

# Computer Vision I

Image Formation - 24.04.2013



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

**Credits:** Slides adapted from Konrad Schindler  
including many slides from:

Svetlana Lazebnik

Steve Seitz

Fredo Durand

Alyosha Efros





# Announcements

- ◆ Remember: No class next week!
  - ◆ May 1<sup>st</sup>, public holiday
- ◆ First homework assignment
  - ◆ out next week
  - ◆ please make sure to be subscribed to the mailing list
- ◆ If you still need a partner to do the homework assignments
  - ◆ meet at the front of the classroom after the lecture

# Computer Vision

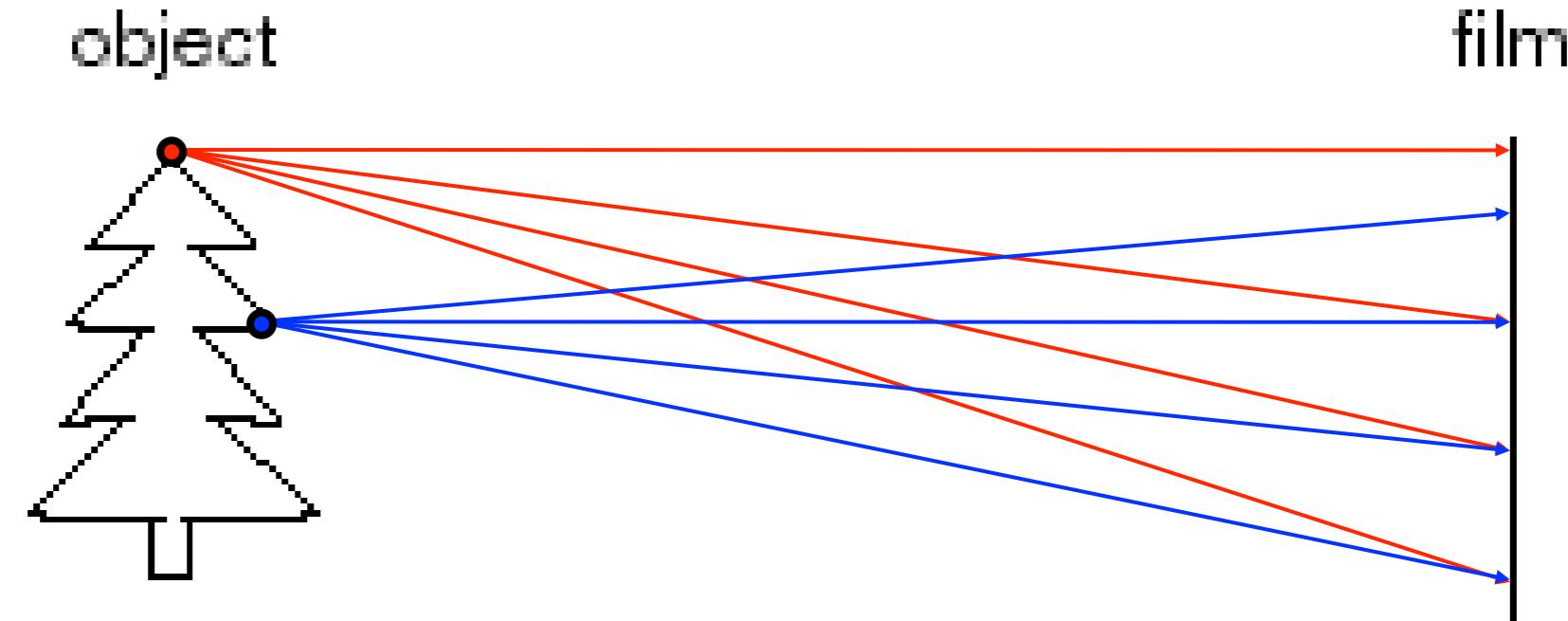
- ◆ We want to devise computer algorithms to understand the visual world much like we do.
- ◆ This means:
  - ◆ We have to use a lot of different cues...
  - ◆ We have to deal with ambiguity...
  - ◆ We have to exploit what is a plausible and meaningful interpretation...
  - ◆ ... in order to extract information about the 3D visual world from a small amount of data.
- ◆ CV is an **inverse problem**.



# Overview

- ◆ The **pinhole projection** model
  - ◆ Properties of pinhole cameras
  - ◆ Algebraic model: Perspective projection matrix
- ◆ **Single view geometry**
  - ◆ Projection from world to image coordinates
  - ◆ Camera calibration
- ◆ Cameras with **lenses**
  - ◆ Depth of focus, field of view, lens aberrations
- ◆ **Digital cameras**
  - ◆ Types of sensors, color

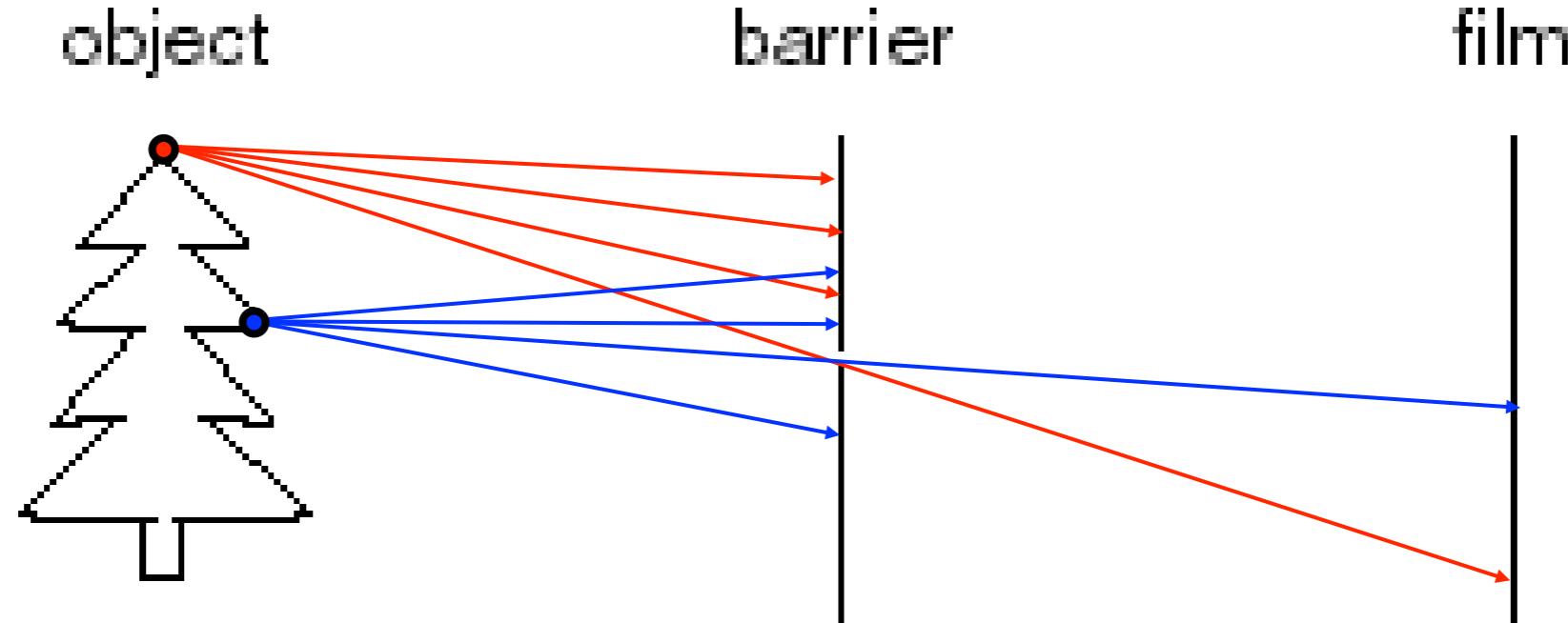
# How do we see the world?



- ◆ Let's design a camera:
  - ◆ Idea 1: Put a piece of film (or a CCD) in front of an object
  - ◆ Do we get a reasonable image?

Slide by Steve Seitz

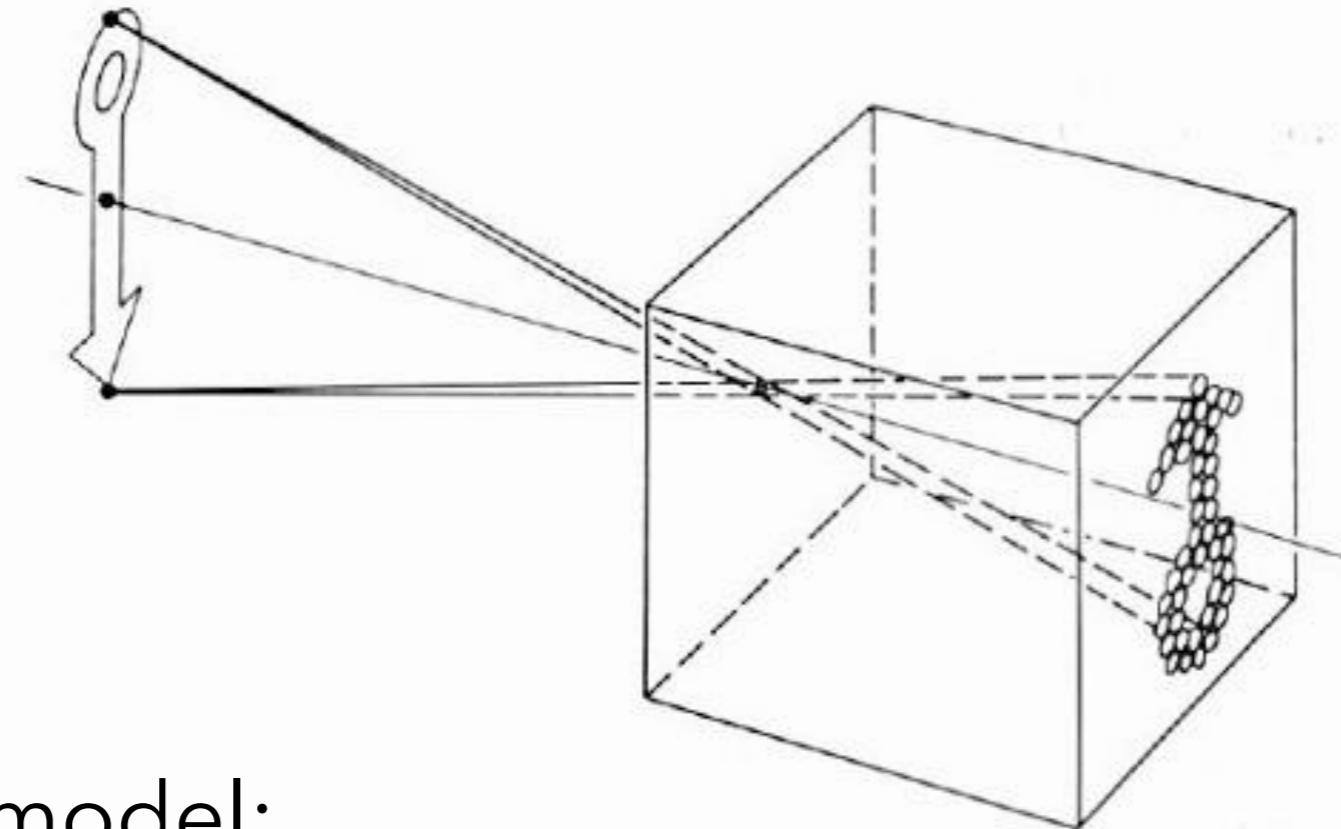
# Pinhole camera



- ◆ Add a barrier to block off most of the rays:
  - ◆ This reduces blurring
  - ◆ The opening is known as the **aperture**

Slide by Steve Seitz

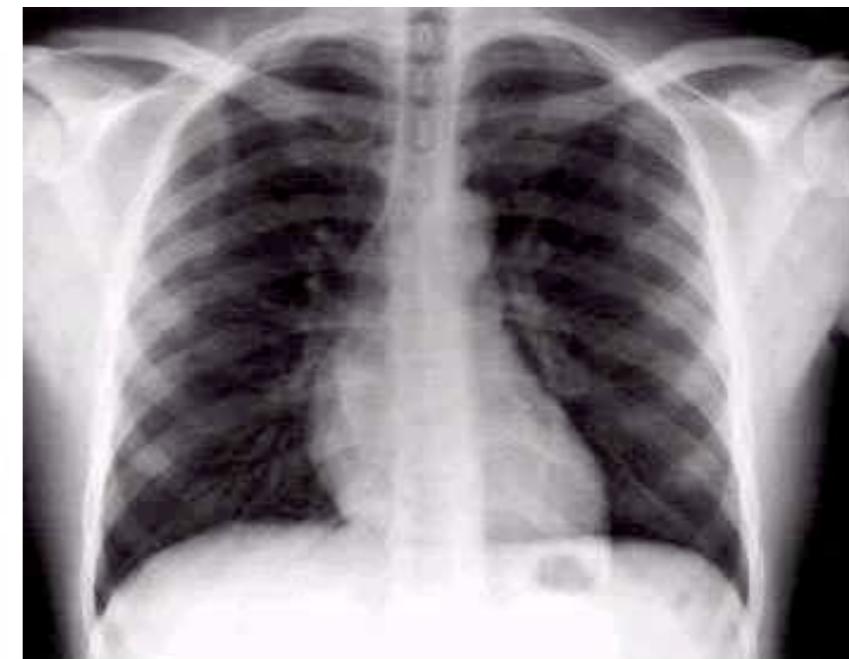
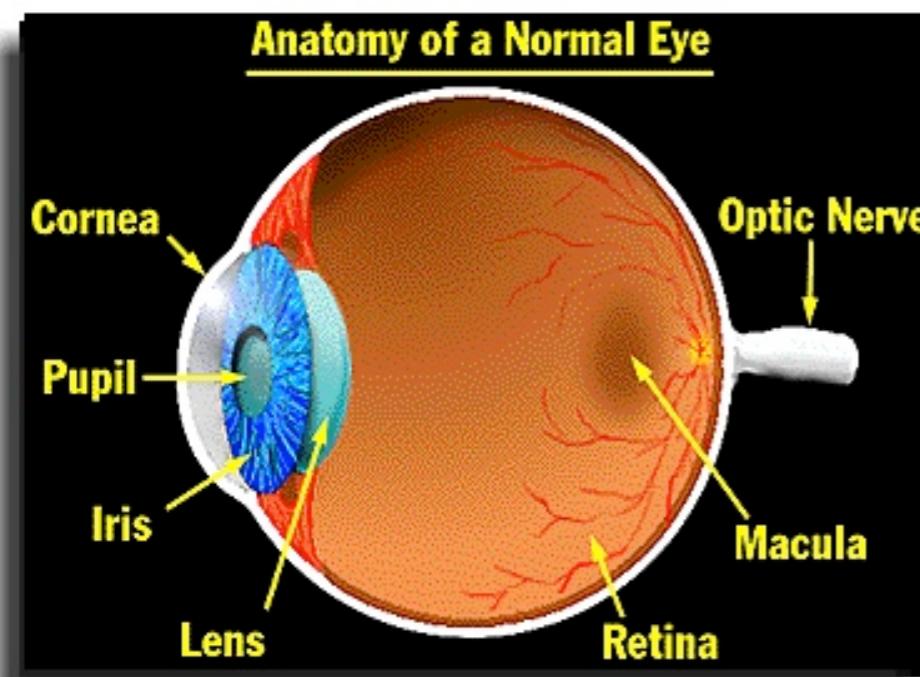
# Pinhole camera model



- ◆ Pinhole model:
  - ◆ Captures **pencil of rays** - all rays through a single point
  - ◆ Projection rays are **straight** lines
  - ◆ The point is called **center of projection (focal point)**
  - ◆ The image is formed on the **image plane**
  - ◆ Two equivalent projections: **positive, negative**

# Pinhole camera

- ◆ A model for many common sensors:
  - ◆ Photographic cameras
  - ◆ human eye
  - ◆ X-ray machines

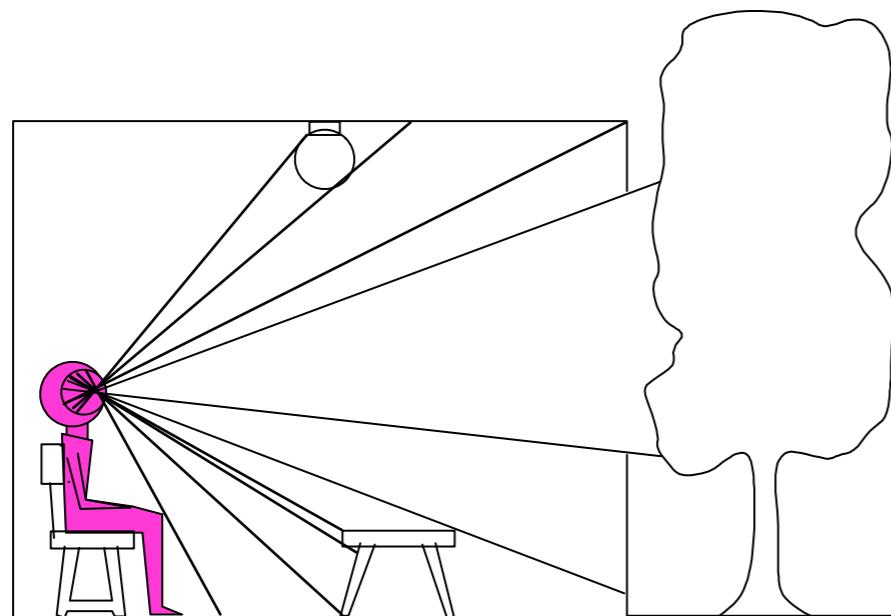


# Dimensionality Reduction Machine (3D to 2D)



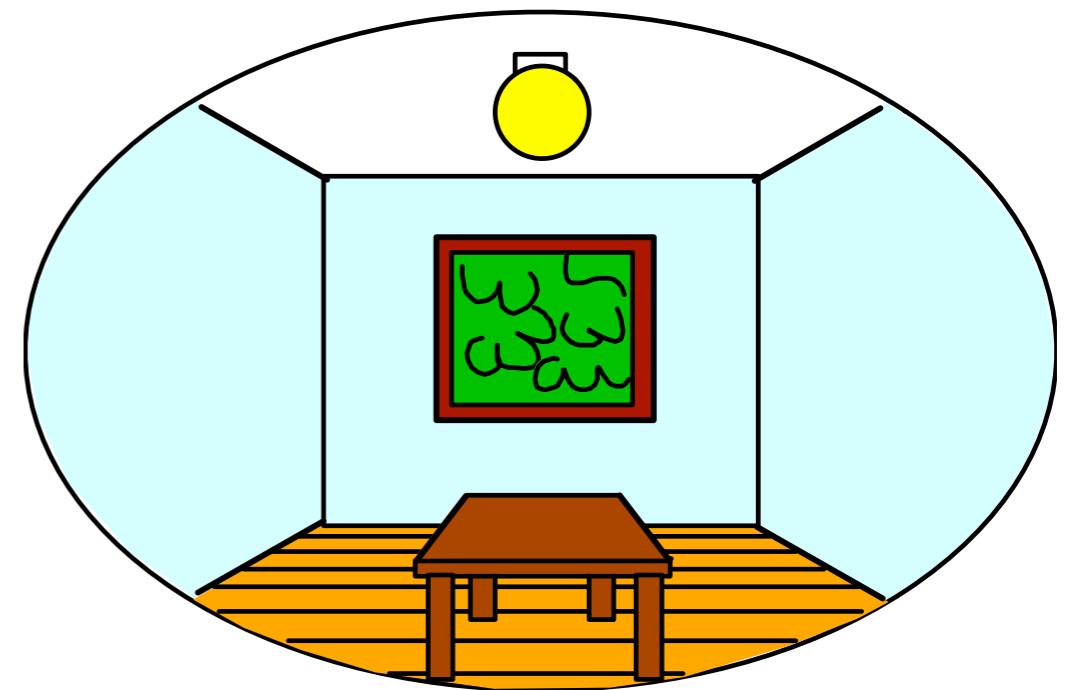
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3D world



Point of observation

2D image



- ◆ What have we lost?
  - ◆ Angles
  - ◆ Distances (lengths)

Slide by A. Efros

Figures © Stephen E. Palmer, 2002

# Projection properties



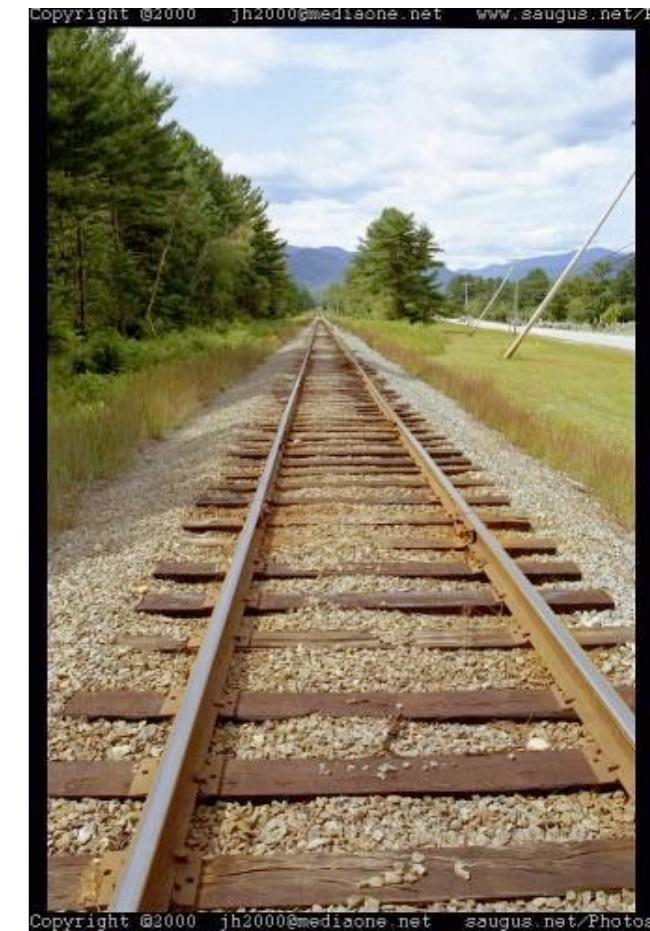
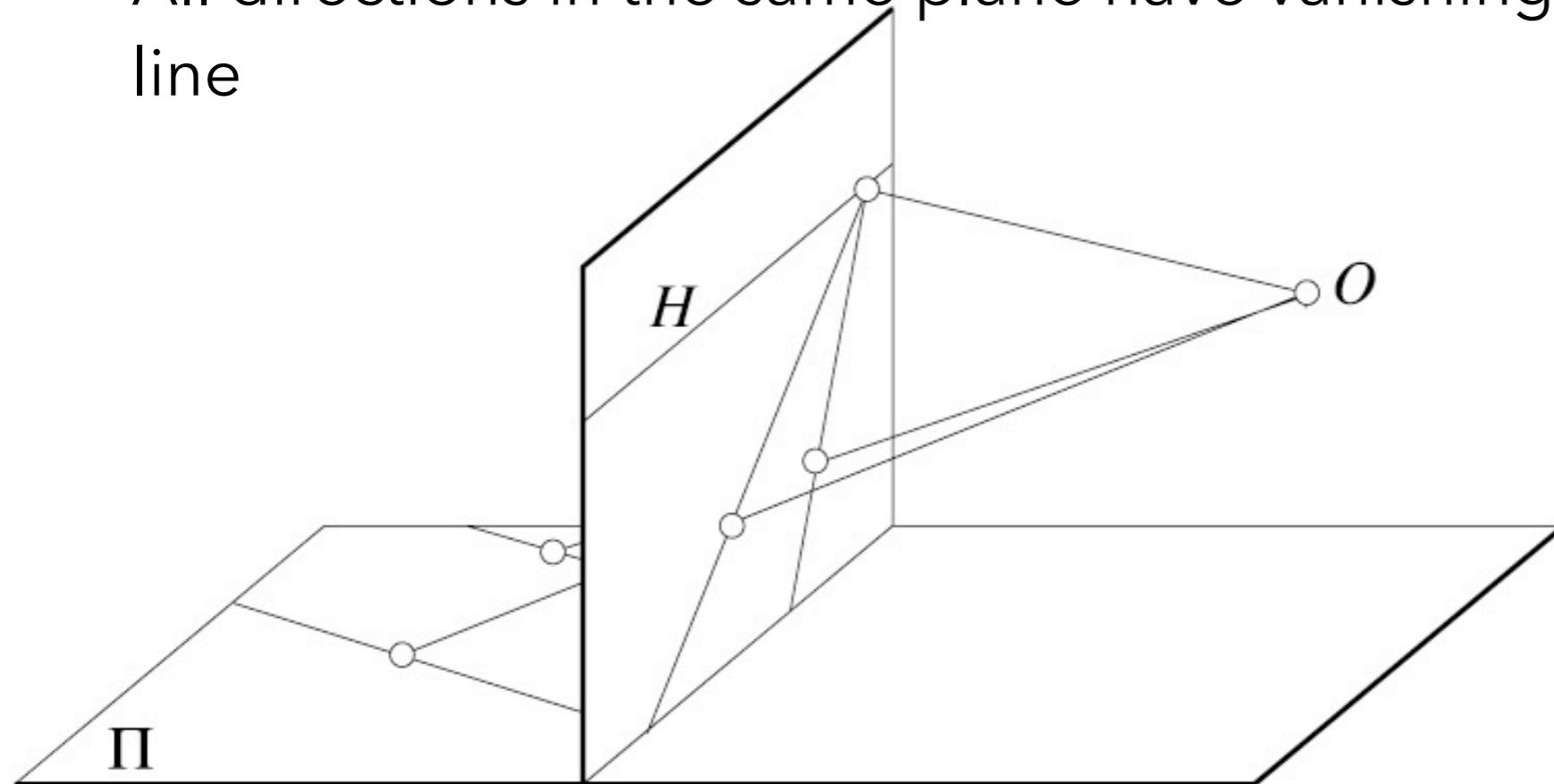
- ◆ **Many-to-one:**

- ◆ any points along the same ray map to the same point in image
- ◆ Points → points
  - ◆ But projection of points on focal plane is undefined
- ◆ Lines → lines (collinearity is preserved)
  - ◆ But line through focal point projects to a point
- ◆ Planes → planes (or half-planes)
  - ◆ But plane through focal point projects to line

Slide by Lana Lazebnik

# Projection properties

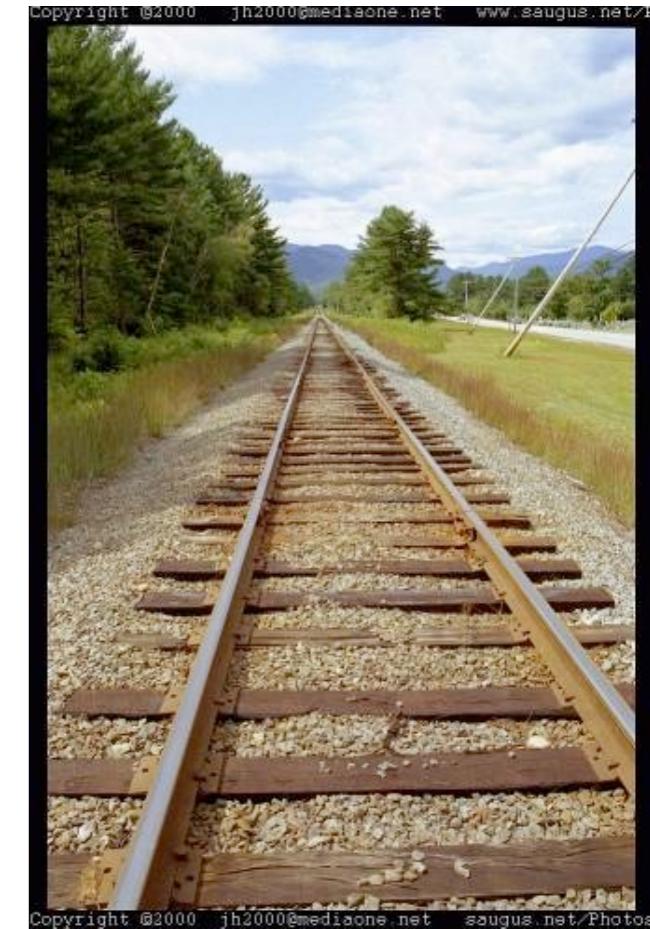
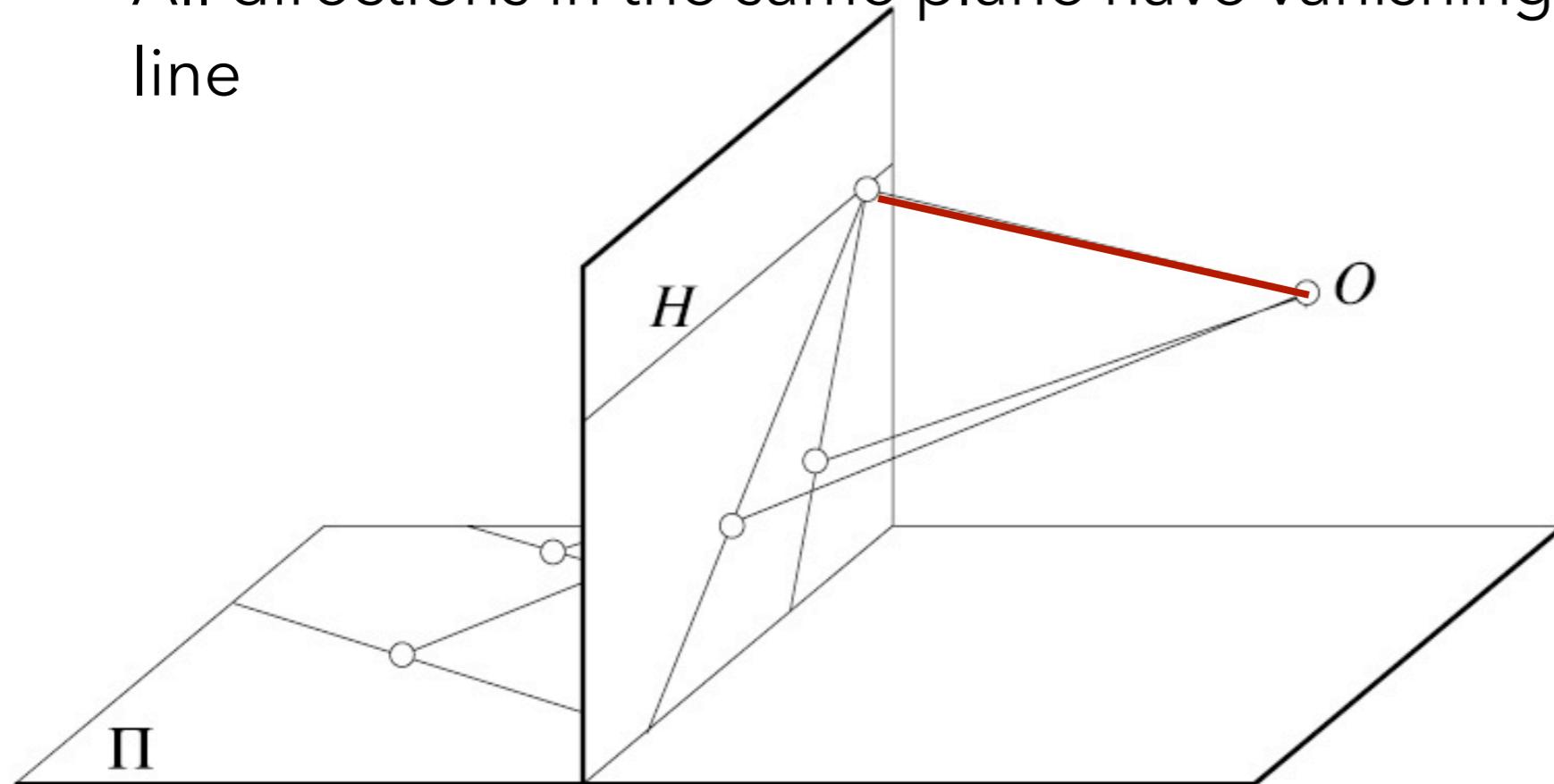
- ◆ Parallel lines converge at a **vanishing point**
  - ◆ Each direction in space has its own vanishing point
  - ◆ But parallels also parallel to the image plane remain parallel
  - ◆ All directions in the same plane have vanishing points on the same line



How do we construct the vanishing point/line?

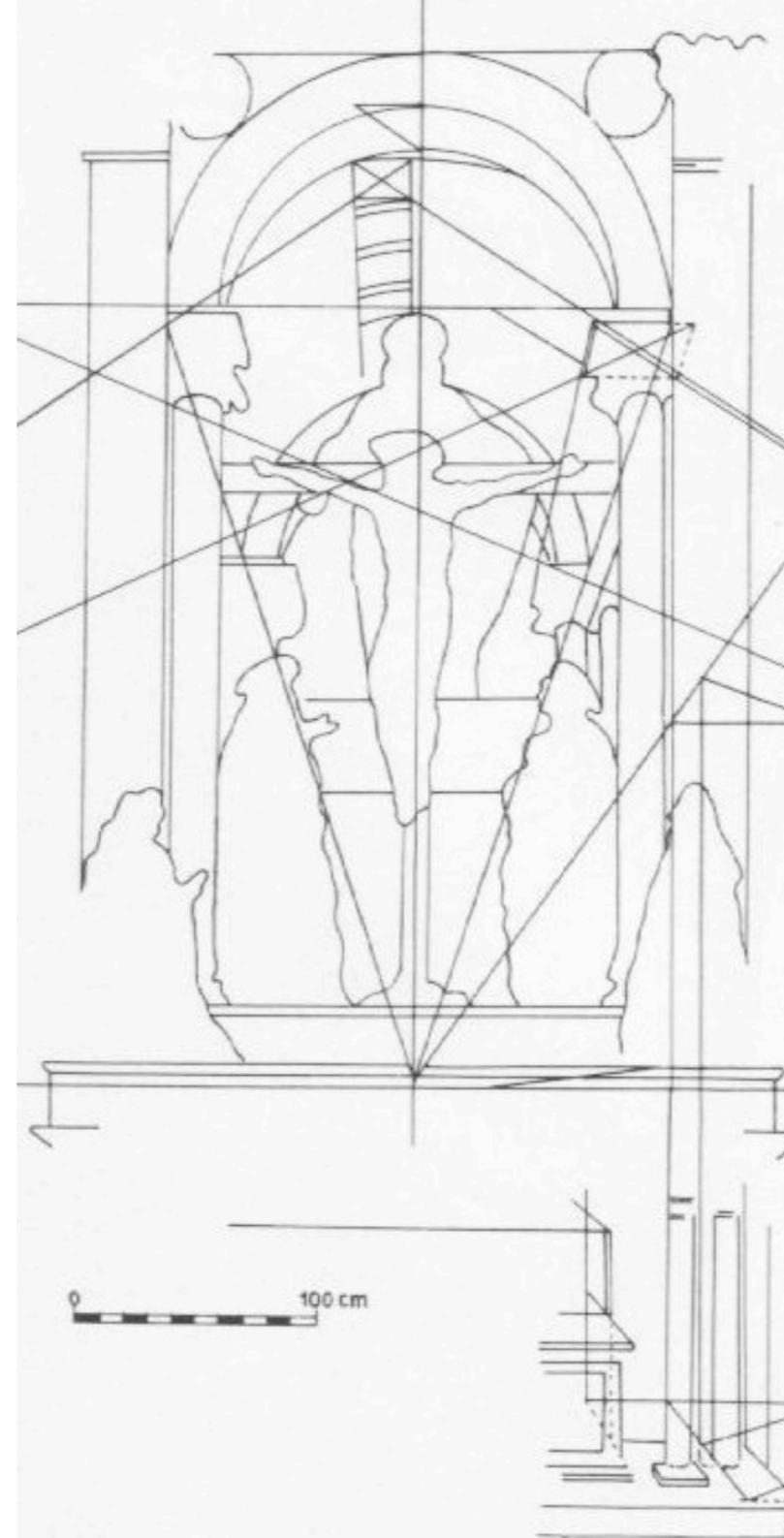
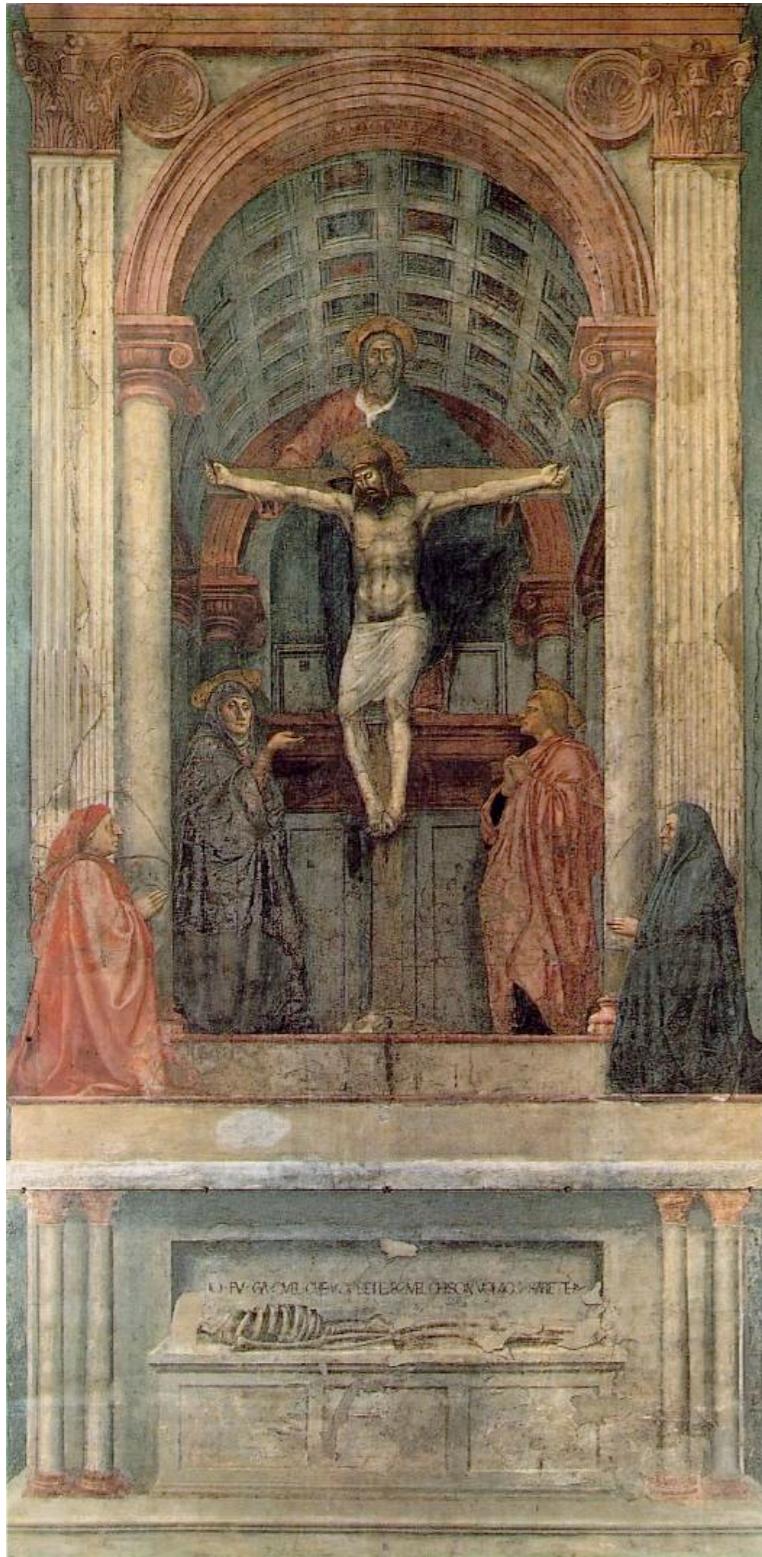
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How do we construct the vanishing point/line?

# One-point perspective



- ◆ Masaccio, [Trinity](#), Santa Maria Novella, Florence, 1425-28
- ◆ First consistent use of perspective in Western art?

# Perspective distortion

- ◆ What does a **sphere** project to?

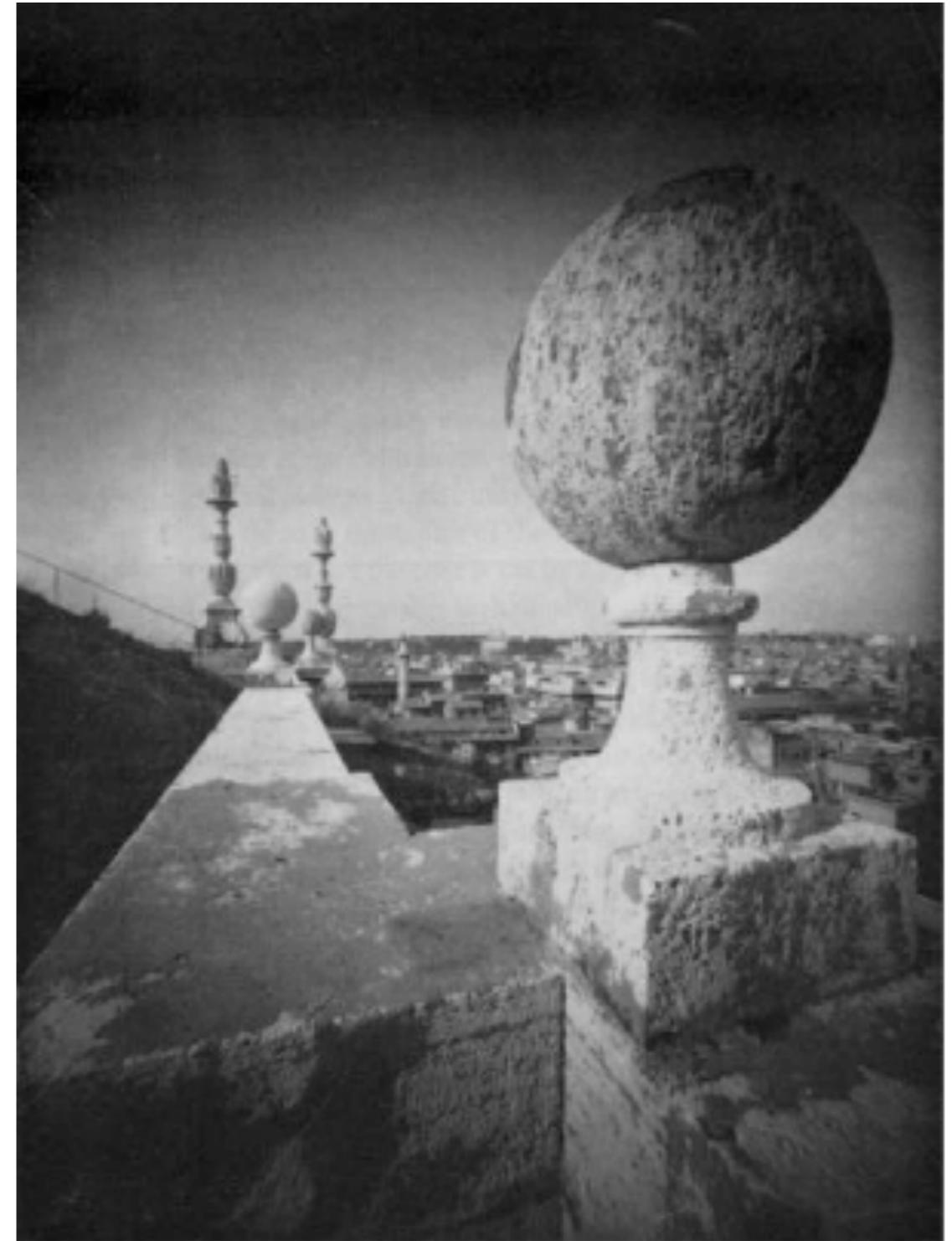
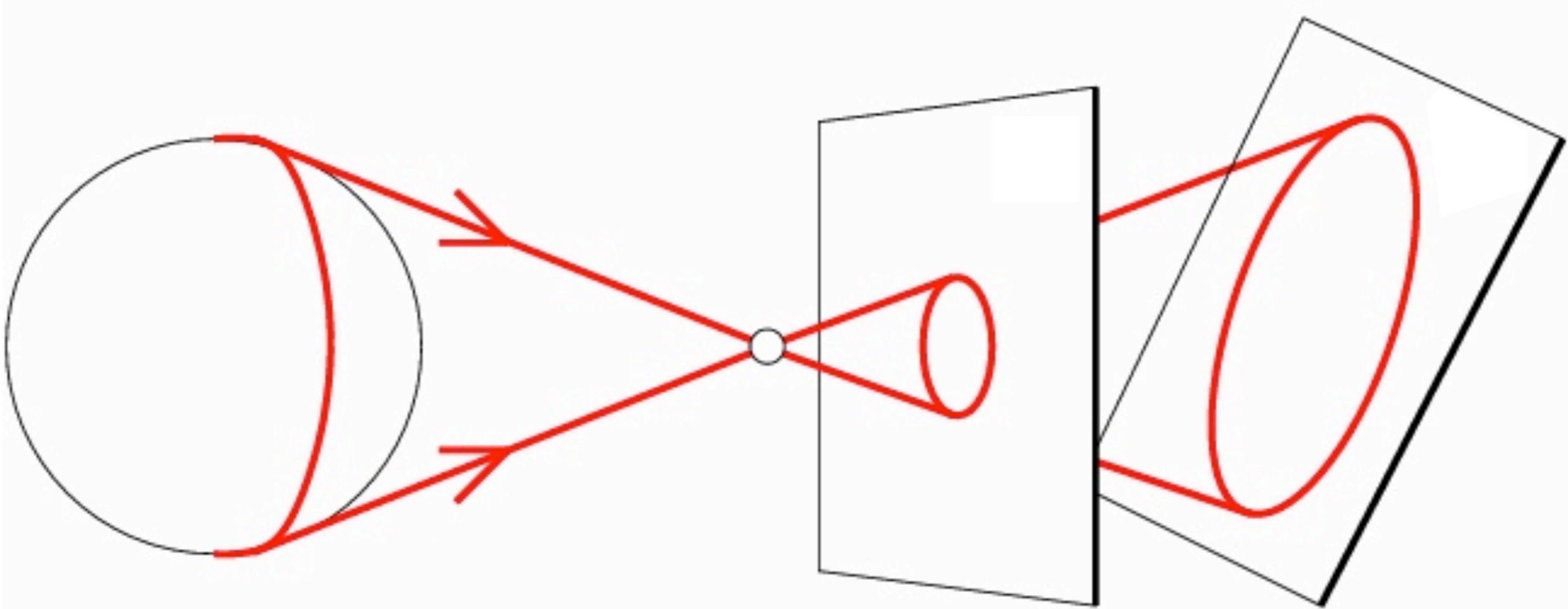


Image source: F. Durand

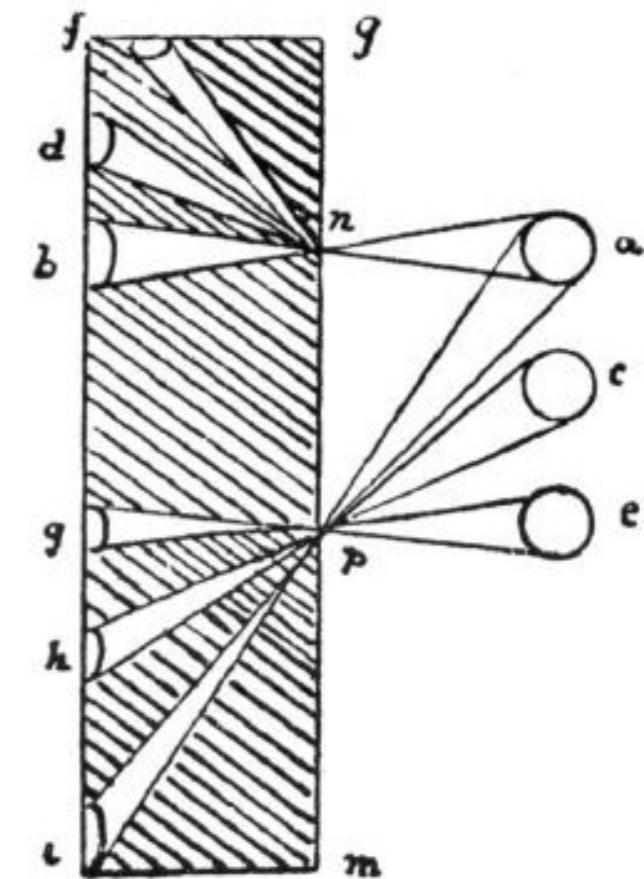
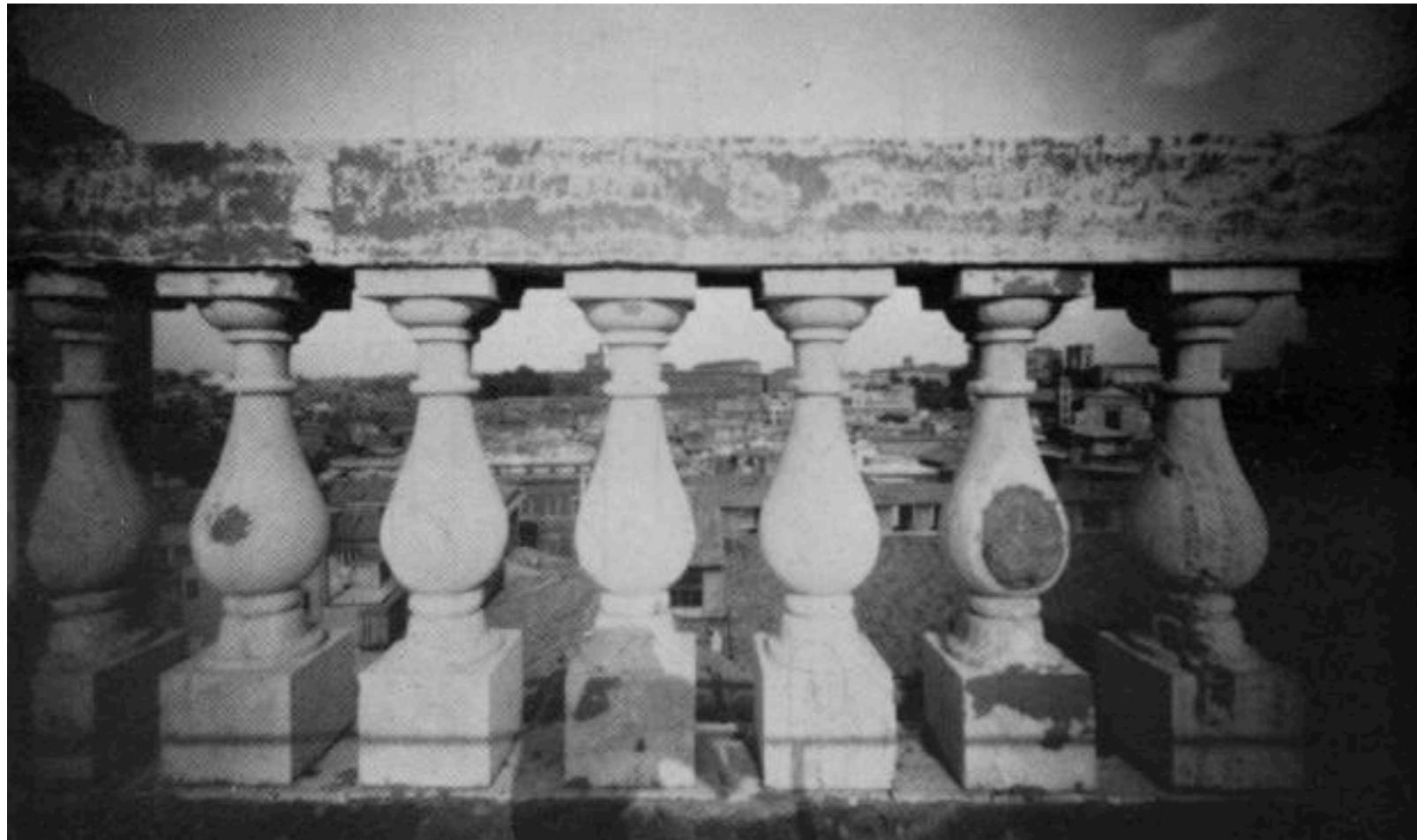
# Perspective distortion

- ◆ What does a **sphere** project to?



# Perspective distortion

- ◆ The exterior columns appear bigger
- ◆ The distortion is **not** due to lens flaws
- ◆ Problem pointed out by Da Vinci

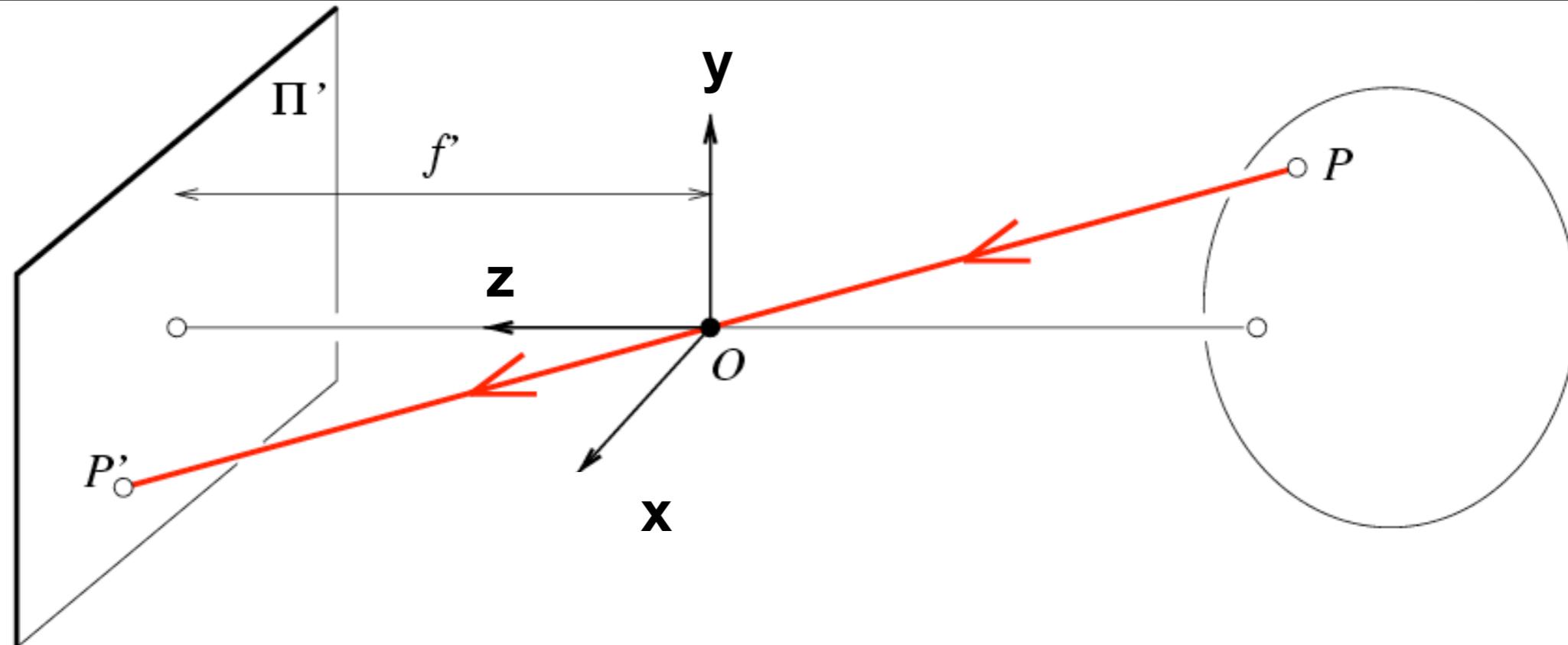


Slide by F. Durand

# Perspective distortion: People



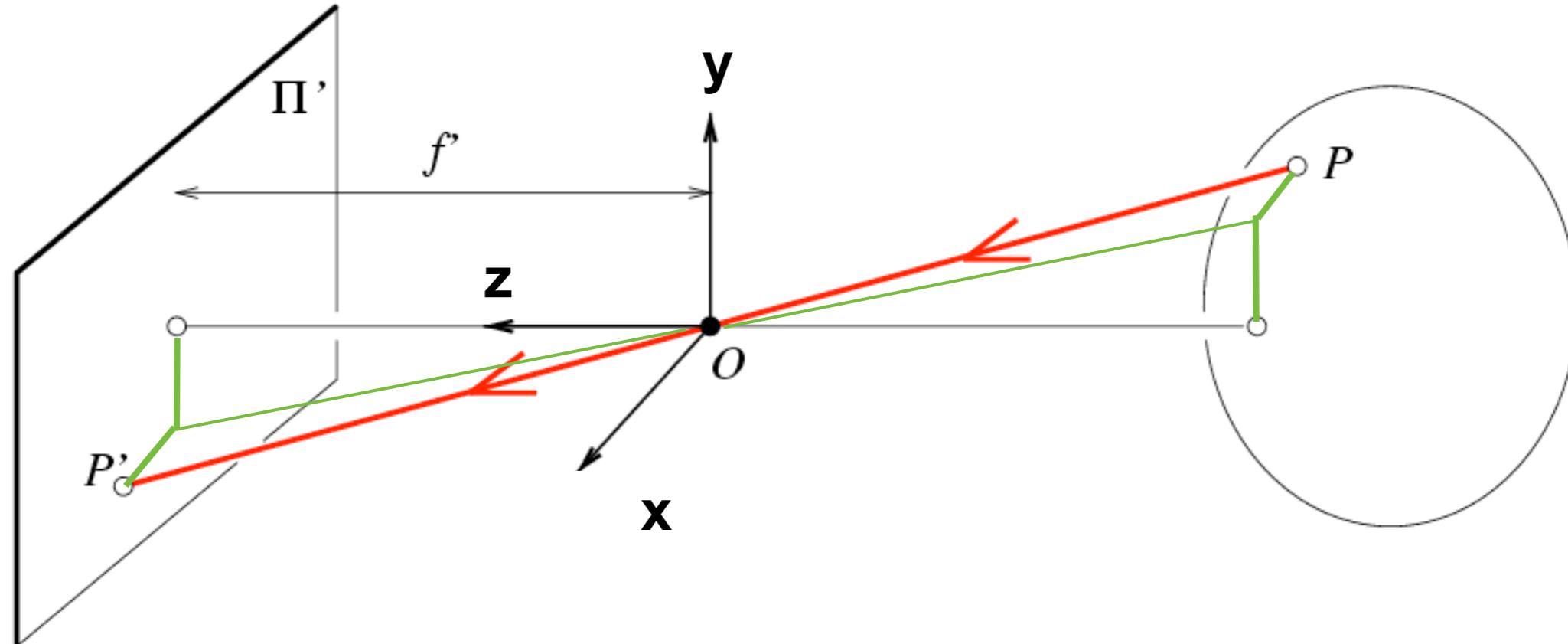
# Modeling projection



- ◆ The coordinate system
  - ◆ We will use the pinhole model as an approximation
  - ◆ Put the optical center ( $O$ ) at the origin
  - ◆ Put the image plane ( $\Pi'$ ) in front of  $O$

Source: J. Ponce, S. Seitz

# Modeling projection



- ◆ Projection equations
  - ◆ Compute intersection with  $\Pi'$  of ray from  $P = (x, y, z)$  to  $O$
  - ◆ Derived using similar triangles  $(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z}, f')$
  - ◆ To get the projection, throw away last coordinate  $(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z})$

Source: J. Ponce, S. Seitz



# Homogeneous coordinates

$$(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z})$$

- ◆ Is this a linear transformation?

Slide by Steve Seitz

# Homogeneous coordinates

$$(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z})$$

- ◆ Is this a linear transformation?
  - ◆ no - division by  $z$  is **nonlinear**
- ◆ Trick: add one more coordinate

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image  
coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene  
coordinates

- ◆ Converting **from** homogeneous  
coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$
$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

# Perspective Projection Matrix

- ◆ Projection is a matrix multiplication in homogeneous coordinates:

$$\begin{pmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} f'x \\ f'y \\ z \end{pmatrix} \Rightarrow \left( f' \frac{x}{z}, f' \frac{y}{z} \right)$$

divide by the third coordinate

Slide by Lana Lazebnik

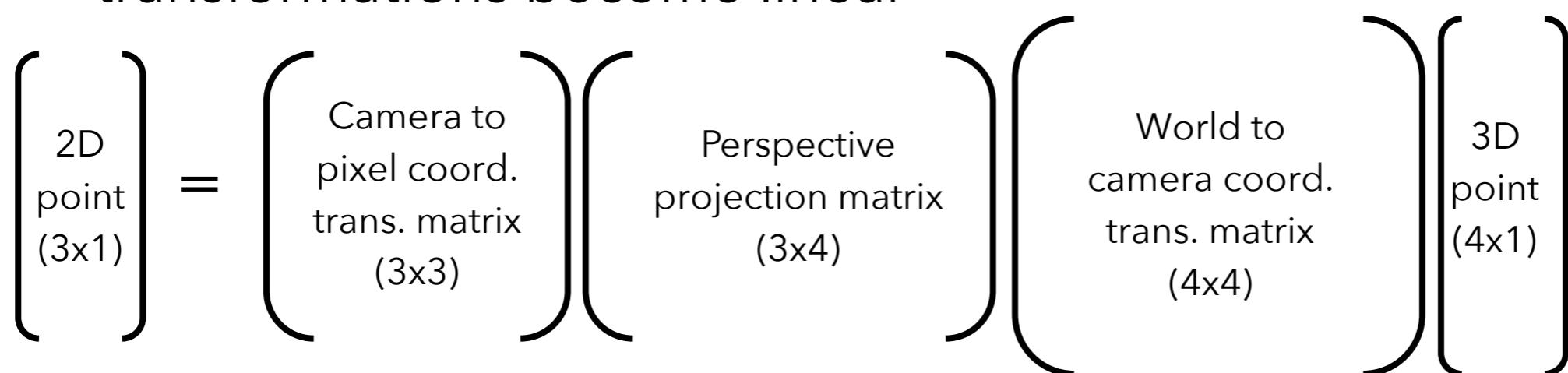
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divide by the third coordinate

- ◆ By changing the algebraic representation all relevant transformations become linear



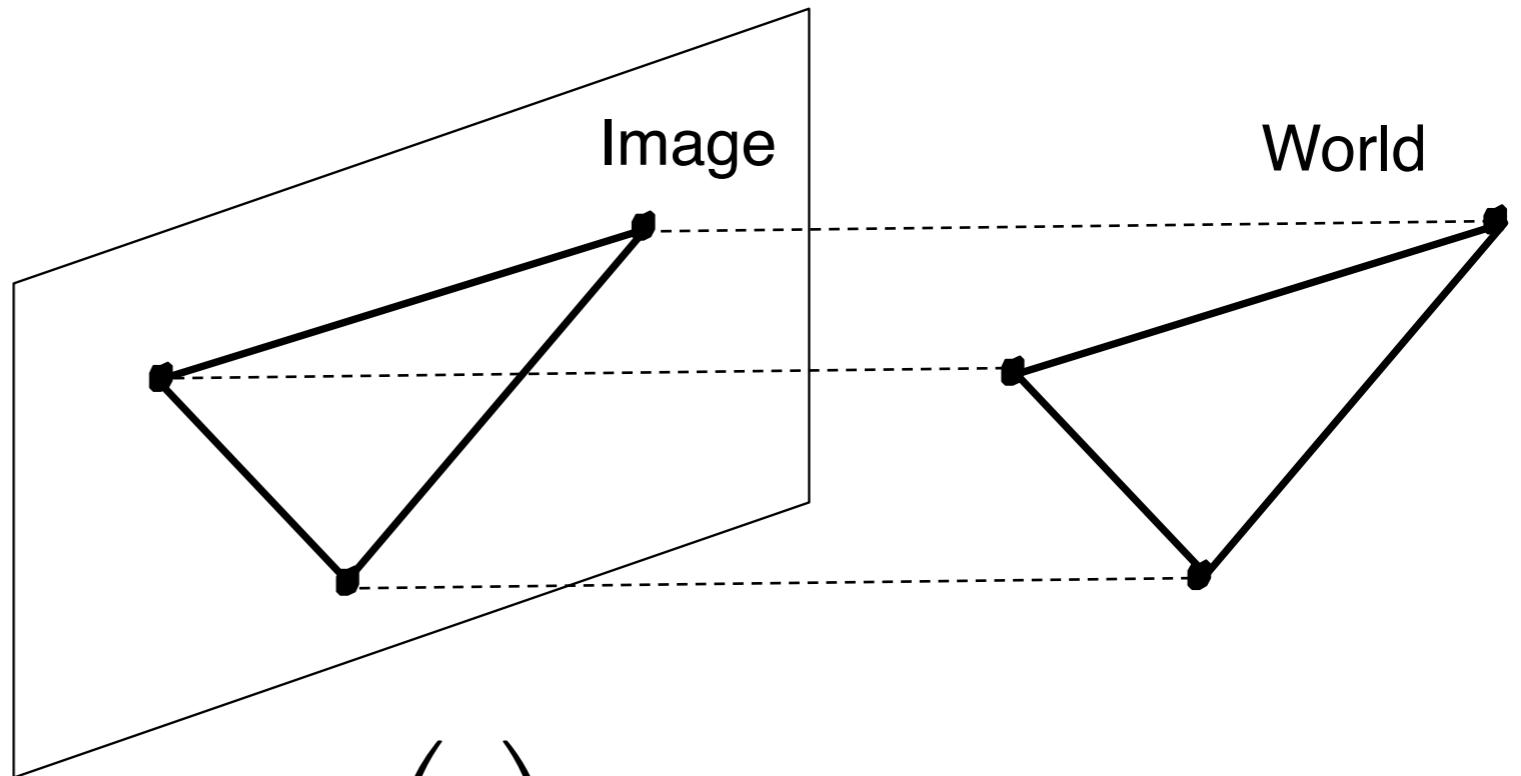
Slide by Lana Lazebnik

# Orthographic Projection

- ◆ Special case of perspective projection
  - ◆ Distance from center of projection to image plane is infinite

- ◆ Also called  
“parallel projection”

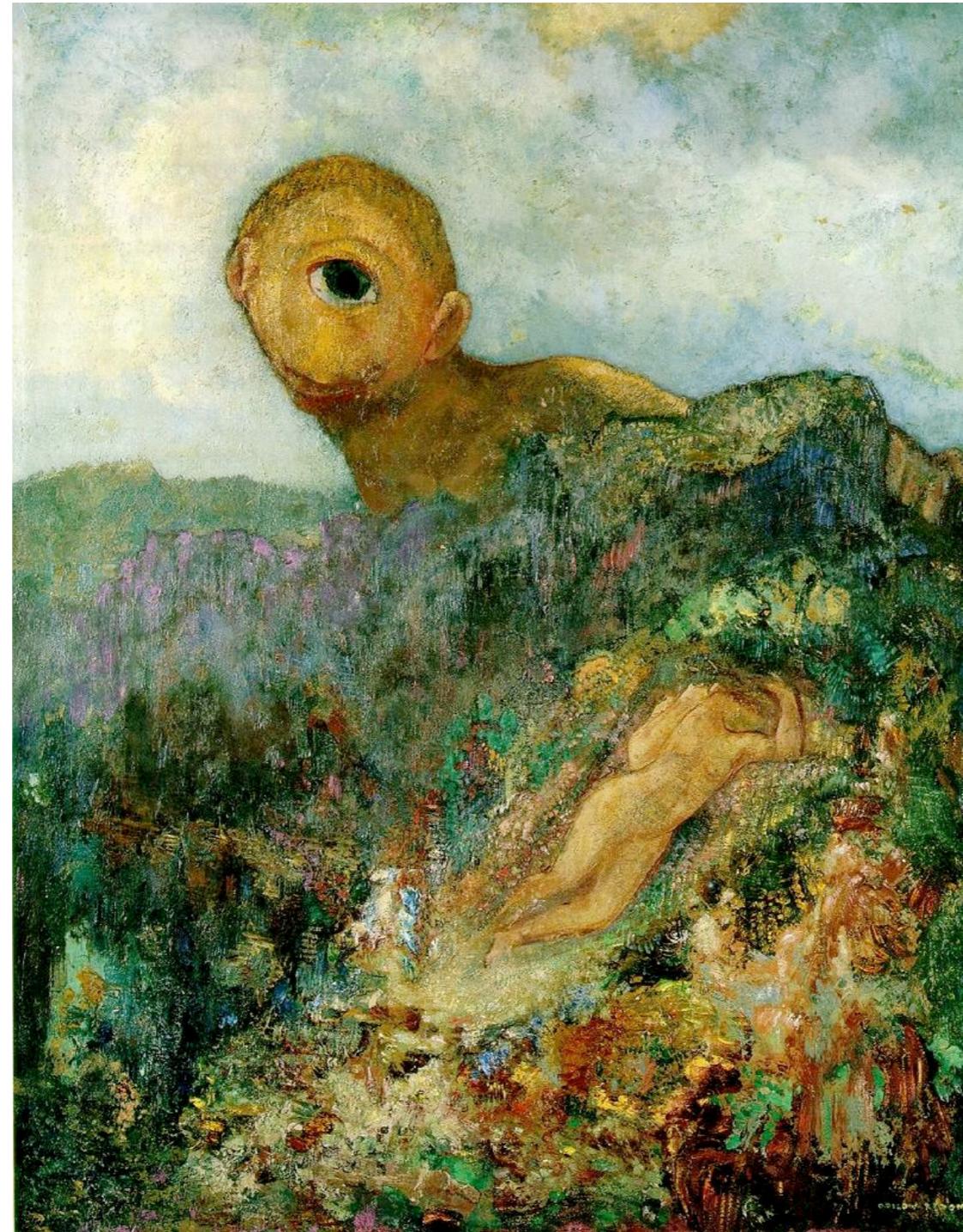
- ◆ What's the  
projection  
matrix?



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \Rightarrow (x, y)$$

Slide by Steve Seitz

# Single-view geometry

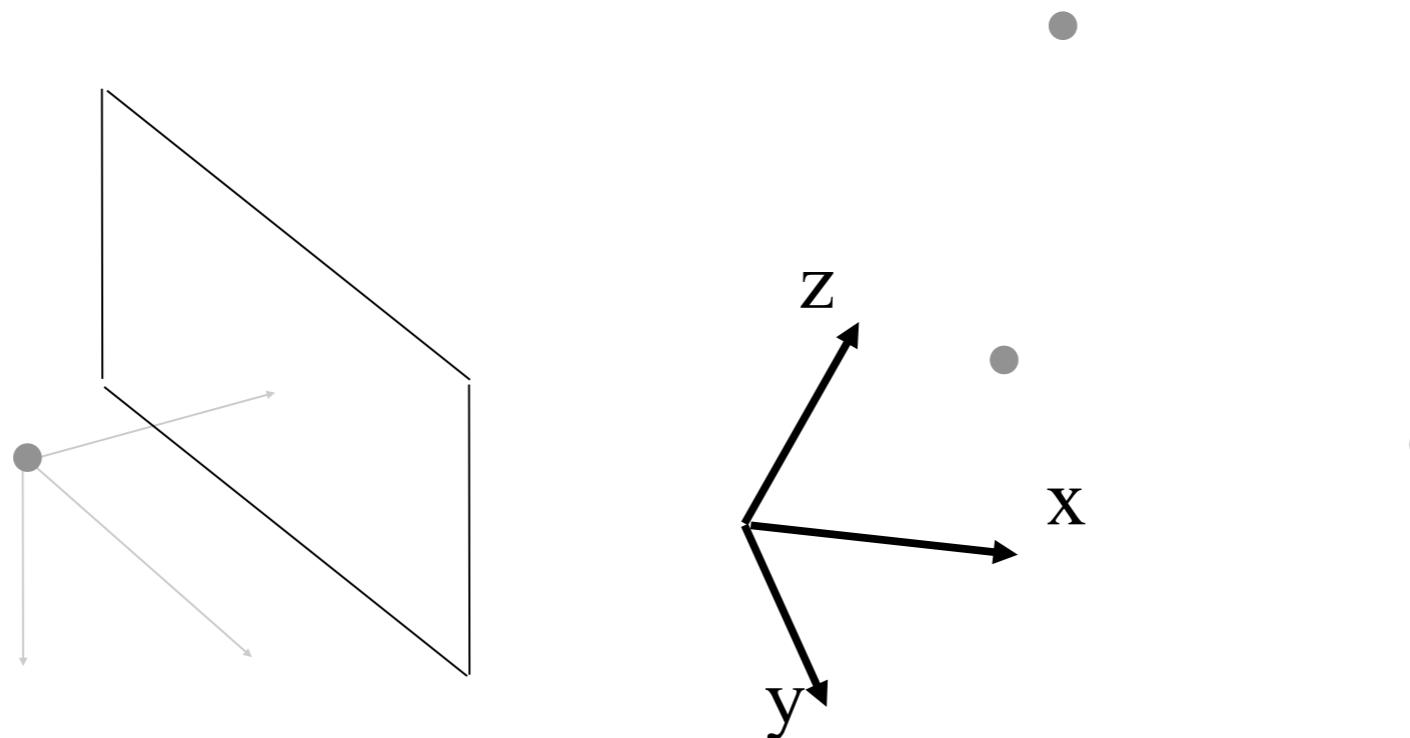


Odilon Redon, Cyclops, 1914

Slide by Lana Lazebnik

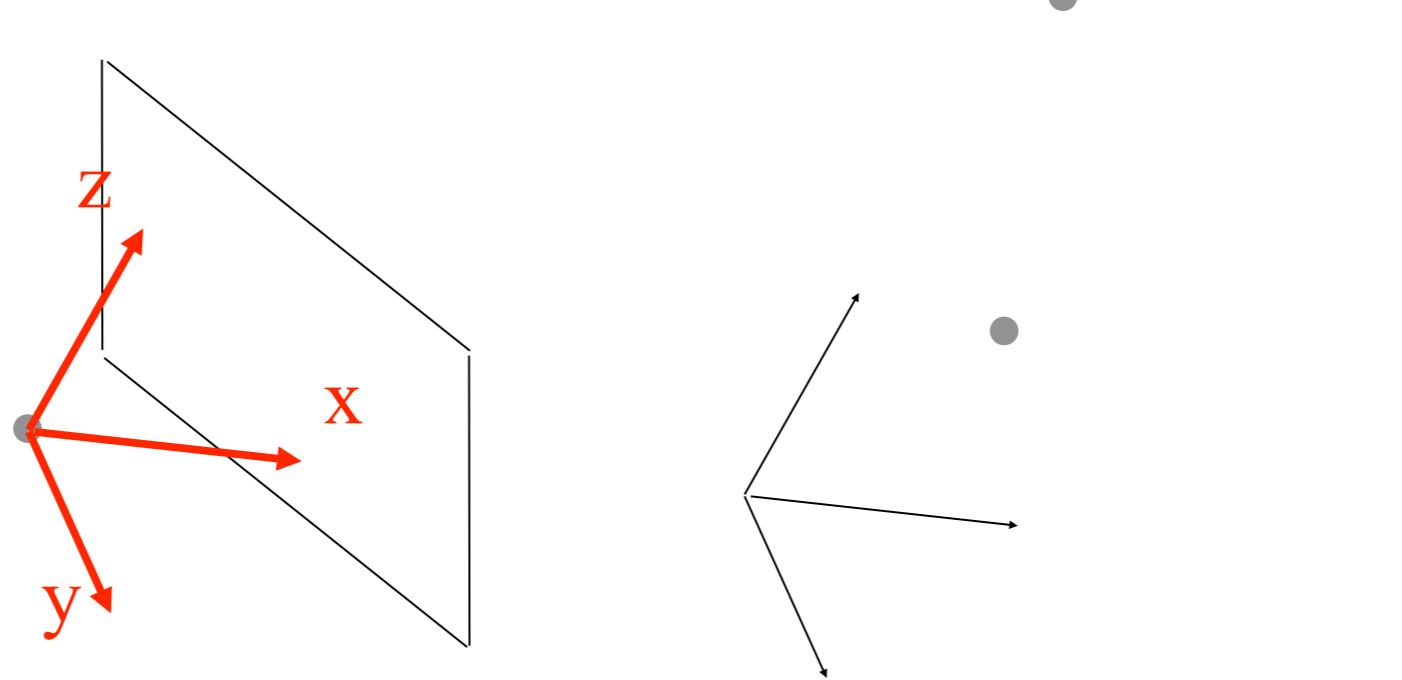
# Camera calibration

- ◆ Let's place the camera in the world
  - ◆ Translation
  - ◆ Rotation
  - ◆ Pinhole projection



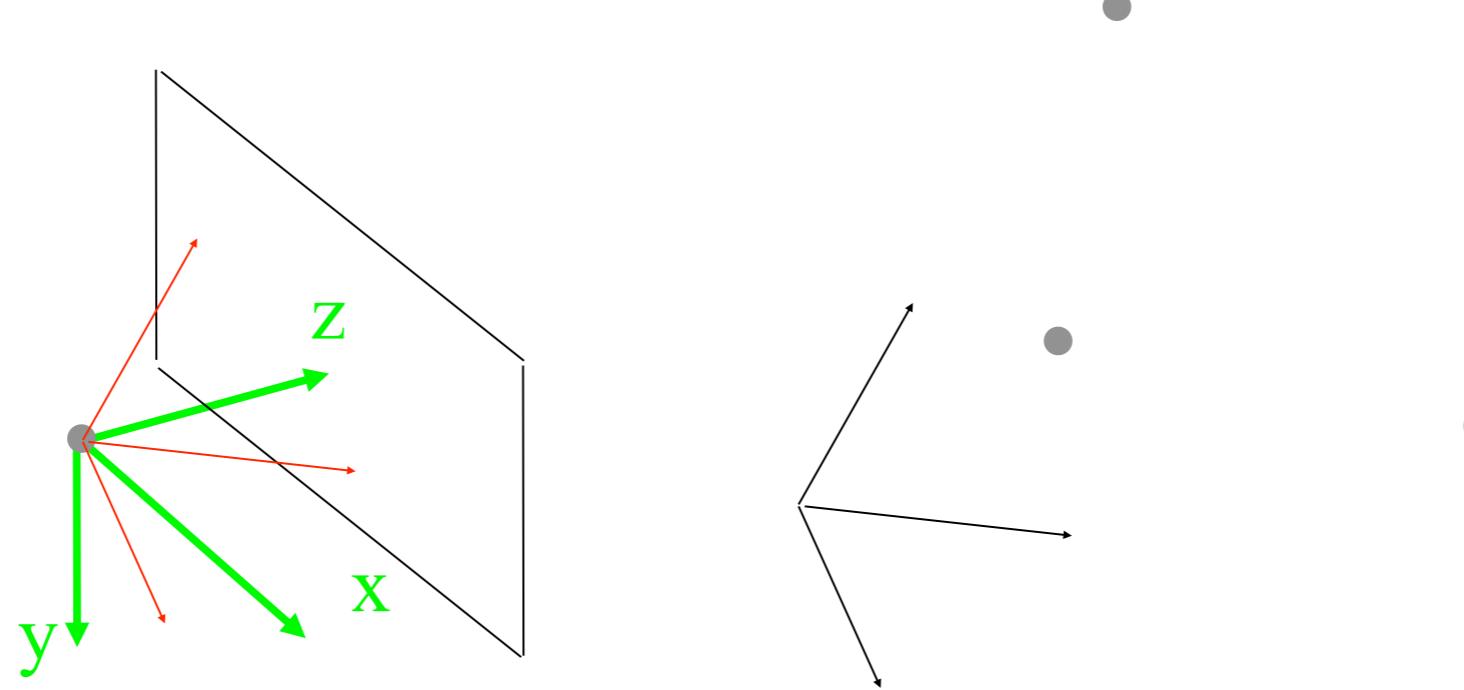
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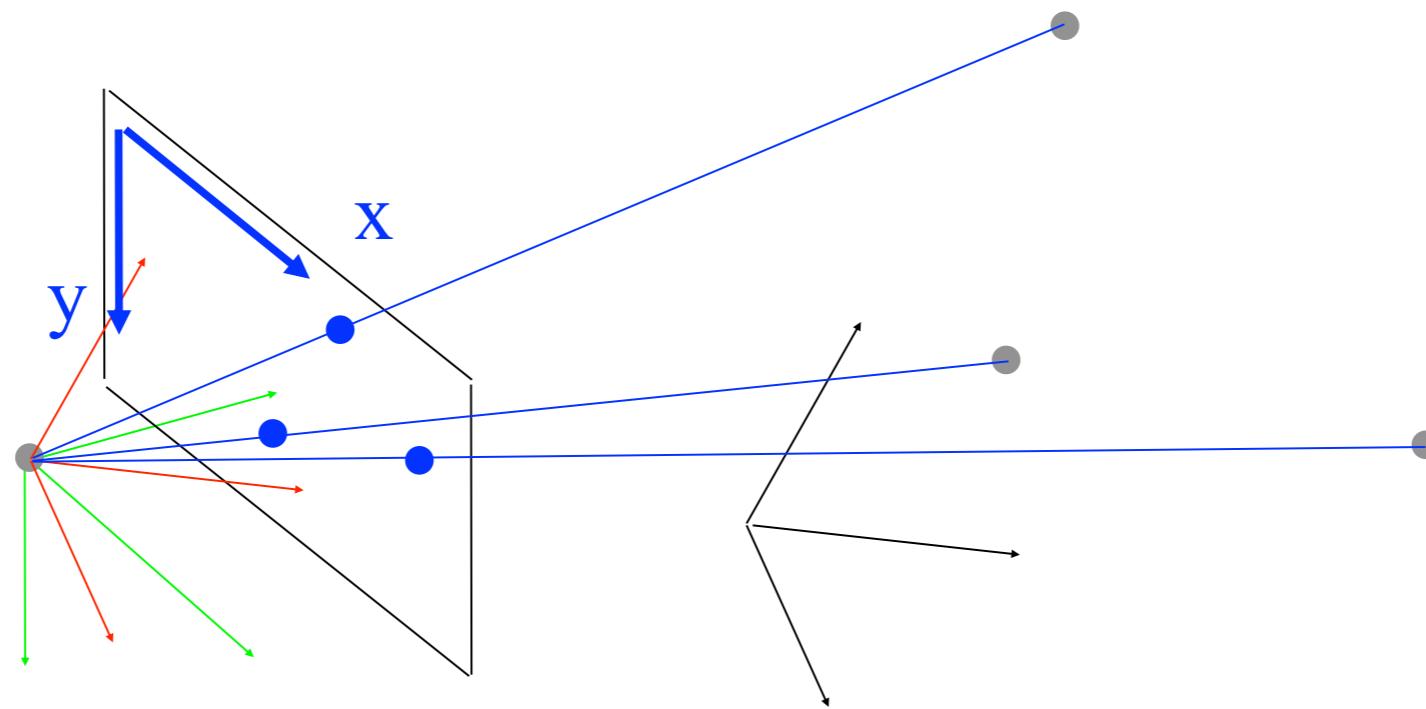
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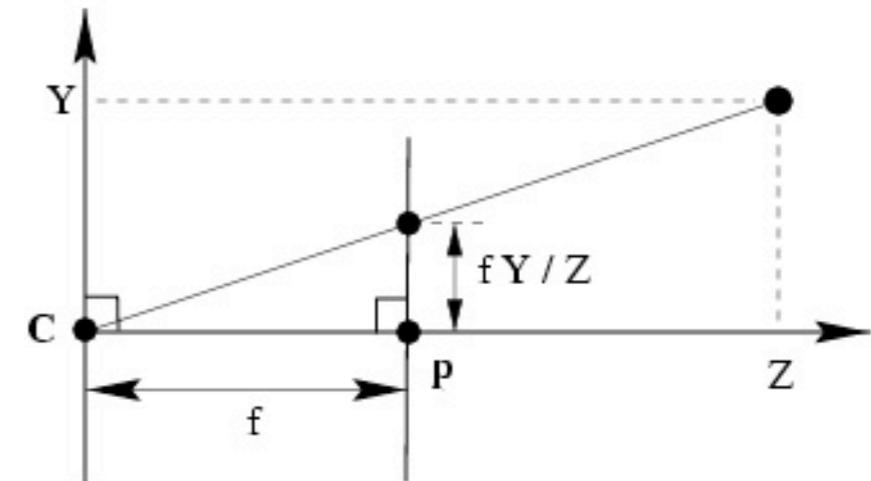
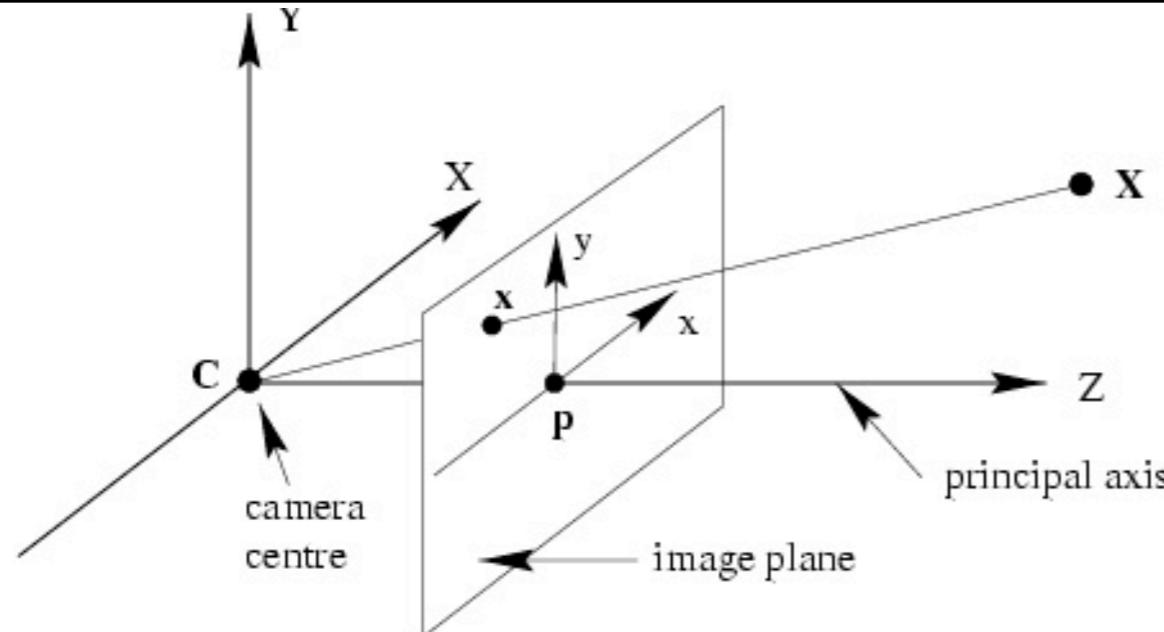


# Camera calibration

- ◆ Let's place the camera in the world
  - ◆ Translation
  - ◆ Rotation
  - ◆ Pinhole projection



# Recall: Pinhole camera model

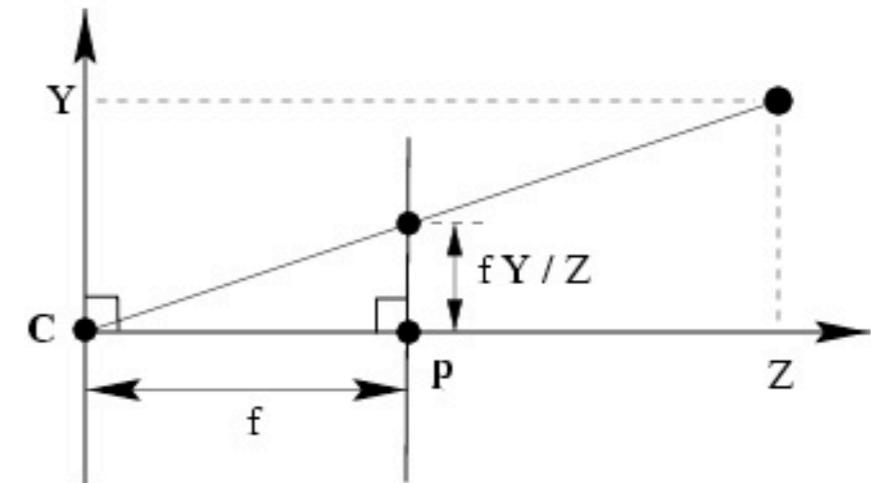
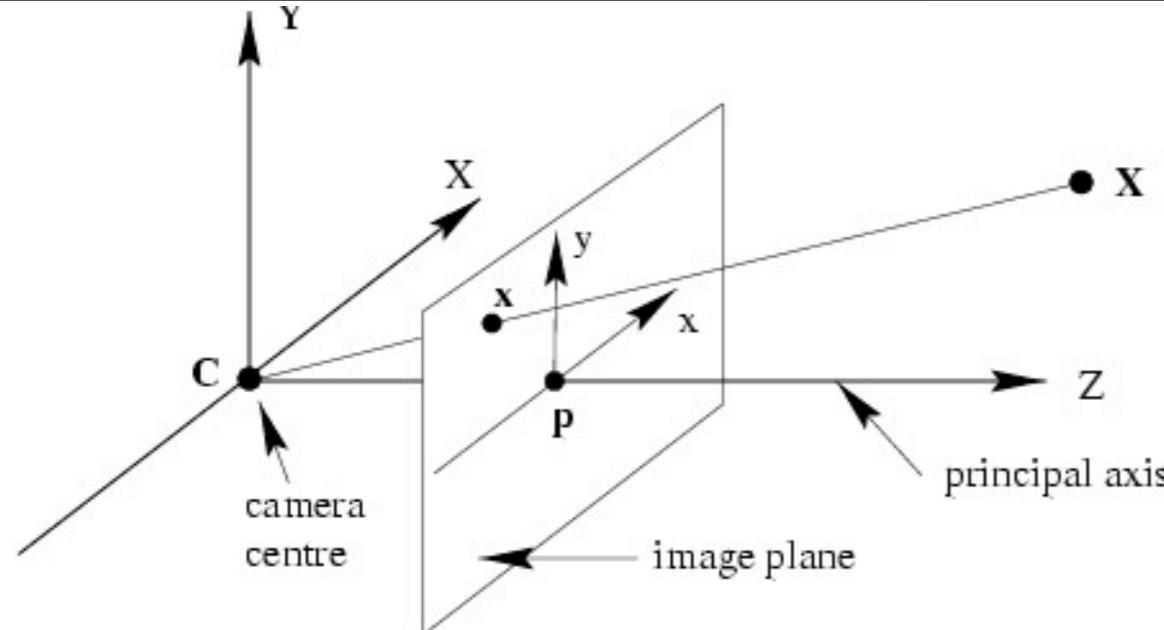


$$(X, Y, Z) \rightarrow (fX/Z, fY/Z)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} fX \\ fY \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathbf{x} = P\mathbf{X}$$

# Recall: Pinhole camera model

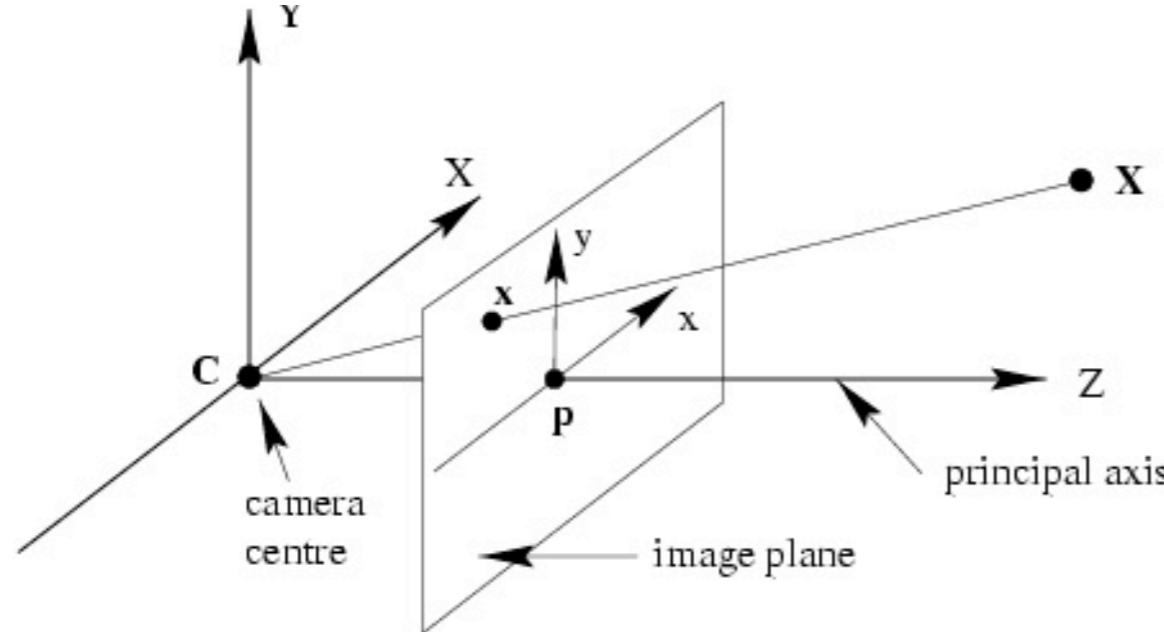


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$$\mathbf{x} = P\mathbf{X} \quad P = \text{diag}(f, f, 1)[I|0]$$

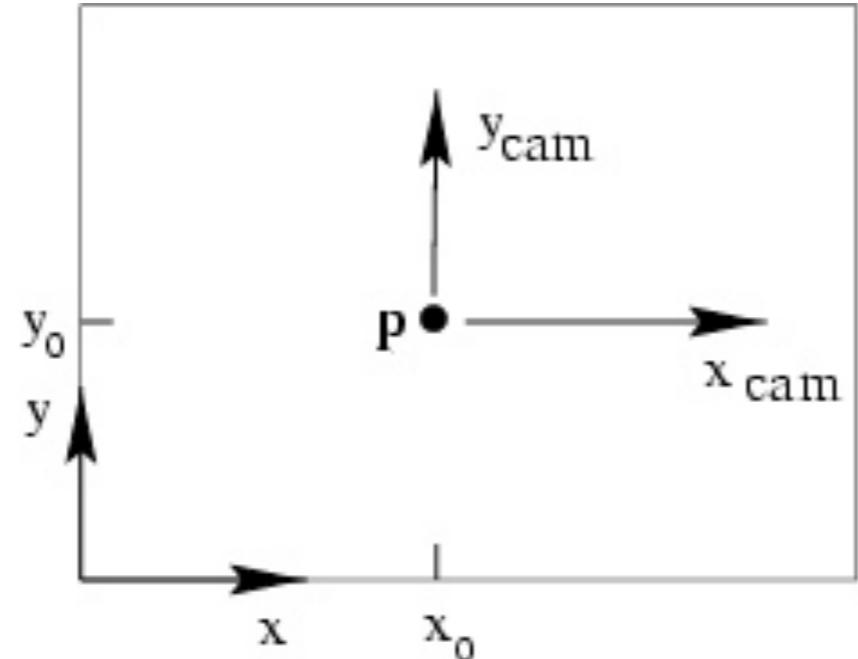
# Camera coordinate system



- ◆ **Principal axis:** line from the camera center perpendicular to the image plane
- ◆ **Normalized (camera) coordinate system:** camera center is at the origin and the principal axis is the z-axis
- ◆ **Principal point (p):** point where principal axis intersects the image plane (origin of normalized coordinate system)

Slide by Lana Lazebnik

# Principal point offset

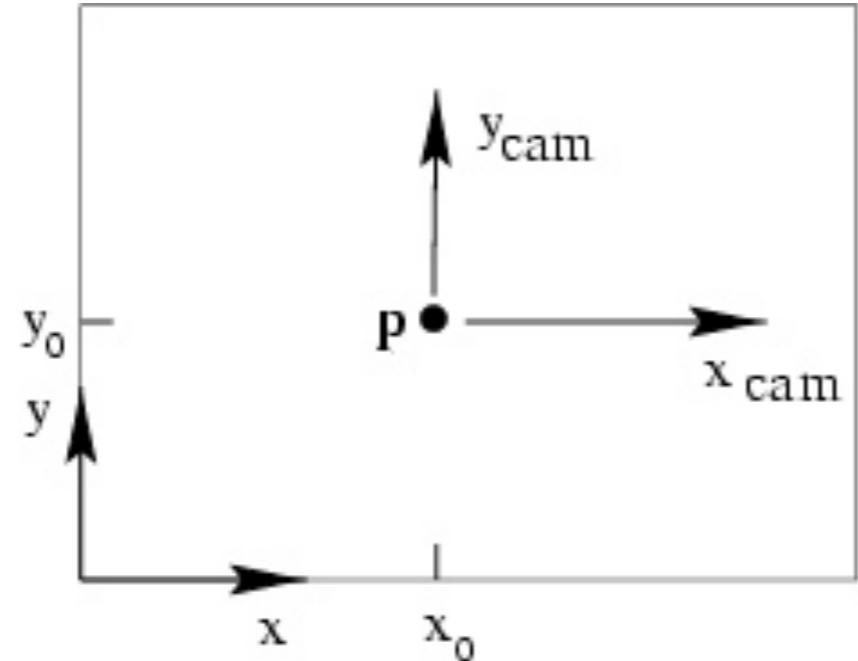


principal point:  $(p_x, p_y)$

- ◆ **Camera coordinate system:** origin is at the principal point
- ◆ **Image coordinate system:** origin is in the corner

Slide by Lana Lazebnik

# Principal point offset

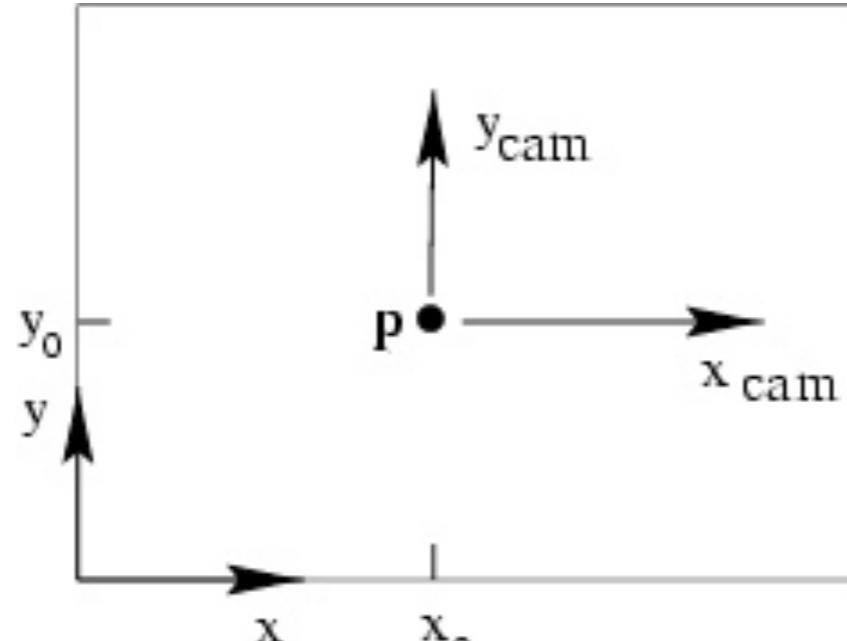


principal point:  $(p_x, p_y)$

$$(X, Y, Z) \rightarrow (fX/Z + p_x, fY/Z + p_y)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

# Principal point offset



principal point:  $(p_x, p_y)$

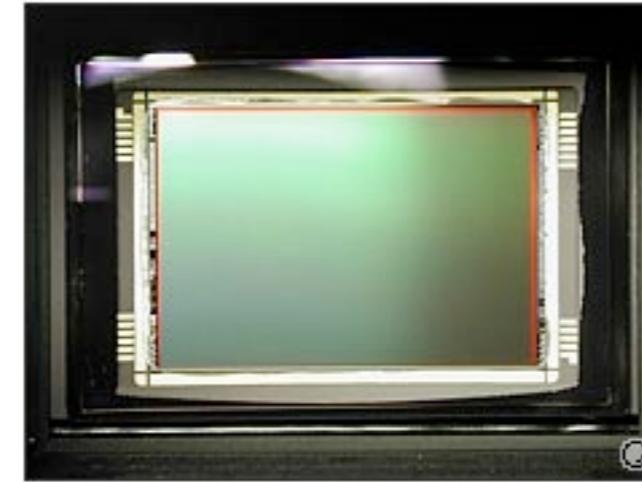
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$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

**calibration matrix**

$$P = K[I|0]$$

# Pixel coordinates



$$\text{Pixel size: } \frac{1}{m_x} \times \frac{1}{m_y}$$

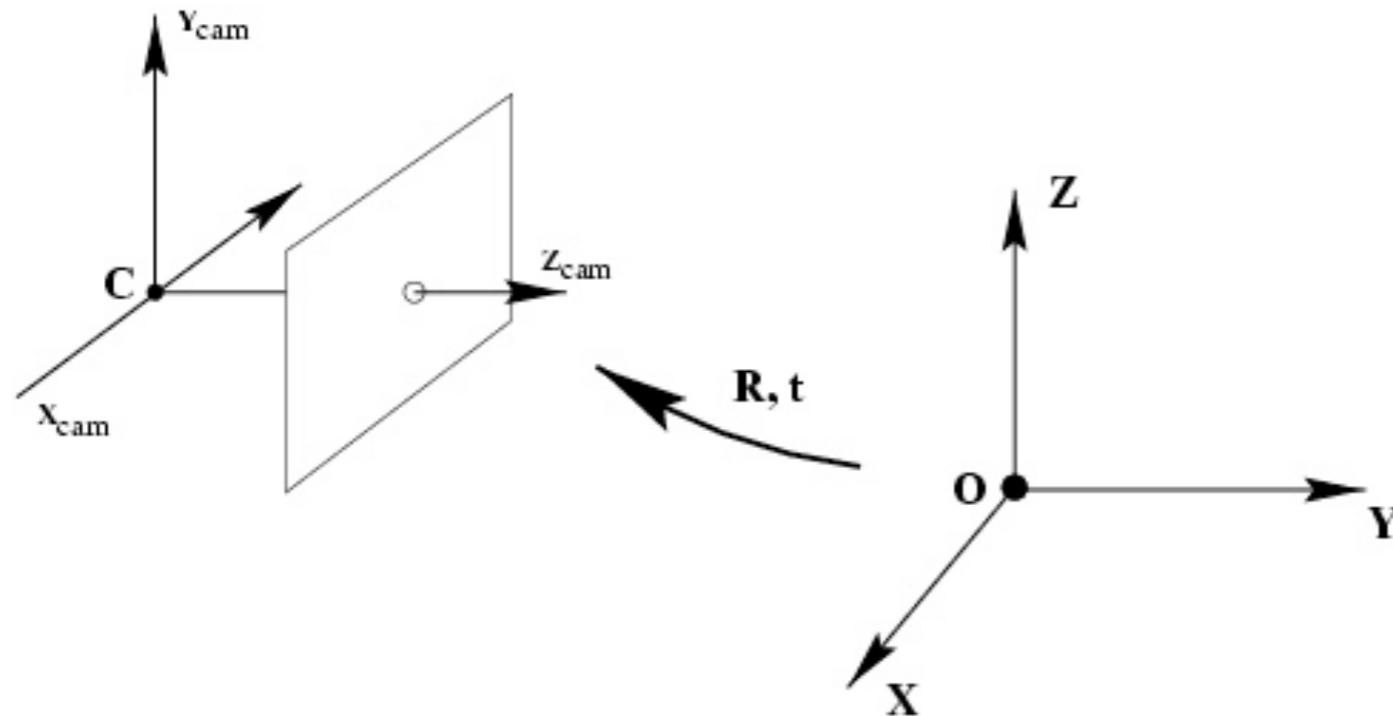
$m_x$  pixels per unit (m, mm, inch, ...) in horizontal direction,  
 $m_y$  pixels per unit in vertical direction

$$K = \begin{bmatrix} m_x \\ m_y \\ 1 \end{bmatrix} \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & \beta_x \\ \alpha_y & \beta_y \\ 1 \end{bmatrix}$$

pixels/unit                    units                    pixels

Slide by Lana Lazebnik

# Camera rotation and translation

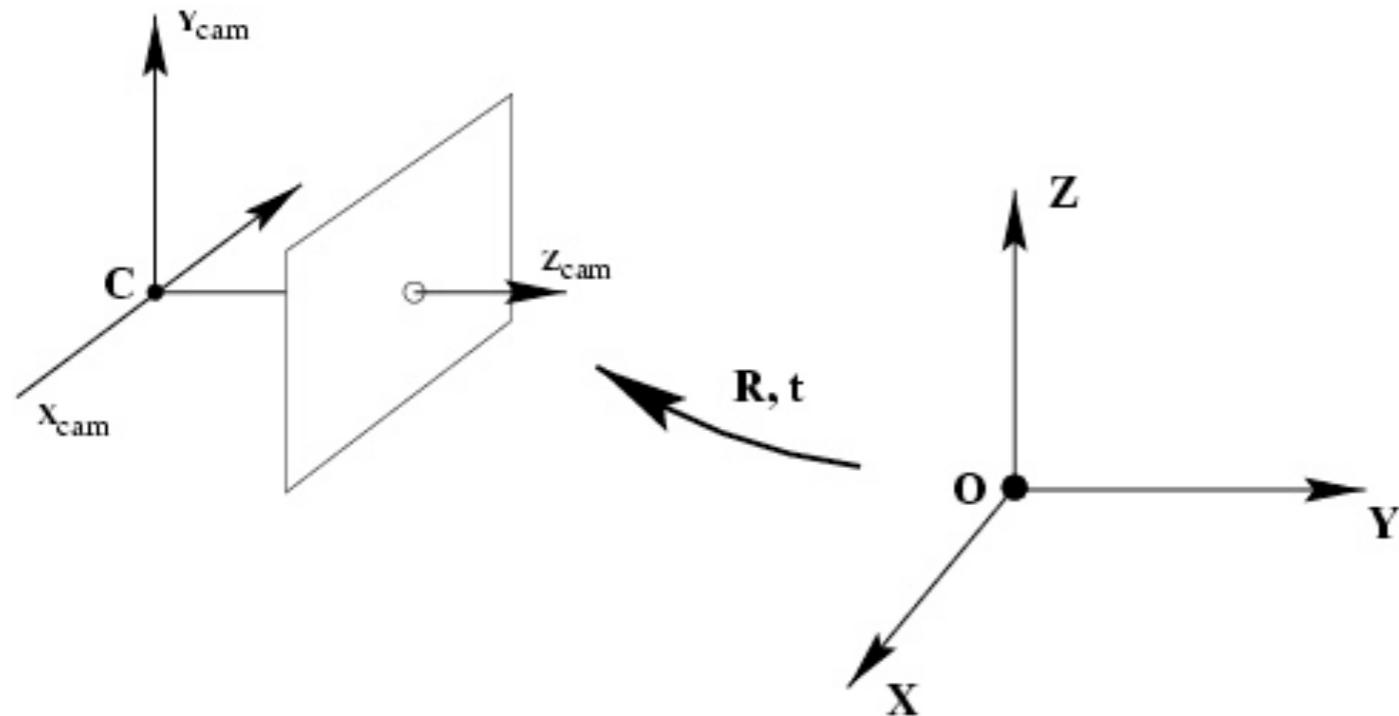


- The camera coordinate frame is related to the world coordinate frame by a rotation and a translation

$$\tilde{X}_{cam} = R(\tilde{X} - \tilde{C})$$

coords. of point in camera frame      coords. of a point in world frame (non-homogeneous)      coords. of camera center in world frame

# Camera rotation and translation



In non-homogeneous  
coordinates:

$$\tilde{X}_{cam} = R(\tilde{X} - \tilde{C})$$

$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \tilde{X} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

$$x = K[I|0]X_{cam} = K[R|-R\tilde{C}]X \quad P = K[R|t] \quad t = -R\tilde{C}$$

Note:  $C$  is the null space of the camera projection matrix ( $PC = 0$ )

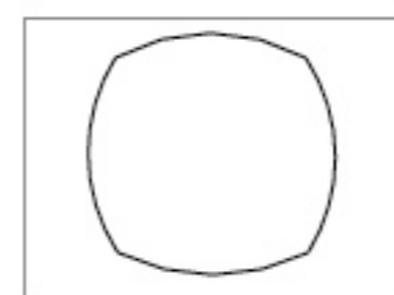
# Camera parameters

- ◆ Intrinsic parameters
  - ◆ Principal point coordinates
  - ◆ Focal length
  - ◆ Pixel magnification factors
  - ◆ Skew (non-rectangular pixels)
  - ◆ Radial distortion

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & \beta_x \\ \alpha_y & \beta_y \\ 1 \end{bmatrix}$$

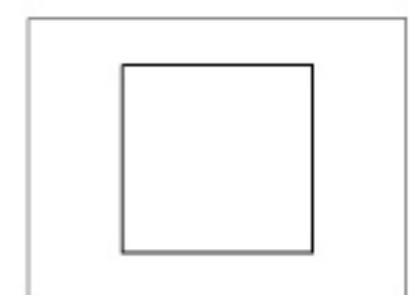


radial distortion



correction

linear image



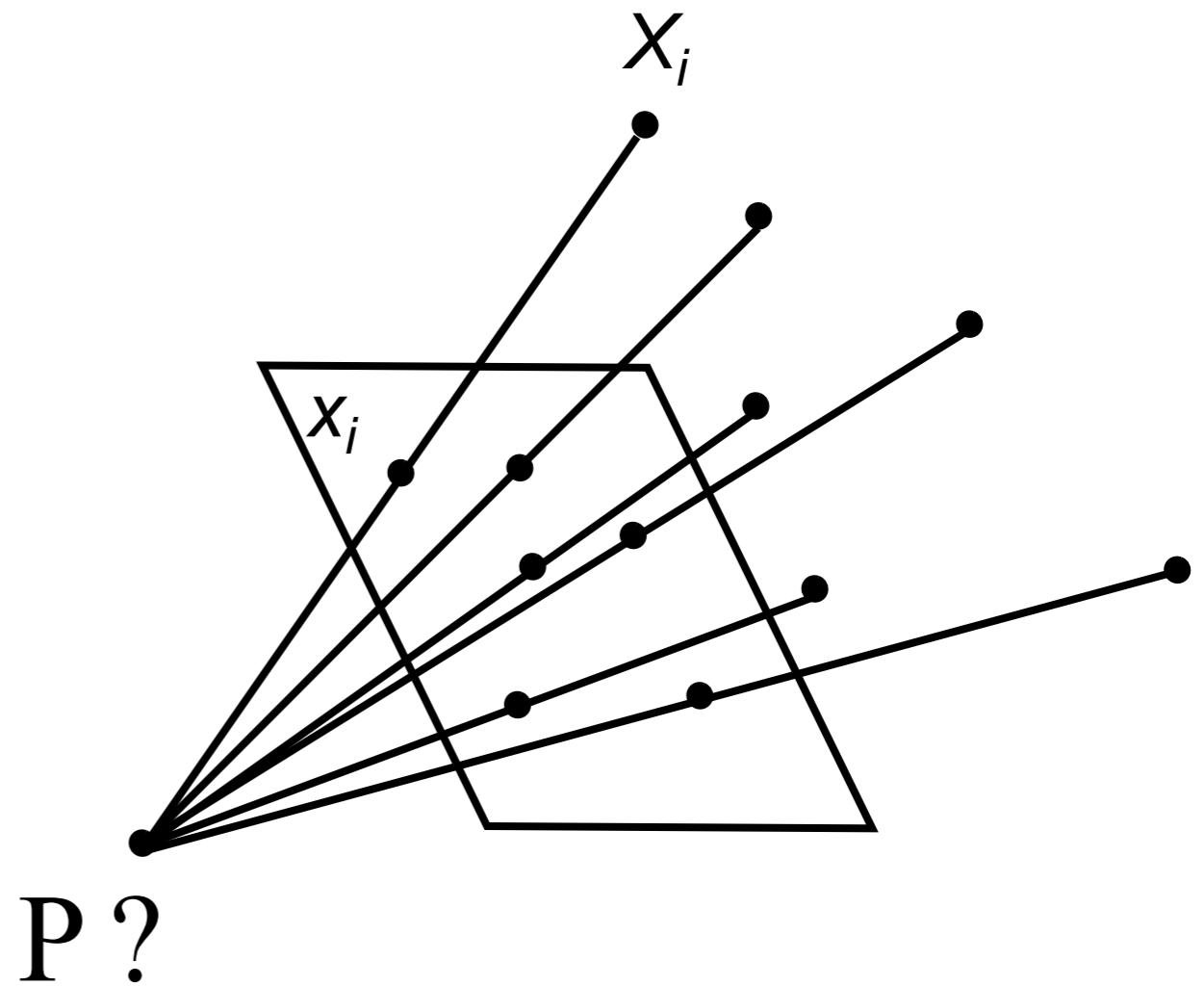
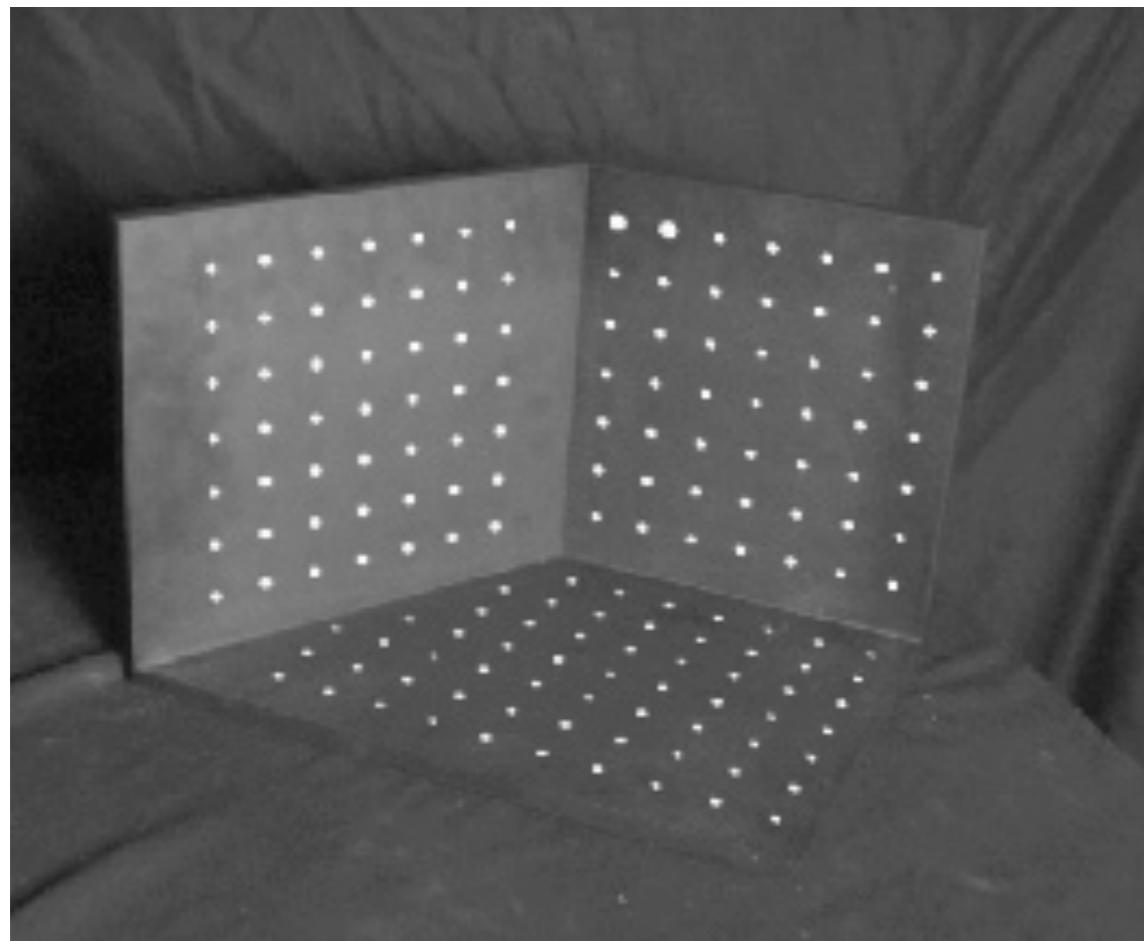
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- ◆ Intrinsic parameters
  - ◆ Principal point coordinates
  - ◆ Focal length
  - ◆ Pixel magnification factors
  - ◆ Skew (non-rectangular pixels)
  - ◆ Radial distortion
- ◆ Extrinsic parameters
  - ◆ Rotation and translation relative to world coordinate system

Slide by Lana Lazebnik

# Camera calibration

- Given  $n$  points with known 3D coordinates  $X_i$  and known image projections  $x_i$ , estimate the camera parameters



Slide by Lana Lazebnik

# Camera calibration

- ◆ What have we got now?
  - ◆ We have a projection matrix  $P$ , which maps any point in the 3D world coordinate frame onto the (infinite) image plane of the camera
  - ◆ Given a 3D point in the world, we can find out where its projection is in the image
  - ◆ One goal of vision: given a point in the images, find out where there **pre-image** is in the world
- ◆ We cannot do this from one image
  - ◆ The depth is lost, the projection matrix can only tell us the ray through an image point  $x$

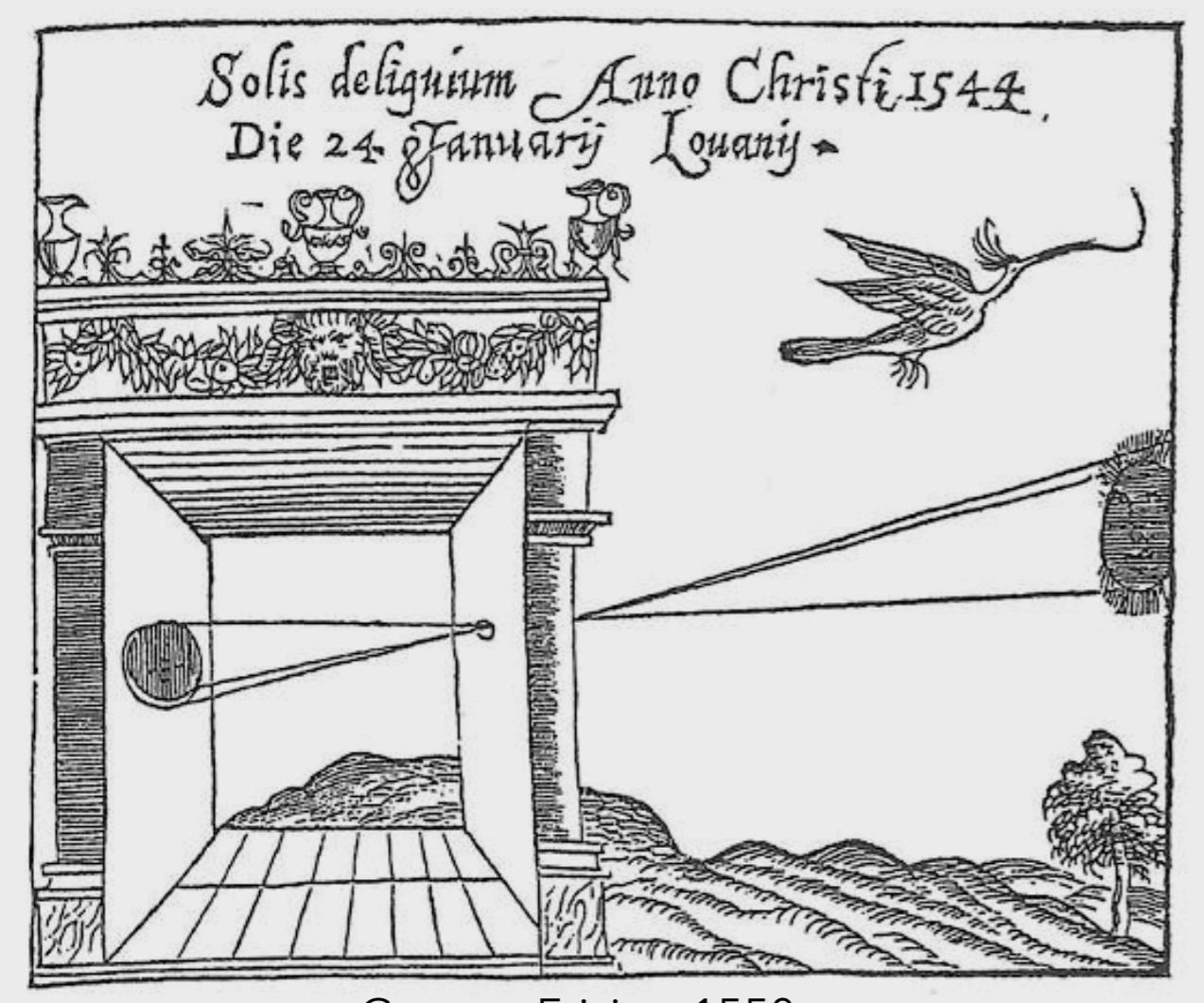


# Building a real camera



Slide by Lana Lazebnik

# Camera Obscura



- ◆ Basic principle known to Mozi (470-391 BC) and Aristoteles (384-322 BC)
- ◆ Described by Ibn al-Haytham (Alhazen) in his "Book of optics" (around 1000AD).
- ◆ Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros

# Abelardo Morell



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- ◆ Camera Obscura Image of Manhattan View  
Looking South in Large Room, 1996

[http://www.abelardomorell.net/camera\\_obscura1.html](http://www.abelardomorell.net/camera_obscura1.html)

After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."

From *Grand Images Through a Tiny Opening*, Photo District News, February 2005

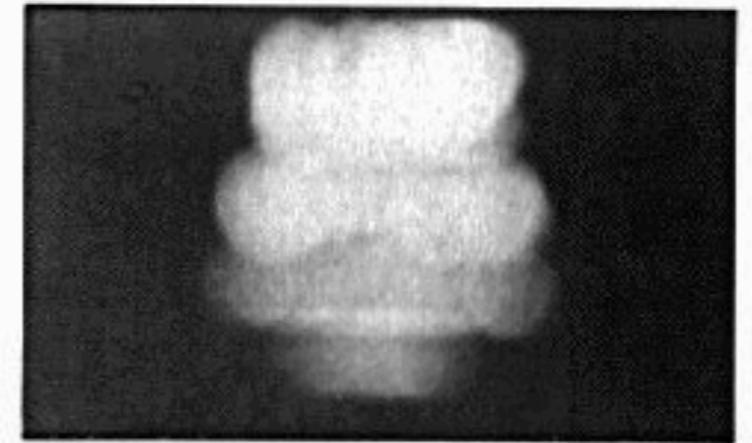
# Home-made pinhole camera



Why so  
blurry?

<http://www.debevec.org/Pinhole/>

# Shrinking the aperture



2 mm



1 mm



0.6mm

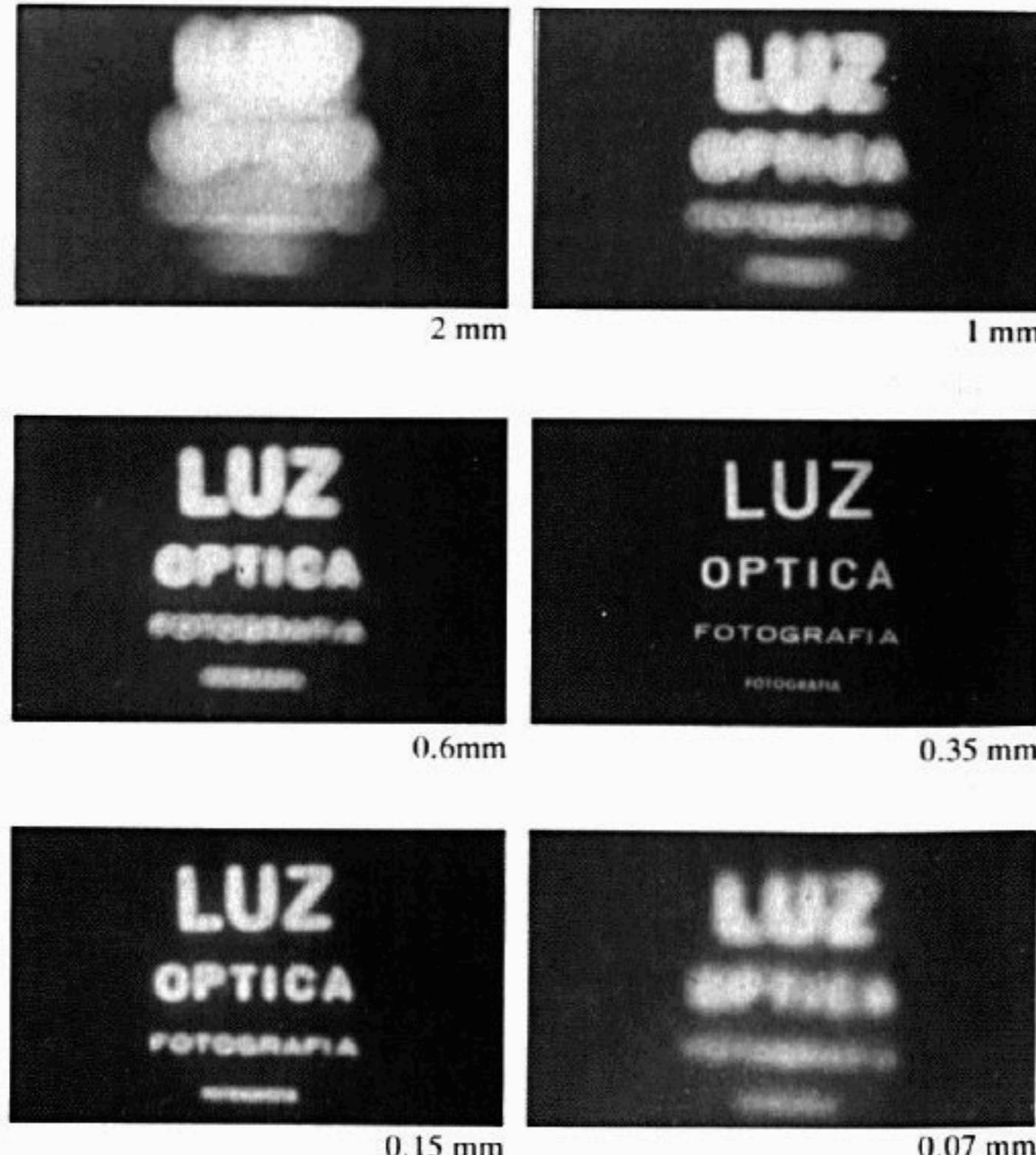


0.35 mm

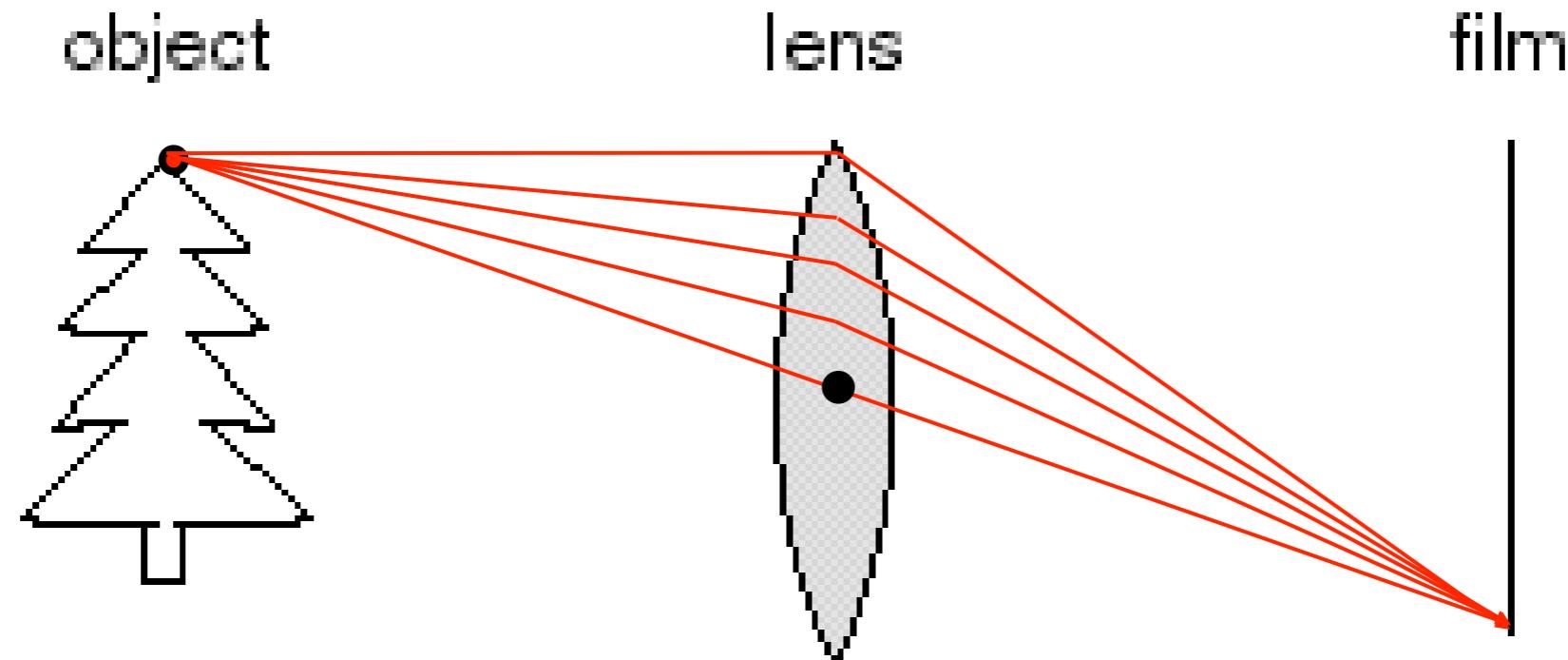
- ◆ Why not make the aperture as small as possible?
  - ◆ Less light gets through
  - ◆ Diffraction effects...

Slide by Steve Seitz

# Shrinking the aperture



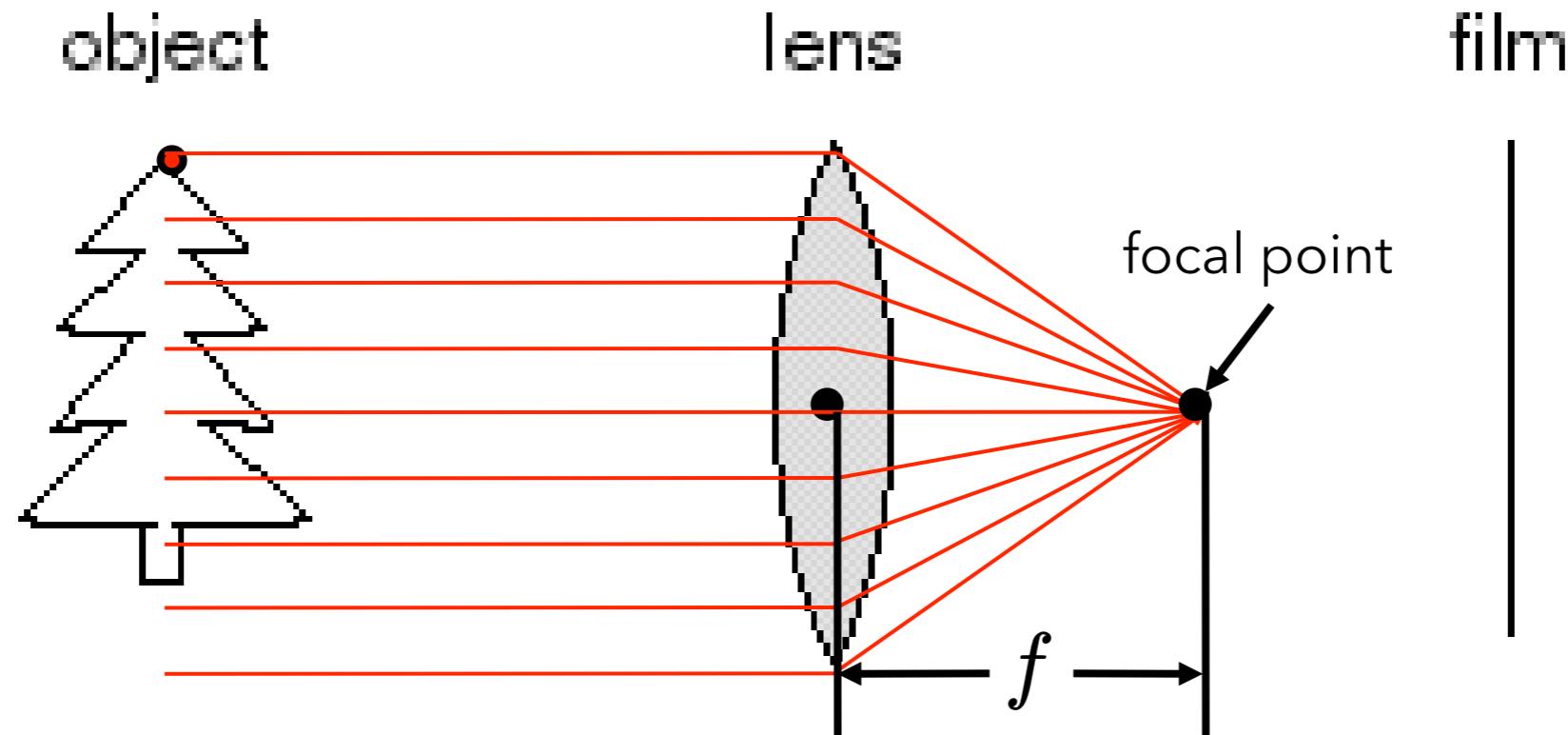
# Adding a lens



- ◆ A lens focuses light onto the film
  - ◆ Rays passing through the center are not deviated

Slide by Steve Seitz

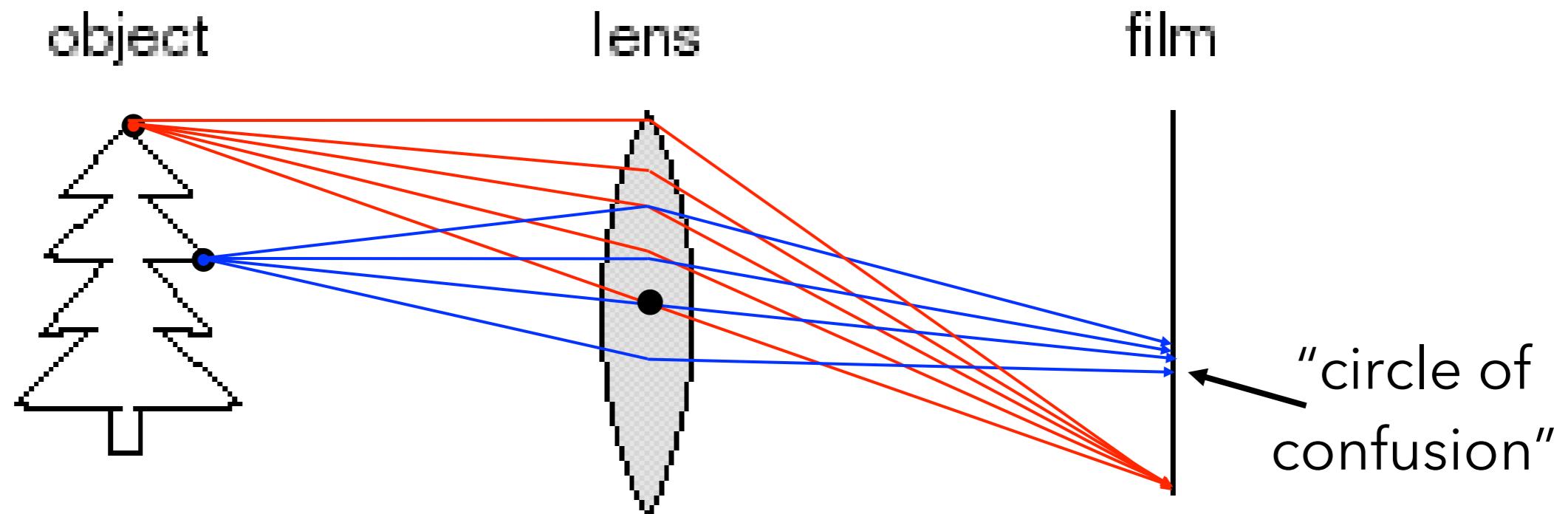
# Adding a lens



- ◆ A lens focuses light onto the film
  - ◆ Rays passing through the center are not deviated
  - ◆ All parallel rays converge to one point on a plane located at the **focal length**  $f$

Slide by Steve Seitz

# Adding a lens



- ◆ A lens focuses light onto the film
  - ◆ There is a specific distance at which objects are “in focus”
  - ◆ other points project to a “circle of confusion” in the image

Slide by Steve Seitz

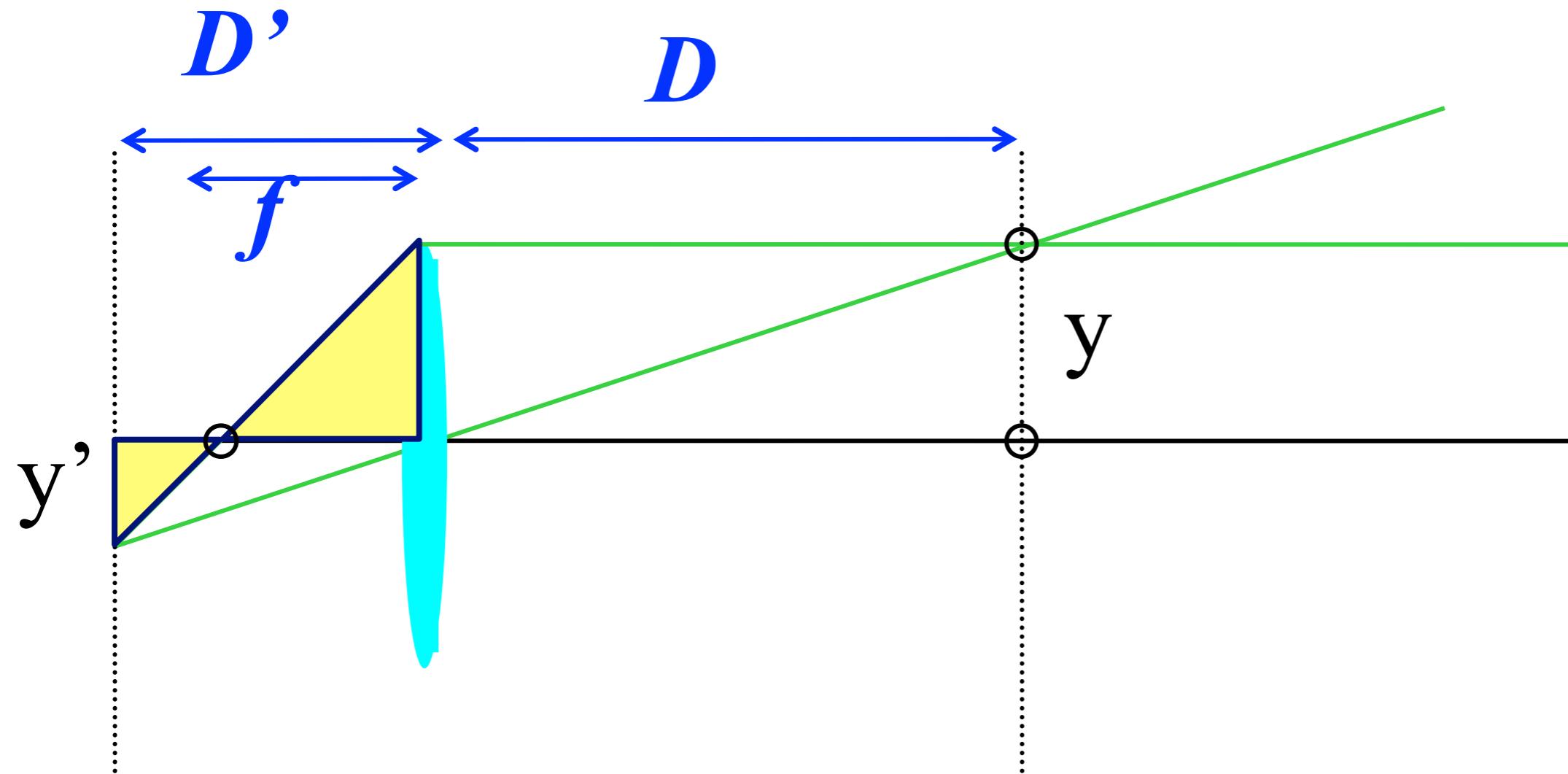
# Thin lens formula



Similar triangles everywhere!

$$\frac{y'}{y} = \frac{D'}{D}$$

$$\frac{y'}{y} = \frac{D' - f}{f}$$

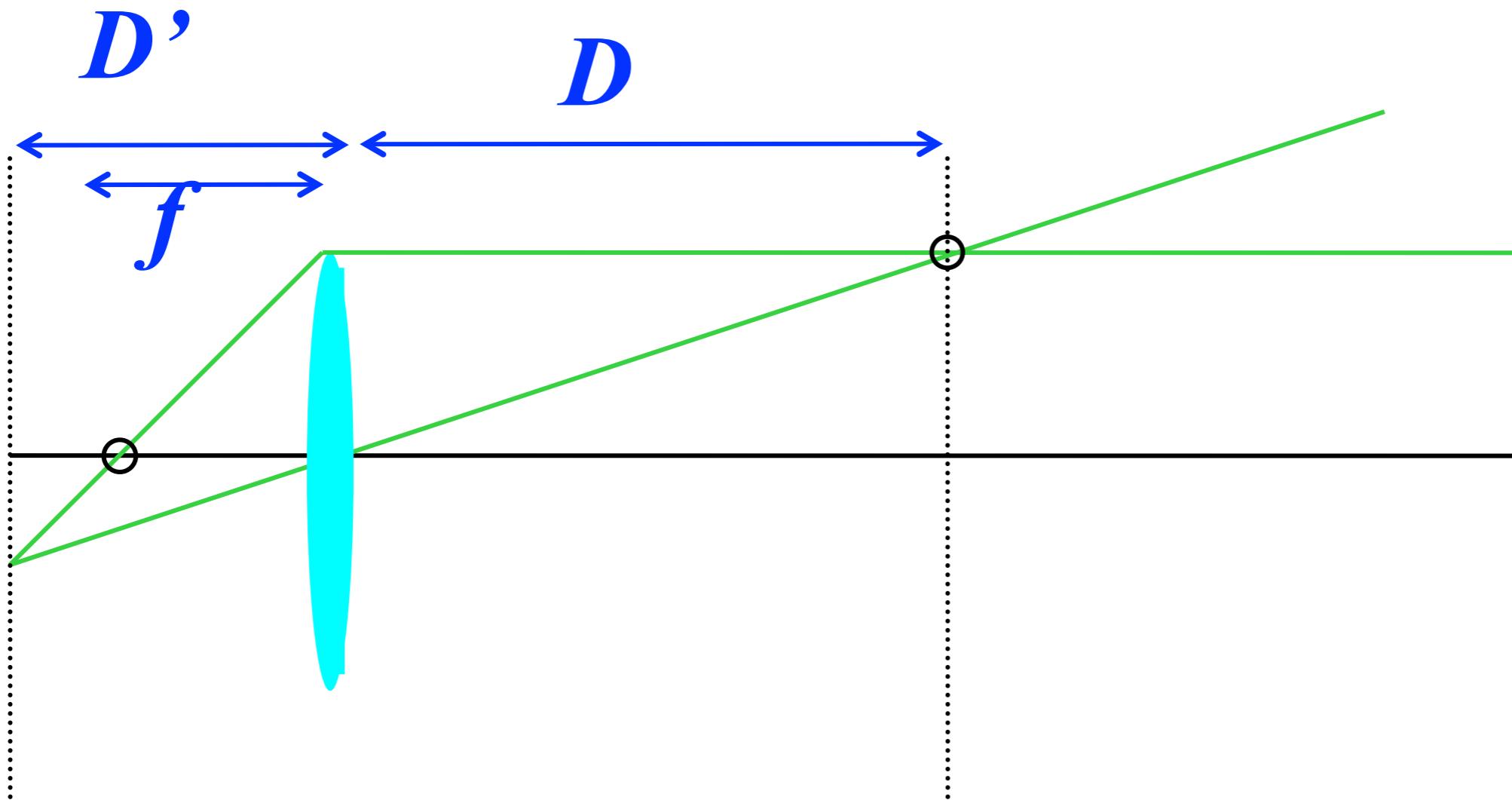


Slide from Frédo Durand

# Thin lens formula

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

Any point satisfying the **thin lens equation** is in focus.



Slide from Frédo Durand

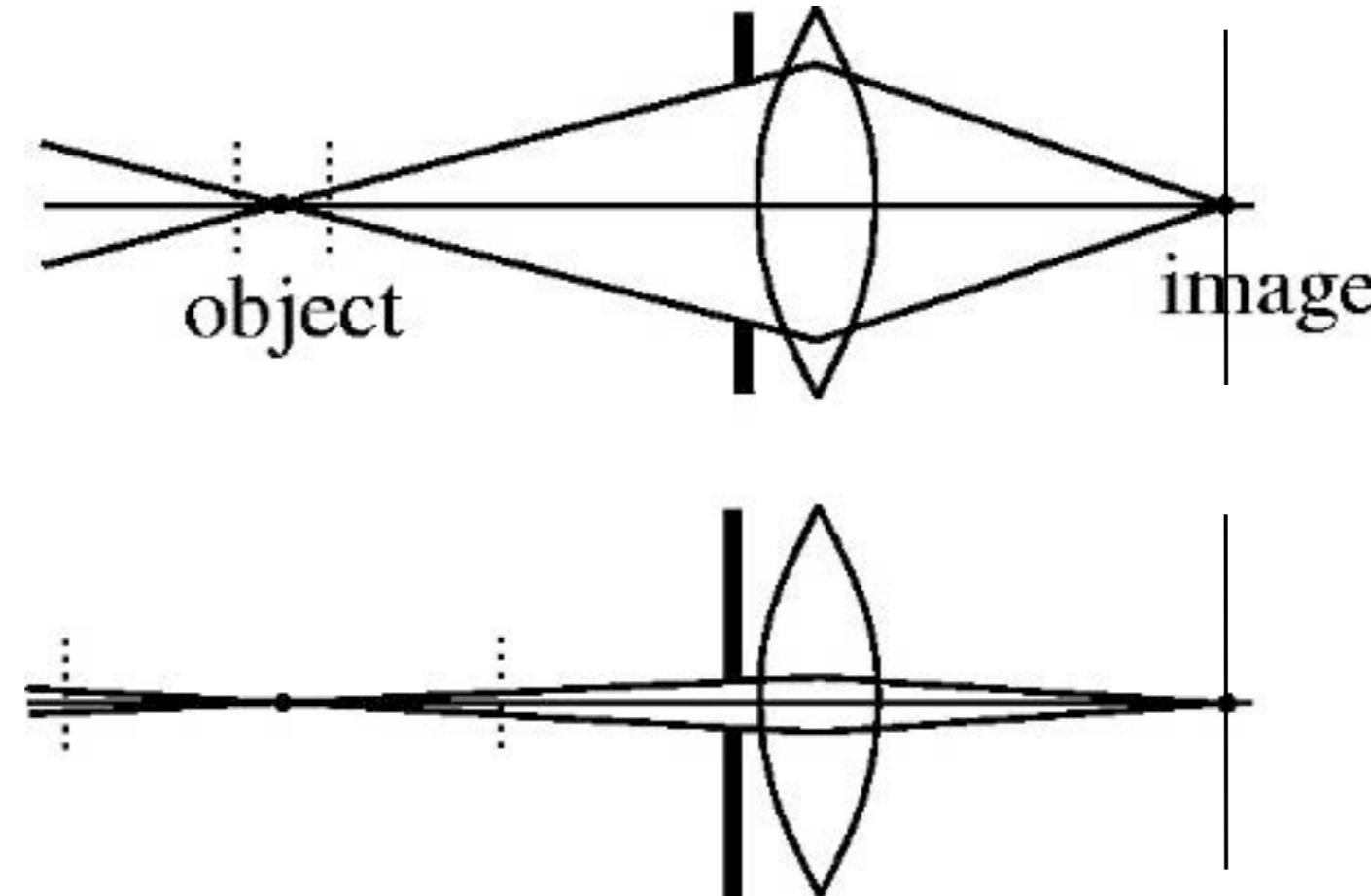
# Depth of Field



<http://www.cambridgeincolour.com/tutorials/depth-of-field.htm>

Slide by A. Efros

# How can we control the depth of field?



- ◆ Changing the aperture size affects depth of field
  - ◆ A smaller aperture increases the range in which the object is approximately in focus
  - ◆ But small aperture reduces amount of light - need to increase exposure

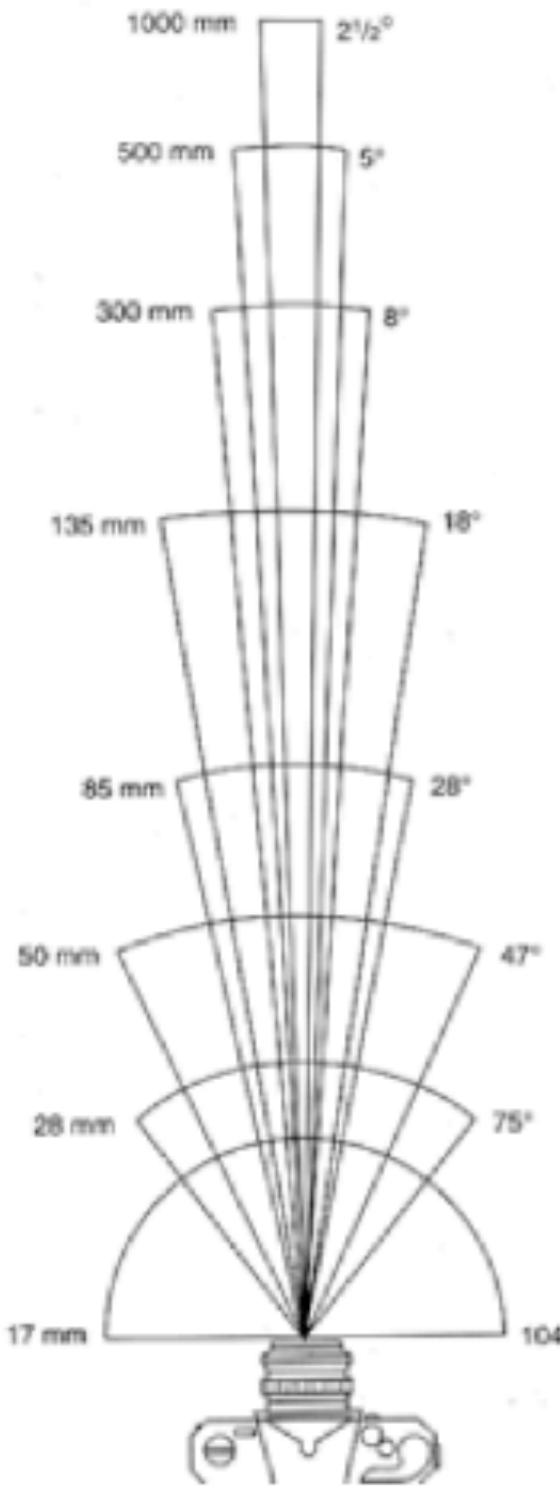
Slide by A. Efros

# Depth of Field Effect



Source: F. Durand

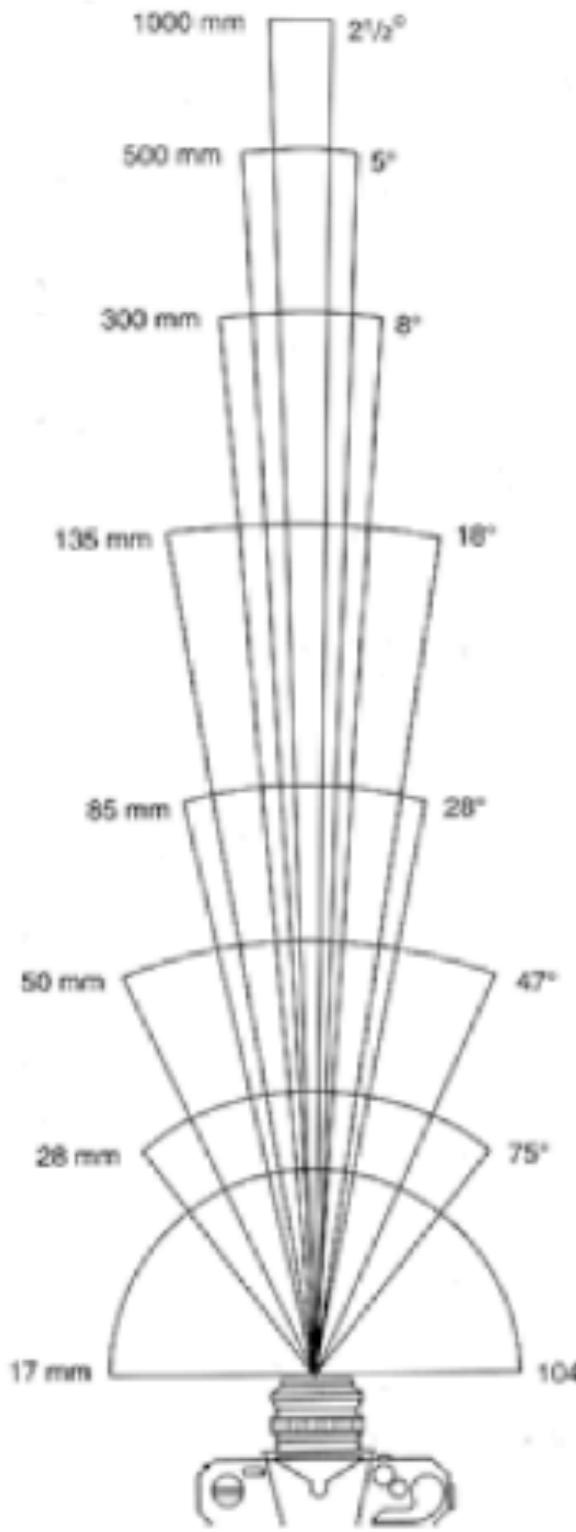
# Field of View (Zoom)



**From London and Upton**

Slide by A. Efros

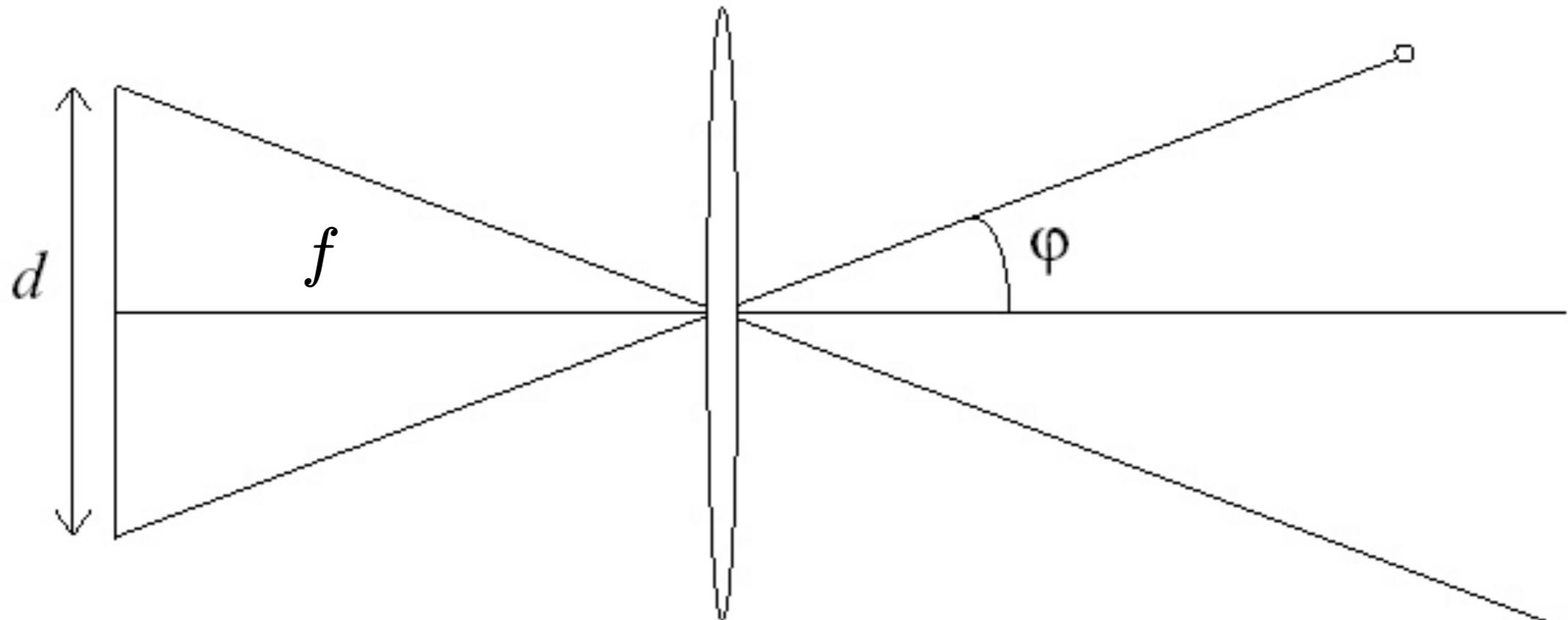
# Field of View (Zoom)



**From London and Upton**

Slide by A. Efros

# Field of View



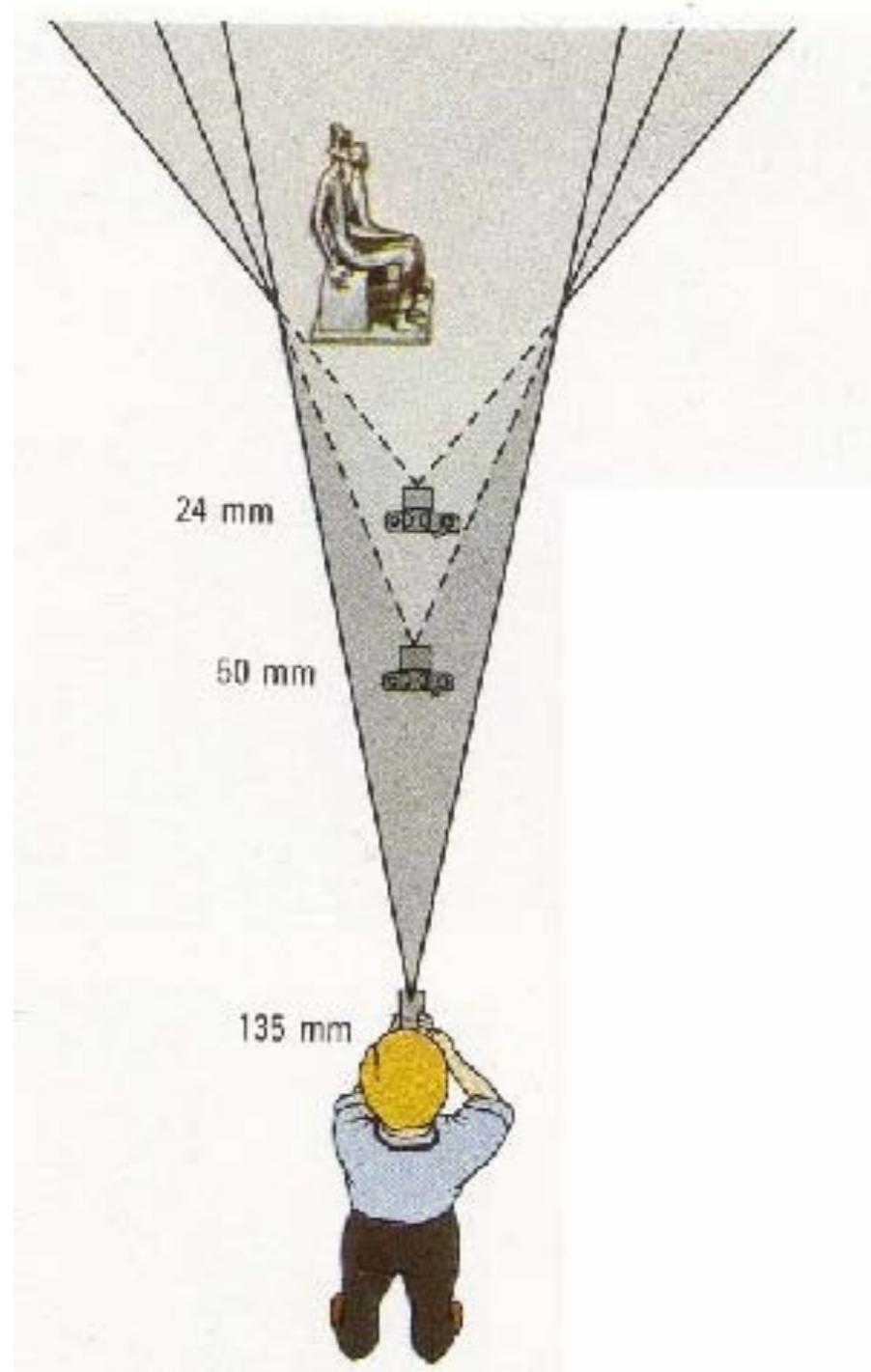
FOV depends on focal length and  
the size of the camera retina (sensor)

$$\phi = \tan^{-1} \left( \frac{d}{2f} \right)$$

Smaller FOV = larger Focal Length

Slide by A. Efros

# Field of View / Focal Length



Large FOV, small  $f$   
Camera close to car



Small FOV, large  $f$   
Camera far from the car

# Same effect for faces



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wide-angle



standard



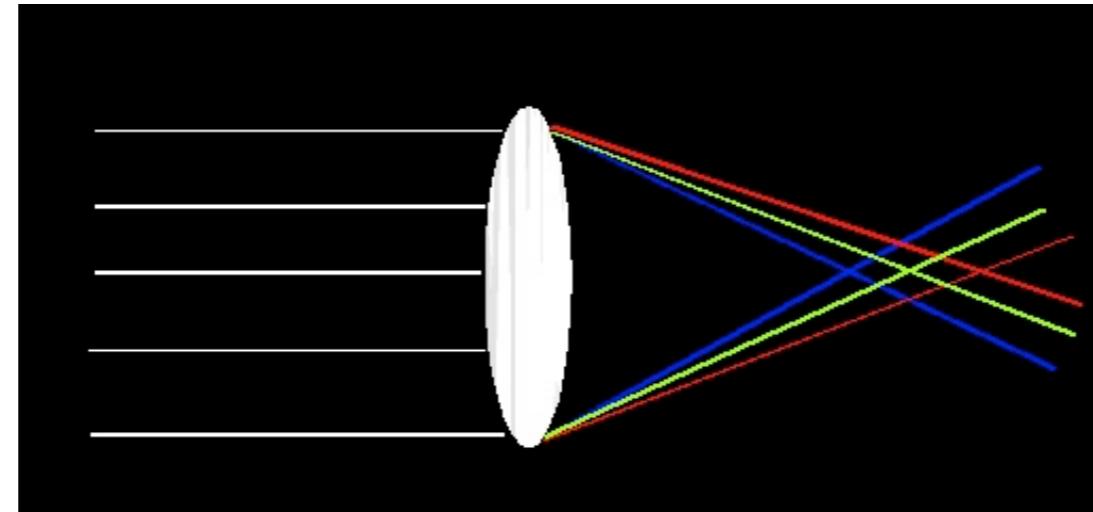
telephoto

Source: F. Durand

# Lens Flaws: Chromatic Aberration



- ◆ Lens has different refractive indices for different wavelengths: causes color fringing



near lens center

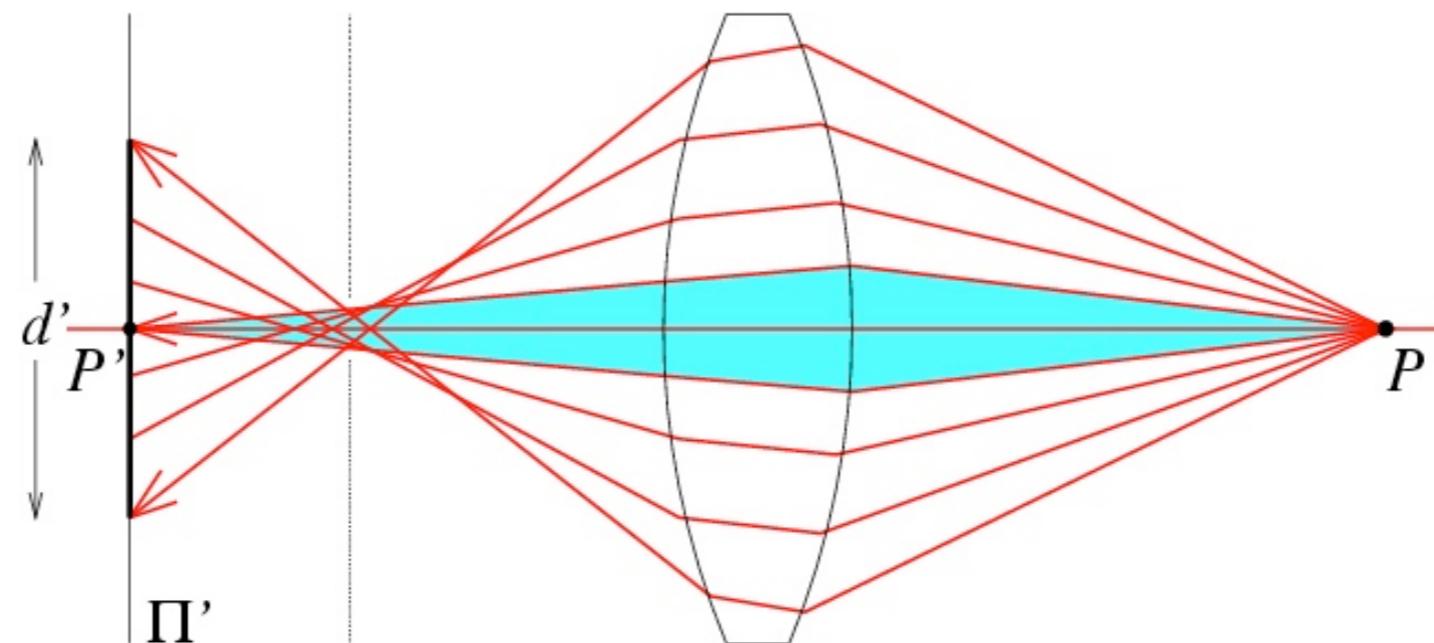


near lens outer edge



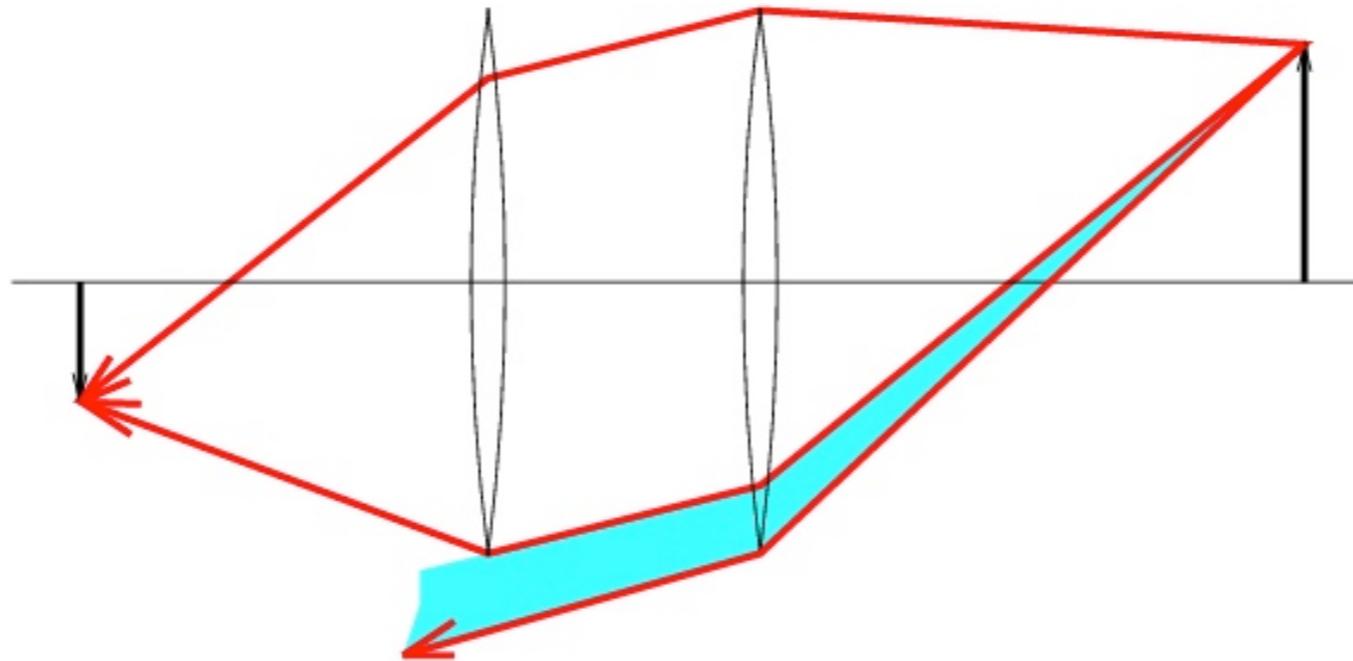
# Lens flaws: Spherical aberration

- ◆ Spherical lenses don't focus light perfectly
  - ◆ Rays farther from the optical axis focus closer



Slide by Lana Lazebnik

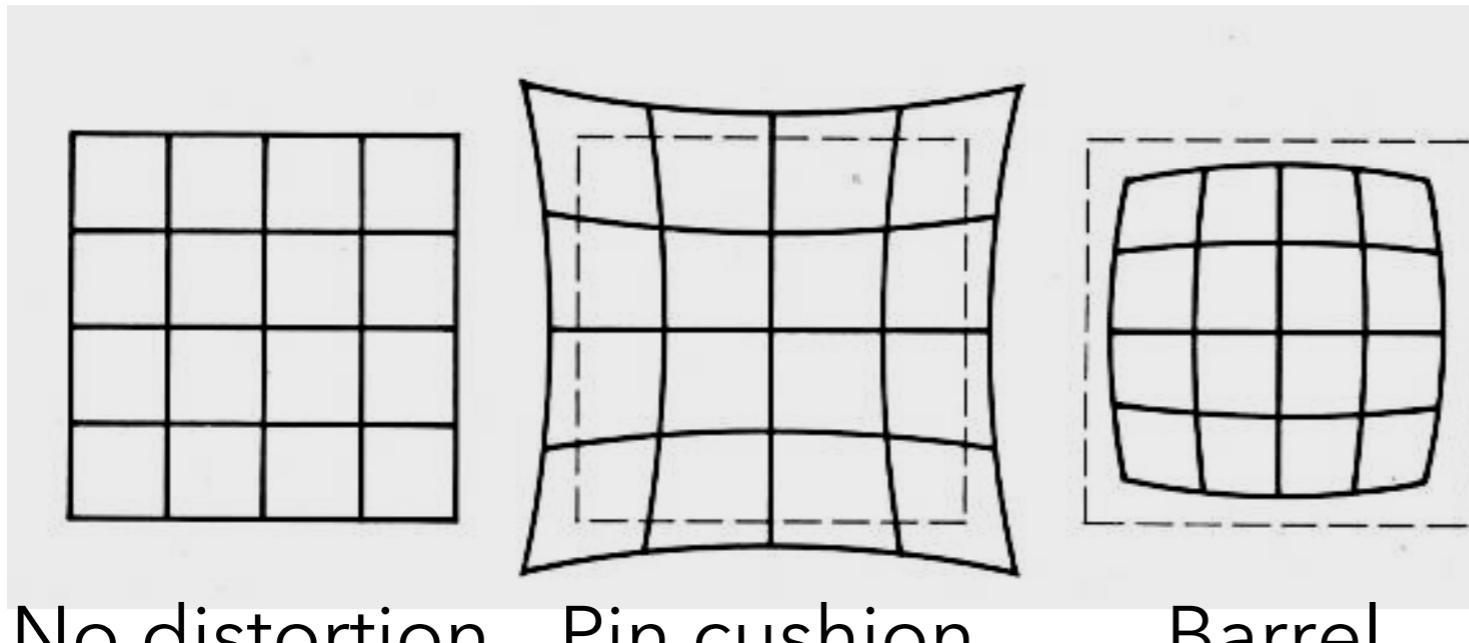
# Lens flaws: Vignetting



Slide by Lana Lazebnik

# Radial Distortion

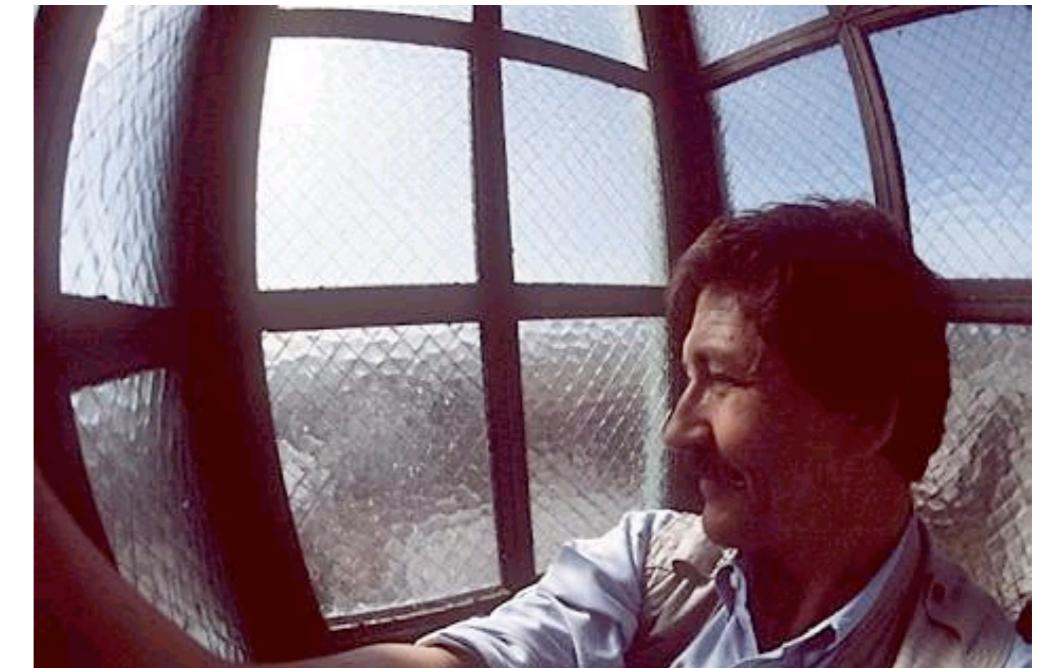
- ◆ Caused by imperfect lenses
- ◆ Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

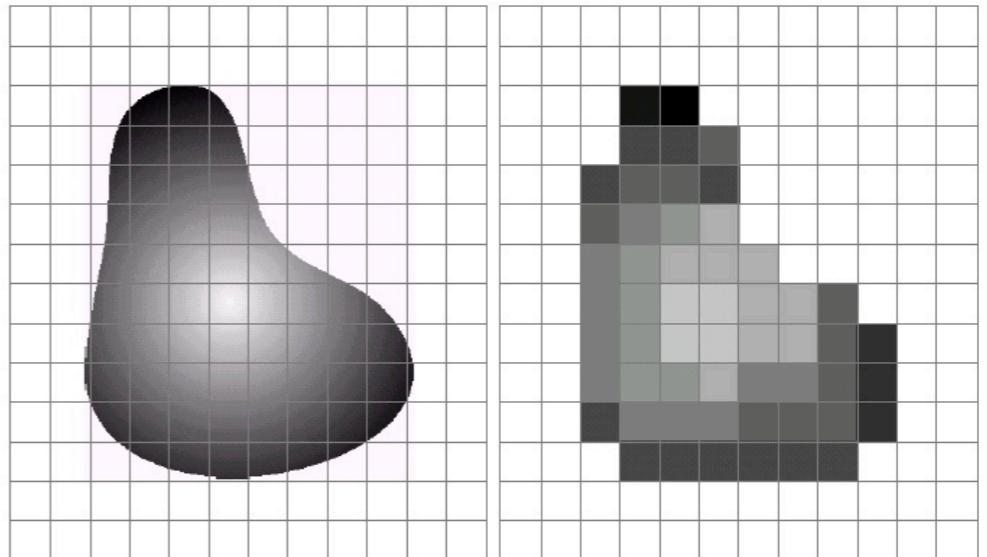
Pin cushion

Barrel



Slide by Lana Lazebnik

# Digital camera



a b

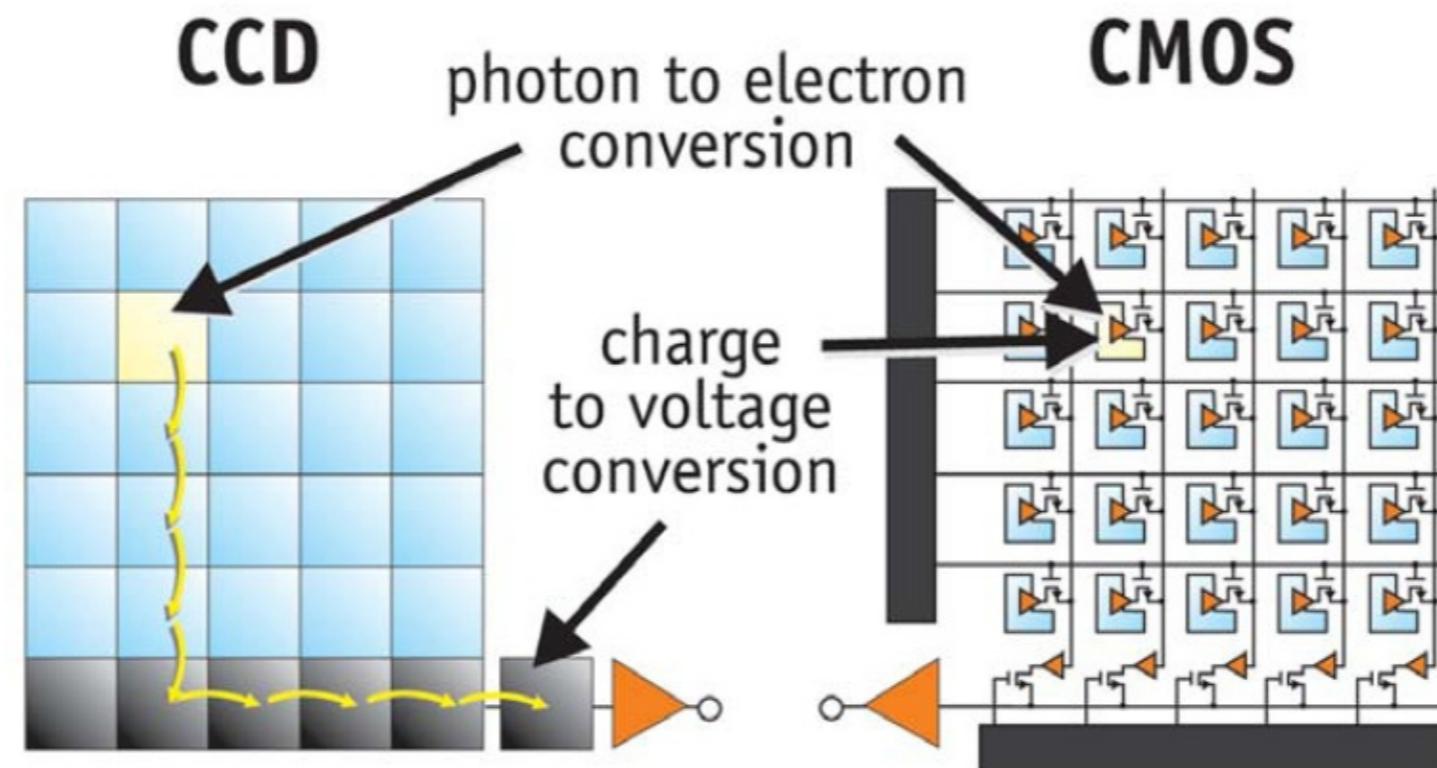
**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

- ◆ A digital camera replaces film with a **sensor array**
  - ◆ Each cell in the array is **light-sensitive diode** that converts photons to electrons
  - ◆ **Two common types**
    - ◆ Charge Coupled Device (CCD)
    - ◆ Complementary metal oxide semiconductor (CMOS)

Slide by Steve Seitz

# CCD vs. CMOS

- ◆ **CCD:** transports the charge across the chip and reads it at one corner of the array. An analog-to-digital converter (ADC) then turns each pixel's value into a digital value by measuring the amount of charge at each photosite and converting that measurement to binary form

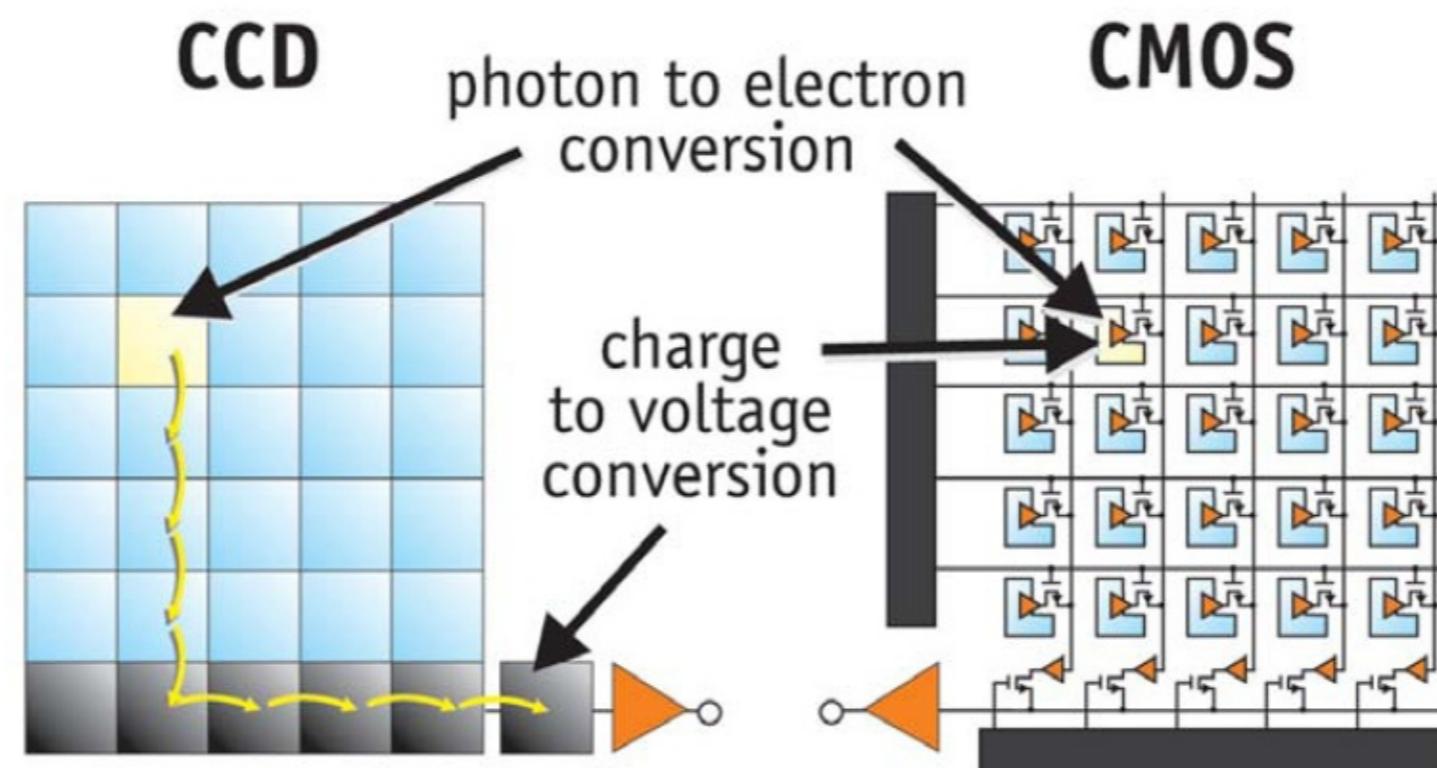


*CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.*

# CCD vs. CMOS



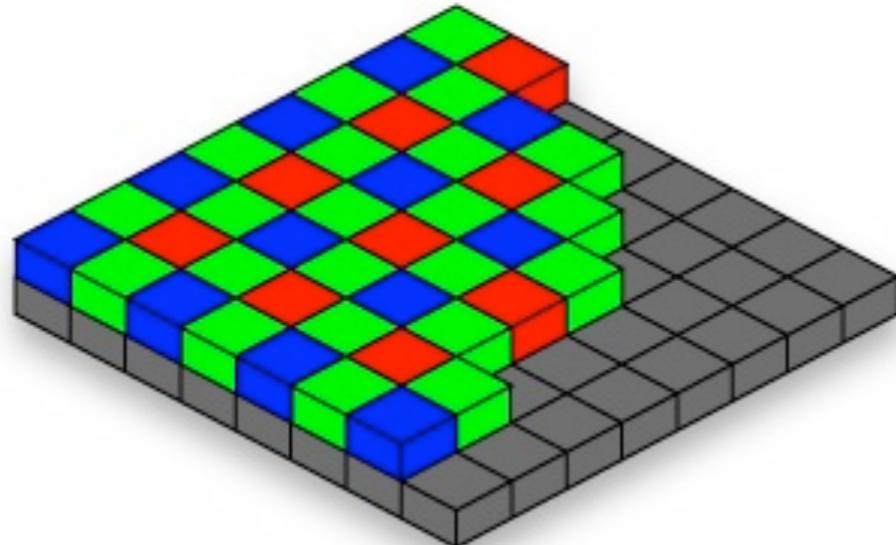
- ◆ **CMOS:** uses several transistors at each pixel to amplify and move the charge using more traditional wires. The CMOS signal is digital, so it needs no ADC.



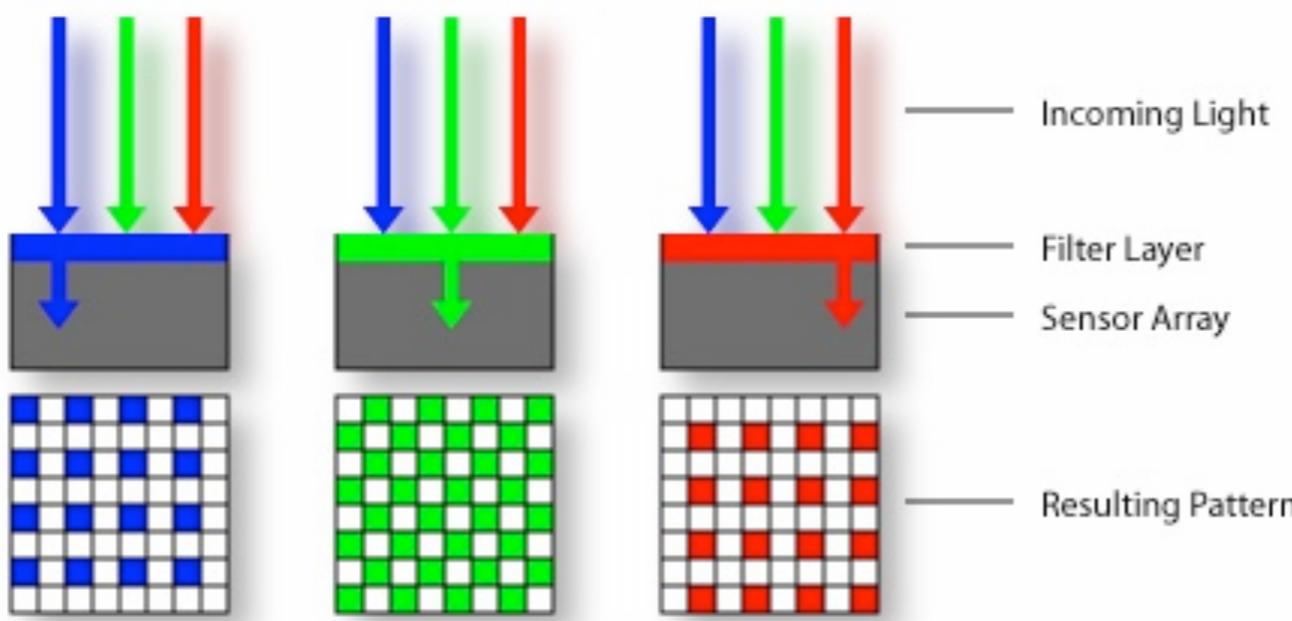
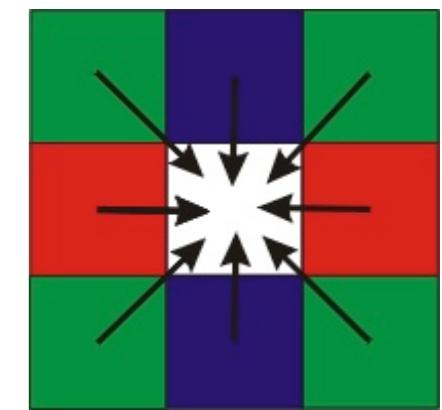
*CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.*

# Color sensing in camera: Color filter array

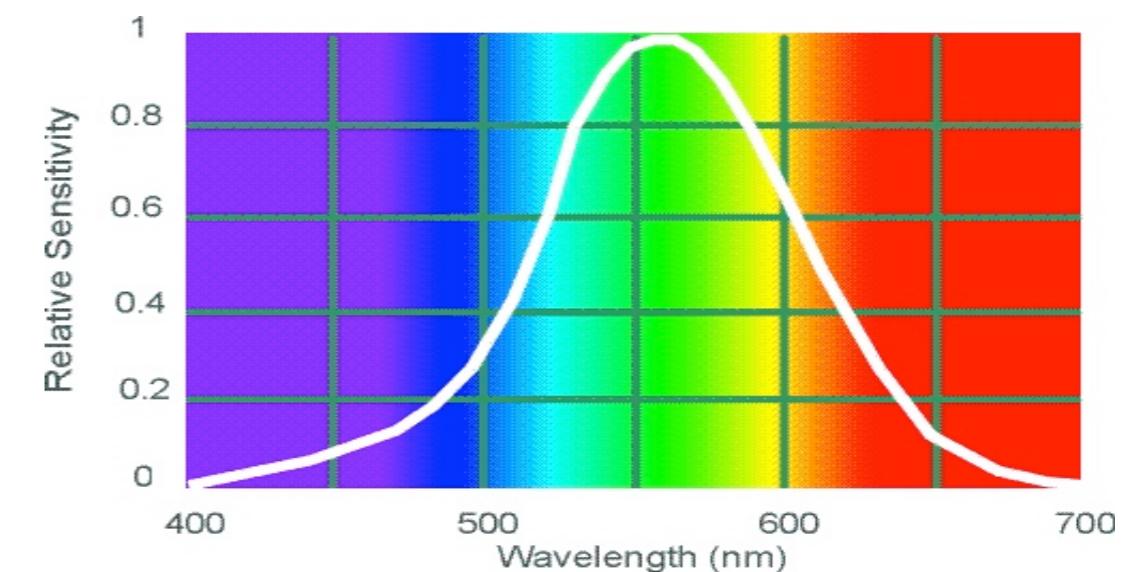
Bayer grid



Estimate missing components from neighboring values (demosaicing)



Why more green?



Human Luminance Sensitivity Function

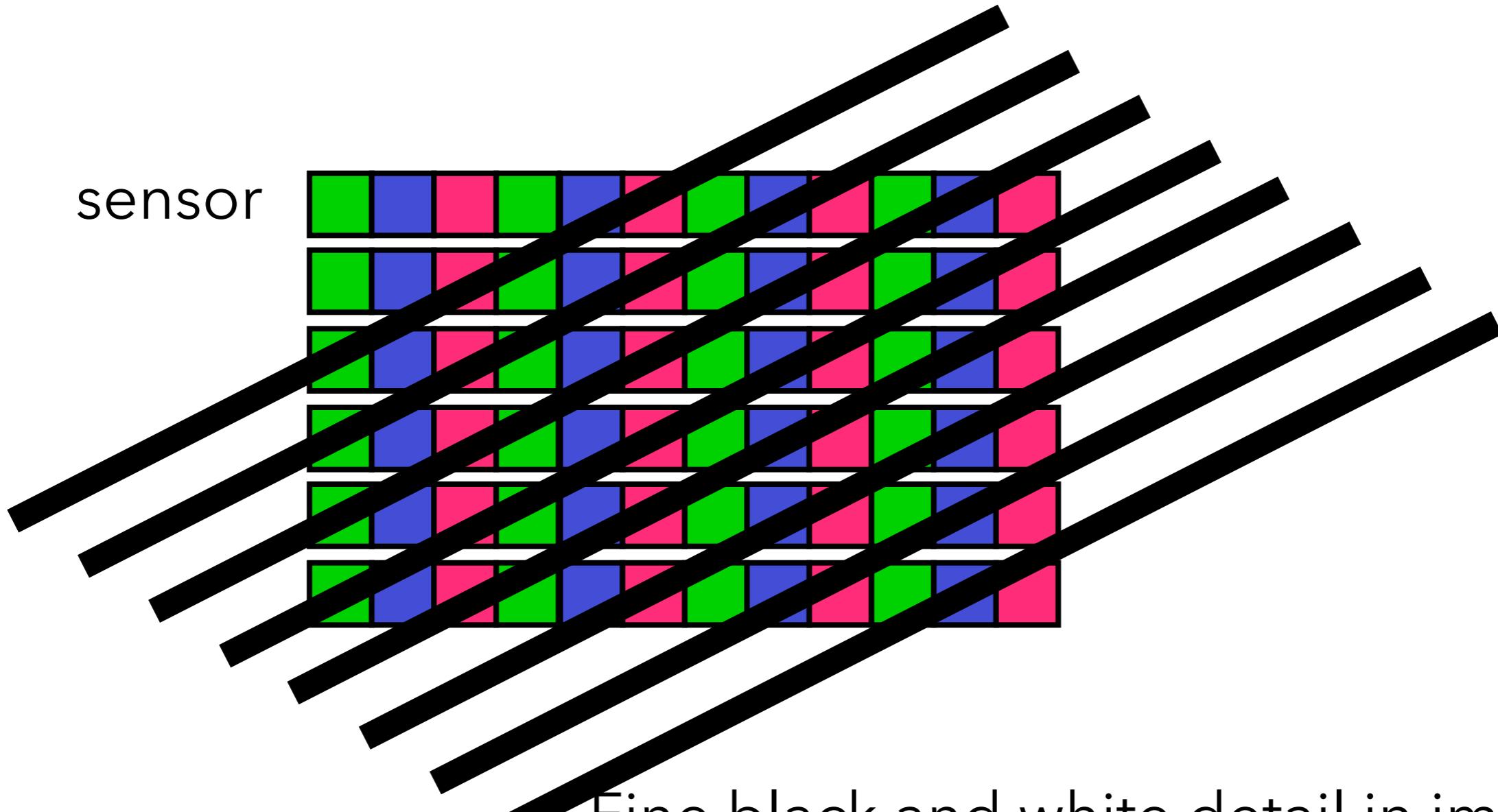
Source: Steve Seitz

# Problem with demosaicing: color moiré



Slide by F. Durand

# The cause of color moiré

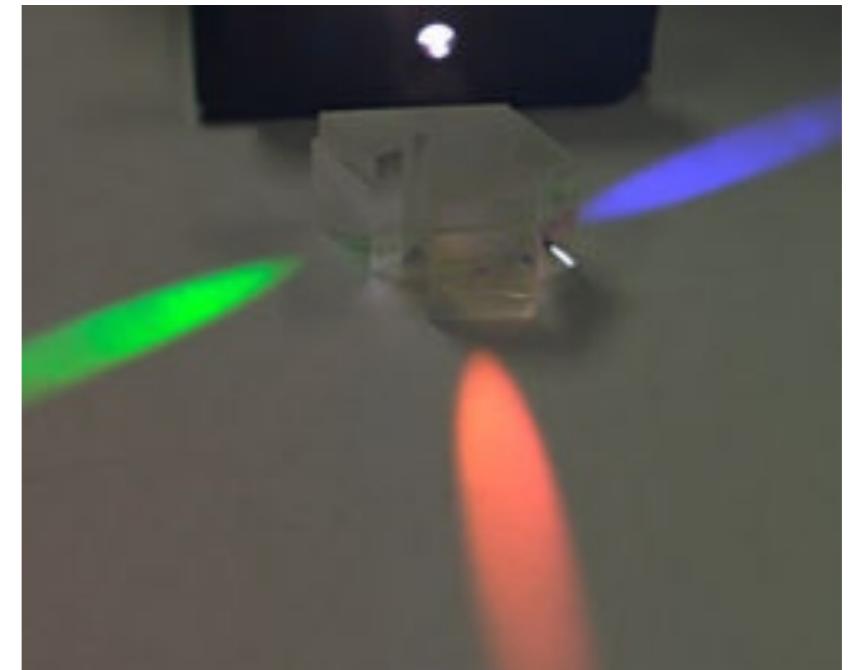
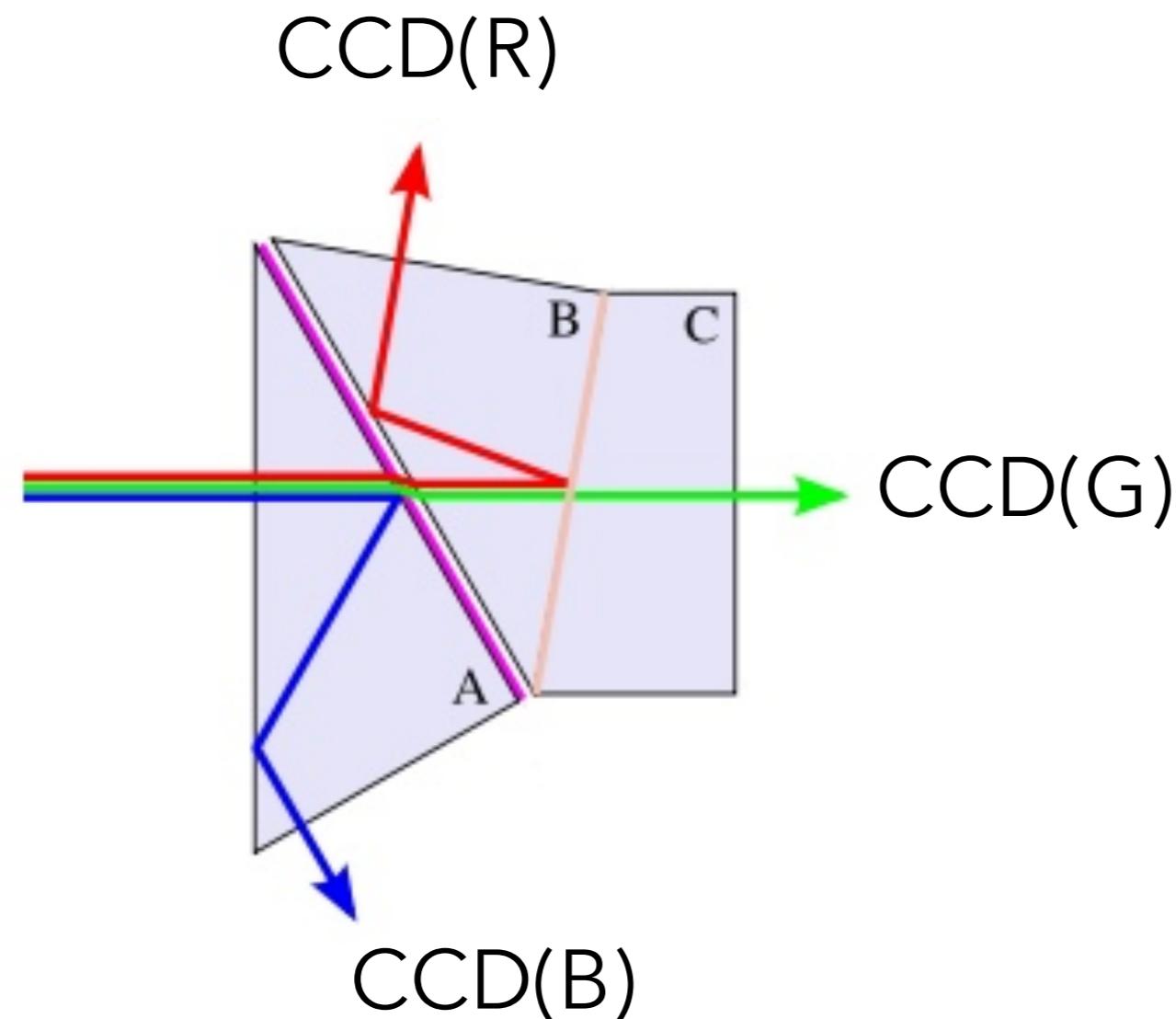


Fine black and white detail in image  
misinterpreted as color information

Slide by F. Durand

# Color sensing in camera: Prism

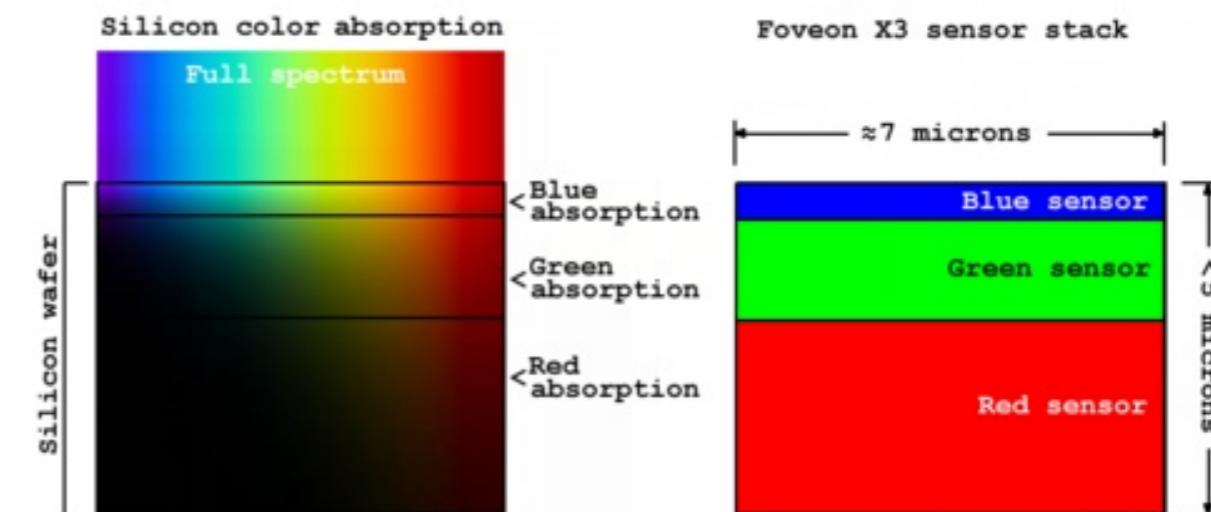
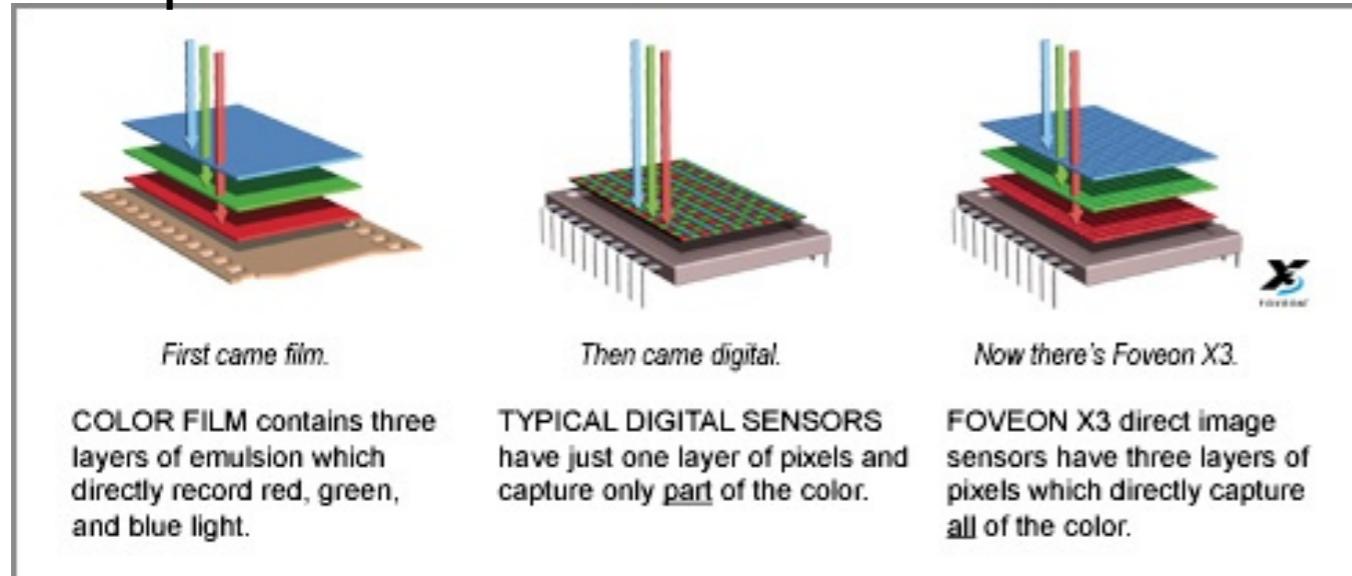
- ◆ Requires three chips and precise alignment
- ◆ More expensive



Slide by Lana Lazebnik

# Color sensing in camera: Foveon X3

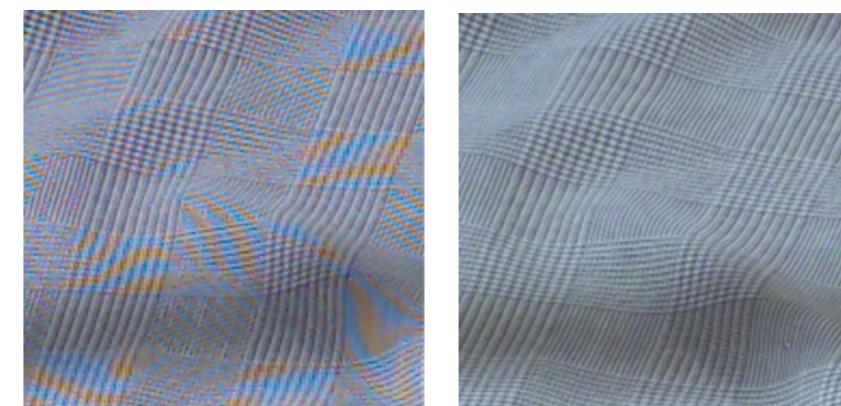
- ◆ CMOS sensor
- ◆ Takes advantage of the fact that red, blue and green light penetrate silicon to different depths



<http://www.foveon.com/article.php?a=67>

[http://en.wikipedia.org/wiki/Foveon\\_X3\\_sensor](http://en.wikipedia.org/wiki/Foveon_X3_sensor)

better image quality



Source: M. Pollefey

# Issues with digital cameras

## ◆ Noise

- ◆ low light is where you most notice noise
- ◆ light sensitivity (ISO) / noise tradeoff
- ◆ stuck pixels



## ◆ Resolution: Are more megapixels better?

- ◆ requires higher quality lens
- ◆ noise issues



## ◆ In-camera processing

- ◆ oversharpening can produce halos

## ◆ RAW vs. compressed

- ◆ file size vs. quality tradeoff

## ◆ Blooming

- ◆ charge overflowing into neighboring pixels
- ◆ white balance

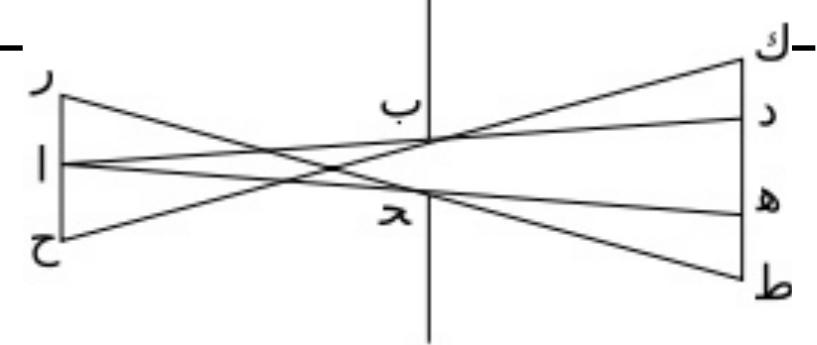
## ■ More info online:

- <http://electronics.howstuffworks.com/digital-camera.htm>
- <http://www.dpreview.com/>
- <http://www.dxomark.com/>

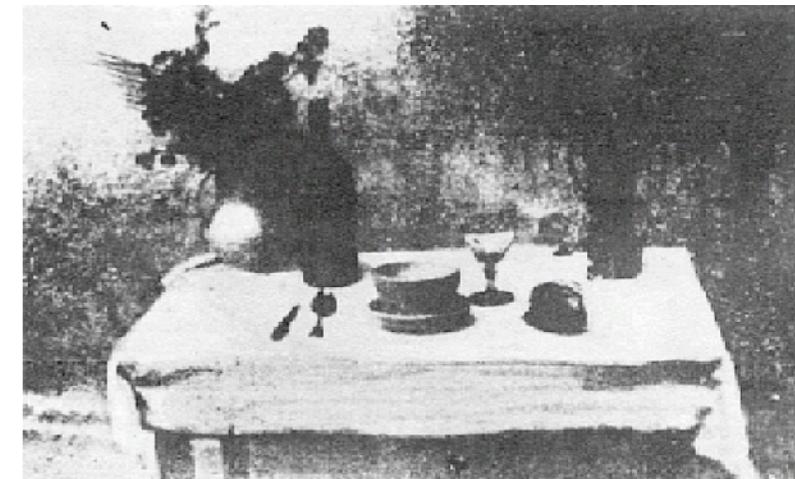
# Historical context



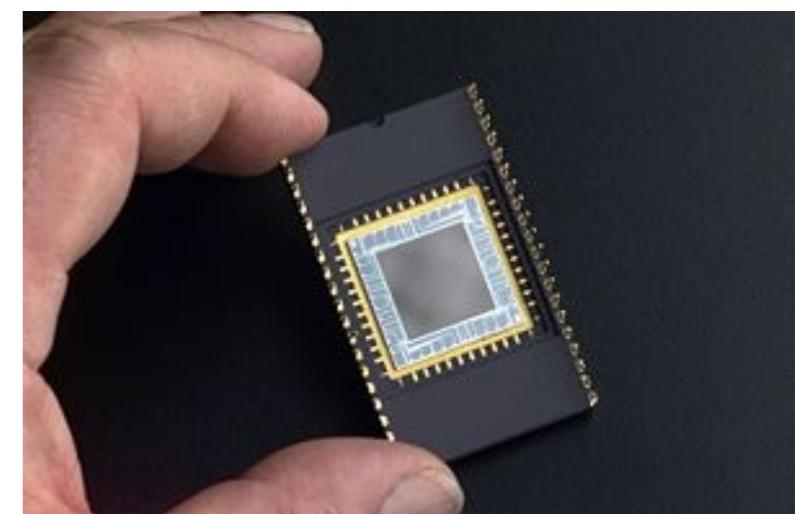
- ◆ Pinhole model: Mozi (470-390=1 BCE), Aristoteles (384-322 BCE)
- ◆ Principles of optics (including lenses): Alhazen (965-1039 CE)
- ◆ Camera obscura: Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- ◆ First photo: Joseph Nicéphore Niépce (1822)
- ◆ Daguerreotypes (1839)
- ◆ Photographic film (Eastman, 1889)
- ◆ Cinema (Lumière Brothers, 1895)
- ◆ Color Photography (Lumière Brothers, 1908)
- ◆ Television (Baird, Farnsworth, Zworykin, 1920s)
- ◆ First consumer camera with CCD: Sony Mavica (1981)
- ◆ First fully digital camera: Kodak DCS100 (1990)



Alhazen's notes



Niépce, "La Table Servie," 1822



CCD chip



# Readings for next week

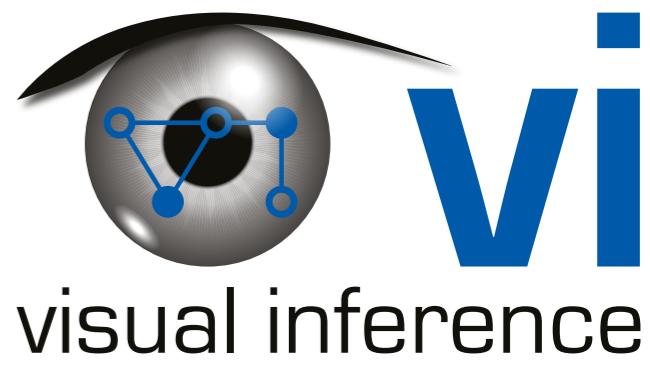
- ◆ Image processing (Ch. 3)
  - ◆ Point operations - 3.1
  - ◆ Linear filters - 3.2
  - ◆ Neighborhood operators - 3.3
  - ◆ Fourier transform - 3.4

# Computer Vision I

Basic Image Processing - 24.04.2013



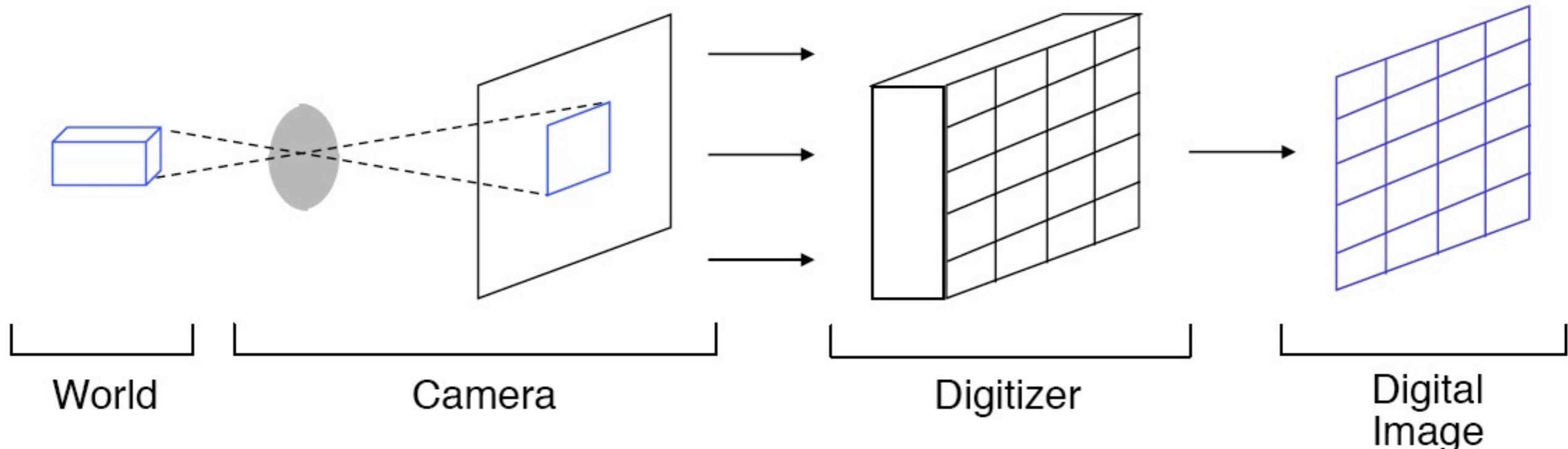
TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



# Overview



- ◆ Computer Vision: “invert” the imaging process
  - ◆ Today: 2D (2-dimensional) digital image processing
  - ◆ Later: ‘pattern recognition’ / 3D image analysis
  - ◆ Later: image understanding



# Digital Image Processing

- ◆ Some Basics
  - ◆ (digital signal processing, FFT, ...)
  - ◆ Image Filtering

- ◆ **Image Filtering**
  - ◆ to reduce noise
  - ◆ to fill-in missing values/information
  - ◆ to extract image features (e.g. edges/corners)
  - ◆ ...

Credits: slides adapted from  
Bernt Schiele  
Michael Black

10	5	3
4	5	1
1	1	7

Local image data

Some function  
→

		7

Modified image data



# Today - Basics of Digital Image Processing

- ◆ Images
- ◆ Filtering
  - ◆ Linear filtering
  - ◆ Non-linear filtering & morphology
- ◆ Multi-scale image representation
  - ◆ Gaussian pyramid
  - ◆ Laplacian pyramid
- ◆ Edge detection
  - ◆ 'Recognition using line drawings'
  - ◆ Image derivatives (1st and 2nd order)

# Linear Operations / Systems

- ◆ Basic properties:

- ◆ homogeneity

$$T[a \cdot X] = a \cdot T[X]$$

- ◆ additivity

$$T[X + Y] = T[X] + T[Y]$$

- ◆ superposition

$$T[a \cdot X + b \cdot Y] = a \cdot T[X] + b \cdot T[Y]$$

- ◆ Examples:

- ◆ matrix-vector operations
  - ◆ convolutions

# Convolution

- ◆ Replace each pixel by a linear combination of its neighbors (and itself).
- ◆ 2D convolution (discrete):

$$g = f * h \quad g(i, j) = \sum_{k, l} f(i - k, j - l)h(k, l) = \sum_{k, l} f(k, l)h(i - k, j - l),$$

45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120

image

$$f(i, j)$$

filter (kernel)

$$h(i, j)$$

\*

0.1	0.1	0.1
0.1	0.2	0.1
0.1	0.1	0.1

=

69	95	116	125	129	132
68	92	110	120	126	132
66	86	104	114	124	132
62	78	94	108	120	129
57	69	83	98	112	124
53	60	71	85	100	114

smaller  
output?

filtered image

$$g(i, j)$$

# Convolution / Correlation

- ◆ Convolution is
  - ◆ linear  $h * (f_0 + f_1) = h * f_0 + h * f_1$
  - ◆ associative  $(f * g) * h = f * (g * h)$
  - ◆ commutative  $f * h = h * f$
  - ◆ shift-invariant  $g(i, j) = f(i + k, j + l) \Leftrightarrow (h * g)(i, j) = (h * f)(i + k, j + l)$
  - ◆ can be represented as matrix-vector product  $\mathbf{g} = \mathbf{H}\mathbf{f}$
  - ◆ Continuous versions exist, skip them here...

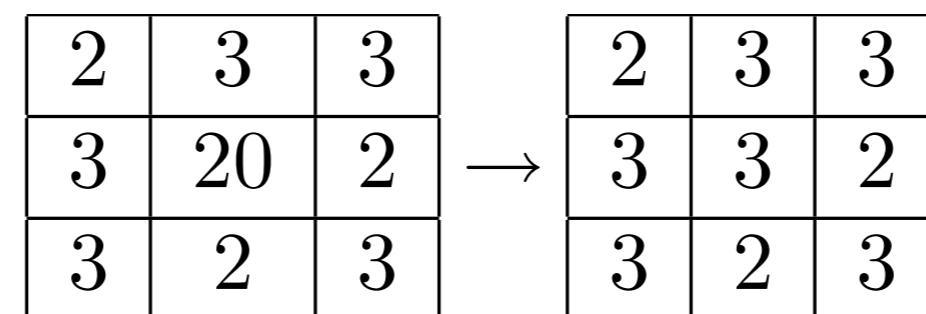
## ◆ Correlation:

- ◆ Closely related, but do not mirror filter

$$g = f \otimes h \quad g(i, j) = \sum_{k,l} f(i + k, j + l)h(k, l)$$

# Filtering to Reduce Noise

- ◆ “Noise” is what we are not interested in
  - ◆ low-level noise: light fluctuations, sensor noise, quantization effects, finite precision, ...
  - ◆ complex noise (not today): shadows, extraneous objects.
- ◆ Assumption:
  - ◆ The pixel's neighborhood contains information about its intensity



The diagram illustrates a 3x3 neighborhood of pixels being processed by a filter. On the left, the input neighborhood is shown as a 3x3 grid of values: the top row is [2, 3, 3], the middle row is [3, 20, 2], and the bottom row is [3, 2, 3]. An arrow points from this input to the output neighborhood on the right. The output neighborhood is also a 3x3 grid, where each value is the result of applying a filter to the corresponding 3x3 input window. The output values are: the top row is [2, 3, 3], the middle row is [3, 3, 2], and the bottom row is [3, 2, 3]. This demonstrates how a filter processes the input to produce a smoother output by averaging or applying a specific rule to each pixel's neighborhood.

2	3	3
3	20	2
3	2	3

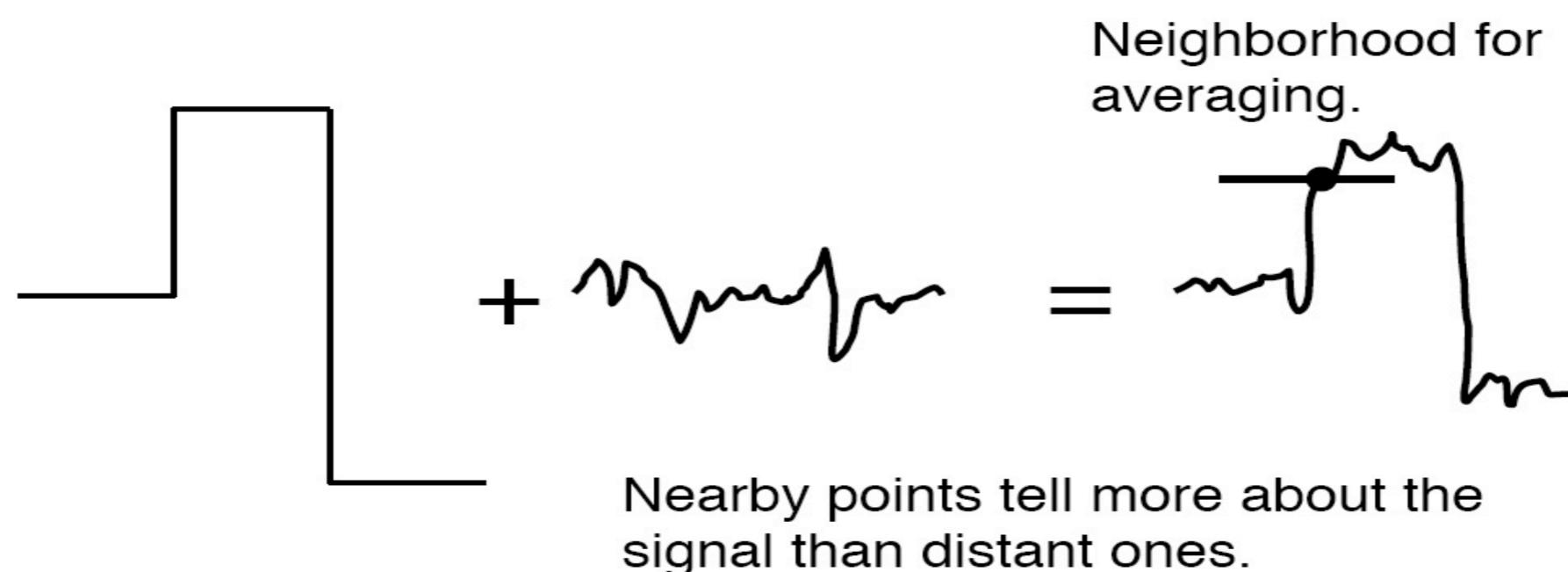
→

2	3	3
3	3	2
3	2	3

# Model: Additive Noise

Signal + Noise = Image

$S$  +  $N$  =  $I$



# Average Filter

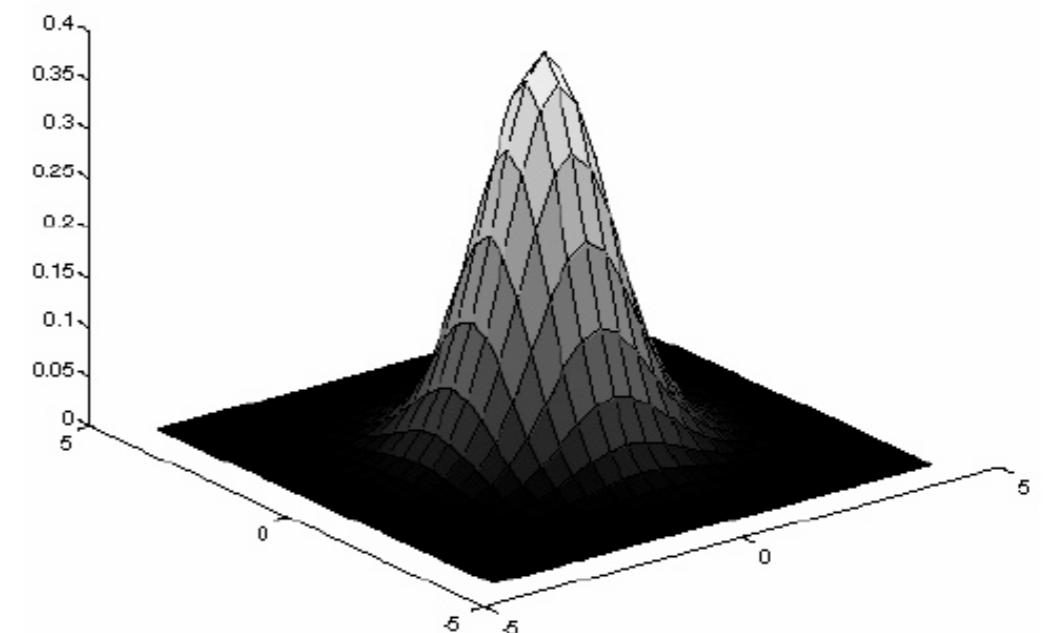
- ◆ Average Filter
  - ◆ replaces each pixel with an average of its neighborhood
  - ◆ Mask with positive entries that sum to 1
- ◆ If all weights are equal, it is called a **box filter**

$$\frac{1}{9} \cdot \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$



# Gaussian Averaging

- ◆ Isotropic Gaussian (rotationally symmetric)
- ◆ Weighs nearby pixels more than distant ones



- ◆ Smoothing kernel proportional to

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

