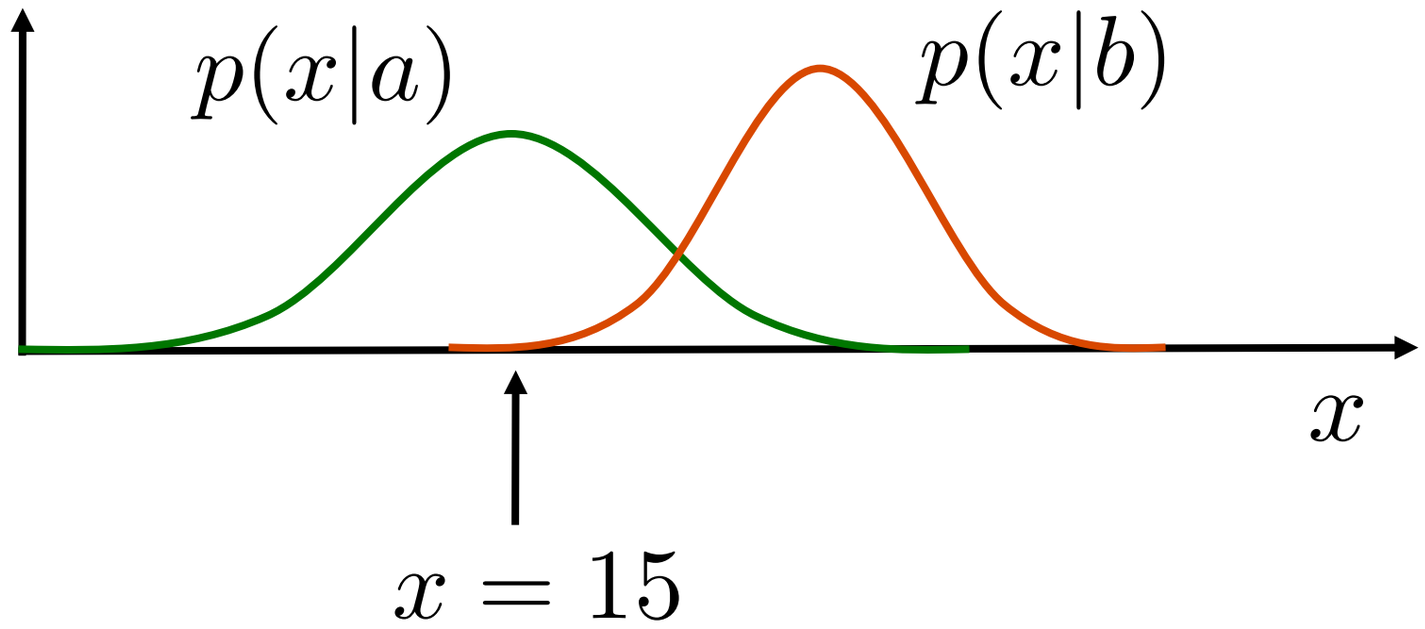
- Example:



Question:

- How do we decide which class the data point belongs to?
- Here, we should decide for class a .

Bayesian Decision Theory

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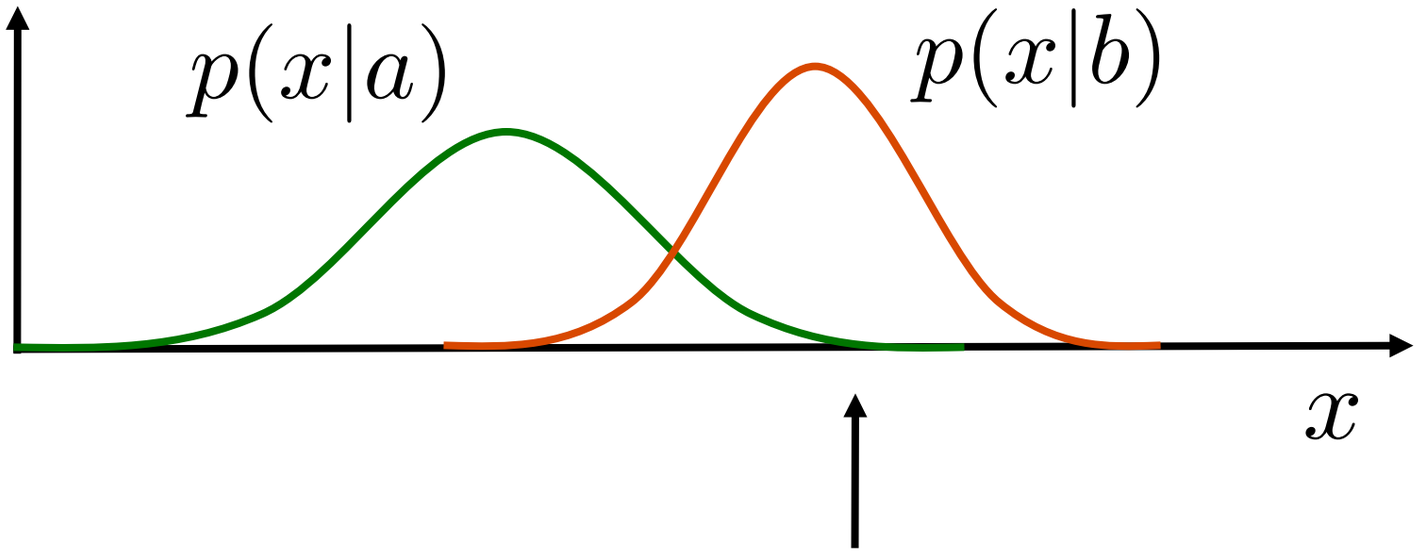
- Example:



- Question:
  - How do we decide which class the data point belongs to?
  - Since $p(x|a)$ is a lot smaller than $p(x|b)$ we should now decide for class b.

Bayesian Decision Theory

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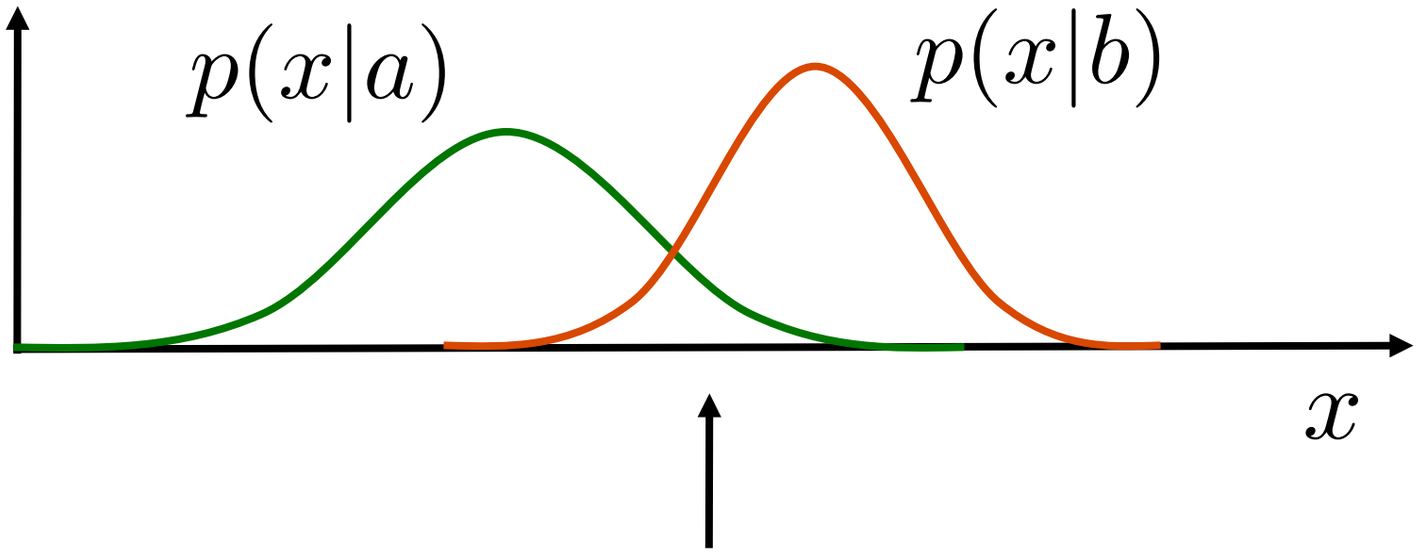
- Example:



- Question:
  - How do we decide which class the data point belongs to?
  - Remember that $p(a) = 0.75$ and $p(b) = 0.25$
  - This means we should decide class a.

Bayesian Decision Theory

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- Formalize this using Bayes' theorem:
  - We want to find the a-posteriori probability (posterior) of the class C_k given the observation (feature) x

class-conditional probability (likelihood)

class posterior

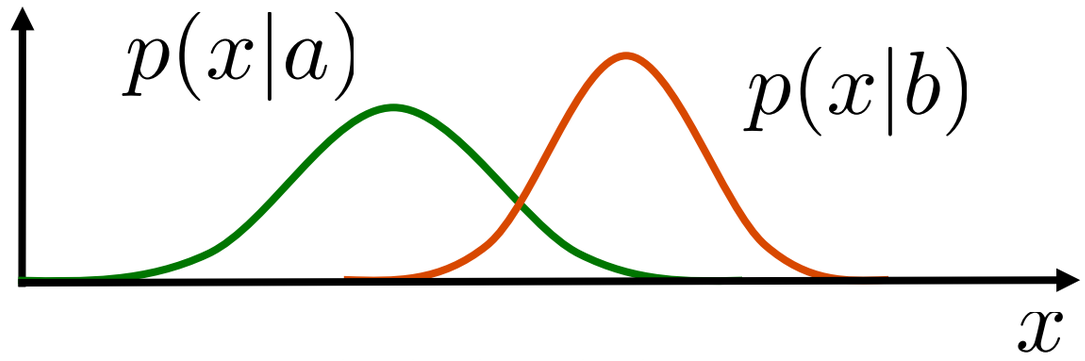
class prior

$$p(C_k|x) = \frac{p(x|C_k)p(C_k)}{p(x)}$$

normalization term

$$p(C_k|x) = \frac{p(x|C_k)p(C_k)}{p(x)} = \frac{p(x|C_k)p(C_k)}{\sum_j p(x|C_j)p(C_j)}$$

Bayesian Decision Theory

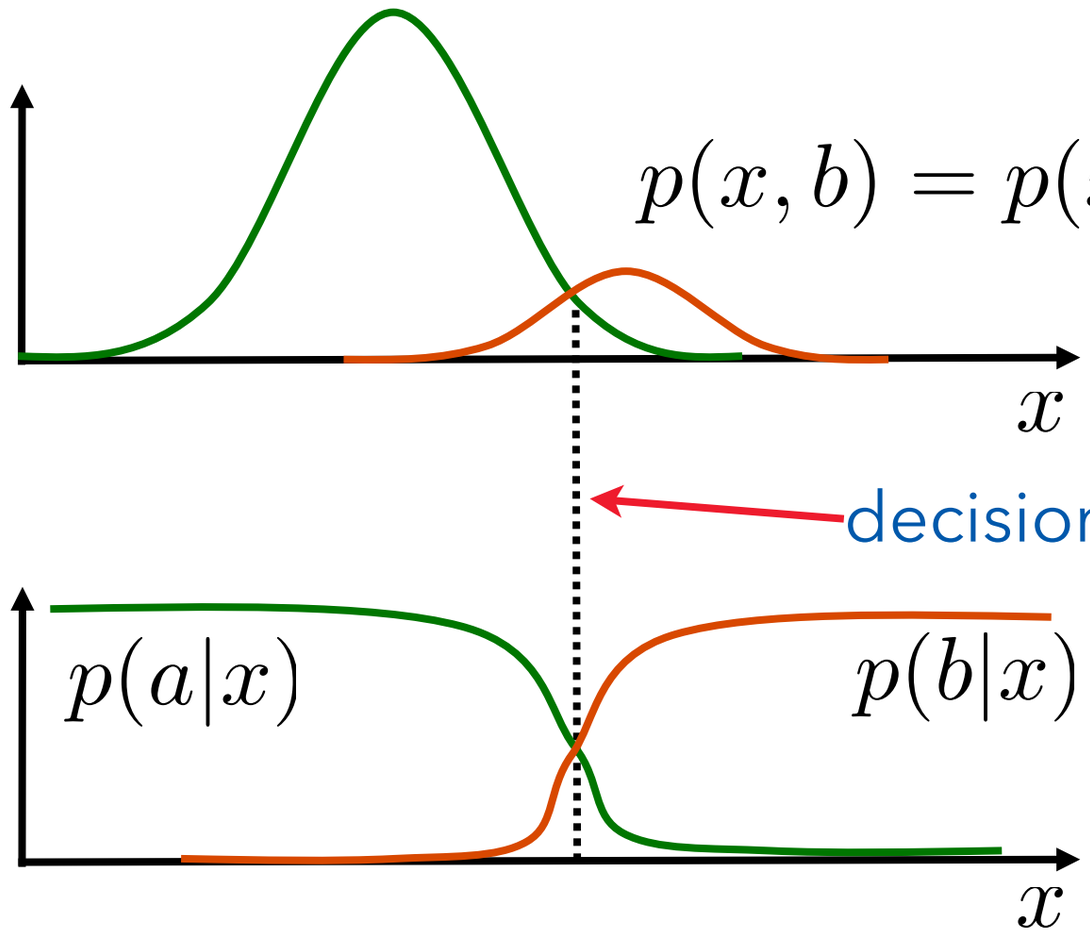


Likelihood

$$p(x,a) = p(x|a)p(a)$$

$$p(x,b) = p(x|b)p(b)$$

Likelihood × *Prior*



$$Posterior = \frac{Likelihood \times Prior}{Normalization\ factor}$$

Bayesian Decision Theory

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- Decision rule:
  - Decide C_1 if $p(C_1|x) > p(C_2|x)$

We do not need the normalization!

  - This is equivalent to

$$p(x|C_1)p(C_1) > p(x|C_2)p(C_2)$$

  - Which is equivalent to

$$\frac{p(x|C_1)}{p(x|C_2)} > \frac{p(C_2)}{p(C_1)}$$

- Bayes optimal classifier:
  - A classifier obeying this rule is called a Bayes optimal classifier.

Naïve Bayes Classifier

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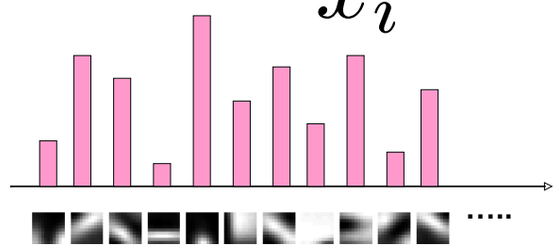
- Classify image using histograms of occurrences on visual words:

$$\mathbf{x} = \quad x_i \qquad x_i \in [0, 1]$$



- simpler alternative: only take presence/absence of a word into account: $x_i \in \{0, 1\}$
- Naïve Bayes classifier assumes that visual words are conditionally independent given object class

$$P(\mathbf{x}|c) = \prod_{i=1}^{m} P(x_i|c)$$

[T. Hofmann]

Naive Bayes Classifier

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- Multinomial model for each object class:

$$P(\mathbf{x}|c) = \prod_{i=1}^{m} P(x_i|c)$$

- Class priors: $P(c)$, with $\sum_c P(c) = 1$

- Posterior probabilities:

$$P(c|\mathbf{x}) = \frac{P(c)\prod_{t=1}^{n} P(x_t|c)}{\sum_{c'} P(c')\prod_{t=1}^{n} P(x_t|c')}$$

Naive Bayes Classifier: Decision

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- Bayes optimal decision:

$$c^* = \text{argmax}_c \; P(c|\mathbf{x})$$

$$= \text{argmax}_c \left[\log P(c) + \sum_{t=1}^{n} \log P(x_t|c) \right]$$

Naive Bayes Classifier: Estimation

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- Count empirical frequencies

$$\hat{P}(c) = \frac{1}{n}|\{i : y_i = c\}|$$

$$\hat{P}(w|c) = \frac{\sum_{i:y_i=c} |\{t : x_{it} = w\}|}{\sum_i n_i}$$

Training Set:

$$\{(\mathbf{x}_i, y_i) : i = 1, \ldots, n\}$$

$$\mathbf{x}_i \in \{w_1, \ldots, w_m\}^{n_i}$$

- Smoothing (e.g. Laplace)

$$\hat{P}(w|c) = \frac{1 + \sum_{i:y_i=c} |\{t : x_{it} = w\}|}{m + \sum_i n_i}$$

Image Classification with Naive Bayes

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- Image dataset: 7 object categories, arbitrary views, partial occlusions

[Csurka]

Example of feature extraction

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All features detected in the image

Features corresponding to two different visual words

[Csurka]

Recognition results:

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Table 1. Confusion matrix and the mean rank for the best vocabulary (*k=1000*).

True classes →	*faces*	*buildings*	*trees*	*cars*	*phones*	*bikes*	*books*
faces	**76**	4	2	3	4	4	13
buildings	2	**44**	5	0	5	1	3
trees	3	2	**80**	0	0	5	0
cars	4	1	0	**75**	3	1	4
phones	9	15	1	16	**70**	14	11
bikes	2	15	12	0	8	**73**	0
books	4	19	0	6	7	2	**69**
Mean ranks	1.49	1.88	1.33	1.33	1.63	1.57	1.57

Examples of correctly classified images:




Learning and Recognition

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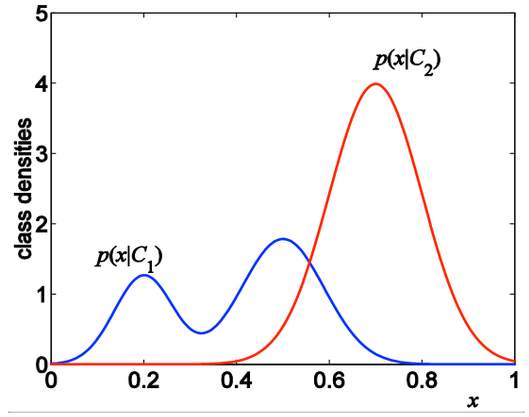
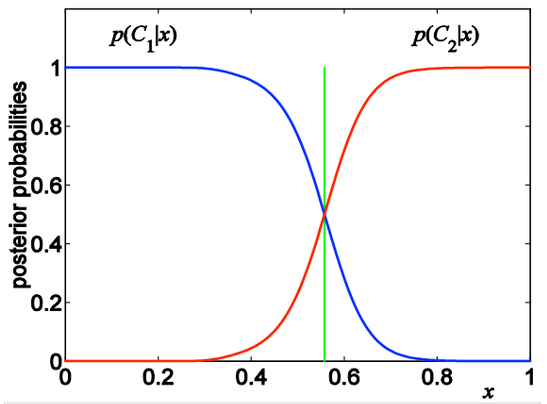
- Generative approach:
  - graphical models



- **Discriminative approach:**
  - Support Vector Machine (SVM)



category models (and/or) classifiers

Discriminative vs. generative

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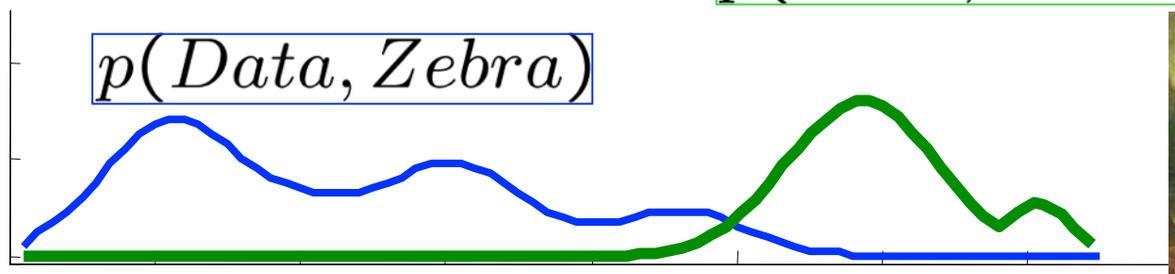
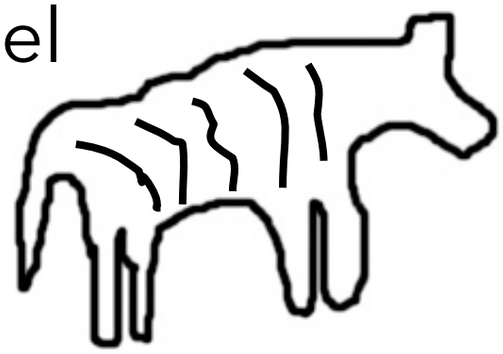
Generative model

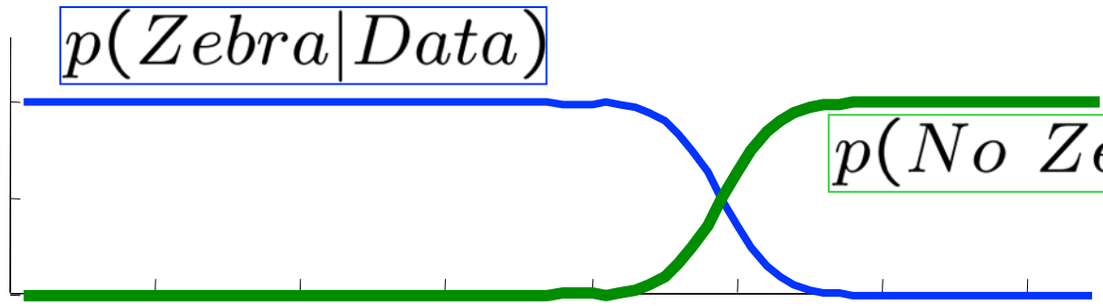
(The artist)

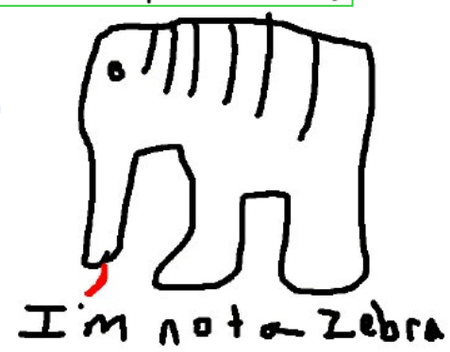


Discriminative model

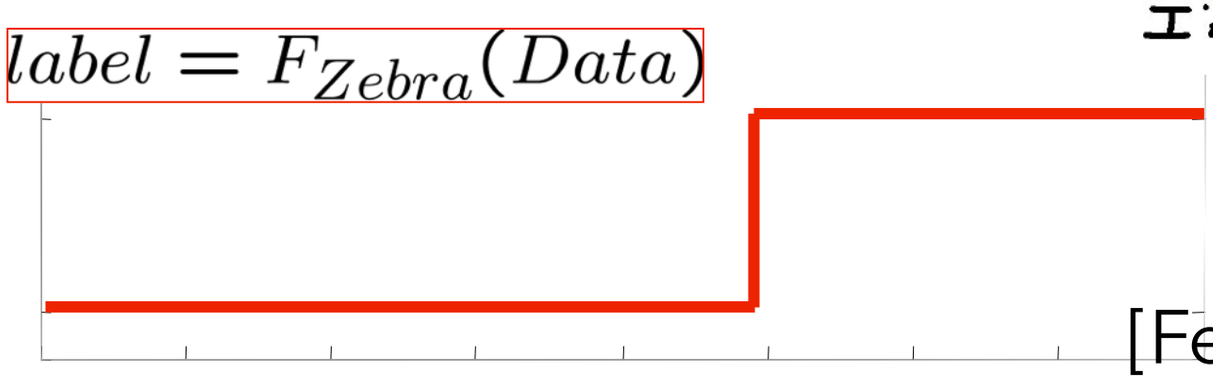
(The lousy painter)







Discriminant function



[Fergus]

Discriminant Functions

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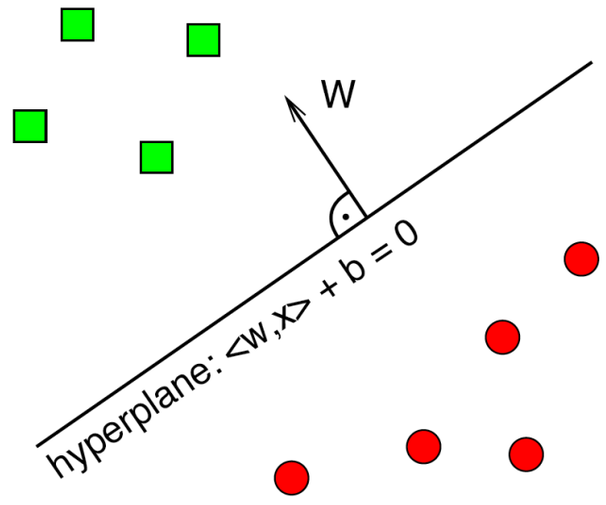
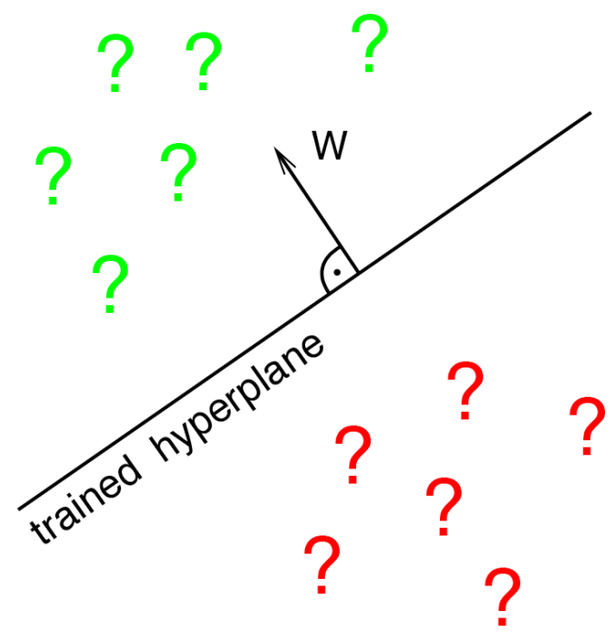
- Linear discriminant function:
  - Linear hyperplane:
    $$y(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\mathbf{x} + b$$
  - trained on samples of both classes
    C_1 (■) and C_2 (●)



- Classification:
  - decide class C_1 (?) when $y(\mathbf{x}) > 0$
  - decide class C_2 (?) when $y(\mathbf{x}) < 0$



Which Hyperplane is Best? and Why?

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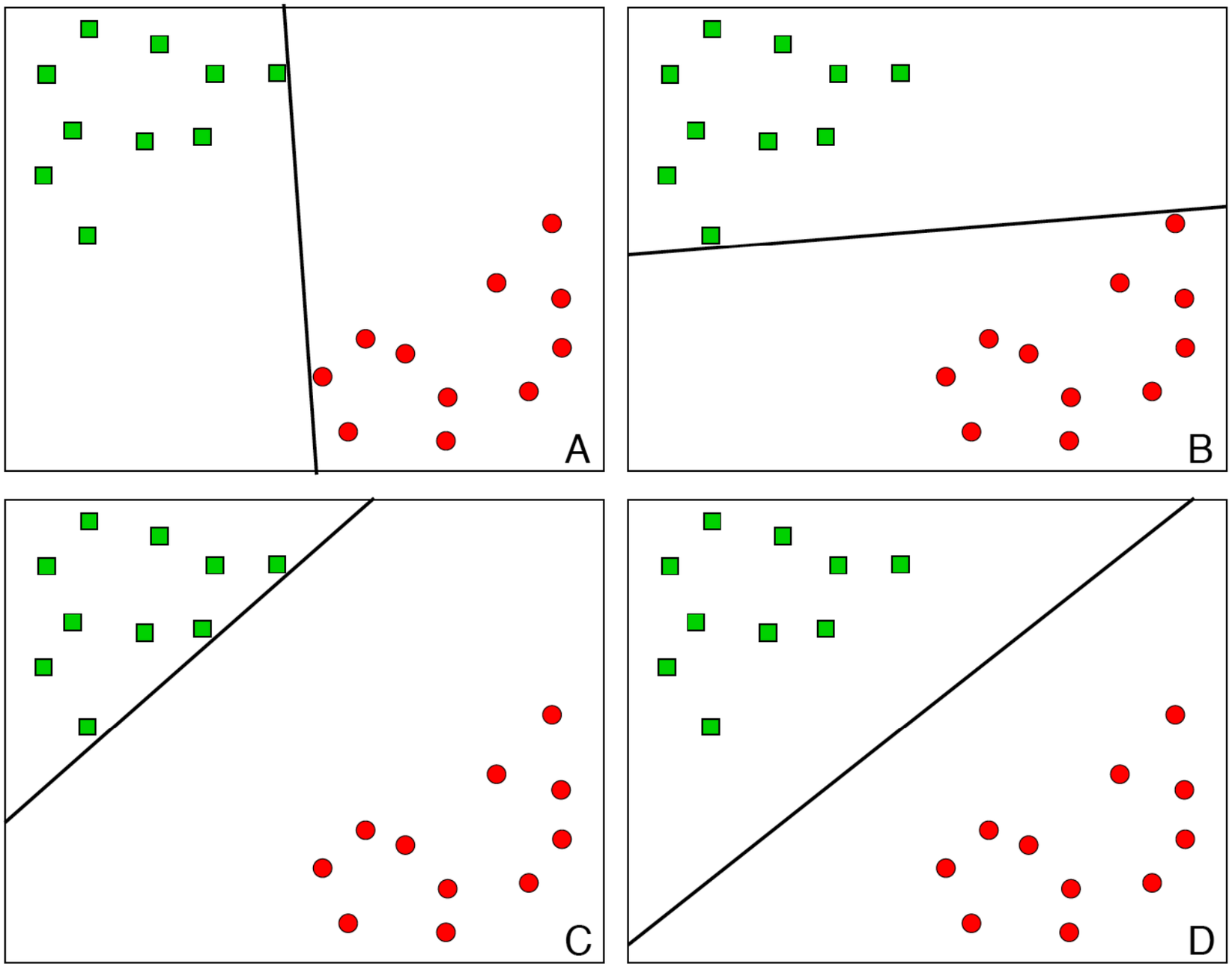

"Learning Power"

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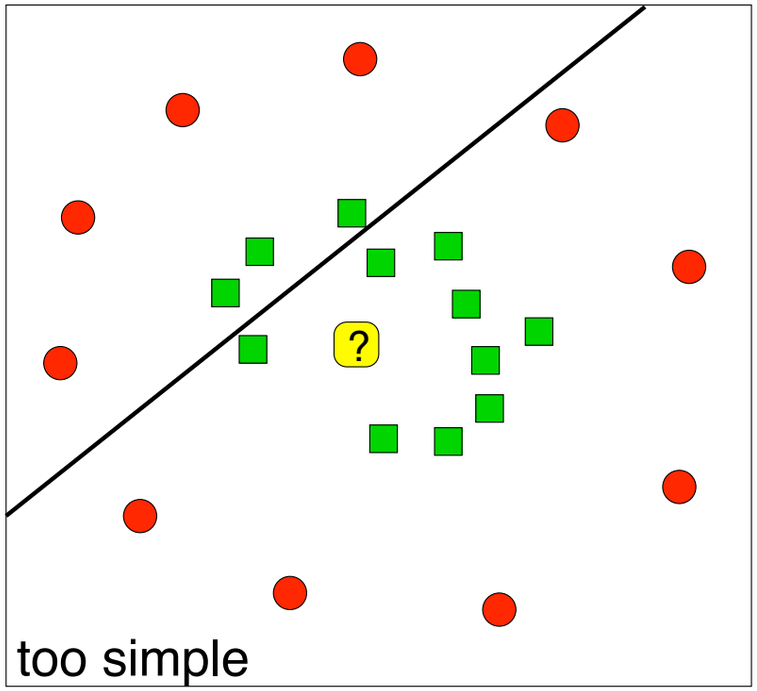
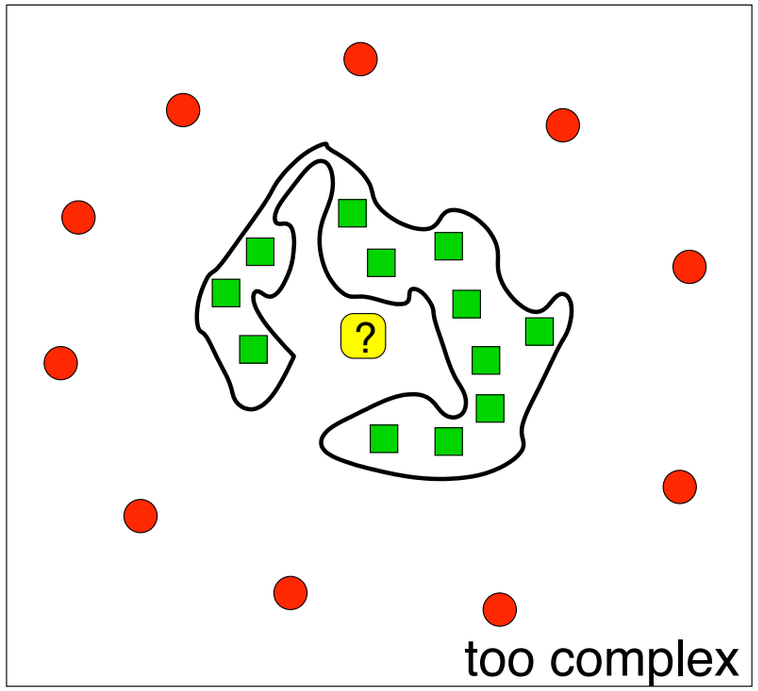
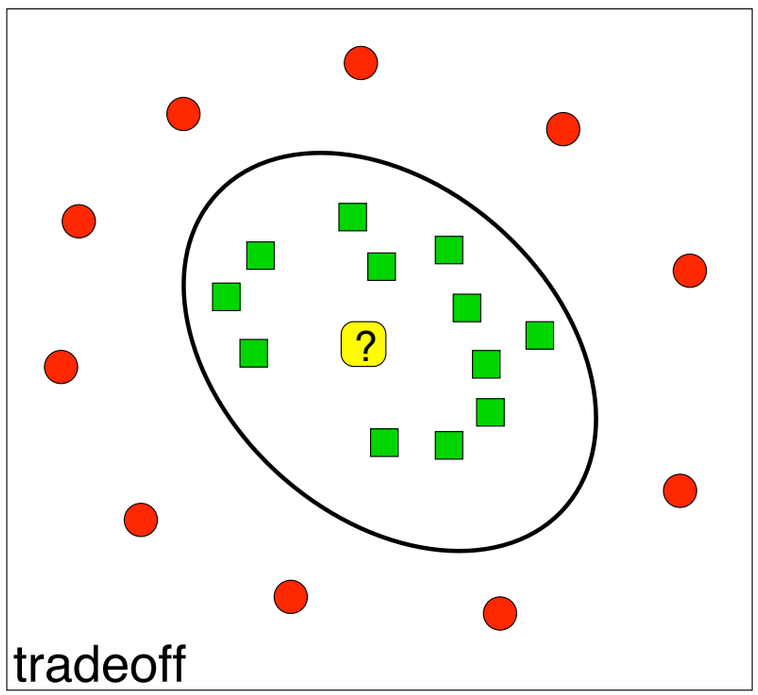




negative example
positive example
? new patient



[Florian Markowetz]

Support Vector Machines

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- Basic idea
  - Limit the “learning power”
  - Minimize an upper bound on the true risk (generalization ability)

- Key result (Vapnik):
  - Assume that the data points lie in a sphere of fixed radius
  - The linear classifier with maximal margin minimizes the learning power (VC dimension)
  - This in turn lowers a bound on the true risk
  - Leads to good generalization to unseen data

Support Vector Machines

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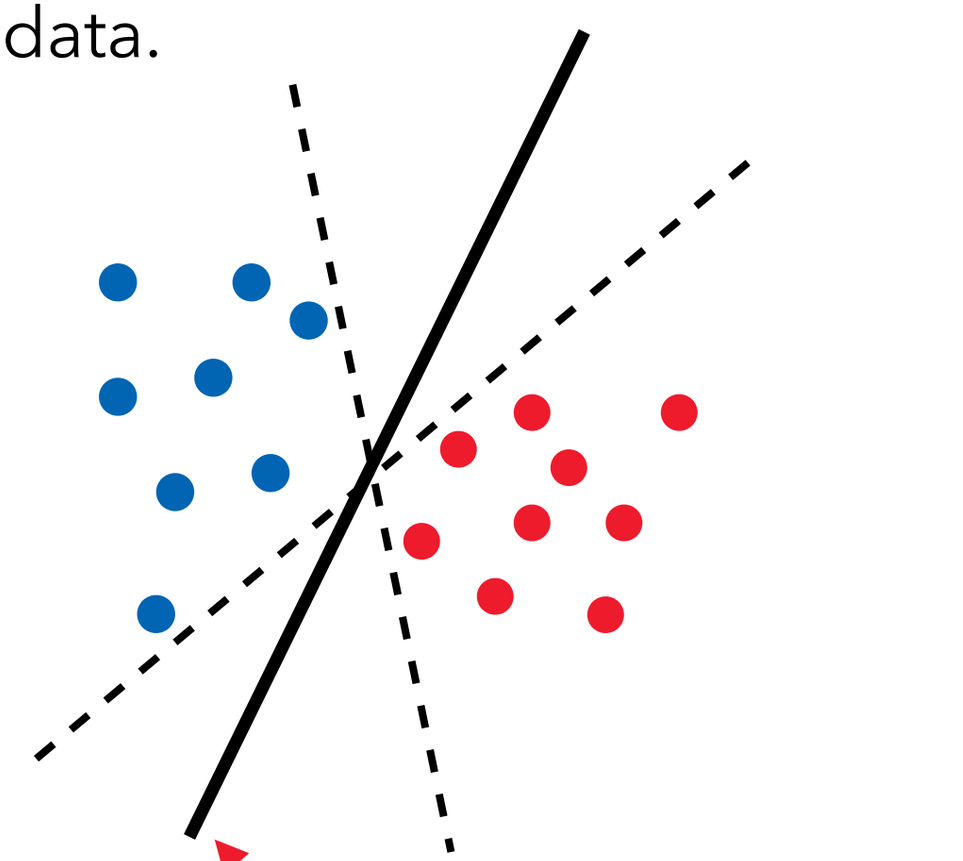
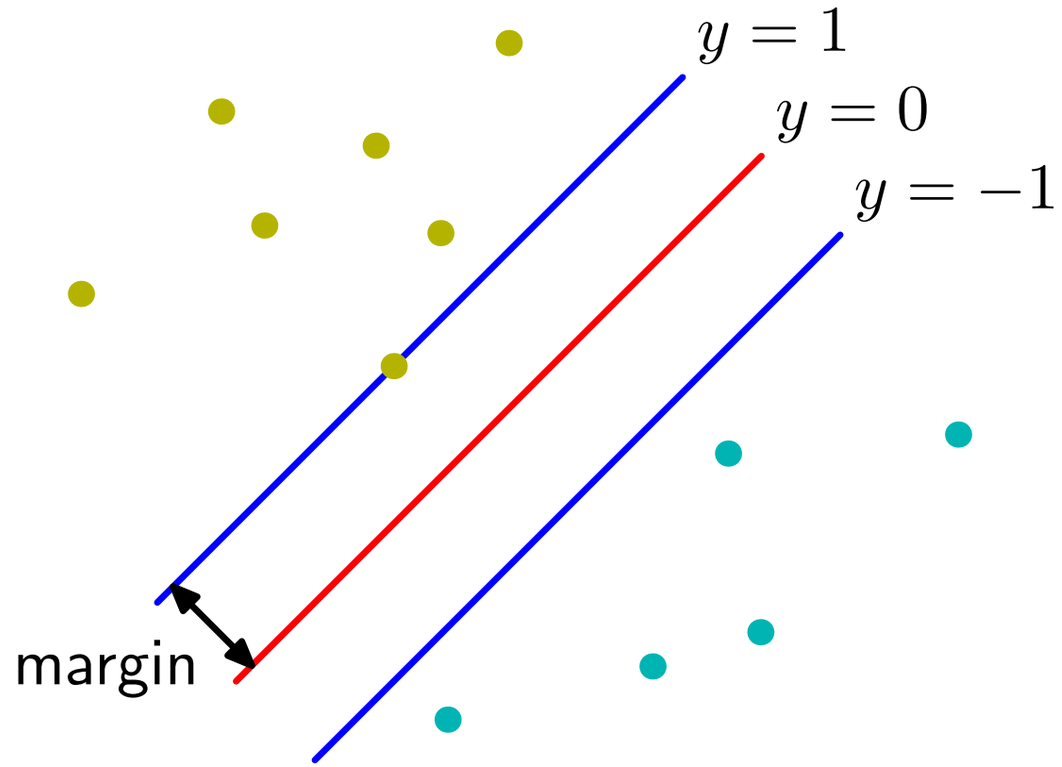
- Intuitively:
  - We should find the hyperplane with the maximum "distance" to the data.



generalizes best to unseen data



Maximize the margin
(distance to the closest data point)

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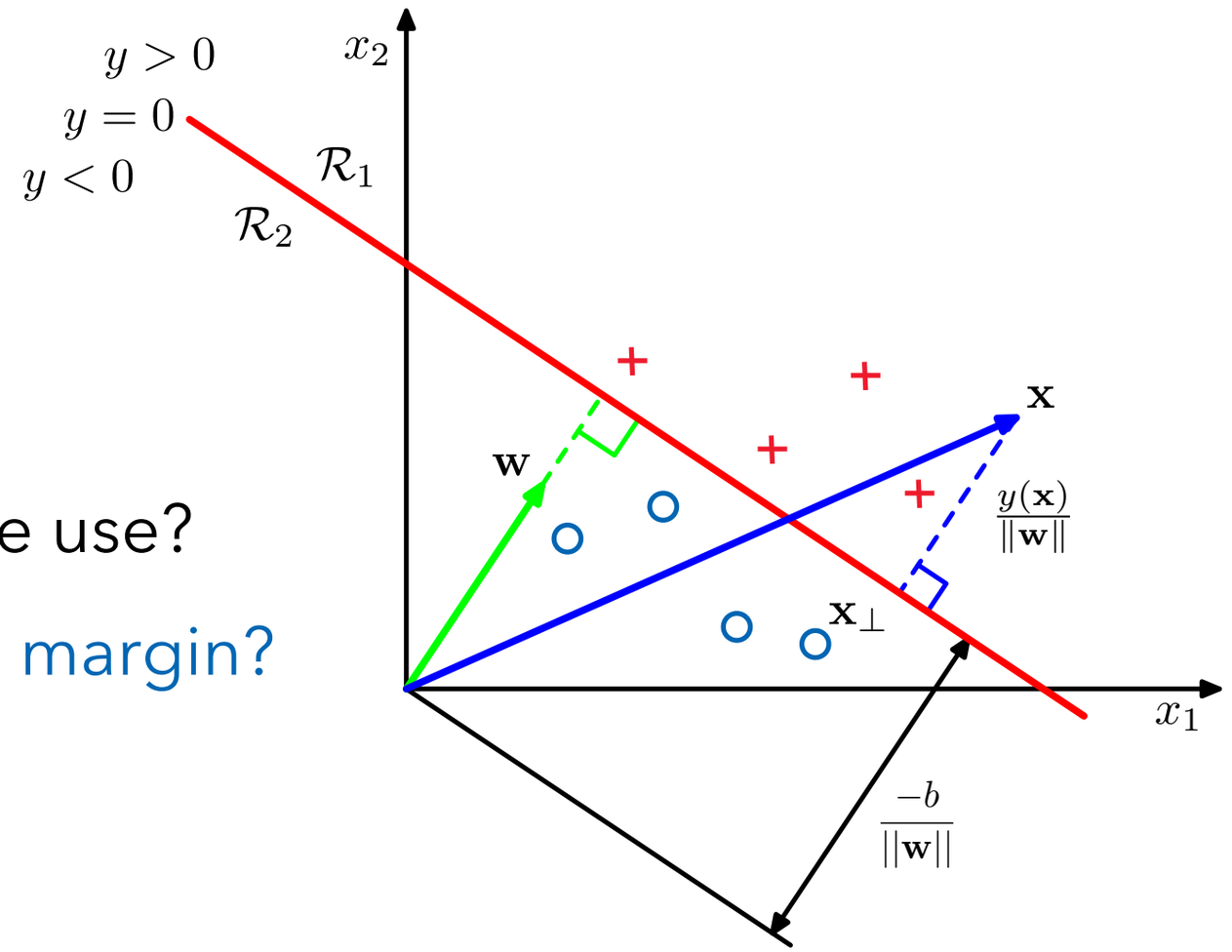
- For now: linearly separable data $x_i \in \mathbb{R}^d$
  - N training data points: $\{x_i, y_i\}_{i=1}^N$ $y_i \in \{-1, 1\}$
  - Hyperplane that separates the data:

$$y(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\mathbf{x} + b$$



  - Which hyperplane shall we use?
  - How can we maximize the margin?

Support Vector Machines

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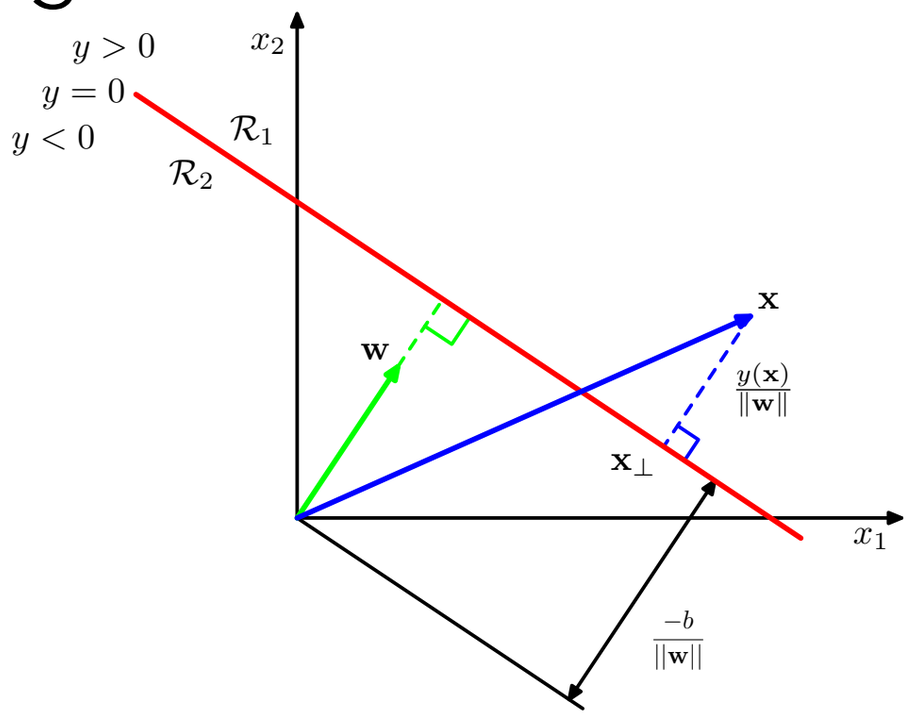
- Want to find a hyperplane so that the data is linearly separated:

$$y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) \geq 1, \qquad \forall i$$

  - Enforce $y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) = 1$ for at least one data point.

- Then we can easily express the margin:
  - The distance to the hyperplane is: $\dfrac{y(\mathbf{x}_i)}{||\mathbf{w}||} = \dfrac{\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b}{||\mathbf{w}||}$
  - This means that the margin is $\dfrac{1}{||\mathbf{w}||}$



Support Vector Machines

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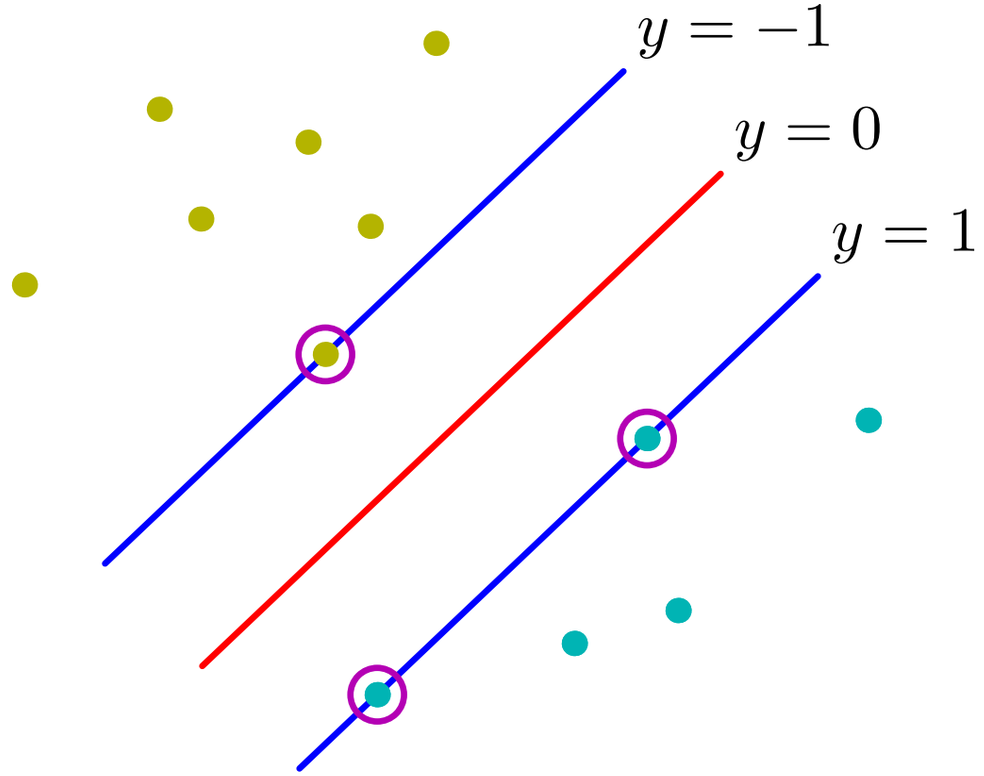
- Illustration:



- Support vectors:
  - All data points that lie on the margin (i.e $y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) = 1$)

Support Vector Machines

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- Maximizing the margin $1/||\mathbf{w}||$ is equivalent to minimizing $||\mathbf{w}||^2$.
- Formulate as constrained optimization problem:

$$\arg\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2$$
$$\text{s.t.} \quad y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) - 1 \geq 0$$

- Lagrangian formulation: $\alpha_i \geq 0$

$$L(\mathbf{w}, b, \alpha) = \frac{1}{2}||\mathbf{w}||^2 - \sum_{i=1}^{N} \alpha_i (y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) - 1)$$

  - Minimize Lagrangian

Support Vector Machines

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- Minimize $L(\mathbf{w}, b, \alpha) = \frac{1}{2}||\mathbf{w}||^2 - \sum_{i=1}^{N} \alpha_i (y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) - 1)$

- Set the gradient to zero:

$$\frac{\partial L(\mathbf{w}, b, \alpha)}{\partial b} = 0 \quad \Rightarrow \quad \sum_{i=1}^{N} \alpha_i y_i = 0$$

$$\frac{\partial L(\mathbf{w}, b, \alpha)}{\partial \mathbf{w}} = 0 \quad \Rightarrow \quad \mathbf{w} = \sum_{i=1}^{N} \alpha_i y_i \mathbf{x}_i$$

- The separating hyperplane is a linear combination of the input data.
- Solving this quadratic program is relatively easy.
  - Standard implementations exist.

Sparsity

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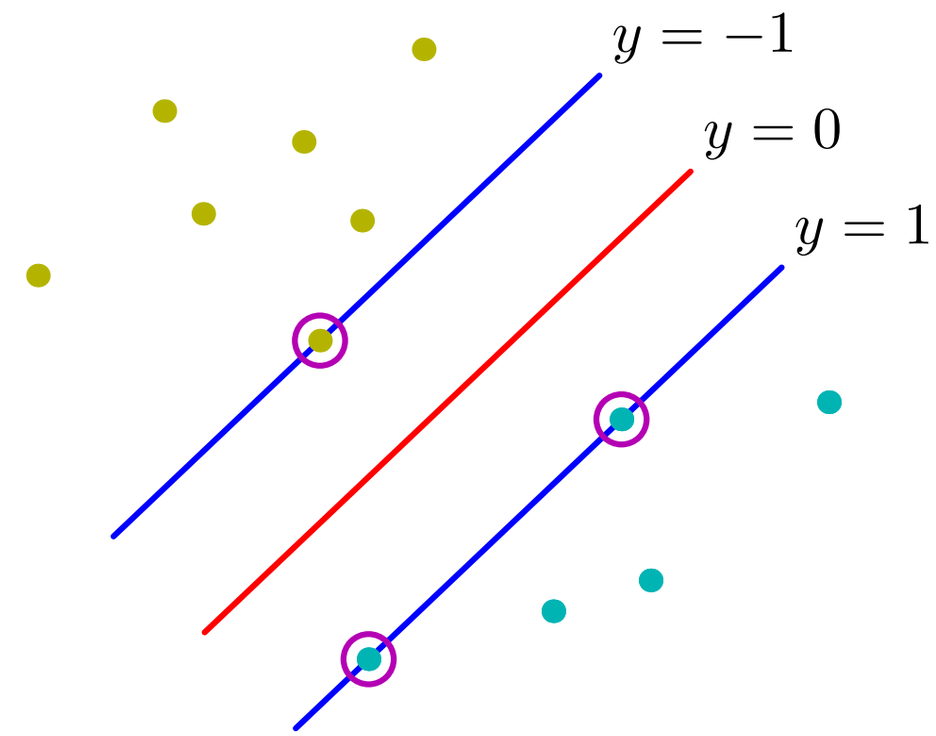
- Important property:
  - Almost all the α_i are zero.
  - There are only a few support vectors.
  - But the hyperplane was written as:

$$\mathbf{w} = \sum_{i=1}^{N} \alpha_i y_i \mathbf{x}_i$$



- SVMs are sparse learning machines:
  - The classifier only depends on a few data points.

Non-separable data

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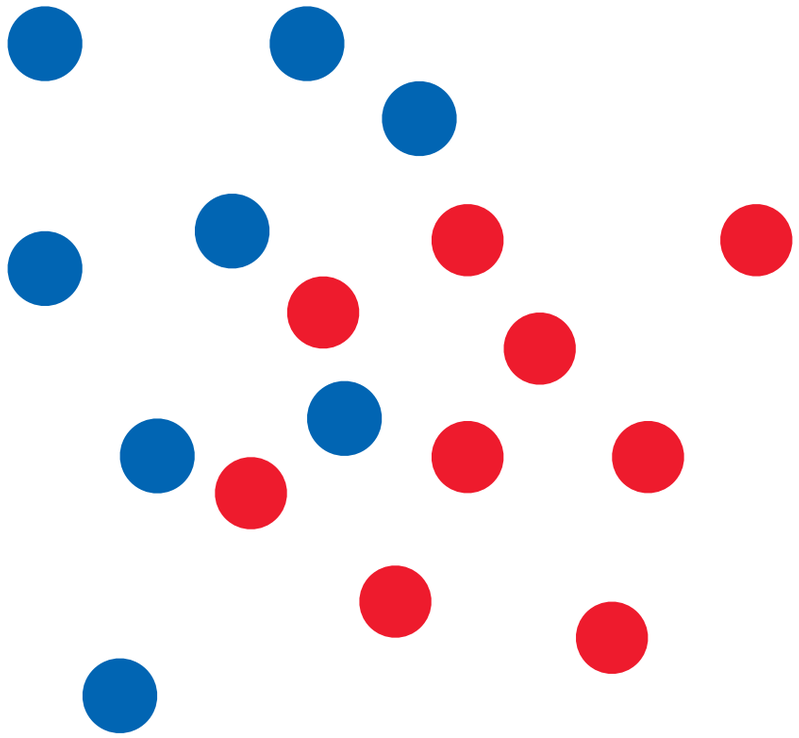
- What if our data is not linearly separable?



- Solution: Allow for data points to “violate the margin”.

Support Vector Machines

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- Instead of requiring that the data is perfectly linearly separable:

$$\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b \geq +1 \qquad \text{for} \quad y_i = +1$$

$$\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b \leq -1 \qquad \text{for} \quad y_i = -1$$

- we allow for small violations ξ_i from perfect separation:

$$\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b \geq +1 - \xi_i \qquad \text{for} \quad y_i = +1$$

$$\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b \leq -1 + \xi_i \qquad \text{for} \quad y_i = -1$$

$$\xi_i \geq 0 \qquad \forall i$$

Support Vector Machines

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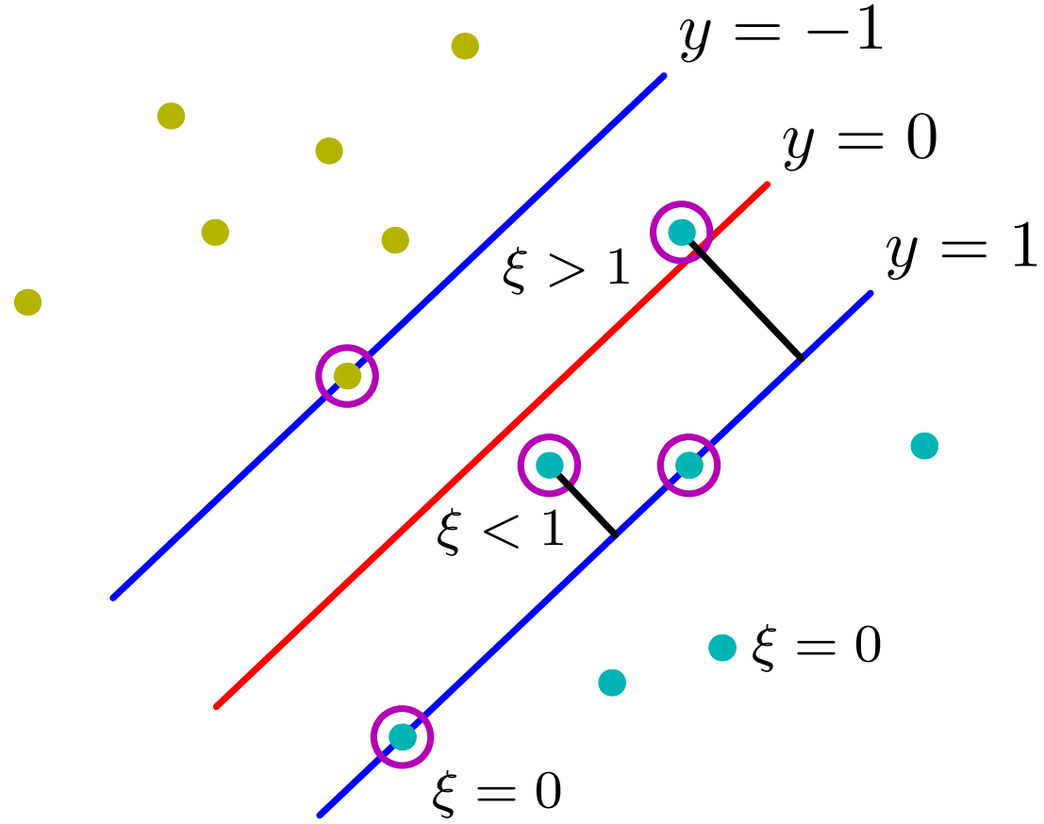
- More concisely, we require that:

$$y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) \geq 1 - \xi_i, \qquad \xi_i \geq 0 \qquad \forall i$$

  - The ξ_i are called "slack variables".

- Illustration:



Support Vector Machines

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- We have to penalize the deviations:

$$\arg\min_{\mathbf{w},b,\boldsymbol{\xi}} \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{N}\xi_i$$

$$\text{s.t.} \quad y_i(\mathbf{w}^{\mathrm{T}}\mathbf{x}_i + b) - 1 + \xi_i \geq 0$$

$$\xi_i \geq 0$$

- Maximize the margin while minimizing the penalty for all data points that are not outside the margin.
- The weight C allows us to specify a trade-off.
  - Typically determined through cross-validation.
- Even if the data is separable, it may be better to allow for an occasional penalty.

Nonlinear Separation of Data

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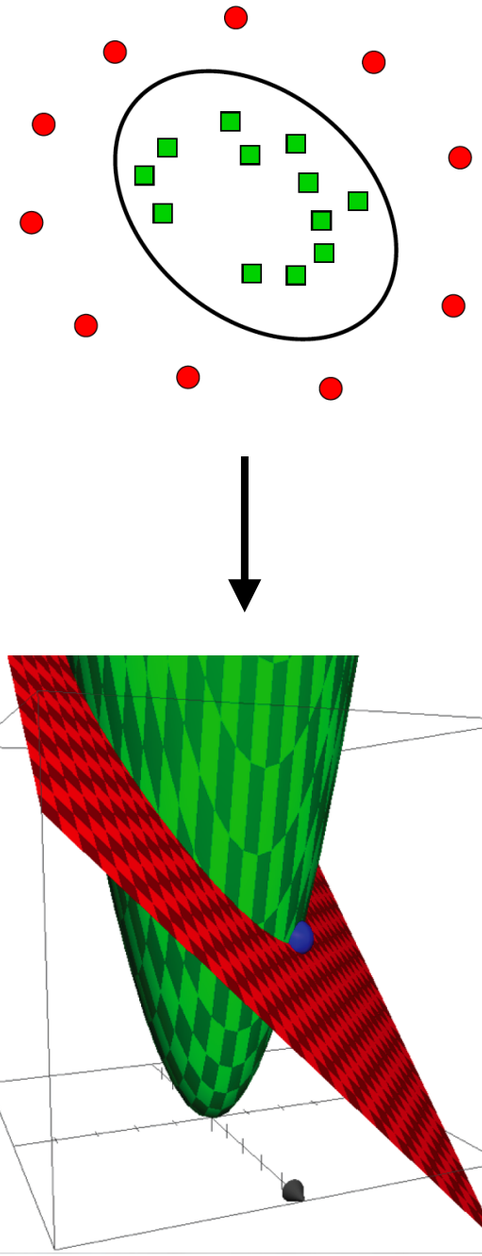
- Example:
  - Linear separation in 2D not possible
  - Linear separation in 3D however possible
  - Nonlinear transformation from 2D to 3D...



$$\begin{aligned} \Phi: \quad \mathbb{R}^2 &\rightarrow \mathbb{R}^3 \\ (g_1, g_2) &\mapsto (z_1, z_2, z_3) := (g_1^2, \sqrt{2}g_1g_2, g_2^2) \end{aligned}$$

Nonlinear SVMs

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- Nonlinear transformation ϕ of the data:

$$\mathbf{x} \in \mathbb{R}^d \qquad \phi : \mathbb{R}^d \rightarrow H$$

- Hyperplane in H (linear classifier in H).

$$y(\mathbf{x}) = \mathbf{w}^{\mathrm{T}} \phi(\mathbf{x}) + b$$

- Nonlinear classifier in $\mathbb{R}^d$.
- Remember:
  - The hyperplane can be written in terms of the input features

$$\mathbf{w} = \sum_{i=1}^{N_S} \alpha_i y_i \phi(\mathbf{x}_i)$$

Nonlinear SVMs with Kernels

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- Nonlinear discriminant function:

$$y(\mathbf{x}) = \sum_{i=1}^{N_S} \alpha_i y_i \phi(\mathbf{x}_i)^{\mathrm{T}} \phi(\mathbf{x}) + b$$

  - Still maximize the margin of the classifier!
    - Built-in generalization properties.
- Kernel "trick":
  - Replace the scalar product of non-linear features with a kernel:

$$K(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^{\mathrm{T}} \phi(\mathbf{x}_j)$$

  - Why?

Radial Basis Function Kernel

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- Gaussian radial basis function (RBF) kernel:

$$K(\mathbf{x}, \mathbf{y}) = \exp\left(-\frac{||\mathbf{x} - \mathbf{y}||^2}{2\sigma^2}\right)$$

  - Measures similarities.

- Interesting property: H is infinite dimensional.
  - Intuition: $$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots$$
  - Since we only use the kernel function, this is no problem.

Histogram Kernels

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- For bags of visual words there are even better kernels.
  - Specialized to histograms.

- Chi-Square kernel

$$K(x,y) = 1 - \sum_{i=1}^{d} \frac{(x_i - y_i)^2}{\frac{1}{2}(x_i + y_i)}$$

- Histogram intersection kernel

$$K(x,y) = \sum_{i=1}^{d} \min(x_i, y_i)$$

Bag-of-Words Model: Done!

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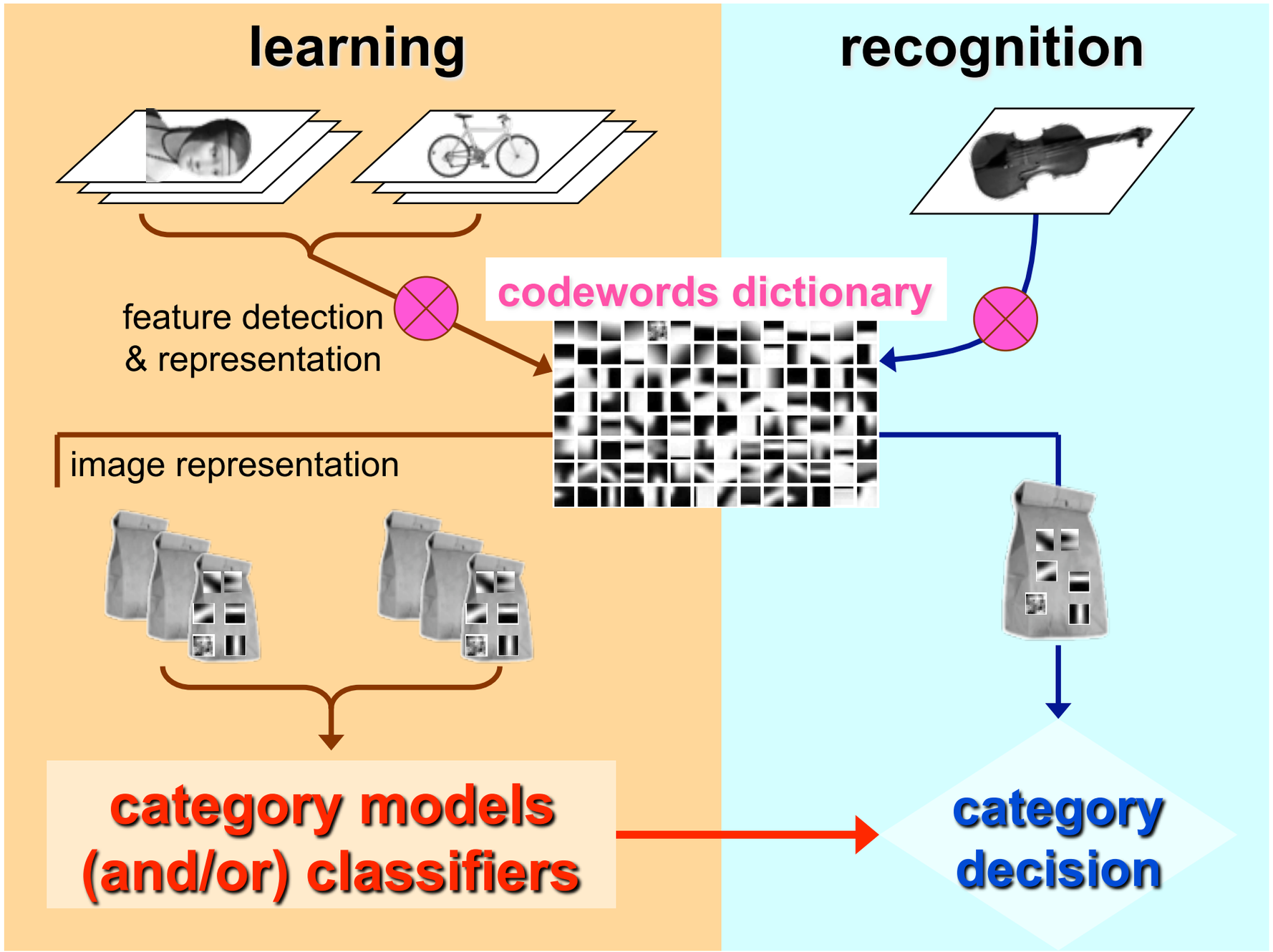

Summary & Discussion

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- Bag-of-words representation:
  - Sparse representation of object category
  - Many machine learning methods are directly applicable.
  - Robust to occlusions
  - Allows sharing of representation between multiple classes
- But there are also problems...

What about spatial information?

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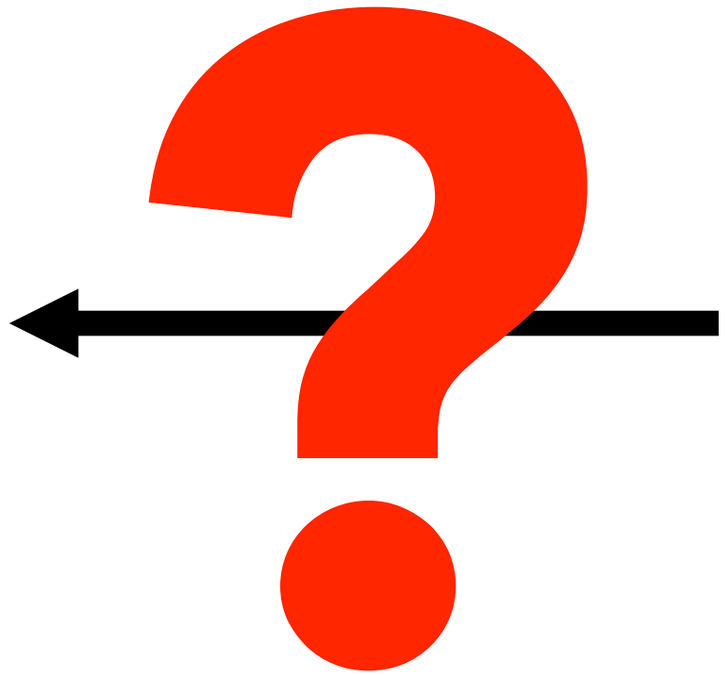

Problem with bag-of-words

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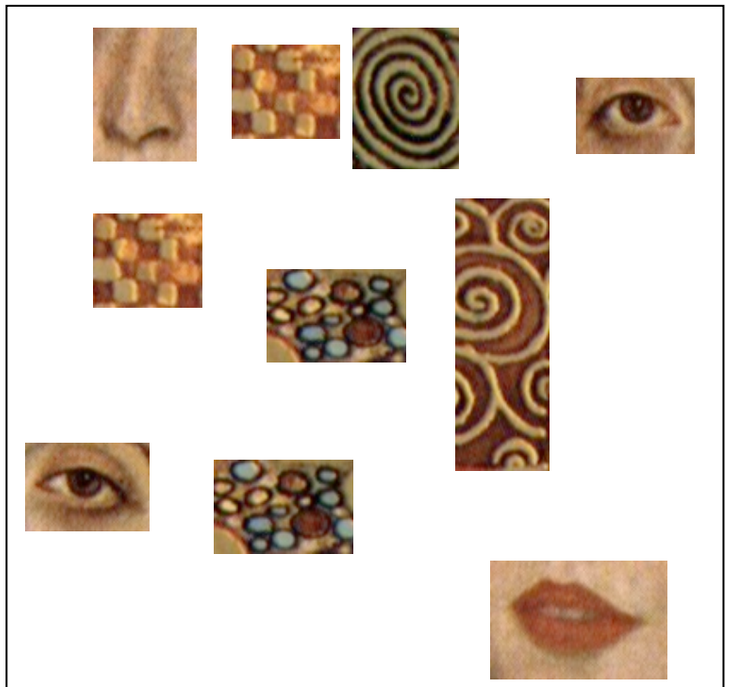

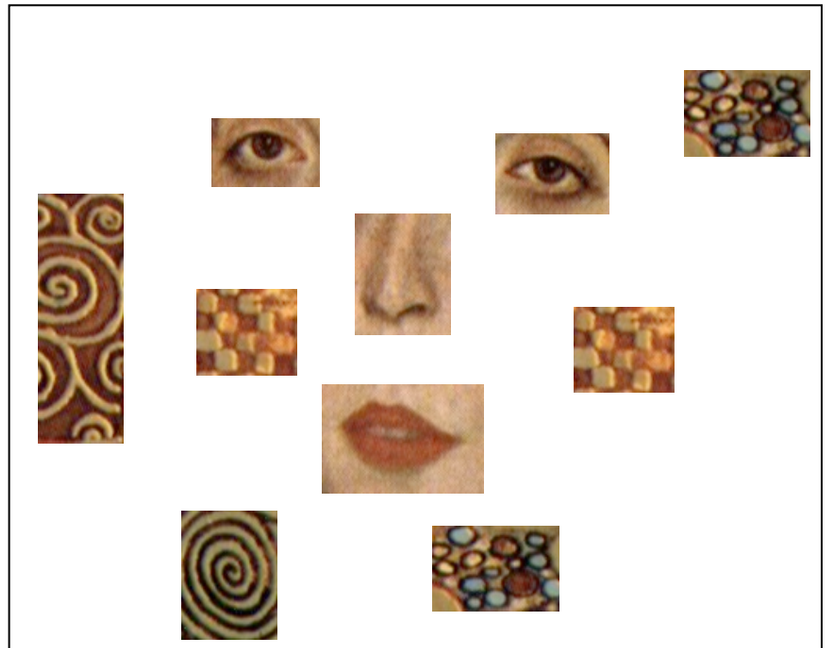

- All have equal probability for bag-of-words methods
- Location information is important!

Model: Parts and Structure

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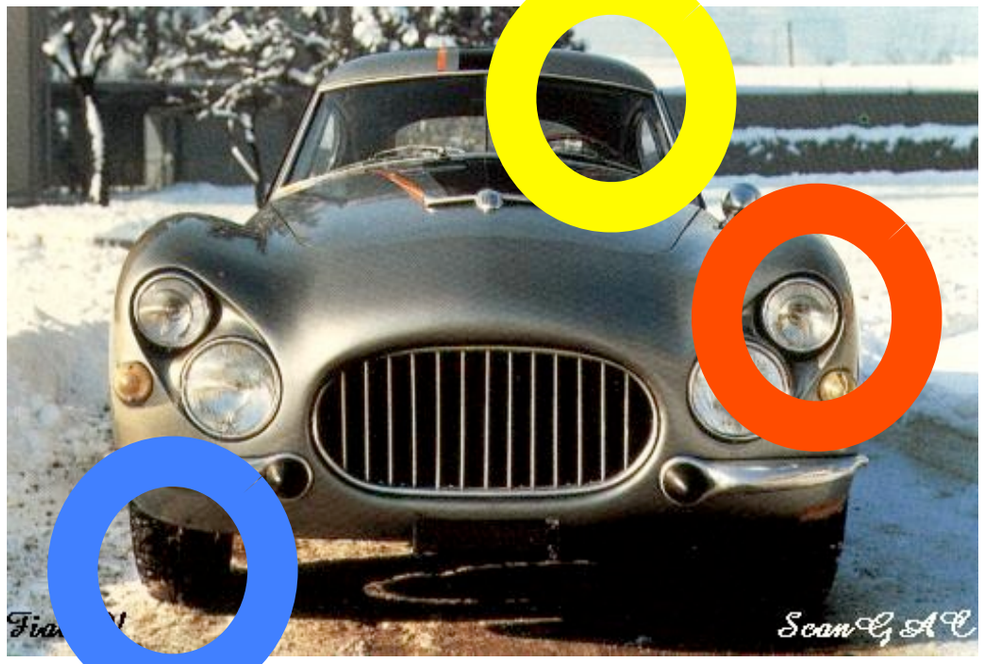

Part-based Representation

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- Object as set of parts
  - Generative representation
- Model:
  - Relative locations between parts
  - Appearance of part
- Issues:
  - How to model location
  - How to represent appearance
  - How to handle occlusion/clutter

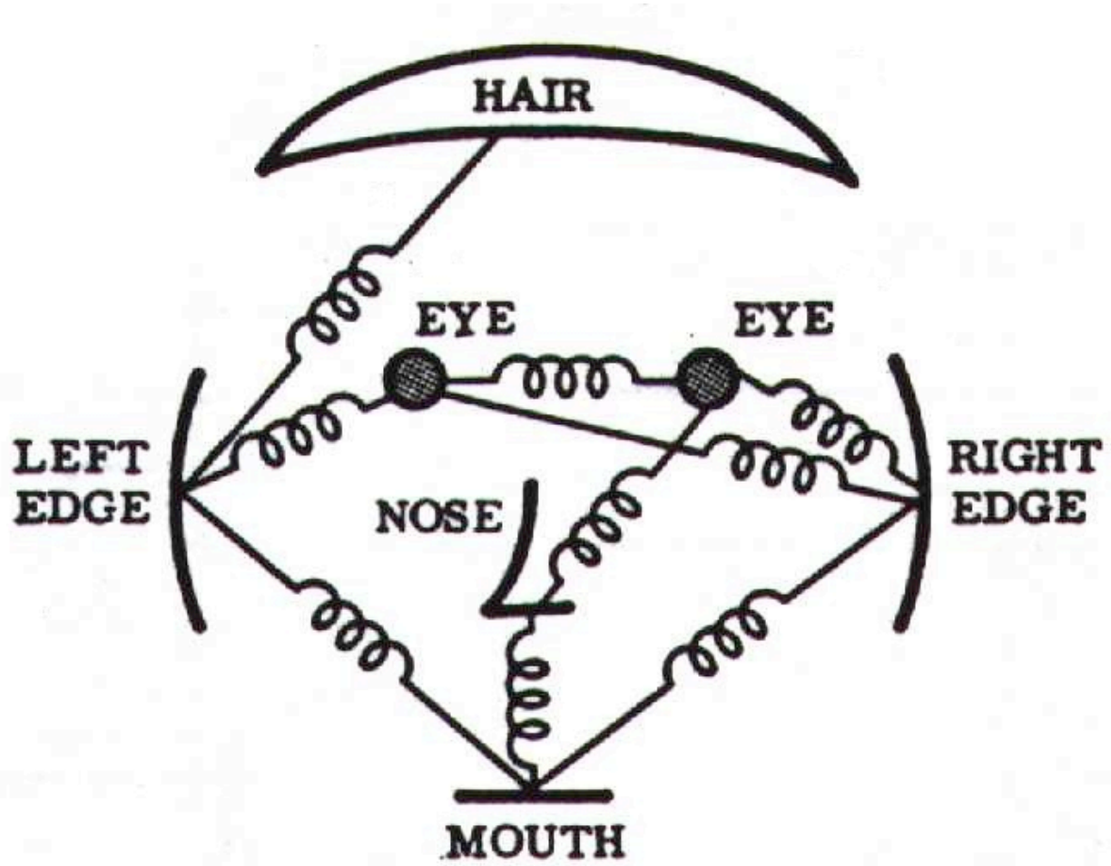

Figure from [Fischler & Elschlager 73]

The correspondence problem

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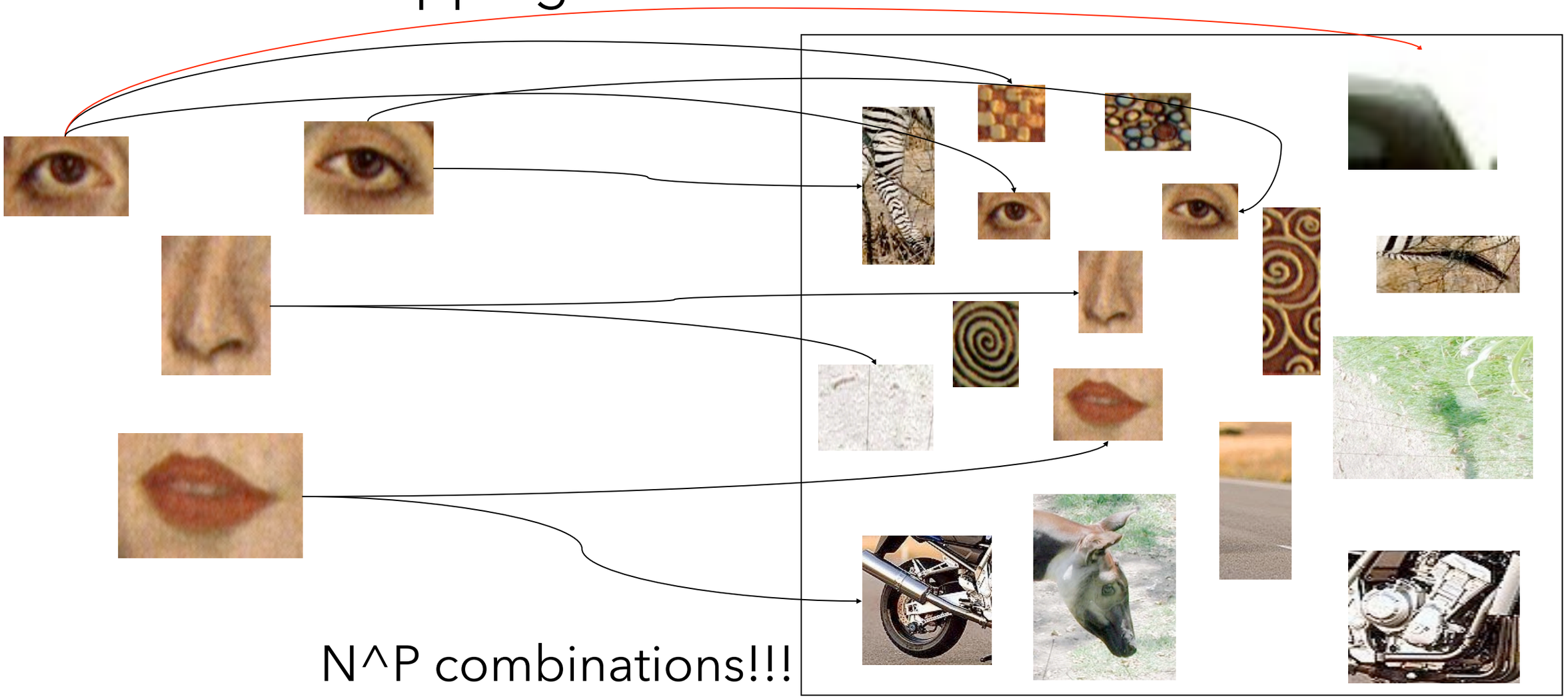
- Model with P parts
- Image with N possible assignments for each part
- Consider mapping to be 1-1



N^P combinations!!!

Pyramid match kernel

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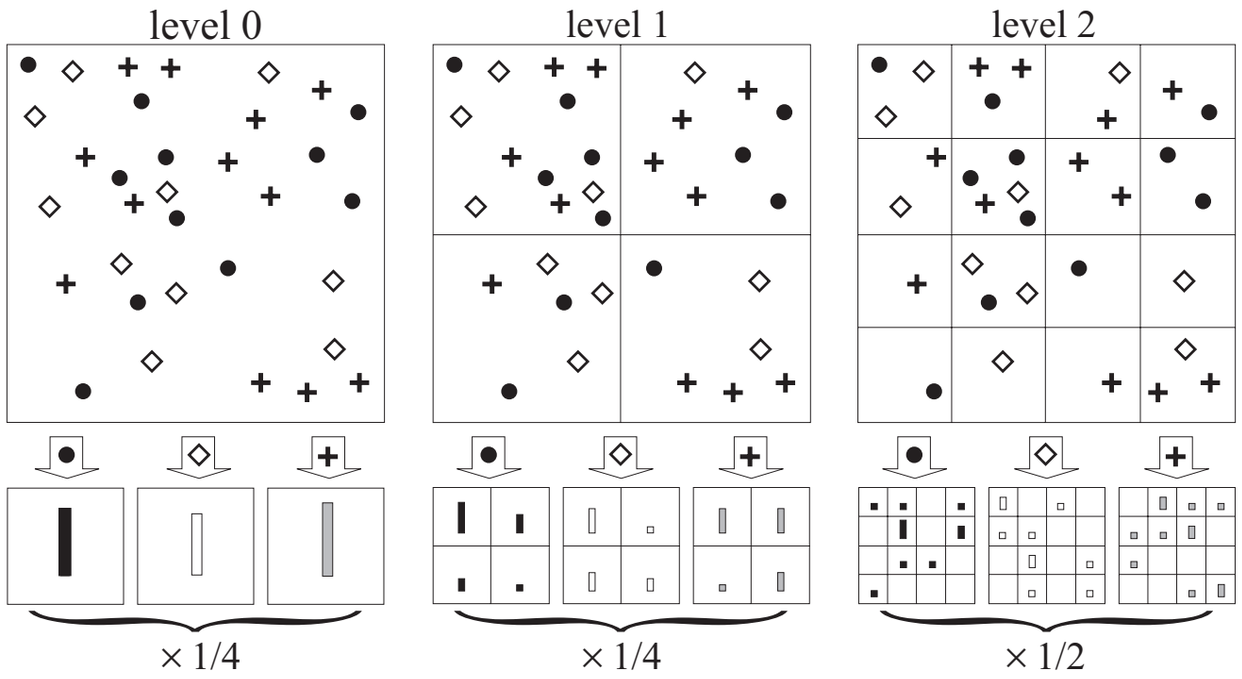
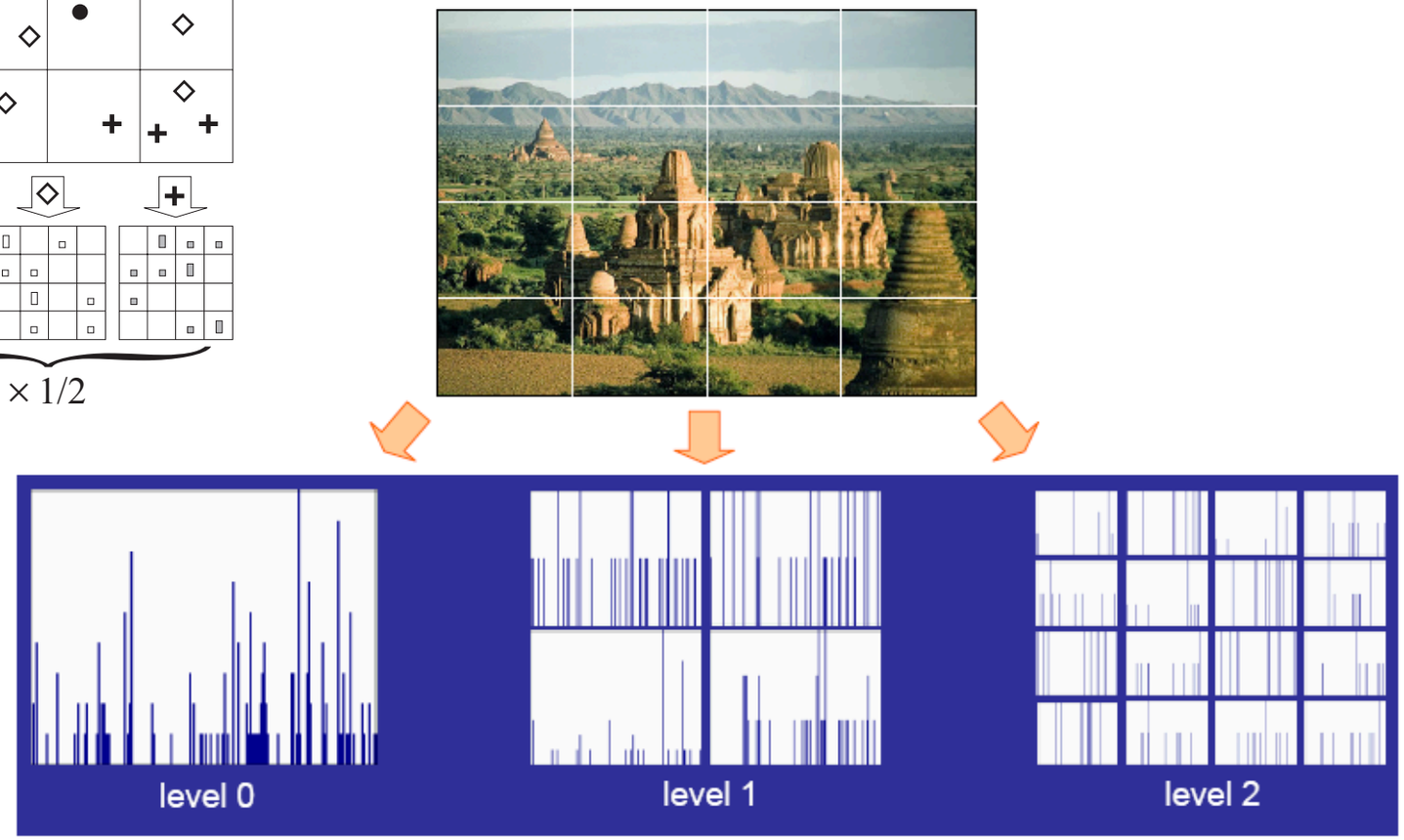
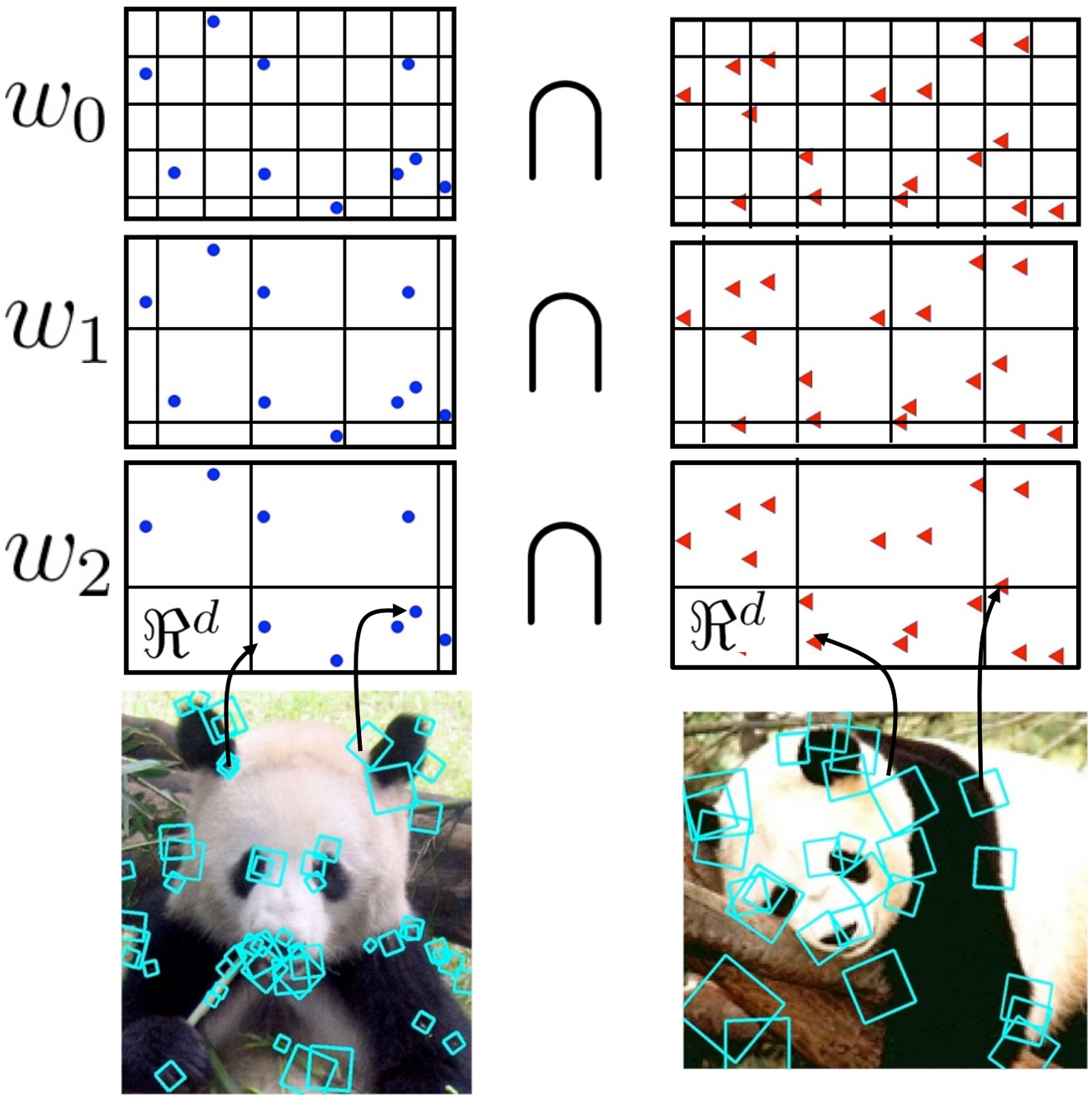
- Combine visual words with spatial information
- Can use SVMs and histogram kernels





[Lazebnik et al.]

Pyramid match kernel



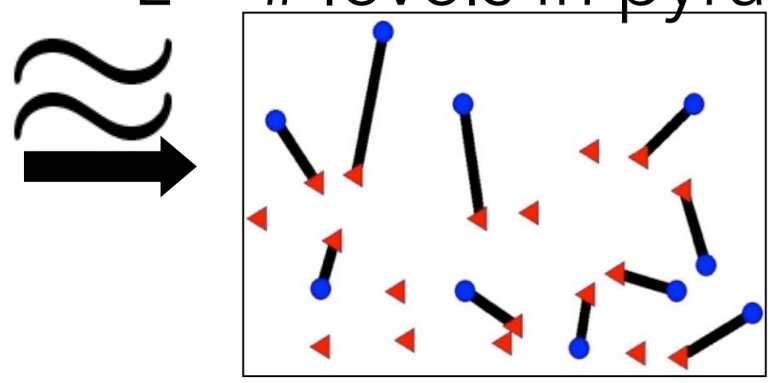
$\mathbf{X} = \{\vec{\mathbf{x}}_1, \ldots, \vec{\mathbf{x}}_m\}$ $\mathbf{Y} = \{\vec{\mathbf{y}}_1, \ldots, \vec{\mathbf{y}}_n\}$

Optimal match: $O(m^3)$

Pyramid match: O(mL)

m = # features

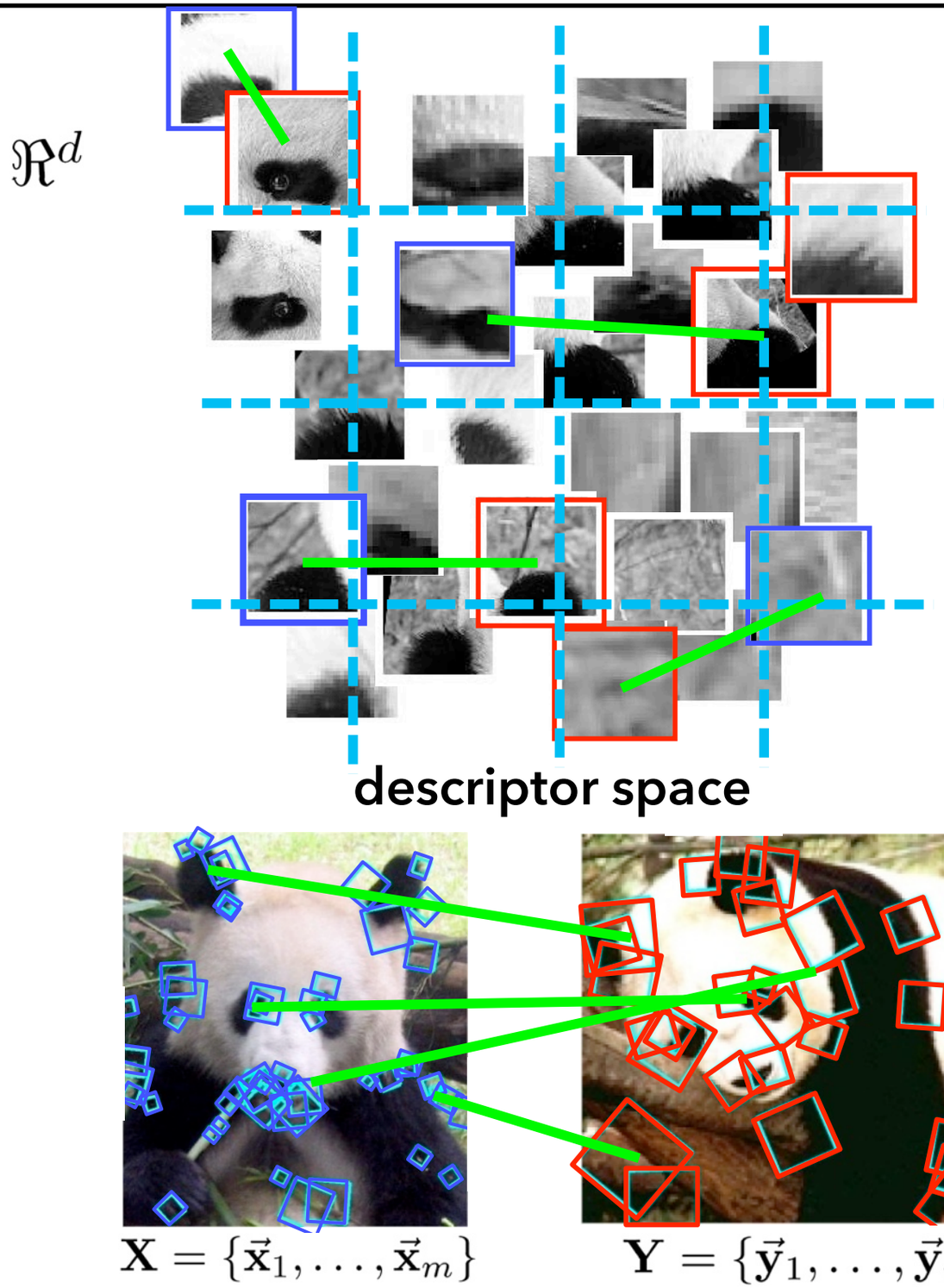
L = # levels in pyramid



optimal partial matching

[Grauman & Darrell]

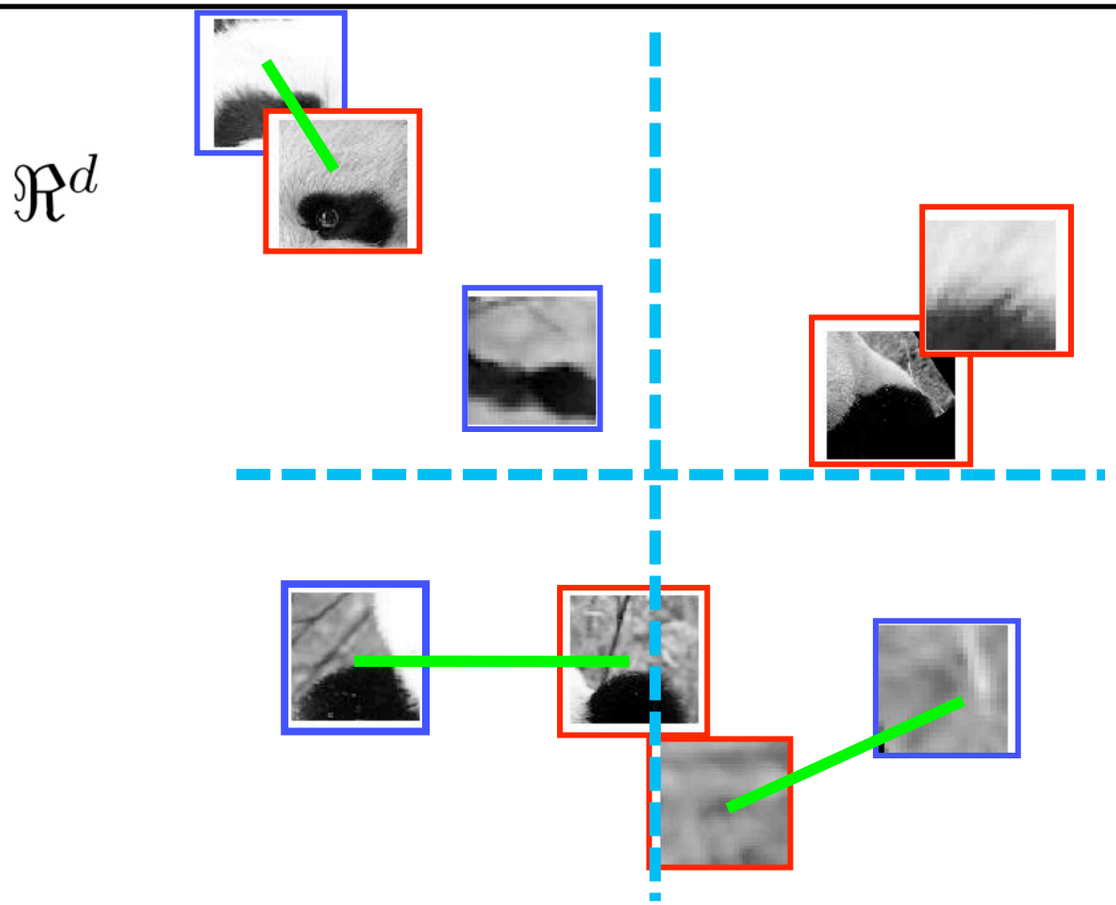
Pyramid match: main idea

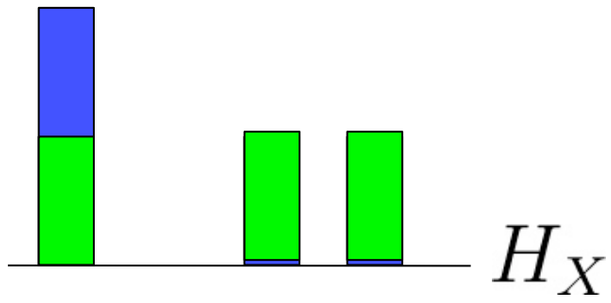


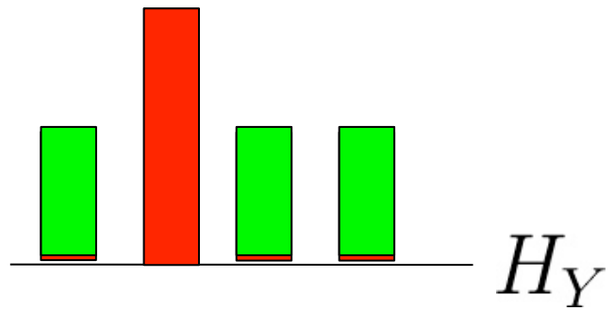
Feature space partitions serve to "match" the local descriptors within successively wider regions.

[Grauman & Darrell]

Pyramid match: main idea





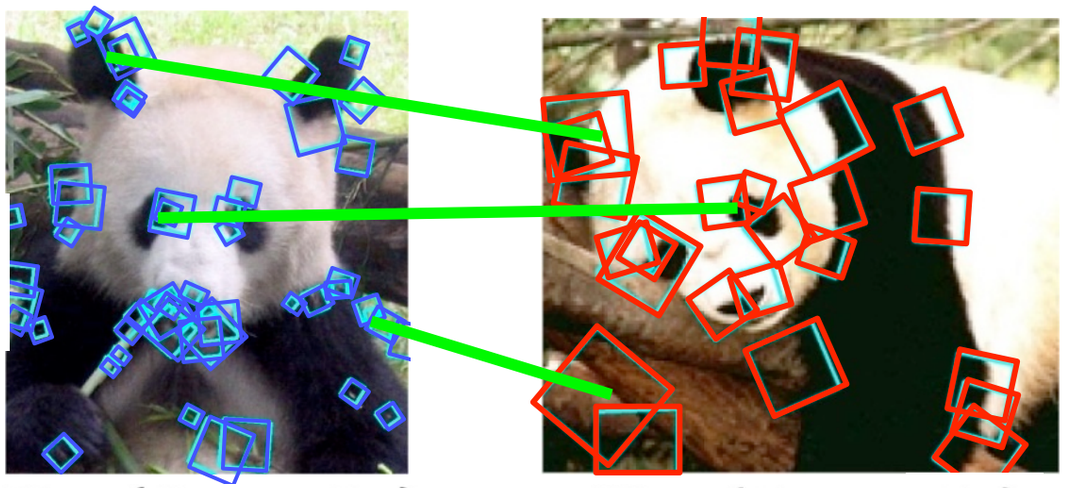


$$\mathcal{I}(H_X, H_Y) = \sum_j \min\left(H_X(j), H_Y(j)\right)$$

$$= 3$$

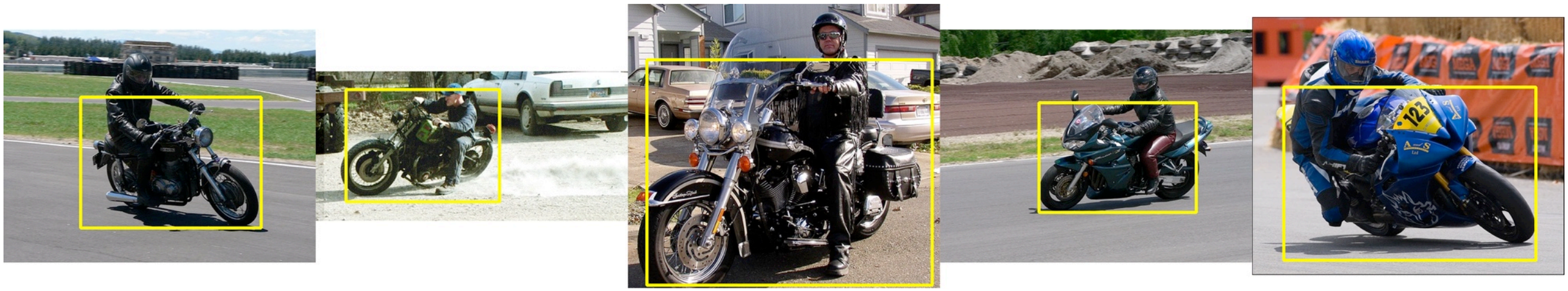
Histogram intersection counts number of possible matches at a given partitioning.

[Grauman & Darrell]



$\mathbf{X} = \{\vec{\mathbf{x}}_1, \ldots, \vec{\mathbf{x}}_m\}$ $\mathbf{Y} = \{\vec{\mathbf{y}}_1, \ldots, \vec{\mathbf{y}}_n\}$

From Categorization to Detection



- We may not only want to recognize if an object is in an image, but rather find it in the image
  - Object detection / localization
- Next time:
  - Sliding window approach
  - Histogram of Oriented Gradients (HOG)
    - global descriptor for object detection