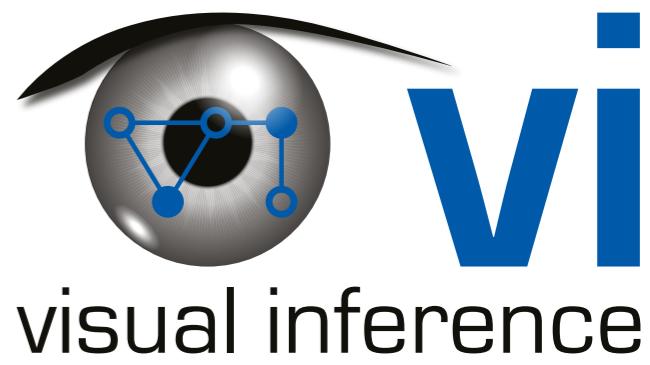


Computer Vision I

Features for Recognition - 29.05.2013



TECHNISCHE
UNIVERSITÄT
DARMSTADT





Announcements

◆ Exam

- ◆ 21. August 2013, 10:00 - 12:00
- ◆ Please double-check that you are registered!

Today

- ◆ Recap
 - ◆ Appearance-based representations
- ◆ Object identification / instance recognition
 - ◆ Global representations
 - ◆ Bag of Visual Words
 - ◆ Color histograms
 - ◆ Receptive field histograms
- ◆ Toward local features
 - ◆ Harris interest point detector

Search and Recognition



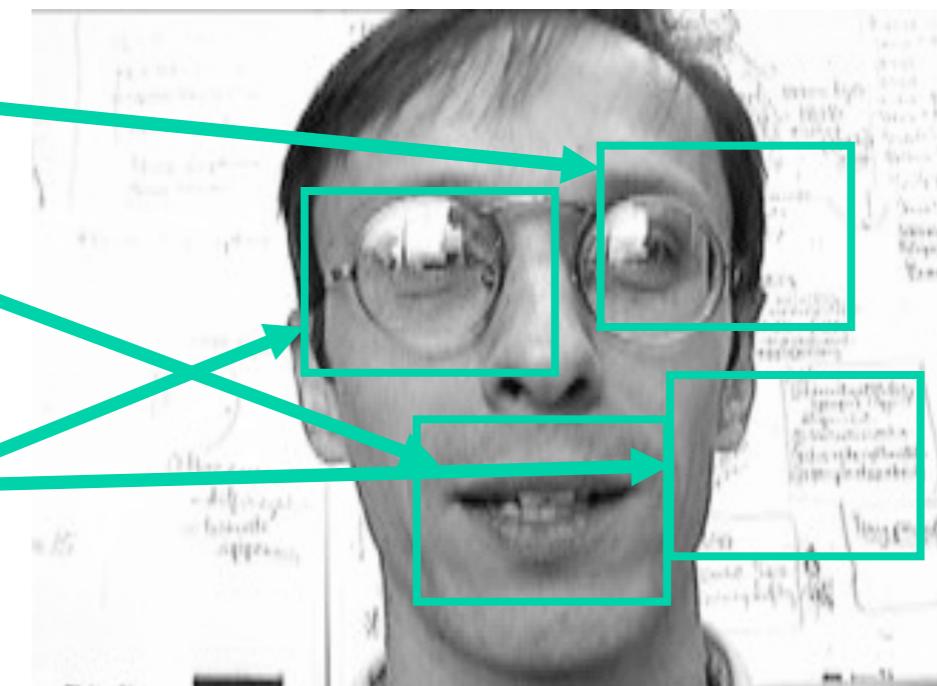
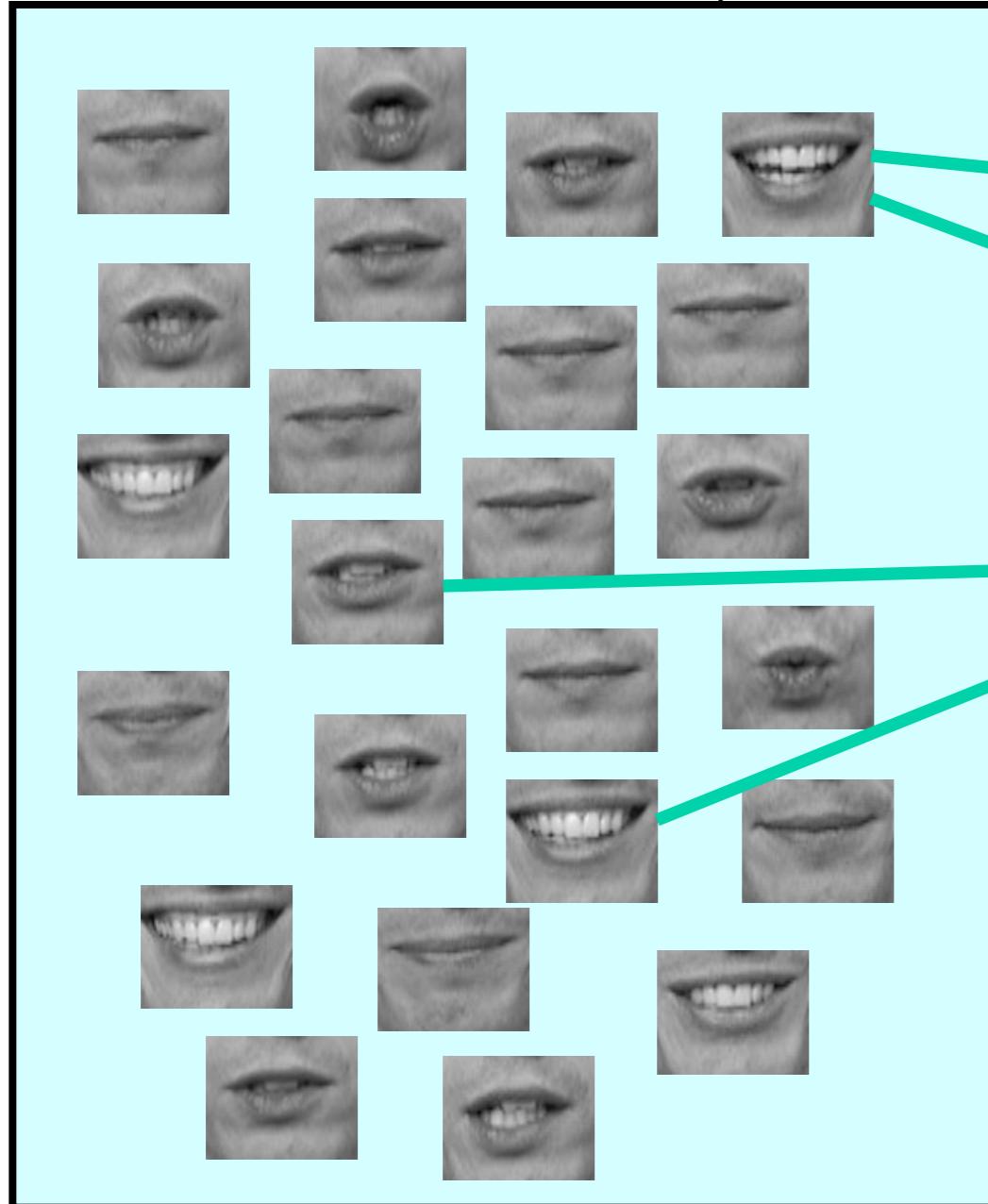
- ◆ What if we do not have a line-based object model?
 - ◆ Edges may not help us.



- ◆ How can we find the mouth?
- ◆ How can we recognize the “expression”?

Naïve View-Based Approach

Database of mouth “templates”



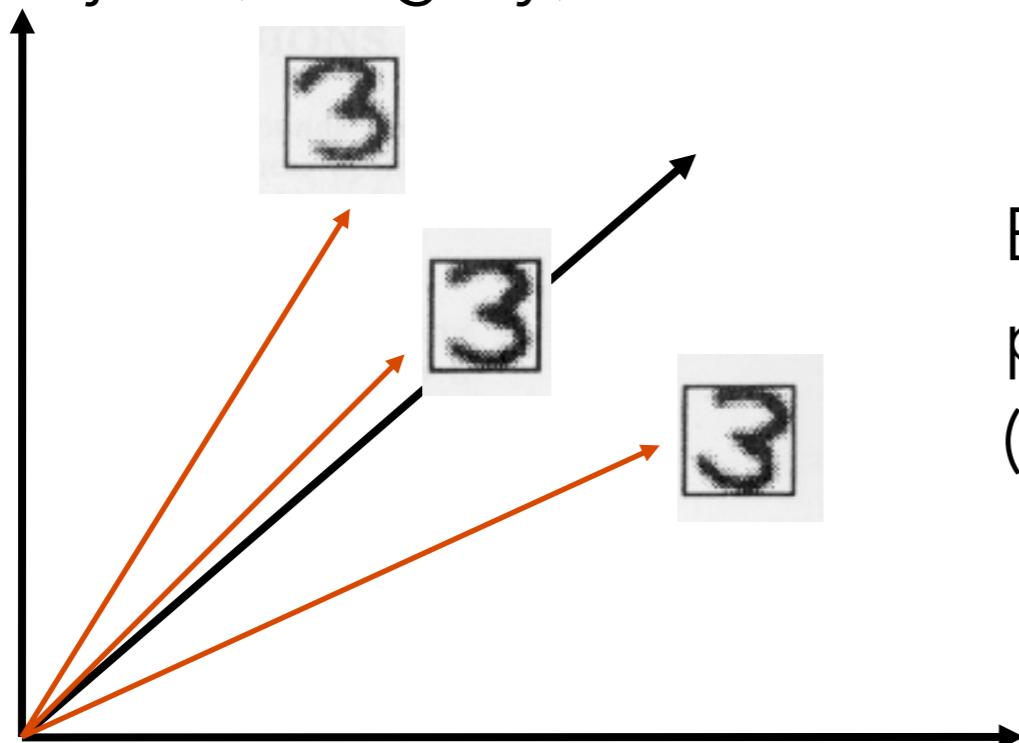
- Search every image region (at every scale).
- Compare each template; chose the best match.

View-Based Methods

- ◆ Idea: Represent objects by their appearance in an ensemble of images, including different poses, illuminations, configurations of shape, ...
- ◆ Approach covered here:
 - ◆ Subspace methods (also called "Eigen methods")

Subspace Methods

- ◆ How can we find **more efficient representations** for the ensemble of views, and more efficient methods for matching?
- ◆ Idea: images are not random... especially images of the same object (category) have similar appearance



E.g., let images be represented as points in a high-dimensional space (e.g., one dimension per pixel)

Fleet & Szeliski

Data point n :

$$\mathbf{x}^n \in \mathbb{R}^M$$

Low-dimensional representation:

$$\mathbf{a}^n \in \mathbb{R}^D \quad D \ll M$$

Mapping: $\mathbf{x}^n \rightarrow \mathbf{a}^n$

◆ Restrict this mapping to be a linear function:

$$\mathbf{a}^n = \mathbf{B}\mathbf{x}^n, \quad \mathbf{B} \in \mathbb{R}^{D \times M}$$

◆ Note:

- ◆ From now on, I will use boldface symbols to denote vectors, e.g.:
 $\mathbf{x}, \mathbf{y}, \dots$

Principal Component Analysis

- ◆ We know how we can represent our data in a lower dimensional space in a principled fashion.
 - ◆ We compute the mean of the data, and subtract it.
 - ◆ We compute the covariance matrix, decompose it, and choose the first D eigenvalues.
 - ◆ This gives us an (eigen)basis for representing the data:

$$\mathbf{a}^n = \mathbf{B}^T(\mathbf{x}^n - \bar{\mathbf{x}}) \quad \text{where} \quad \mathbf{B} = [\mathbf{u}_1, \dots, \mathbf{u}_D]$$
$$\tilde{\mathbf{x}}^n = \bar{\mathbf{x}} + \mathbf{B}\mathbf{a}^n$$

- ◆ It is common to also normalize the variance of each dimension.

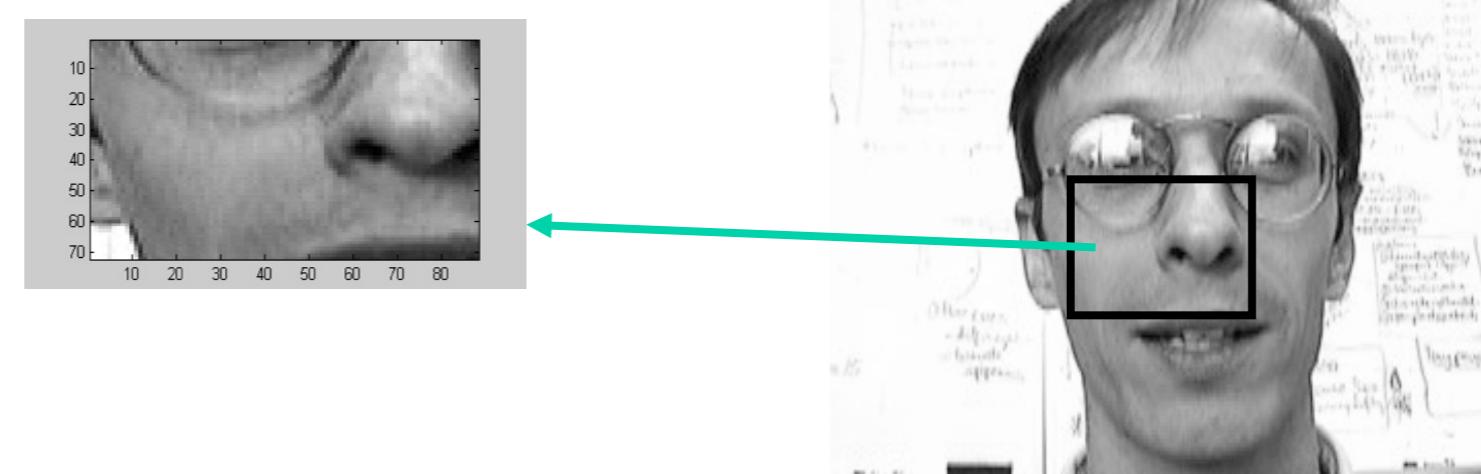
How to use SVD for PCA

- ◆ If we perform SVD on the data matrix $\mathbf{X} = [\mathbf{x}^1, \dots, \mathbf{x}^N]$ (after subtracting the mean) then:
 - ◆ The left-singular vectors give us the eigenvectors of the covariance matrix.
 - ◆ The singular values let us easily compute the eigenvalues of the covariance matrix:

$$\lambda_i = \frac{1}{N} s_i^2$$

- ◆ **Advantage:** We never have to explicitly build and store the covariance matrix!

Simple Search Strategy



- ◆ Project each training image onto the low-dimensional subspace. Store the vectors of coefficients.
- ◆ For each image region:
 - ◆ Project it onto the low-dimensional subspace.
 - ◆ Compare this to each stored coefficient vector (cheap).
 - ◆ If the smallest distance is less than some threshold, then it is a mouth.

Recognition problems

- ◆ What is it?
 - ◆ Object and scene recognition
- ◆ Who is it?
 - ◆ Identity recognition
- ◆ Where is it?
 - ◆ Object detection
- ◆ What are they doing?
 - ◆ Activity recognition
- ◆ All of these are **classification** problems
 - ◆ Choose one class from a list of possible candidates

[Szeliski & Seitz]

What is recognition?

- ◆ A different taxonomy from [Csurka et al. 2006]:
- ◆ Recognition (focus for now)
 - ◆ Where is this particular object?
- ◆ Categorization (next time)
 - ◆ What kind of object(s) is(are) present?
- ◆ Content-based image retrieval
 - ◆ Find me something that looks similar
- ◆ Detection (next time)
 - ◆ Locate all instances of a given class

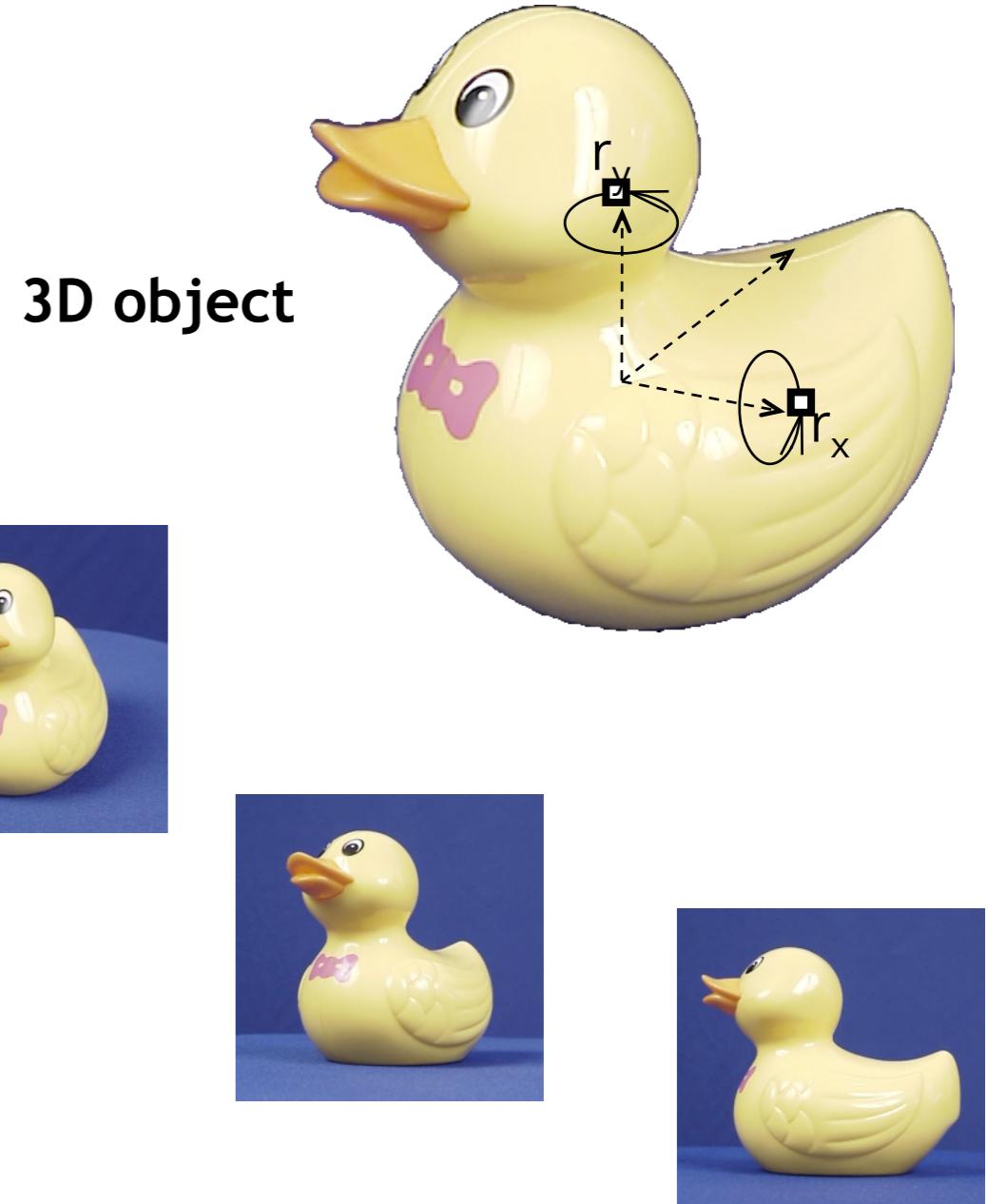
[Szeliski & Seitz]

From faces and mouths to objects



Appearance-Based Instance Recognition

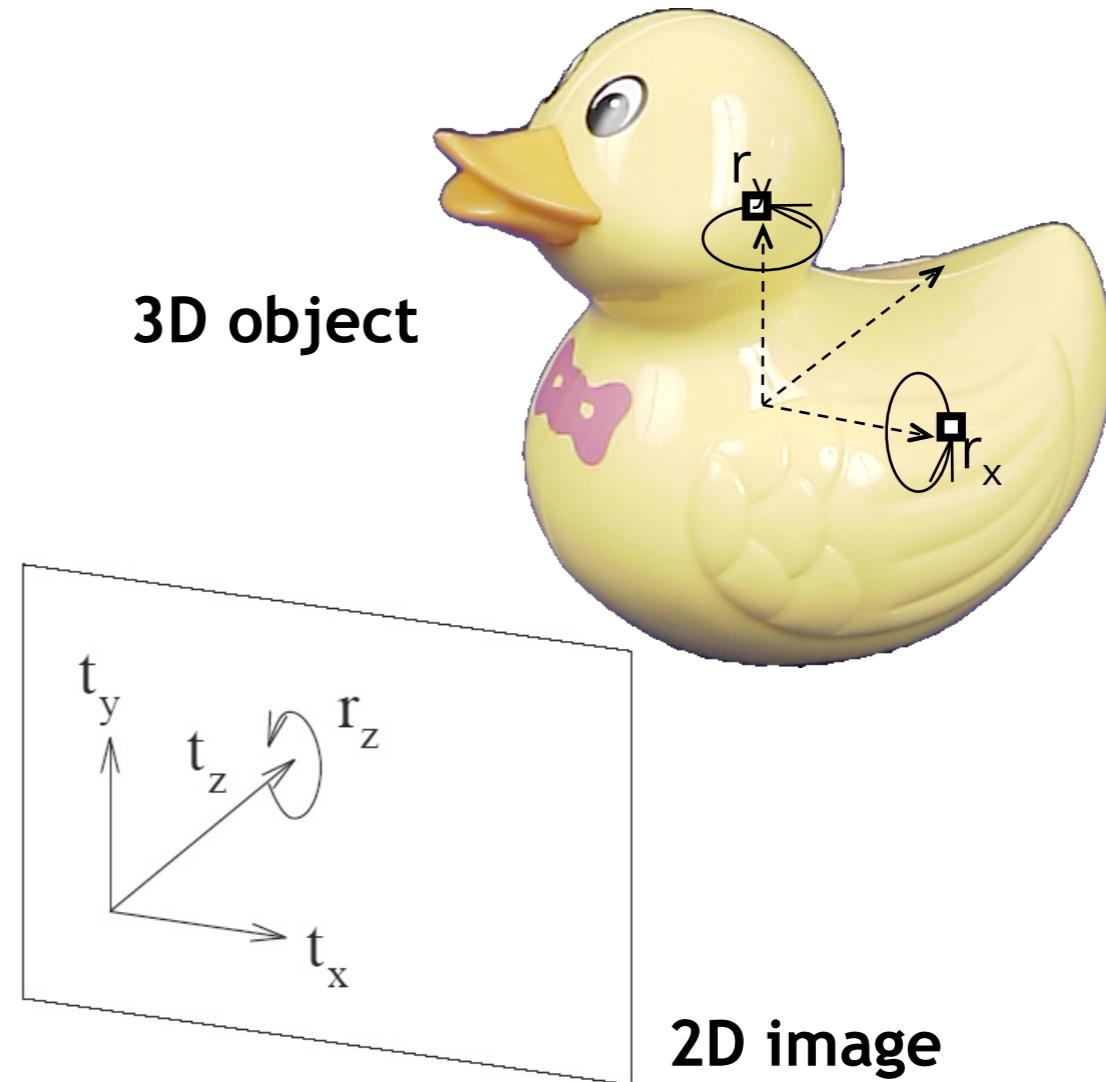
- ◆ Basic assumption:
 - ◆ Objects can be represented by a set of images ("appearances").
 - ◆ For recognition, it is sufficient to just compare the 2D appearances.
 - ◆ No 3D model is needed.



⇒ Fundamental paradigm shift in the 90's

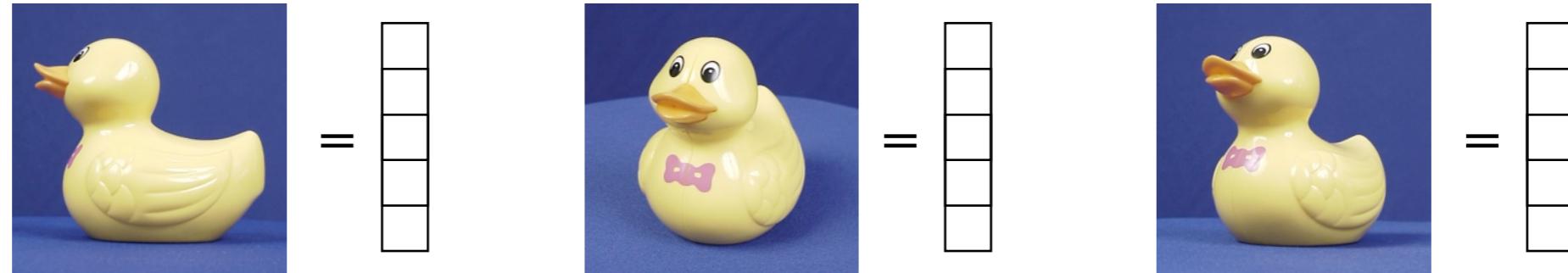
Challenges

- ◆ Viewpoint changes
 - ◆ Translation
 - ◆ Image-plane rotation
 - ◆ Scale changes
 - ◆ Out-of-plane rotation
- ◆ Illumination
- ◆ Clutter
- ◆ Occlusion
- ◆ Noise



Global Representation

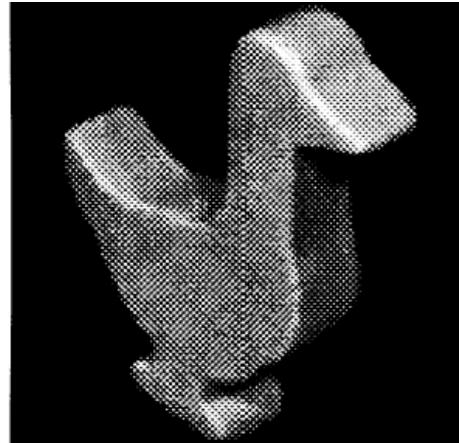
- ◆ Idea:
 - ◆ Represent each object (view) by a global feature descriptor.



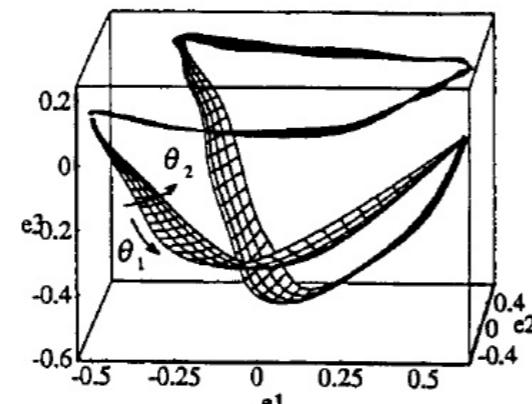
- ◆ For recognizing objects, just match the (global) descriptors.
- ◆ Some modes of variation are built into the descriptor, others have to be incorporated in the training data or the recognition process.
- ◆ View-based representations:
 - ◆ Pixels (or projections onto global basis vectors) are the descriptor.
 - ◆ Very limited amount of invariance!

Appearance Manifolds

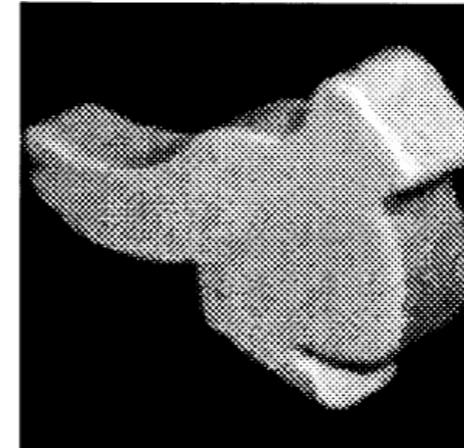
- ◆ Limitations of linear representations:



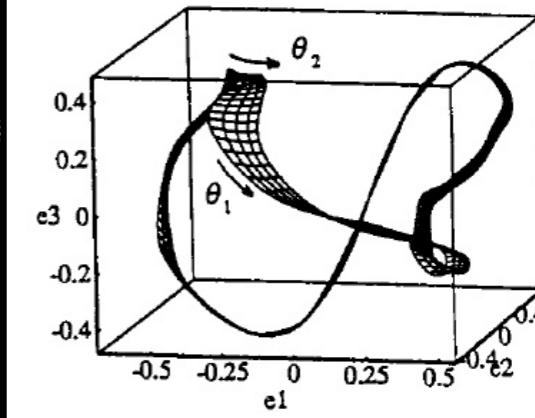
A



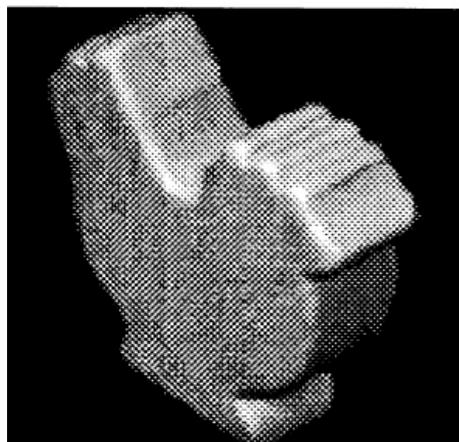
A



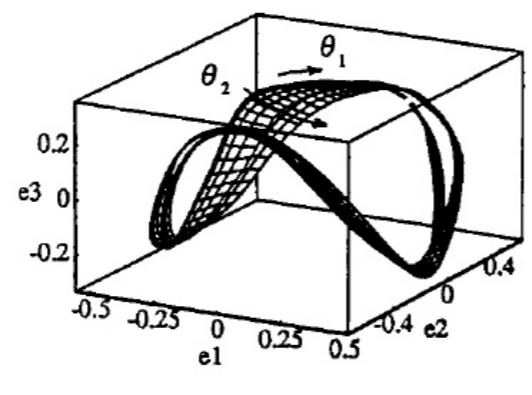
B



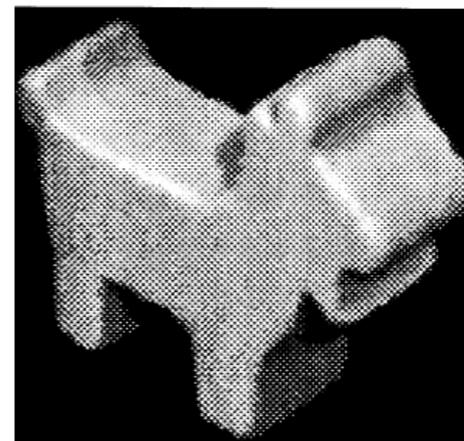
B



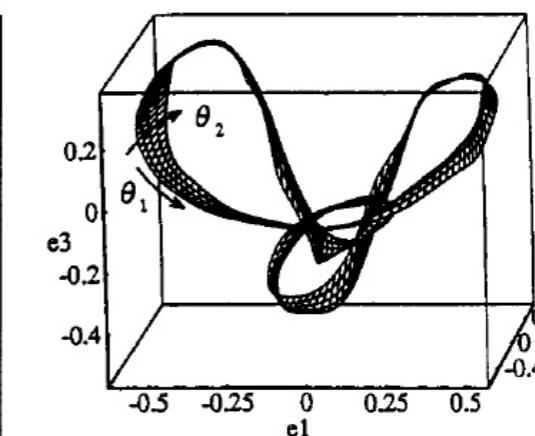
C



C



D



D

[Murase & Nayar, 1996]

- ◆ PCA will not work!

View-Based Approaches

- ◆ ... are **severely challenged** by these common variations
 - ◆ To make them work, we would need an unmanageable amount of examples (training data).
- ◆ ... **do not generalize well**
 - ◆ Almost any variation that hasn't been captured in the training data will not be handled gracefully.
- ◆ **Training data is expensive:**
 - ◆ Humans have to gather and label it.
- ◆ What else can we do?
 - ◆ Move away from representing the object simply by its pixels.
 - ◆ Images as representation are **too rigid**.

“Bag of words” Model

Object → Bag of ‘words’



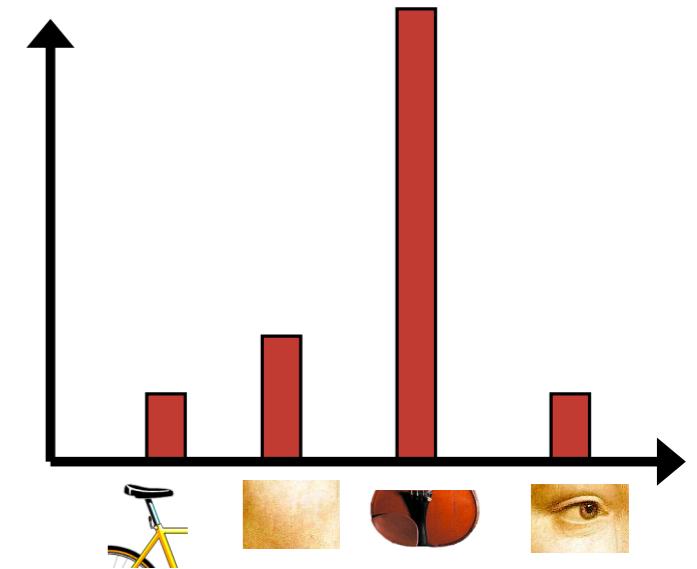
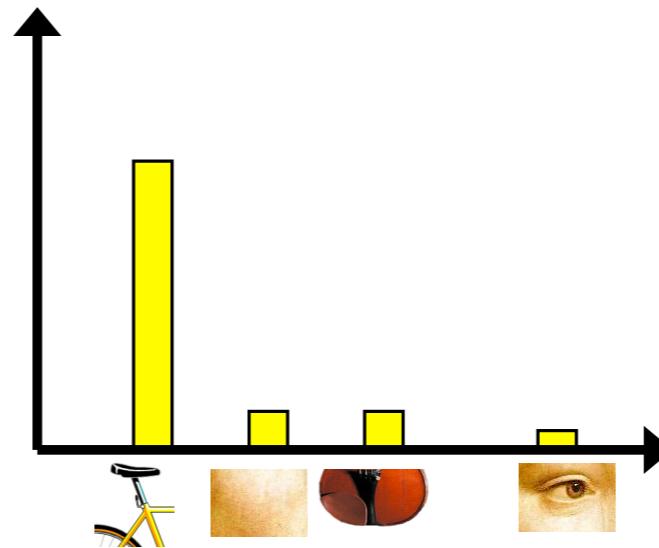
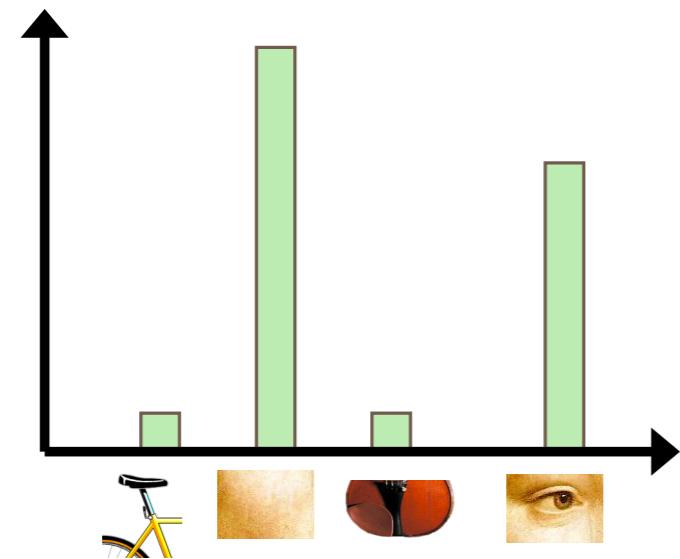
[Fei-Fei]

Analogy to Documents

Of all the sensory impressions proceeding to the brain, the visual experiences are the dominant ones. Our perception of the world around us is based essentially on the messages that reach us through our eyes. For a long time it was believed that the retinal image was processed directly in the visual centers in the brain. In 1960, however, it was discovered that the visual system is a movie screen, displaying a sequence of images. The image is processed by the retina, cerebral cortex, eye, cell, optical nerve, image. **Hubel, Wiesel** following the message about the image falling on the retina undergoes a top-down analysis in a system of nerve cells stored in columns. In this system each column has its specific function and is responsible for a specific detail in the pattern of the retinal image.

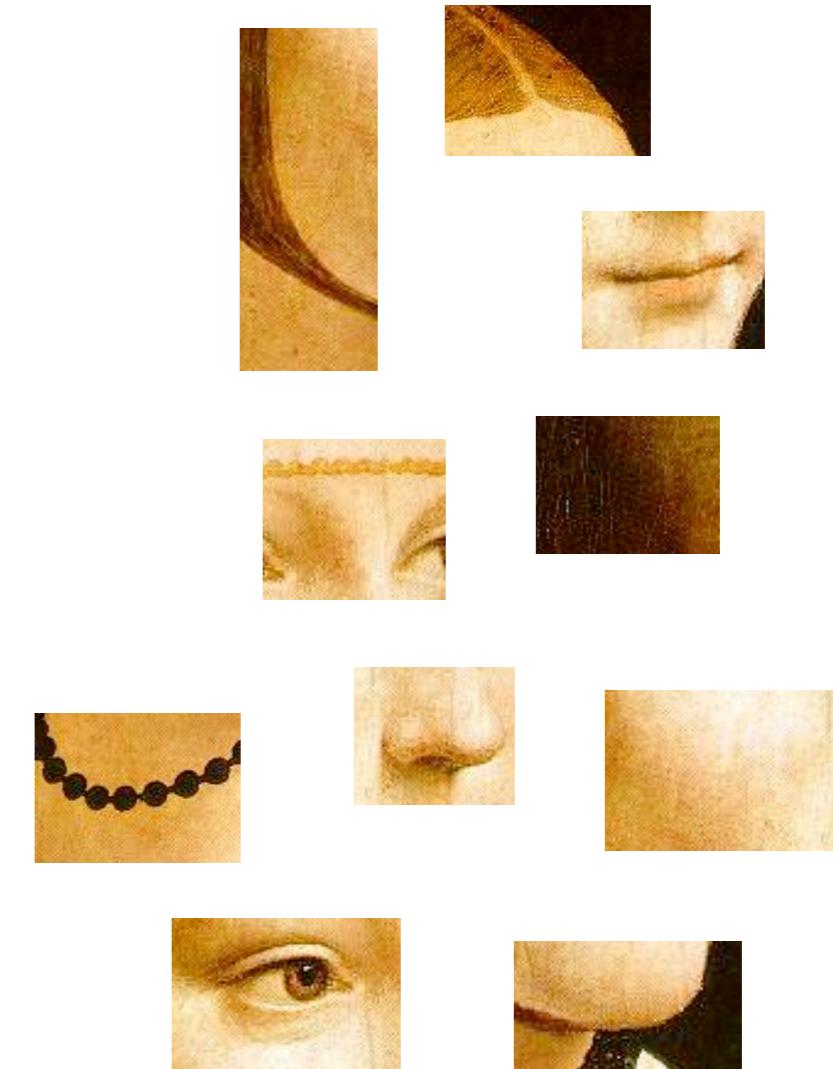
China is forecasting a trade surplus of \$90bn (£51bn) to \$100bn this year, a threefold increase on 2004's \$32bn. The Commerce Ministry said the surplus would be created by a predicted 30% increase in exports to \$750bn, compared with \$660bn. The US has been annoyed by China's trade policies. The Chinese government deliberately agrees to keep the yuan low. The Chinese government also needs to increase domestic demand so that more people buy Chinese-made products. China is also trying to increase the value of the yuan against the dollar. The Chinese government permitted it to trade within a narrow range, but the US wants the yuan to be allowed to trade freely. However, Beijing has made it clear that it will take its time and tread carefully before allowing the yuan to rise further in value.

Visual word distributions



[Fei-Fei]

Which feature representation?

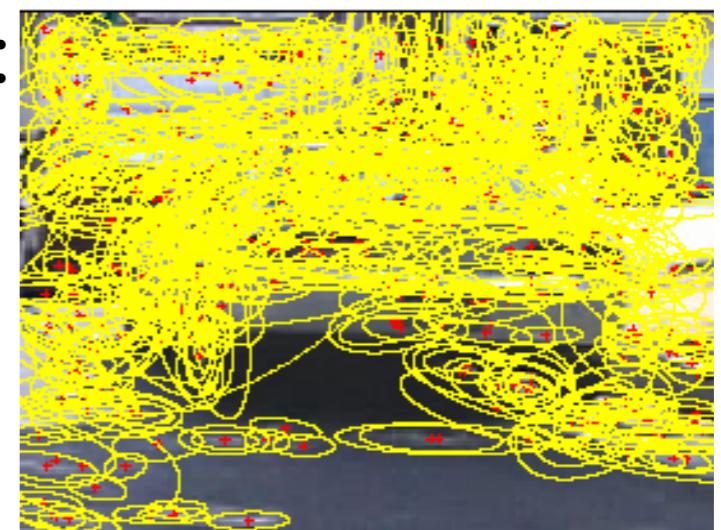
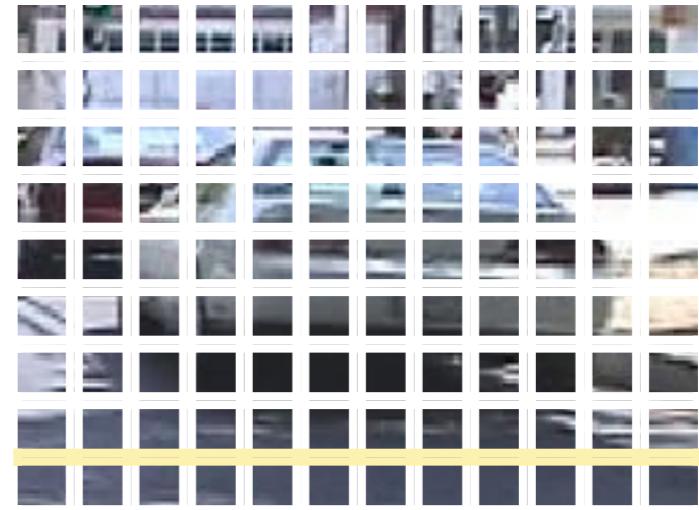


[Fei-Fei]

Feature representation

- ◆ Dense representation (regular grid):
 - ◆ Color histogram approach [Swain & Ballard '91]
 - ◆ Multidimensional receptive field histograms [Schiele & Crowley '96-'00]

- ◆ Sparse feature representations (later):
 - ◆ Find key points with interest point detector
 - ◆ e.g. scale- or affine-invariant
 - ◆ Represent key points with feature vectors
 - ◆ e.g. SIFT, ShapeContext



[Fei-Fei]

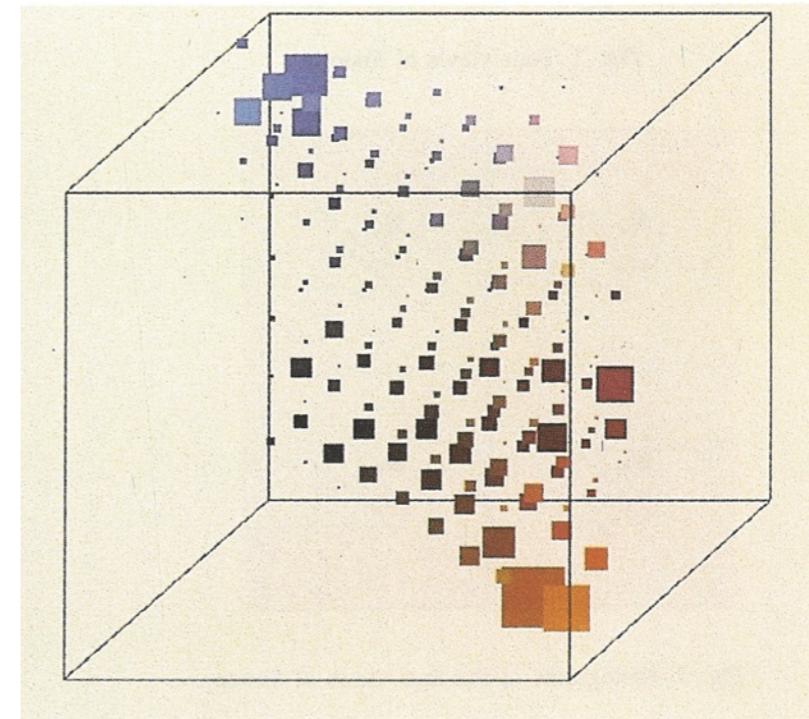
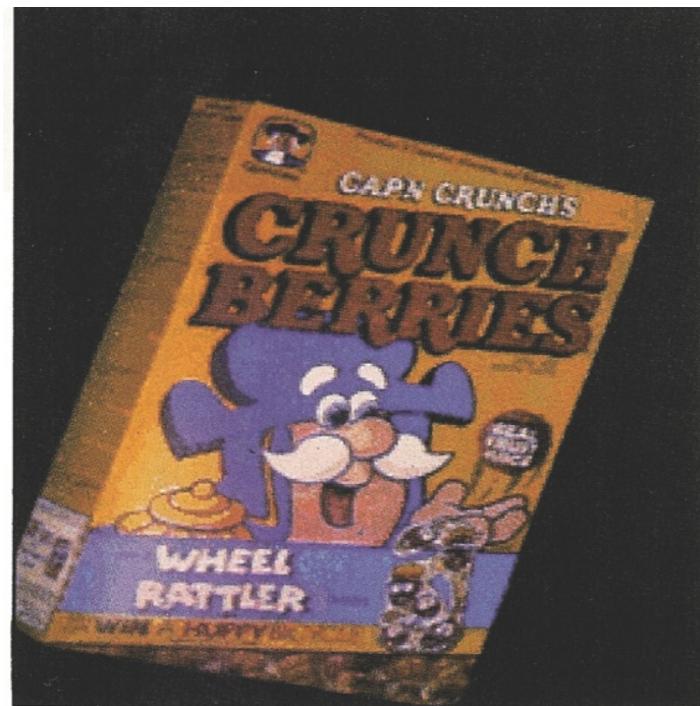
Simplest “visual word”: Color



- ◆ From rigid to very non-rigid representations:
 - ◆ Forget any spatial information!
- ◆ Color:
 - ◆ Color stays constant under geometric transformations
 - ◆ Local feature
 - ◆ Color is defined for each pixel
 - ◆ Robust to partial occlusion
- ◆ Idea
 - ◆ Directly use object colors for (instance) recognition
 - ◆ Better: use statistics of object colors

Color Histograms

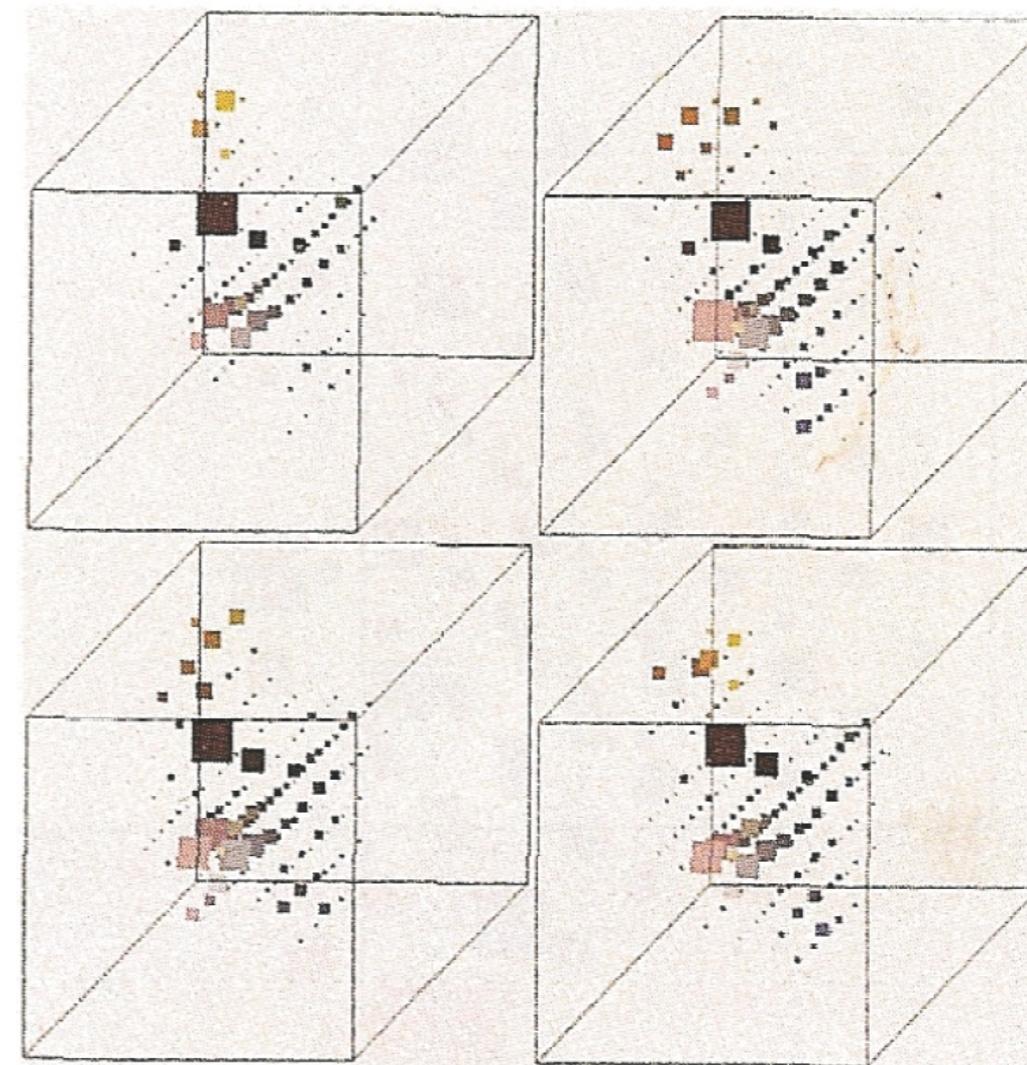
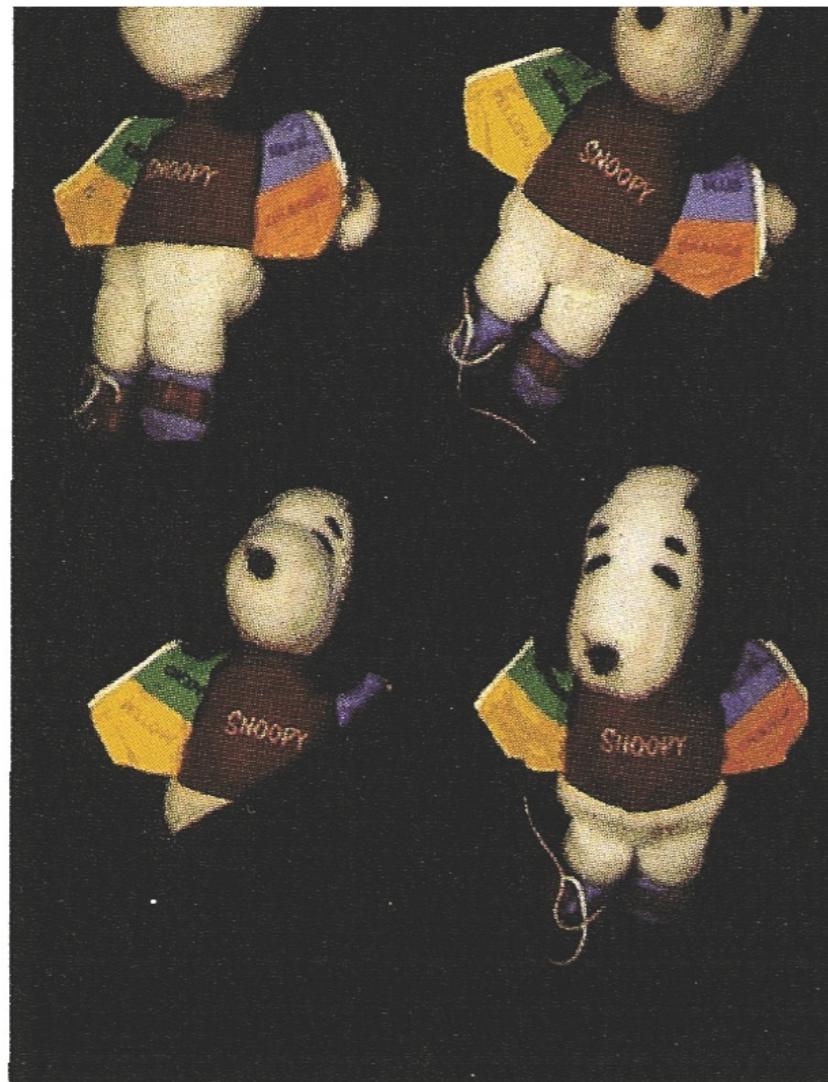
- ◆ Color statistics
 - ◆ Given: tristimulus R,G,B for each pixel
 - ◆ Compute 3D histogram
 - ◆ $H(R,G,B) = \#(\text{pixels with color } (R,G,B))$



[Swain & Ballard, 1991]

Color Histograms

- ◆ Relatively robust representation:
 - ◆ Presence of occlusion, rotation



[Swain & Ballard, 1991]

Intensity normalization

- ◆ One component of the 3D color space is the **intensity**
 - ◆ If a color vector is multiplied by a scalar, the intensity changes, but not the color itself.
 - ◆ This means colors can be **normalized** by the intensity.
 - ◆ Intensity is given by: $I = R + G + B$:
 - ◆ „Chromatic representation“
 - ◆ **Invariant** to changes of the light intensity.

$$r = \frac{R}{R + G + B}$$

$$g = \frac{G}{R + G + B}$$

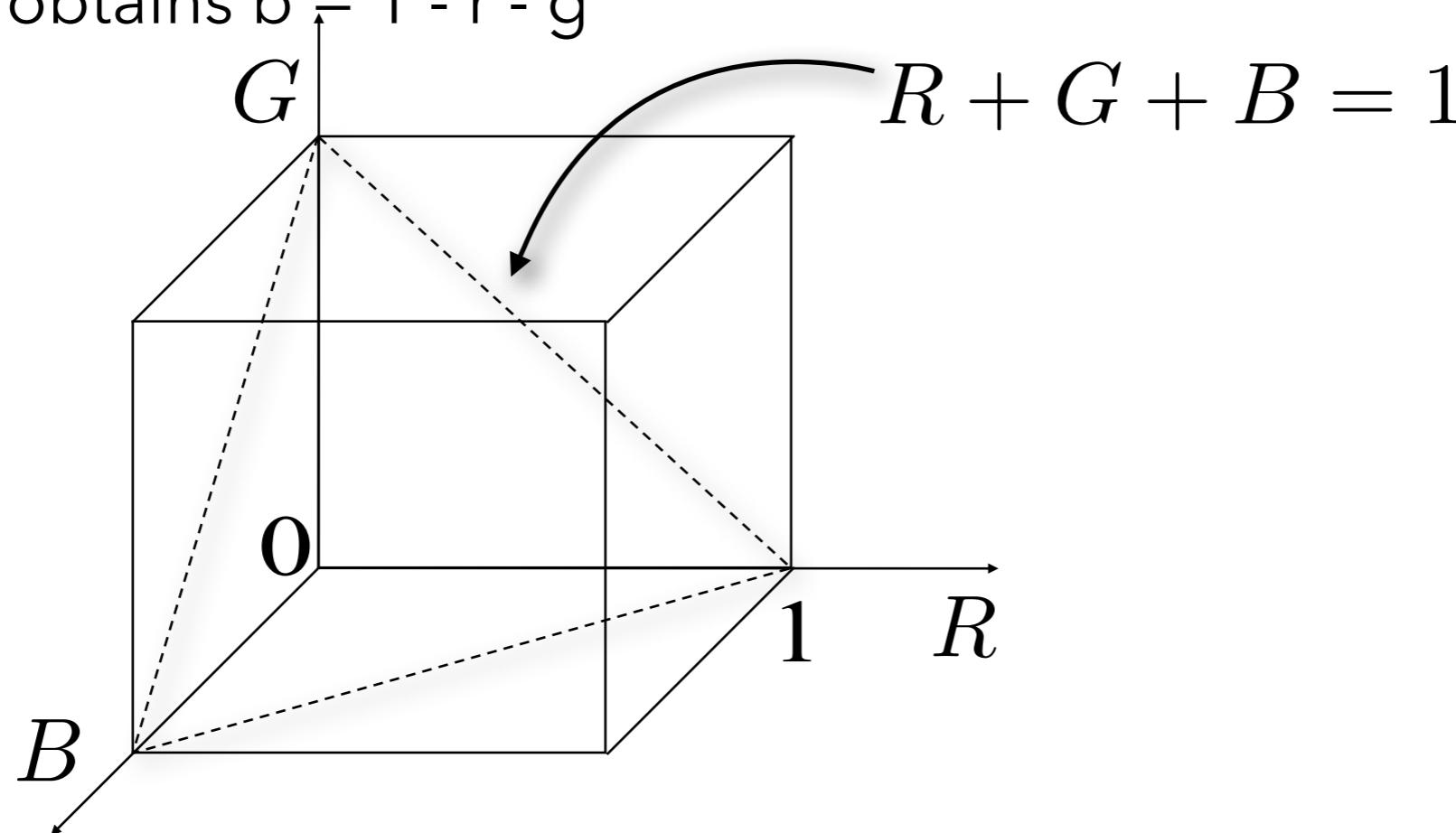
$$b = \frac{B}{R + G + B}$$

- ◆ Observation:

- ◆ Since $r + g + b = 1$, only 2 parameters are necessary
- ◆ E.g. one can use r and g
- ◆ and obtains $b = 1 - r - g$

$$r + g + b = 1$$

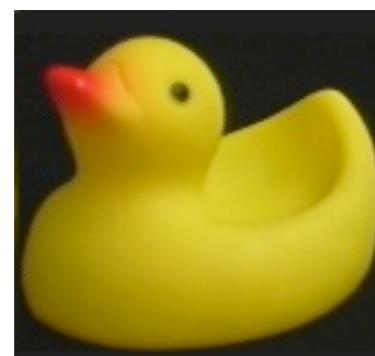
$$\Rightarrow b = 1 - r - g$$



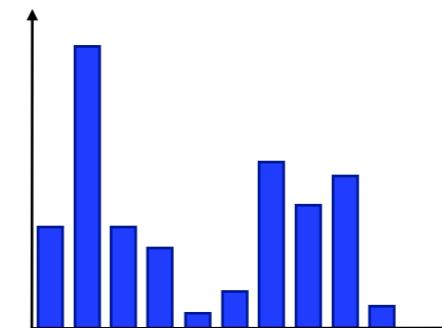
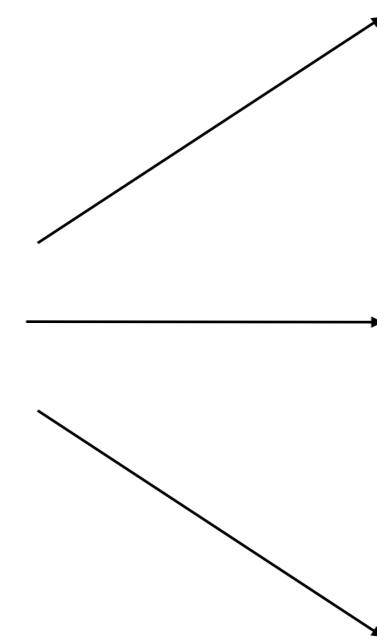
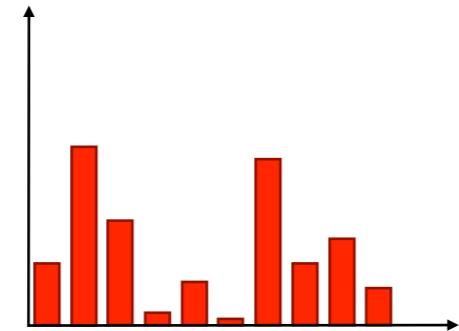
Recognition using Histograms



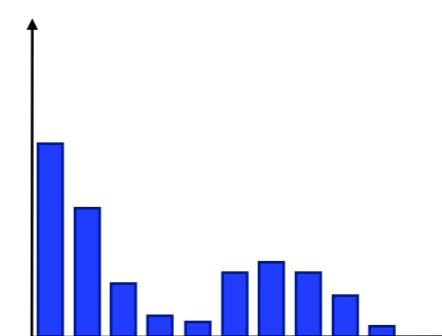
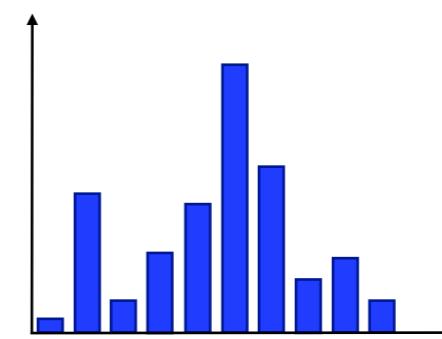
- ◆ Histogram comparison
 - ◆ Database of known objects
 - ◆ Test image of unknown object



test image



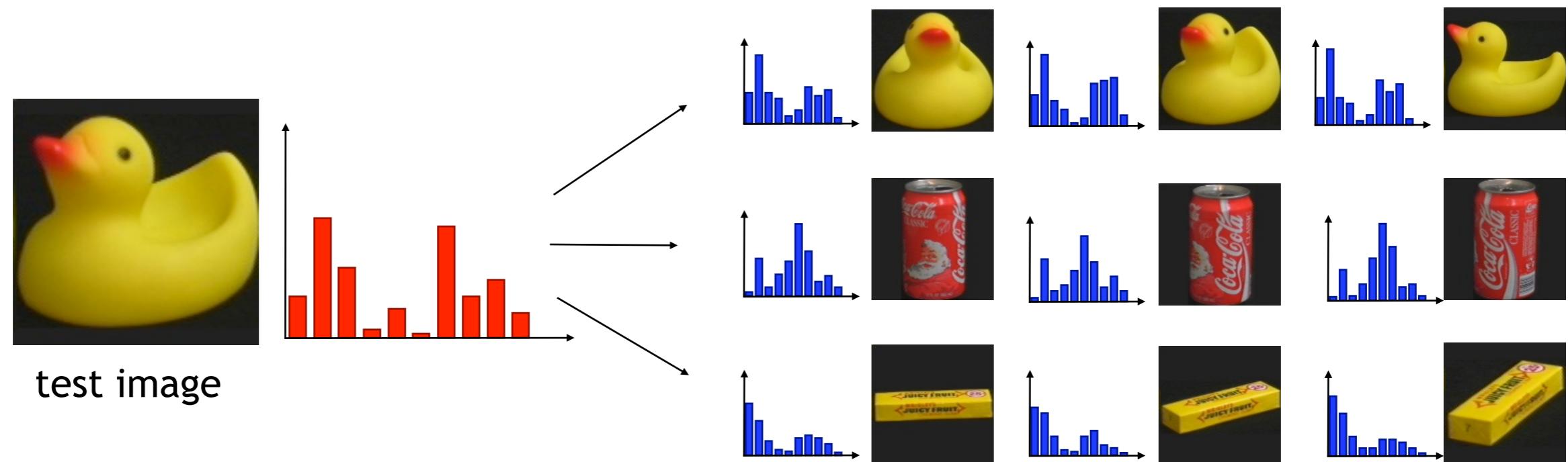
known objects



Recognition using Histograms



- ◆ Database with multiple training views per object



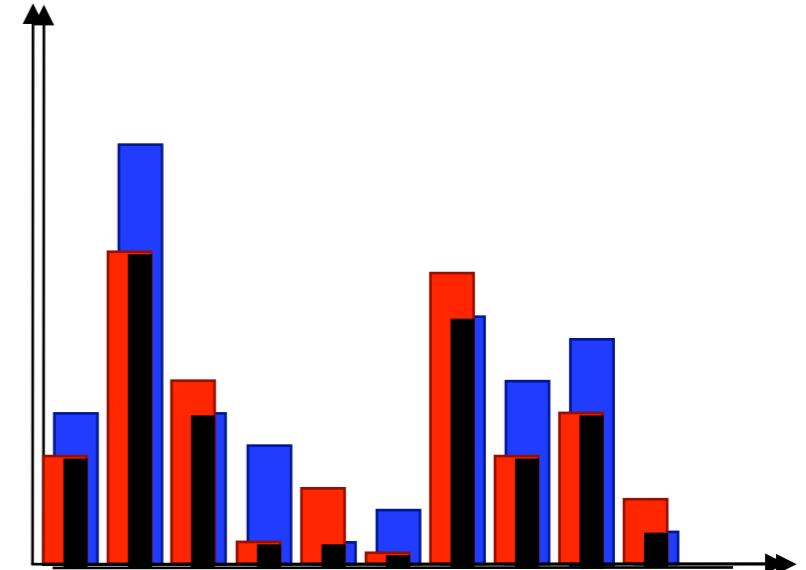
Histogram Comparison

- ◆ Comparison measures
 - ◆ Histogram intersection

$$\cap(Q, V) = \sum_i \min(q_i, v_i)$$

- ◆ Motivation:
 - ◆ Measures the common part of both histograms.
 - ◆ Range: [0,1]
 - ◆ For unnormalized histograms, use

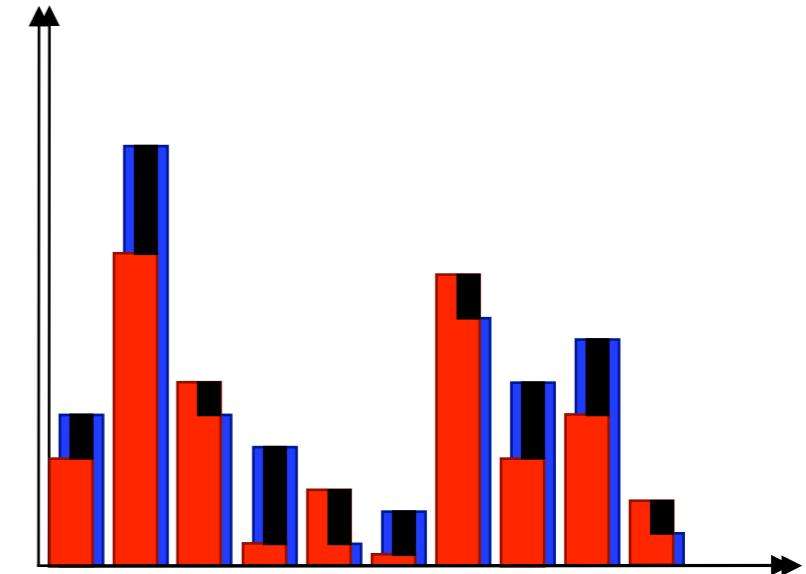
$$\cap(Q, V) = \frac{1}{2} \left(\frac{\sum_i \min(q_i, v_i)}{\sum_i q_i} + \frac{\sum_i \min(q_i, v_i)}{\sum_i v_i} \right)$$



Histogram Comparison

- ◆ Comparison measures
 - ◆ Euclidean distance

$$d(Q, V) = \sum_i (q_i - v_i)^2$$



- ◆ Motivation:
 - ◆ Focuses on the differences between the histograms.
 - ◆ Range: $[0, \infty]$
 - ◆ All cells are weighted equally.
 - ◆ Not very discriminative.

Histogram Comparison

- ◆ Comparison measures

- ◆ Chi-square

$$\chi^2(Q, V) = \sum_i \frac{(q_i - v_i)^2}{q_i + v_i}$$

- ◆ Motivation:

- ◆ Statistical background:

- ◆ Test if two distributions are different.

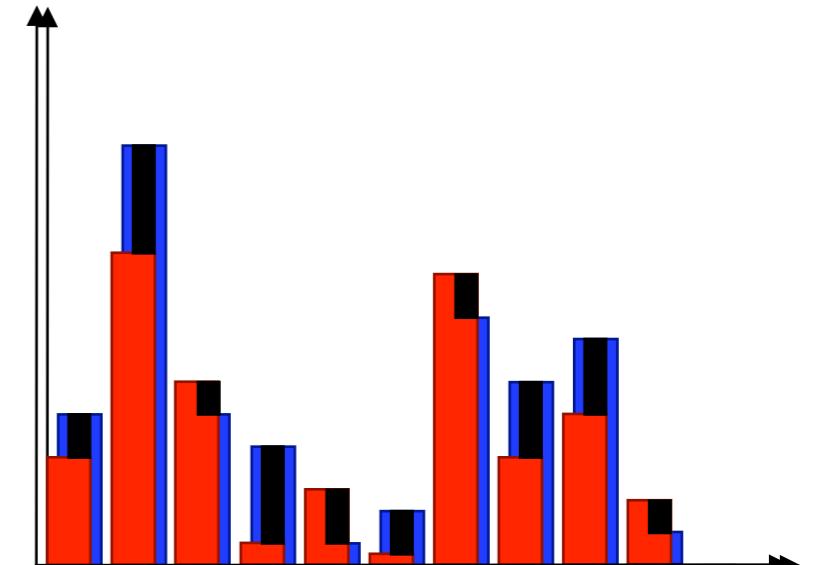
- ◆ Possible to compute a significance score.

- ◆ Range: $[0, \infty]$

- ◆ Cells are not weighted equally!

- ◆ therefore more discriminative

- ◆ may have problems with outliers (therefore assume that each cell contains at least a minimum of samples)



Histogram Comparison

- ◆ Which measure is best?
 - ◆ Depends on the application...
 - ◆ Both intersection and χ^2 give good performance.
 - ◆ Intersection is a bit more robust.
 - ◆ χ^2 is a bit more discriminative.
 - ◆ Euclidean distance is not robust enough.
- ◆ There exist many other measures:
 - ◆ Bhattacharyya distance
 - ◆ Information theoretic: Kullback-Leiber divergence, ...

Recognition using Histograms

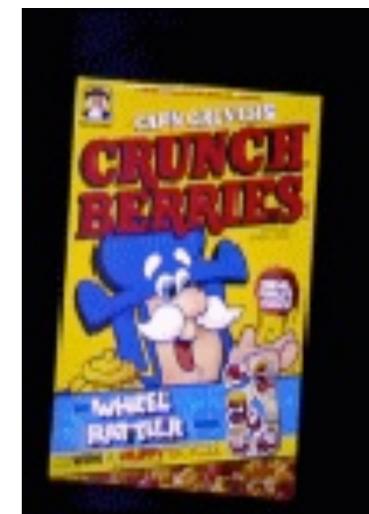
◆ Simple algorithm:

1. Build a set of histograms $H = \{M_1, M_2, M_3, \dots\}$ for each known object
 - ◆ More precisely, for each view of each object.
2. Build a histogram T for the test image.
3. Compare T to each $M_k \in H$
 - ◆ Using a suitable comparison measure.
4. Select the object with the best matching score
 - ◆ Or reject the test image if no object is similar enough.

“Nearest-Neighbor” strategy

Color Histograms

- ◆ Recognition
 - ◆ Works surprisingly well
 - ◆ In the first paper (1991), 66 objects could be recognized almost without errors



[Swain & Ballard, 1991]

Discussion: Color Histograms

◆ Advantages:

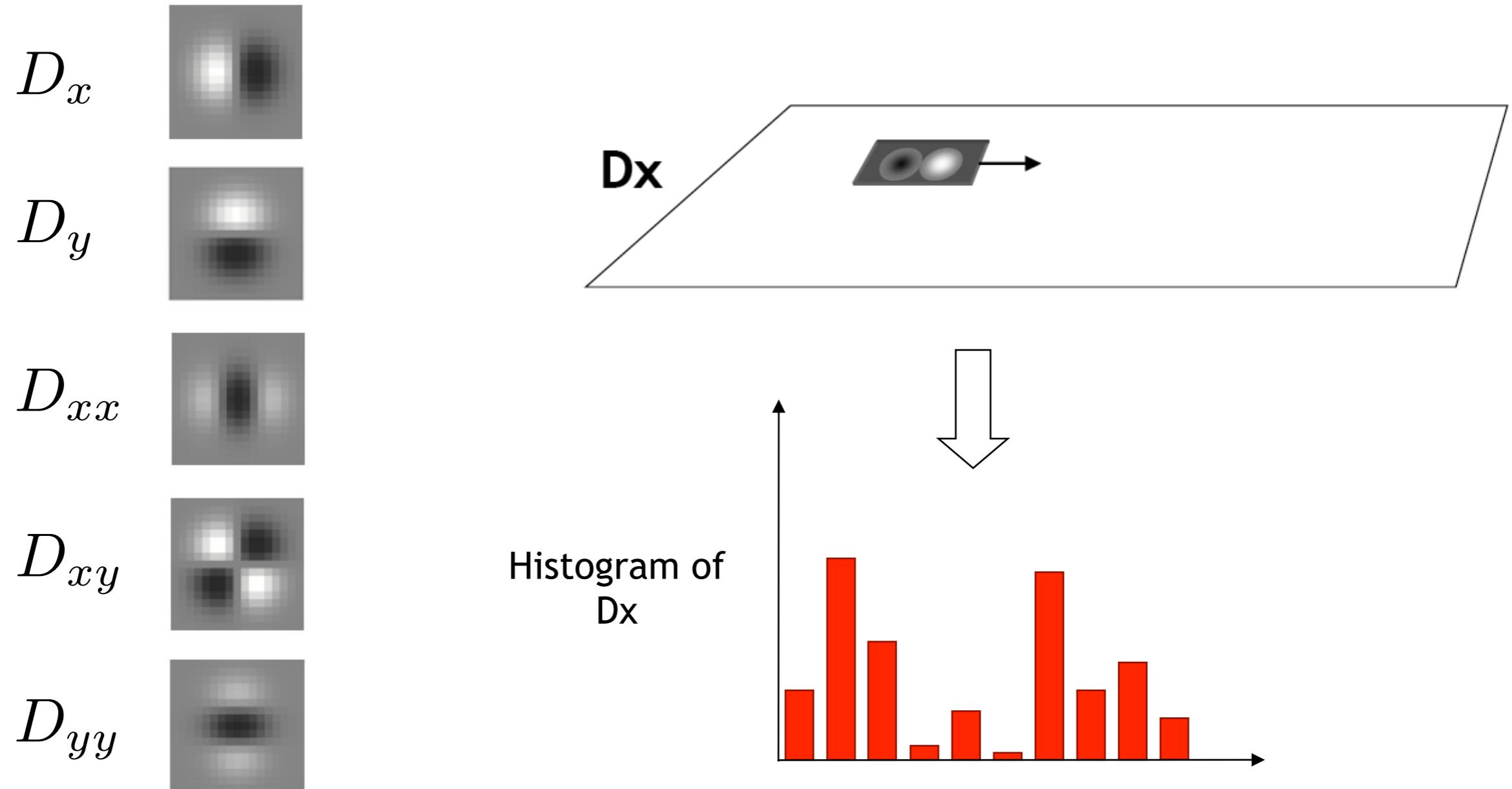
- ◆ Invariant to object translations
- ◆ Invariant to image rotations
- ◆ Slowly changing for out-of-plane rotations
- ◆ No perfect segmentation necessary
- ◆ Histograms change gradually when part of the object is occluded
- ◆ Possible to recognize deformable objects

◆ Problems:

- ◆ The pixel colors change with the illumination
("color constancy problem")
 - ◆ Intensity
 - ◆ Spectral composition (illumination color)
- ◆ Not all objects can be identified by their color distribution.

Generalization of the Idea

- ◆ Histograms of derivatives

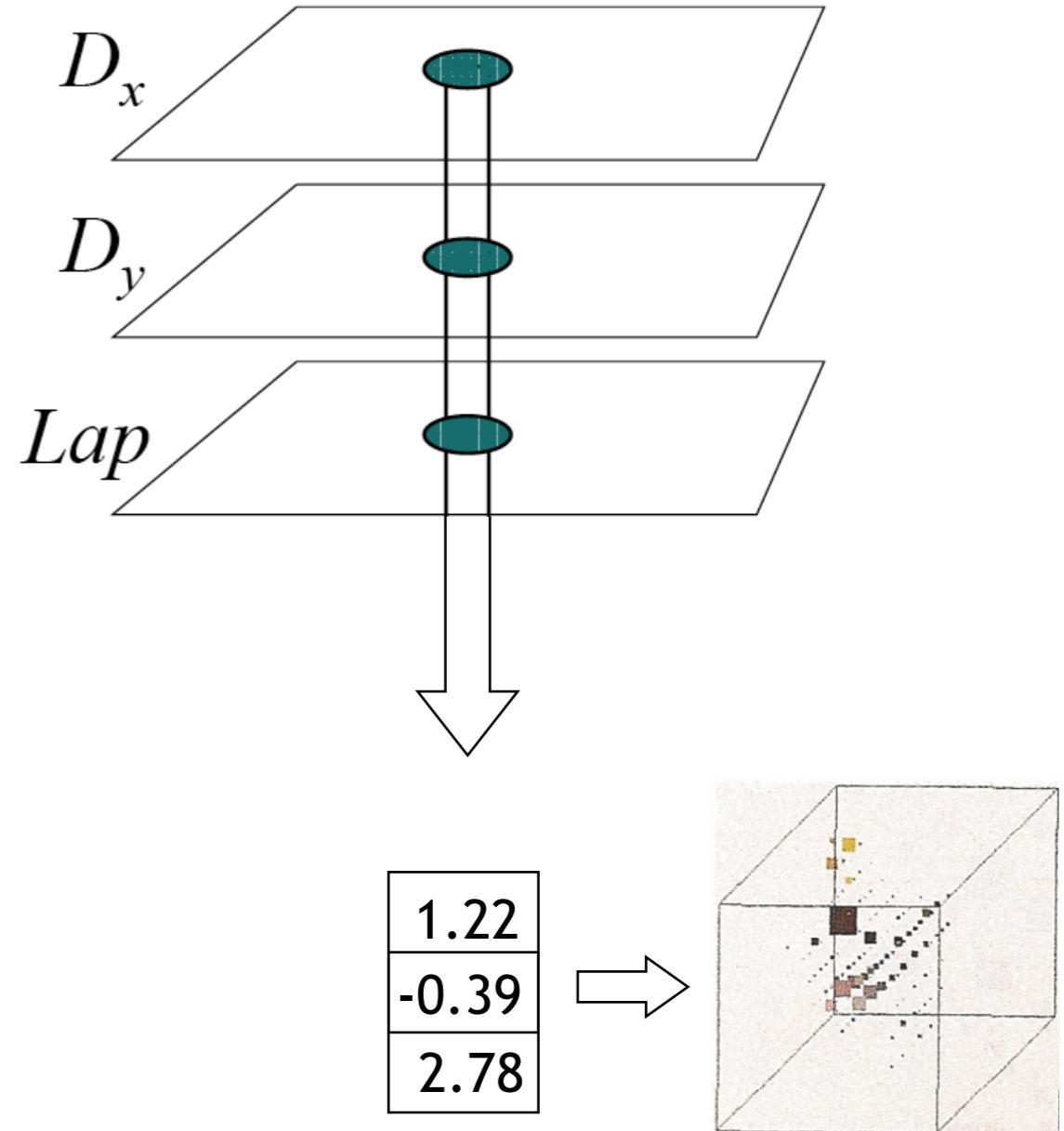


Generalization of the Idea

- ◆ General receptive field histograms
 - ◆ Any local descriptor (e.g. filter, filter combination) can be used to build a histogram.
 - ◆ Examples:
 - ◆ Gradient magnitude $\text{Mag} = \sqrt{D_x^2 + D_y^2}$
 - ◆ Gradient direction $\text{Dir} = \arctan \frac{D_y}{D_x}$
 - ◆ Laplacian $\text{Lap} = D_{xx} + D_{yy}$

Multidimensional Histograms

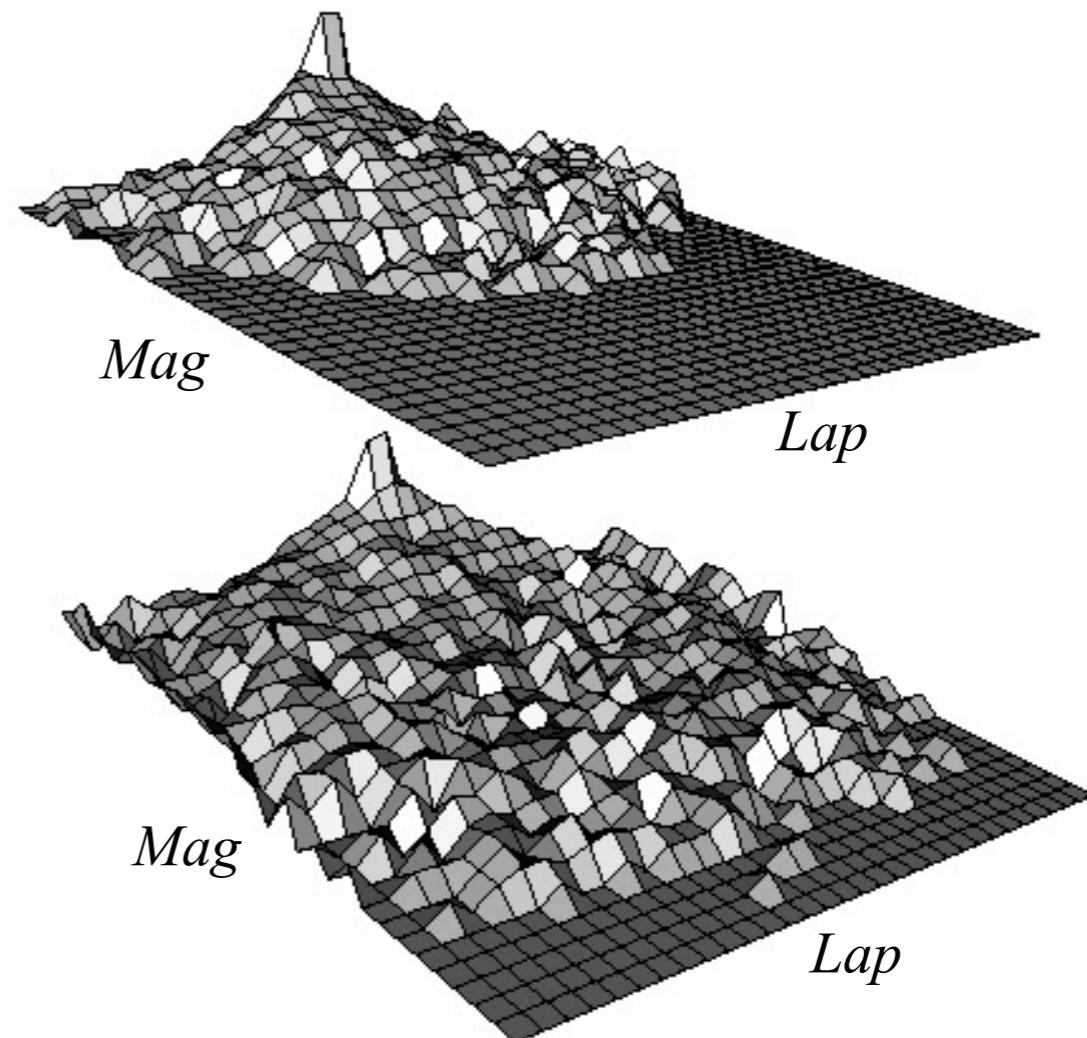
- ◆ Combination of several descriptors
 - ◆ Each descriptor is applied to the whole image.
 - ◆ Corresponding pixel values are combined into one feature vector.
- ◆ Feature vectors are collected in a multidimensional histogram.



Multidimensional Histograms

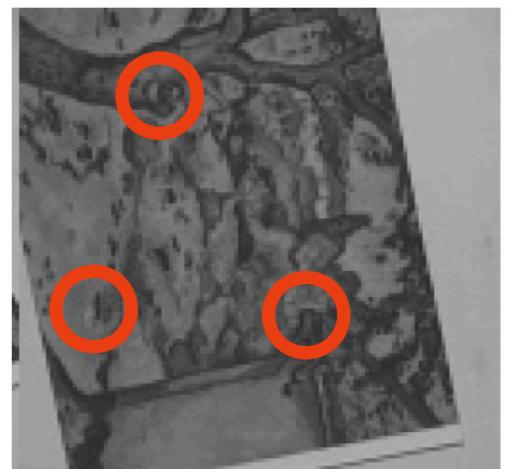
- ◆ Examples:

[Schiele & Crowley, 2000]



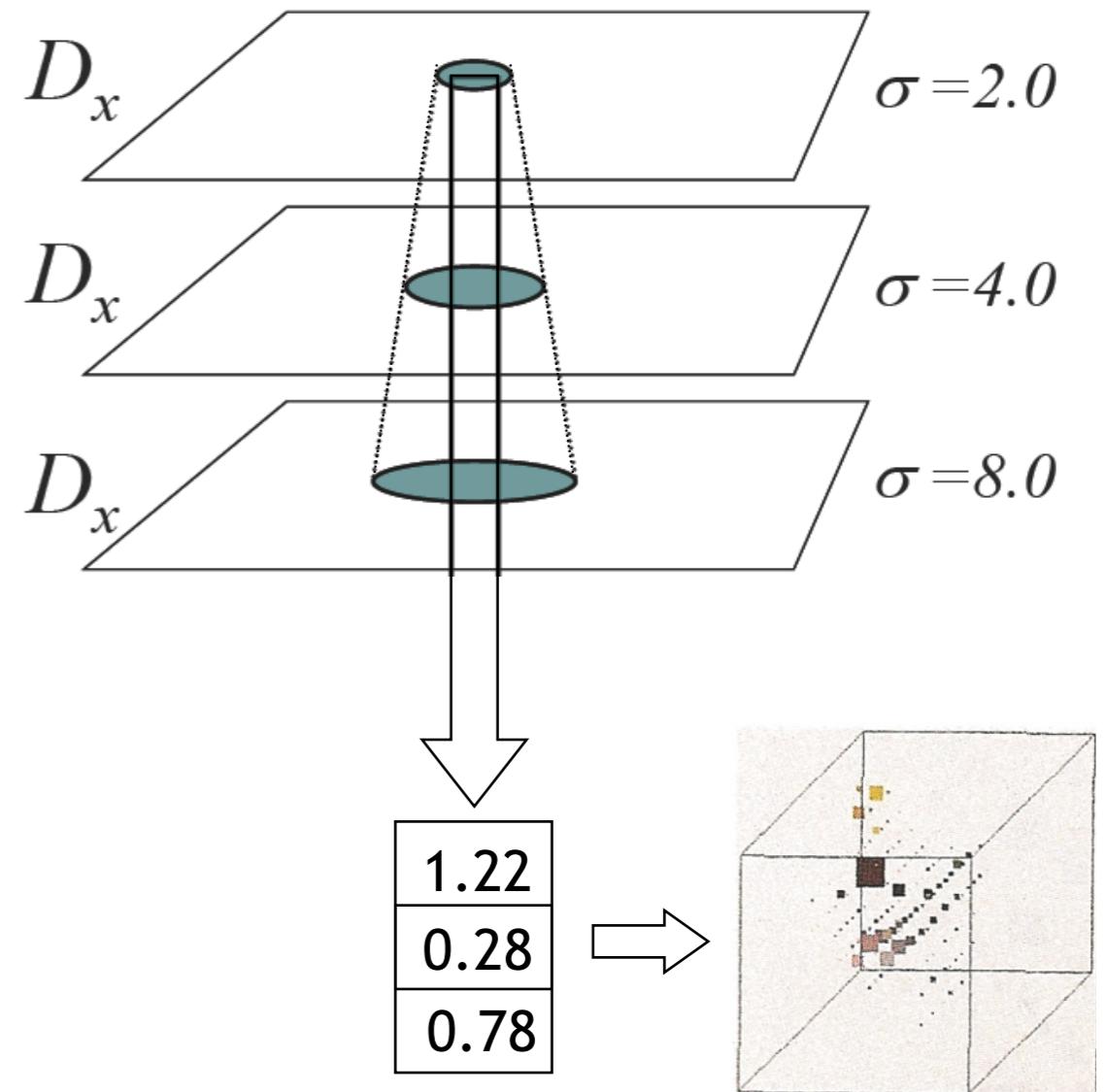
Multidimensional Histograms

- ◆ Useful combinations:
- ◆ D_x-D_y : Rotation-variant
 - ◆ Descriptor changes when image is rotated.
 - ◆ Useful for recognizing oriented structures (e.g. vertical lines)
- ◆ Mag-Lap : Rotation-invariant
 - ◆ Descriptor does not change when image is rotated.
 - ◆ Can be used to recognize rotated objects.
 - ◆ Less discriminative than rotation-variant descriptor.



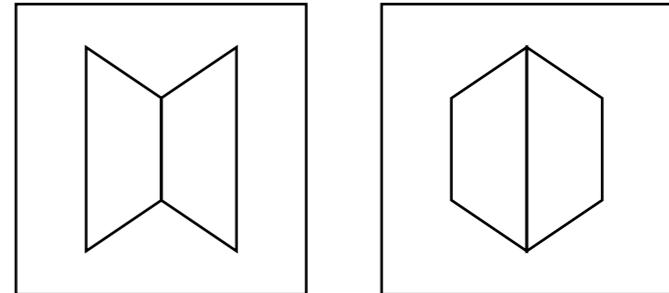
Multidimensional Histograms

- ◆ Combination of several (smoothing) scales
 - ◆ Descriptors are computed at different scales.
 - ◆ Each scale captures different information about the object.
 - ◆ Size of the support region grows with increasing σ .
 - ◆ Feature vectors capture both local details and larger-scale structures.

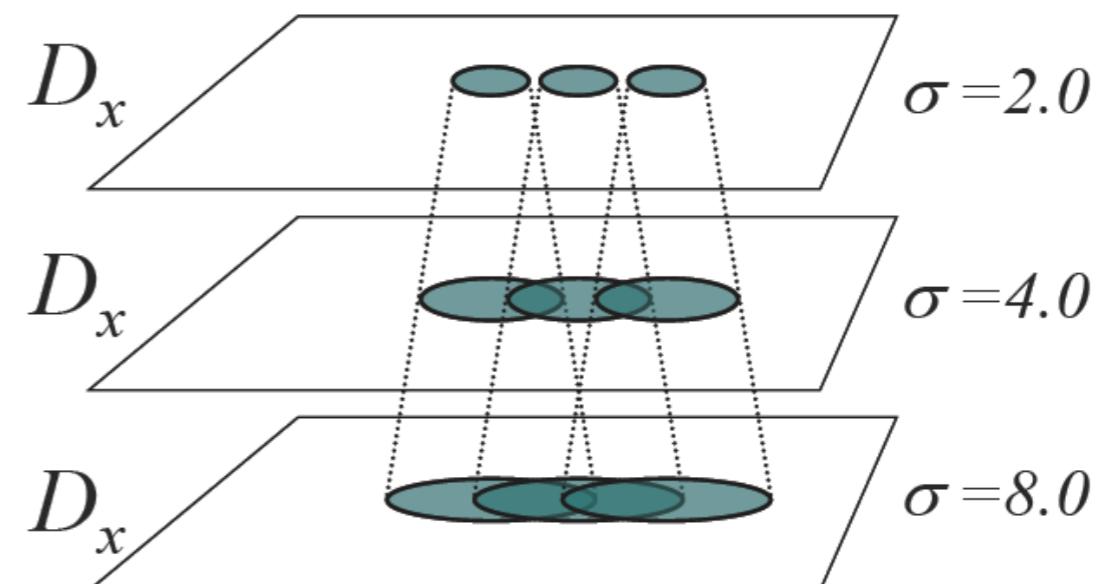


Why multiple scales?

- ◆ Histogram representation
 - ◆ ... contains no structural description.
 - ◆ Many different objects should result in the same histograms.



- ◆ But
 - ◆ Support regions of neighboring descriptors overlap.
 - ◆ Neighborhood relations are captured implicitly.



Multidimensional Histograms

- ◆ Experiences:
 - ◆ Work well for simple recognition problems.
 - ◆ Usually, simple combinations are sufficient (e.g. D_x - D_y , Mag-Lap)
 - ◆ Multiple scales are important!
- ◆ Problems:
 - ◆ High-dimensional histograms ⇒ lots of storage, need many examples
 - ◆ Global representation ⇒ not robust to occlusion

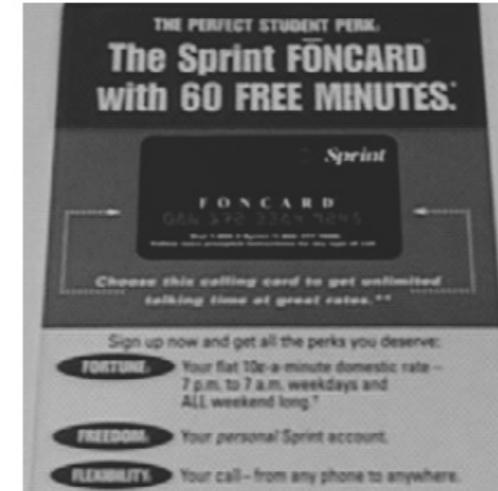
Recognition Results



Test image 1



First Match



Second Match



Third Match



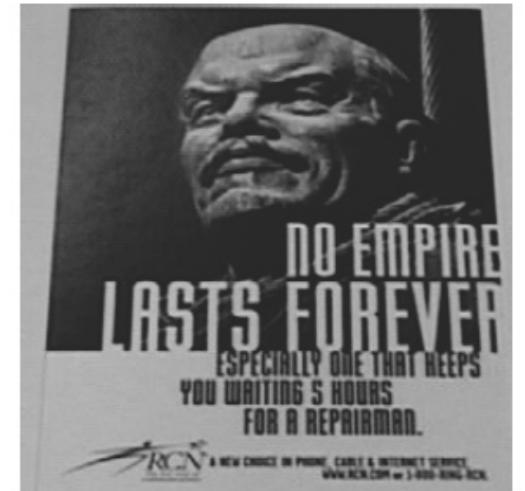
Test image 2



First Match



Second Match



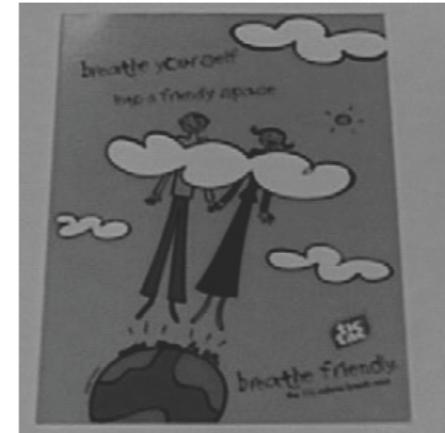
Third Match

[Schiele & Crowley, 2000]

Recognition Results



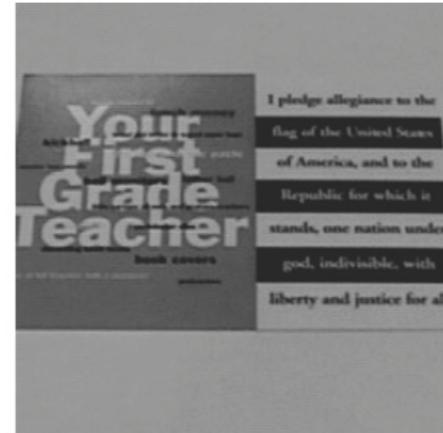
Test image 3



First Match



Second Match



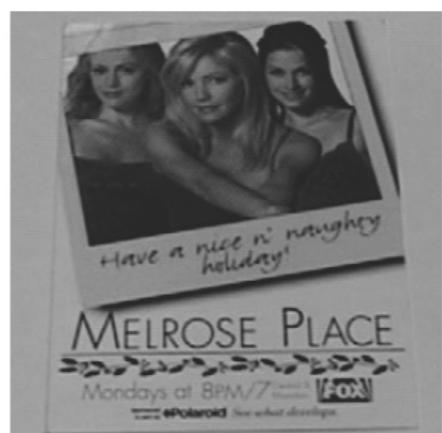
Third Match



Test image 4



First Match



Second Match



Third Match



Fourth Match

[Schiele & Crowley, 2000]



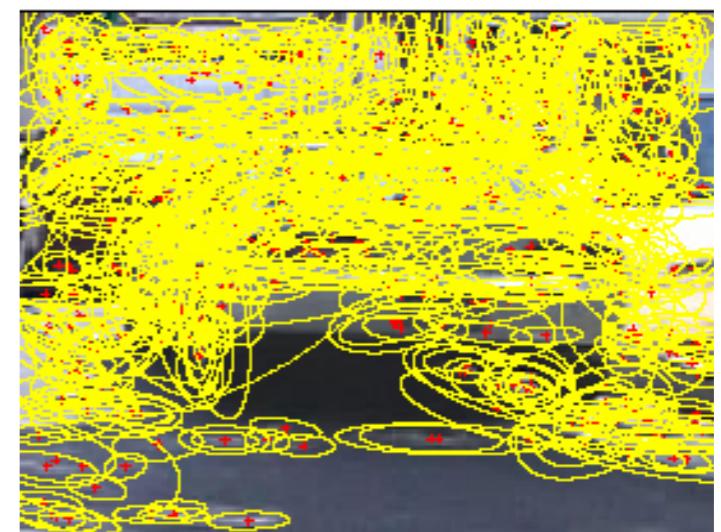
Summary

- ◆ Appearance-based object recognition
 - ◆ Using global representations
 - ◆ With **full** and **without any** spatial information
- ◆ Histograms:
 - ◆ Color histograms
 - ◆ Histogram comparison measures
 - ◆ Multidimensional histograms
- ◆ So far only object identification / instance recognition!
 - ◆ The histogram idea can be also used for object categorization... stay tuned.

Feature representation

- ◆ Dense representation (regular grid):
 - ◆ Color histogram approach [Swain & Ballard '91]
 - ◆ Multidimensional receptive field histograms [Schiele & Crowley '96-'00]

- ◆ Sparse feature representations:
 - ◆ Find key points with interest point detector
 - ◆ e.g. scale- or affine-invariant
 - ◆ Represent key points with feature vectors
 - ◆ e.g. SIFT, ShapeContext



Overview

- ◆ **1 - Local Interest Point Detection (today)**
 - ◆ Finding discriminative points (Harris, Hessian)
 - ◆ Scale invariant interest point detection (Harris-Laplace)
- ◆ 2 - Local Descriptors (= Features)
 - ◆ next time
- ◆ 3 - Bag-of-Words Model (BoW)
 - ◆ next time
 - ◆ BoW for Object Categorization

Motivation for Local Interest Point Detection

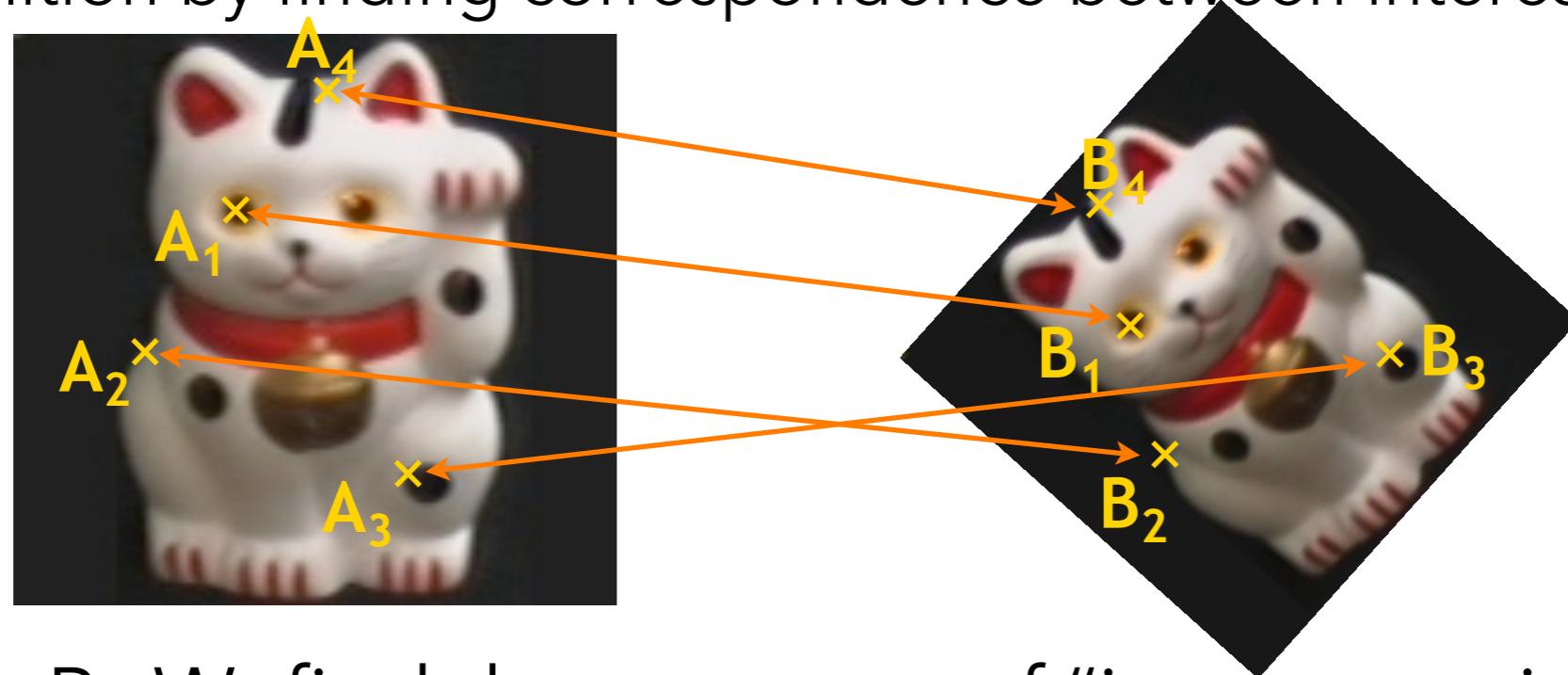
- ◆ General idea of interest point detection:
 - ◆ Recognition by finding correspondence between interest points
- ◆ Goal for BoW: find the same set of “interest points” that are
 - ◆ discriminative for object recognition
 - ◆ can be always found - even in the presence of arbitrary geometric and photometric transformations

Image matching

- ◆ Different application
- ◆ Same goal!



by [Diva Sian](#)



by [swashford](#)

[Szeliski & Seitz]

Image matching - Harder case



- ◆ Different application
- ◆ Same goal!



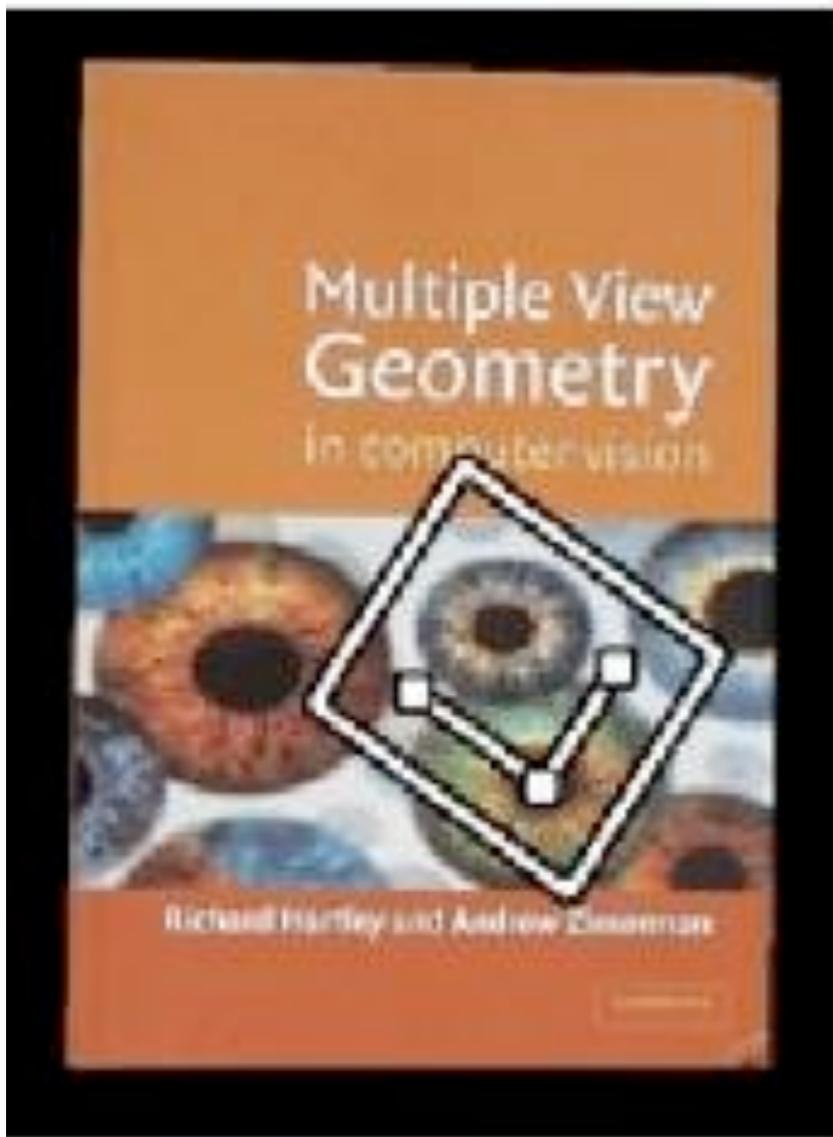
by [Diva Sian](#)



by [scgbt](#)

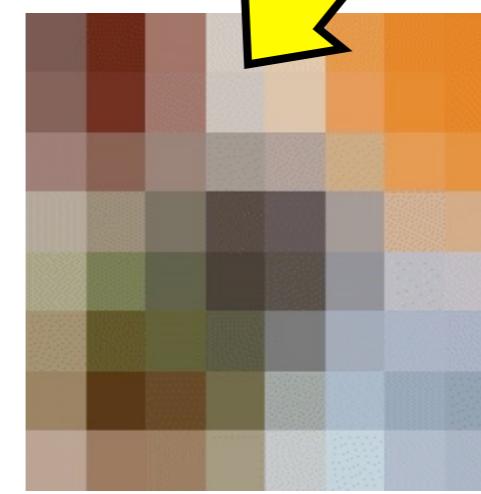
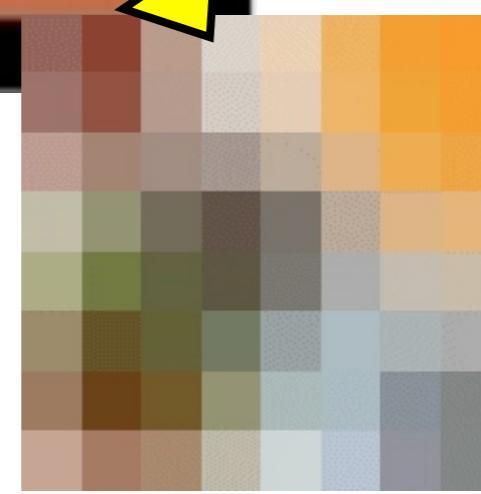
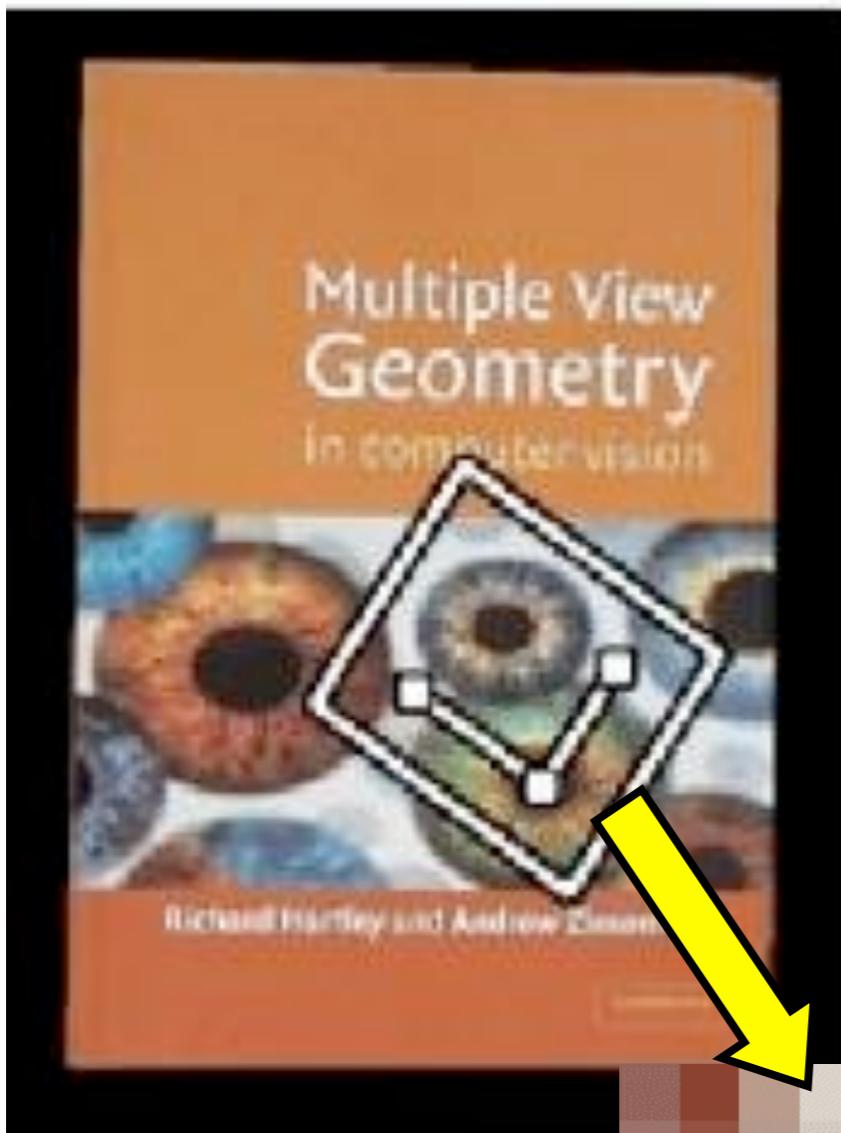
[Szeliski & Seitz]

Image matching



[Szeliski & Seitz]

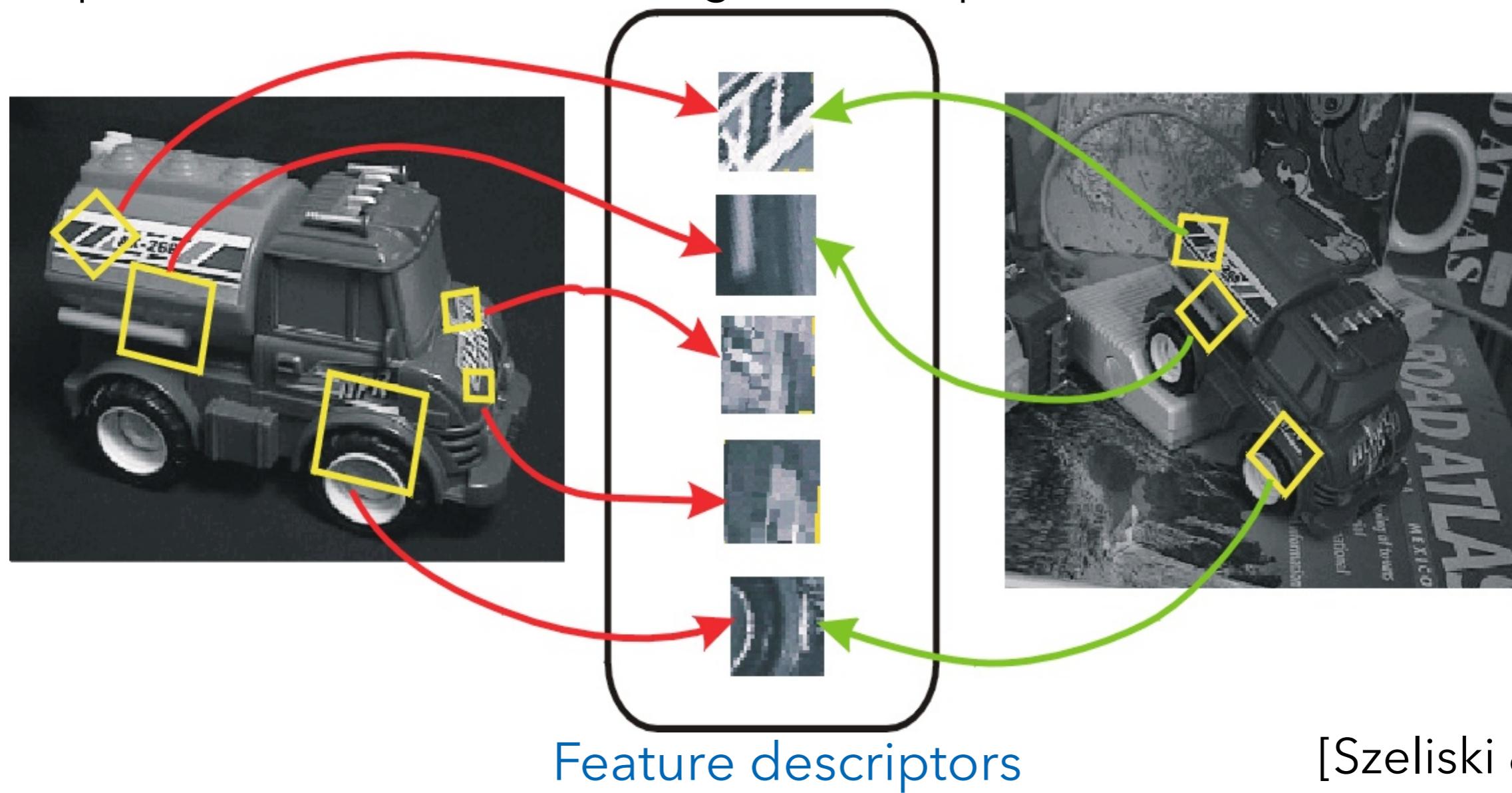
Image matching



[Szeliski & Seitz]

Invariant local features

- ◆ Find features that are **invariant to transformations**
 - ◆ geometric invariance: translation, rotation, scale
 - ◆ photometric invariance: brightness, exposure, ...



[Szeliski & Seitz]



Advantages of local features

- ◆ Locality:
 - ◆ features are local, so robust to occlusion and clutter
- ◆ Distinctiveness:
 - ◆ can differentiate a large database of objects
- ◆ Quantity:
 - ◆ hundreds or thousands in a single image
- ◆ Efficiency:
 - ◆ real-time performance achievable
- ◆ Generality:
 - ◆ exploit different types of features in different situations

[Szeliski & Seitz]

More motivation...

- ◆ Feature points are used for:
 - ◆ Object recognition
 - ◆ Image alignment (e.g., mosaics)
 - ◆ 3D reconstruction
 - ◆ Motion tracking
 - ◆ Indexing and database retrieval
 - ◆ Robot navigation
 - ◆ ... other

[Szeliski & Seitz]

What makes a good feature?



[Szeliski & Seitz]

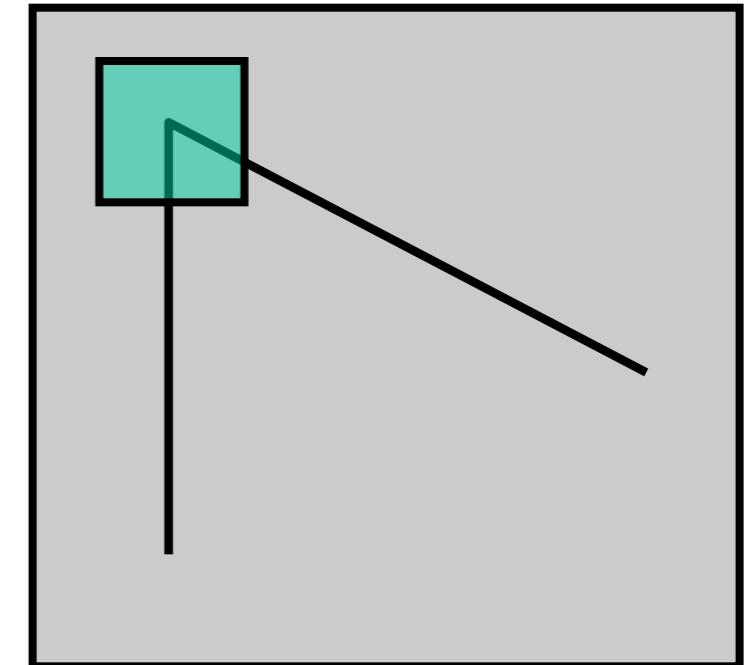
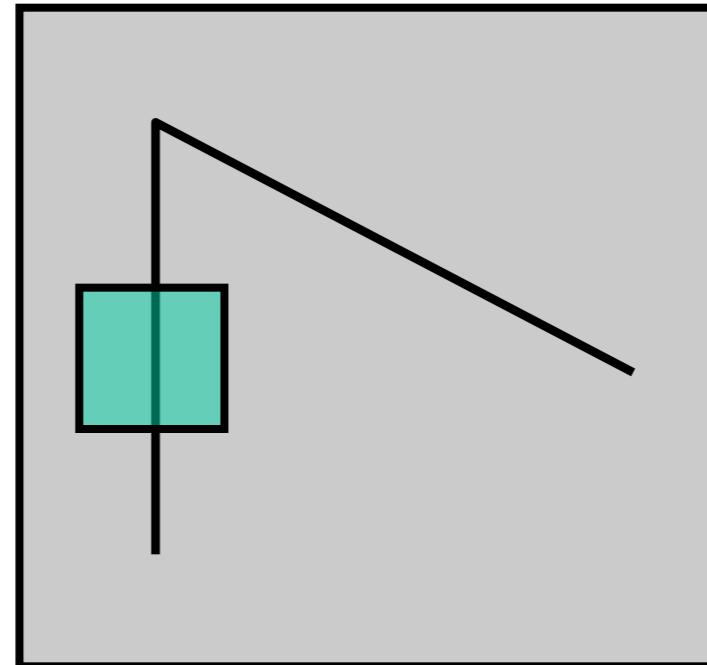
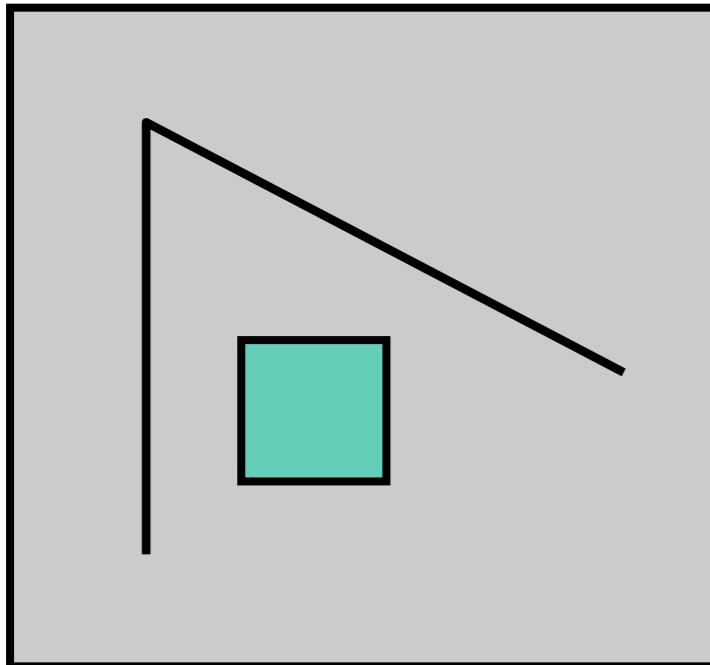
Want uniqueness

- ◆ Look for image regions that are unusual.
 - ◆ Lead to unambiguous matches in other images
- ◆ How to define “unusual”?

[Szeliski & Seitz]

Local measures of uniqueness

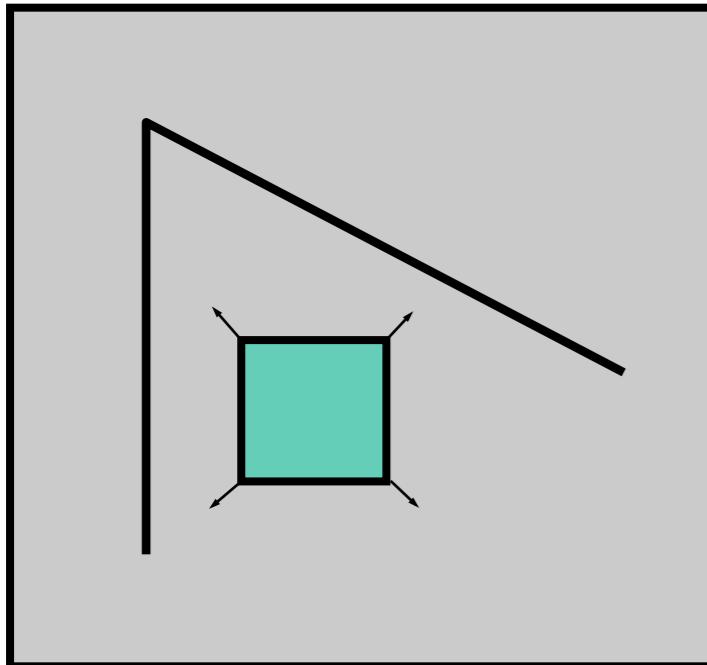
- ◆ Suppose we only consider a **small window of pixels**
 - ◆ What defines whether a feature is a good or bad candidate?



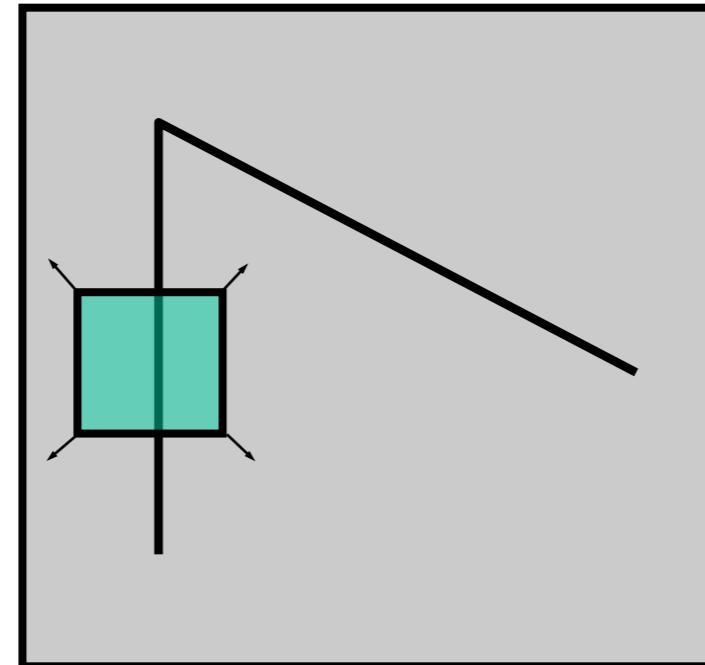
[Szeliski & Seitz]

Feature detection

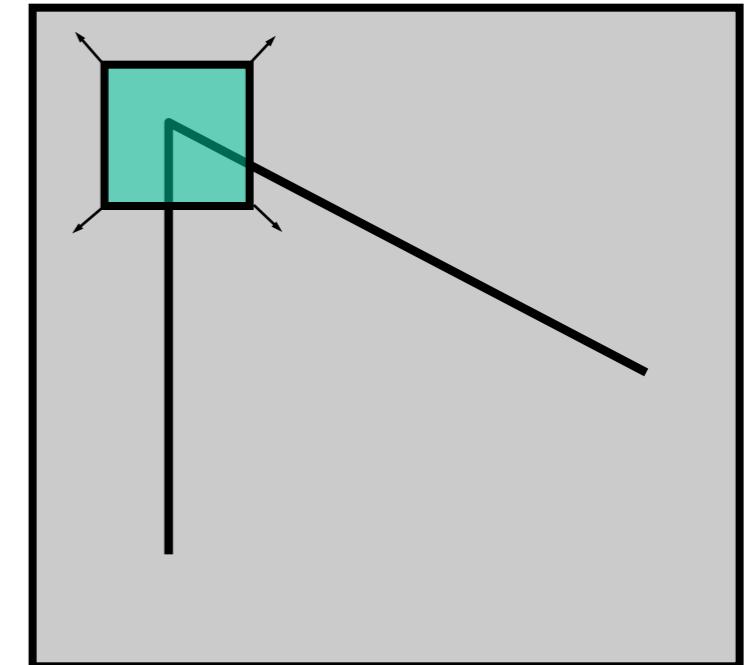
- ◆ Local measure of **feature uniqueness**
 - ◆ How does the window change when you shift it?
 - ◆ Shifting the window in any direction causes a big change



"flat" region:
no change in all
directions



"edge":
no change along the
edge direction

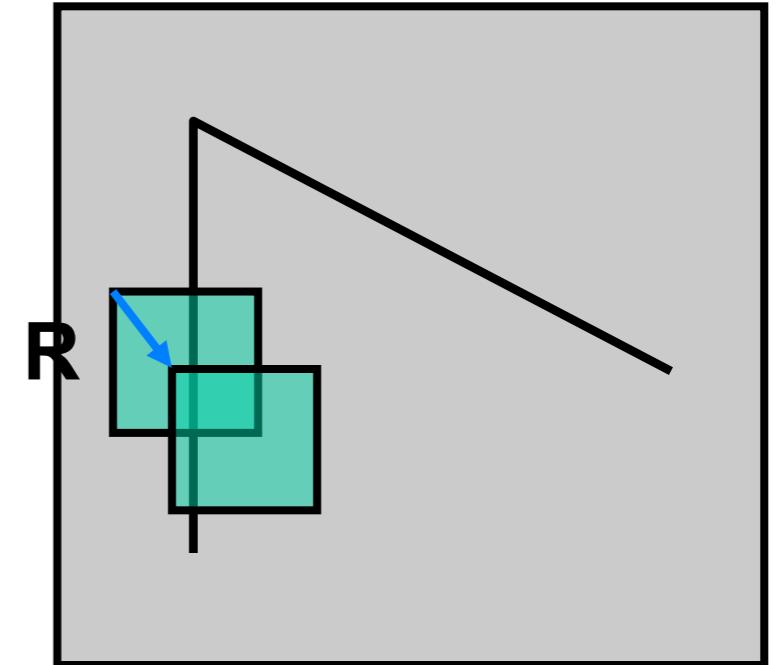


"corner":
significant change in
all directions

[Szeliski & Seitz]

Feature detection: Derivation

- ◆ Consider shifting the window R by (u, v)
 - ◆ how do the pixels in R change?
 - ◆ compare each pixel before and after by summing up the squared differences (SSD)
 - ◆ this defines an SSD "error" of $E(u, v)$:



$$E(u, v) = \sum_{(x,y) \in R} (I(x + u, y + v) - I(x, y))^2$$

[Szeliski & Seitz]

Can we approximate this?

$$E(u, v) = \sum_{(x,y) \in R} (I(x + u, y + v) - I(x, y))^2$$

Taylor series approximation:

$$I(x, y) + u \frac{\partial}{\partial x} I(x, y) + v \frac{\partial}{\partial y} I(x, y) + \epsilon(u^2, v^2)$$

↓

$$E(u, v) \approx \sum_{(x,y) \in R} \left(u \frac{\partial}{\partial x} I(x, y) + v \frac{\partial}{\partial y} I(x, y) \right)^2$$

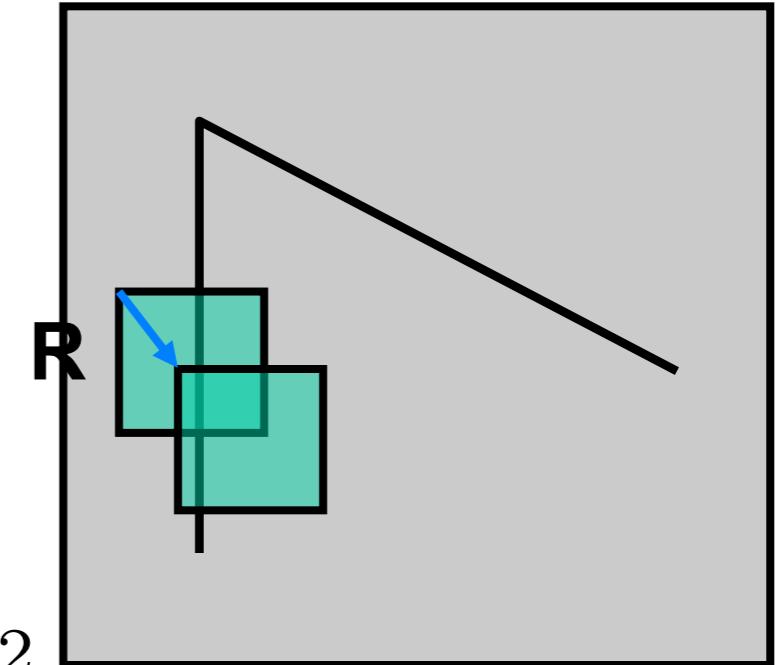
↓

↑
Approximation error

Feature detection: Derivation

- ◆ Consider shifting the window R by (u, v)
 - ◆ how do the pixels in R change?
 - ◆ compare each pixel before and after by summing up the squared differences (SSD)
 - ◆ this defines an SSD "error" of $E(u, v)$:

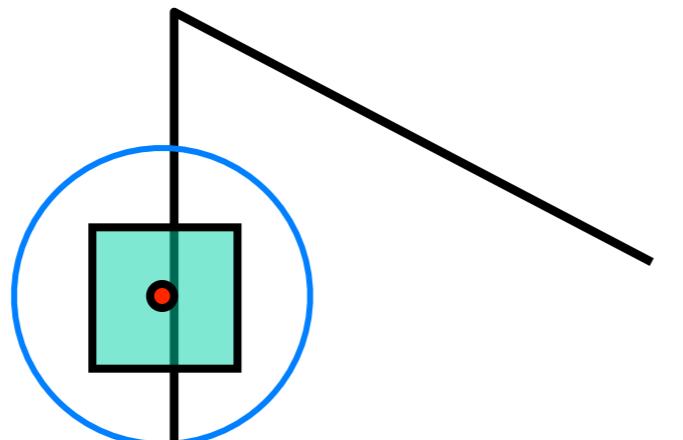
$$\begin{aligned}
 E(u, v) &= \sum_{(x,y) \in R} (I(x + u, y + v) - I(x, y))^2 \\
 &\approx \sum_{(x,y) \in R} (u \cdot I_x(x, y) + v \cdot I_y(x, y))^2 \\
 &= \sum_{(x,y) \in R} (u, v) \nabla I(x, y) \nabla I(x, y)^T \begin{pmatrix} u \\ v \end{pmatrix}
 \end{aligned}$$



$$\nabla I = \begin{pmatrix} I_x \\ I_y \end{pmatrix}$$

[Szeliski & Seitz]

Feature detection: Derivation

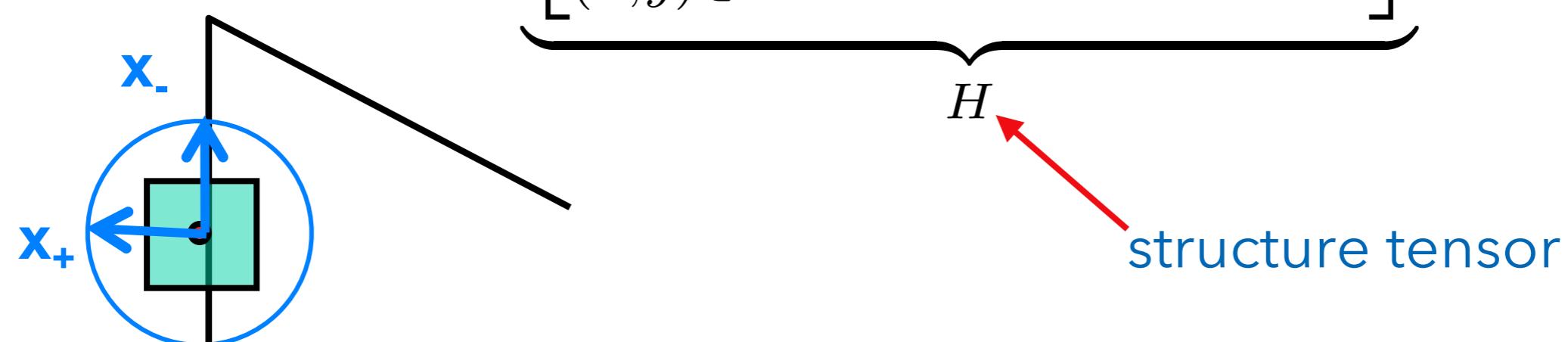
$$E(u, v) \approx (u, v) \left[\sum_{(x,y) \in R} \nabla I(x, y) \nabla I(x, y)^T \right] \begin{pmatrix} u \\ v \end{pmatrix}$$


structure tensor

- ◆ For the example above
 - ◆ Suppose you can move the center of the green window to anywhere on the blue unit circle
 - ◆ Which directions will result in the largest and smallest E values?
 - ◆ We can find these directions by looking at the eigenvectors of H

[Szeliski & Seitz]

Feature detection: Derivation

$$E(u, v) \approx (u, v) \left[\sum_{(x,y) \in R} \nabla I(x, y) \nabla I(x, y)^T \right] \begin{pmatrix} u \\ v \end{pmatrix}$$


- ◆ Eigenvalues and eigenvectors of H

- ◆ Define shifts with the smallest and largest change (E value)
- ◆ x_+ = direction of largest increase in E .
- ◆ λ_+ = amount of increase in direction x_+
- ◆ x_- = direction of smallest increase in E .
- ◆ λ_- = amount of increase in direction x_-

$$Hx_+ = \lambda_+ x_+$$

$$Hx_- = \lambda_- x_-$$

[Szeliski & Seitz]

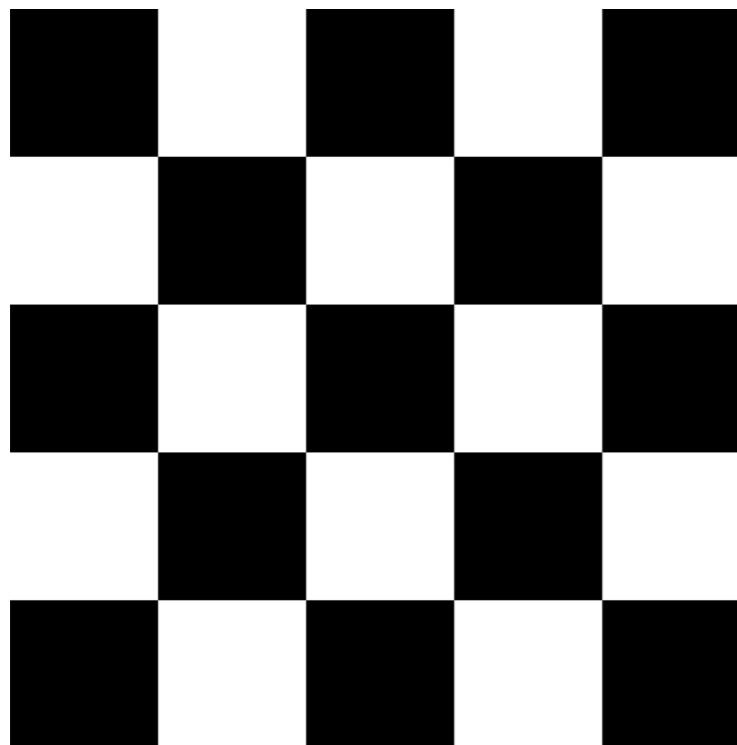
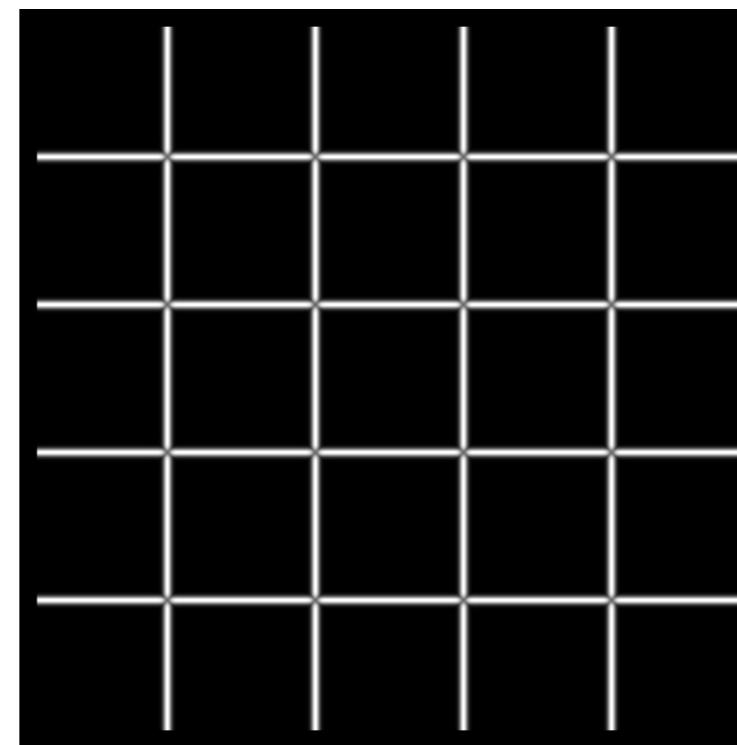
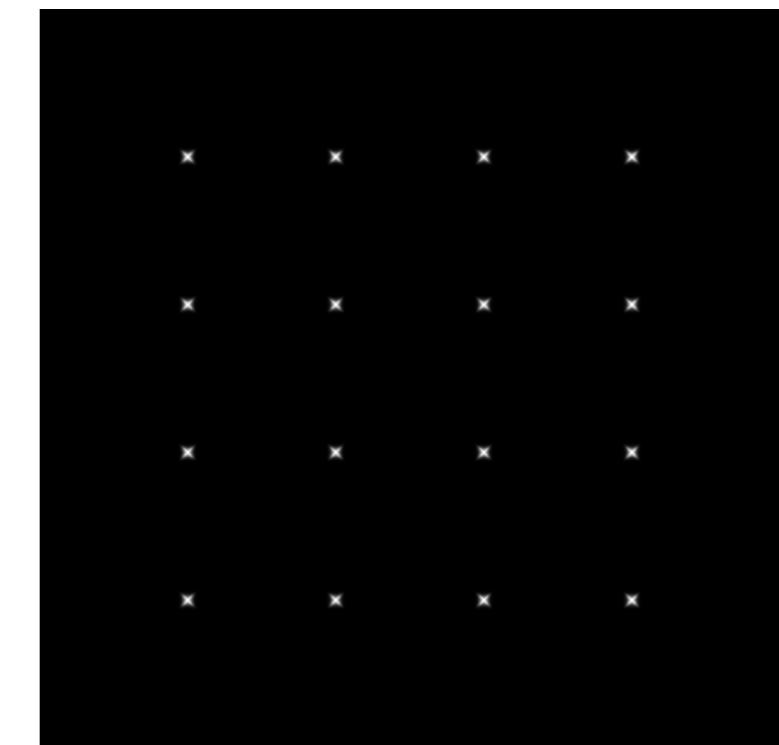
Feature detection

- ◆ How are λ_+ , x_+ , λ_- , and x_- relevant for feature detection
 - ◆ What's our feature scoring function?

[Szeliski & Seitz]

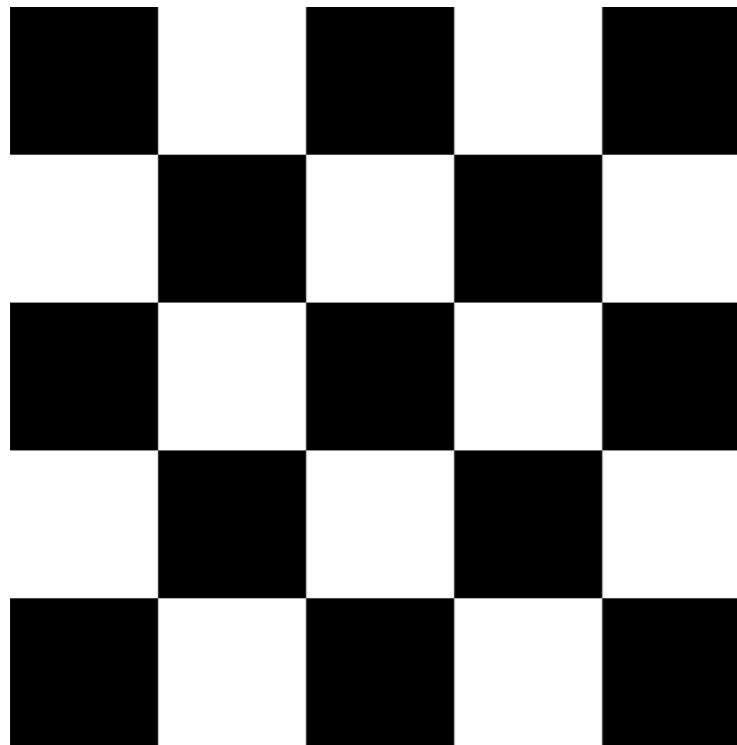
Feature detection

- ◆ How are λ_+ , x_+ , λ_- and x_- relevant for feature detection
 - ◆ What's our feature scoring function?
- ◆ Want $E(u,v)$ to be **large** for small shifts in **all** directions
 - ◆ the *minimum* of $E(u,v)$ should be large, over all unit vectors $[u \ v]$
 - ◆ this minimum is given by the smaller eigenvalue (λ_-) of H

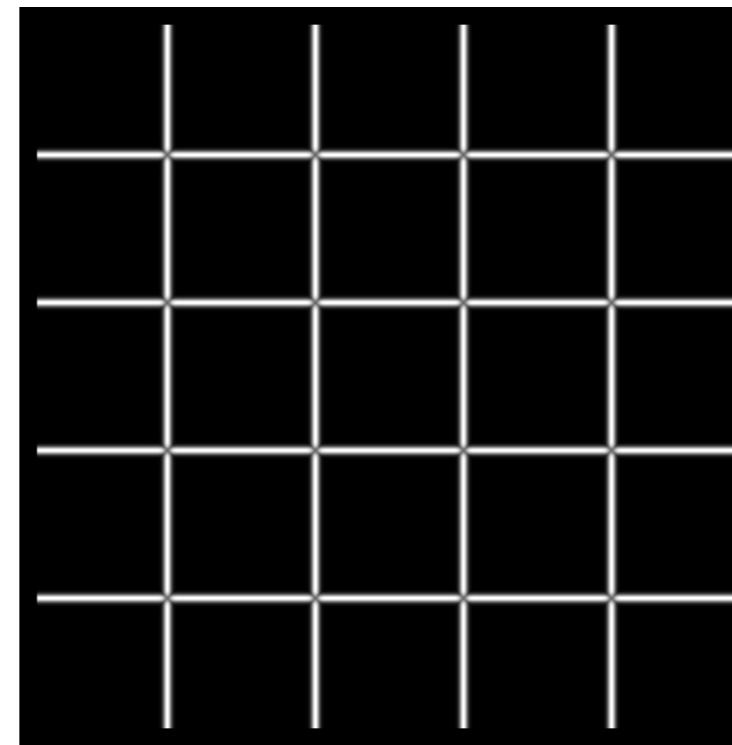

 I

 λ_+

 λ_-

Feature detection

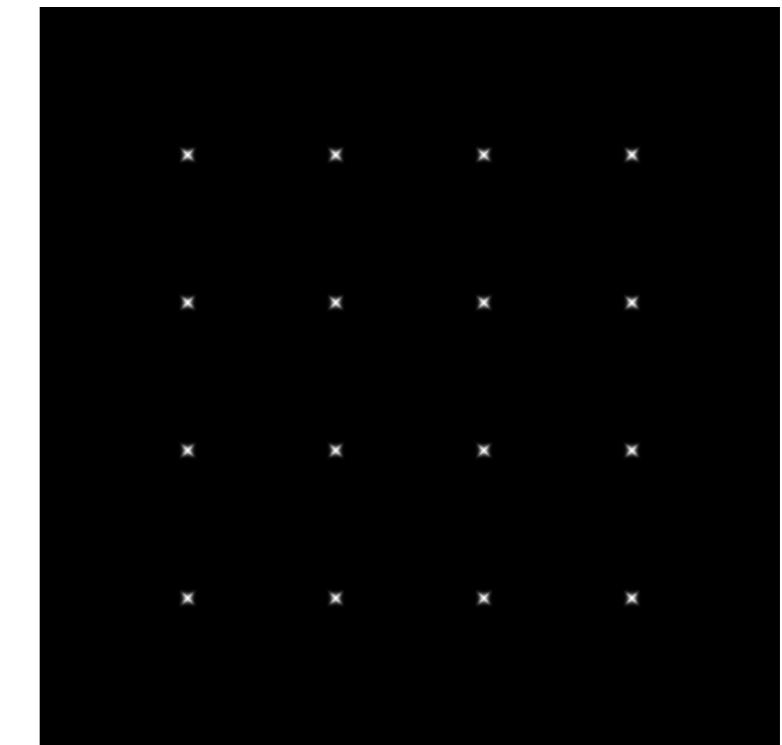
- ◆ Here's what you do
 - ◆ Compute the gradient at each point in the image
 - ◆ Create the H matrix from the entries in the gradient
 - ◆ Compute the eigenvalues.
 - ◆ Find points with large response ($\lambda_- >$ threshold)
 - ◆ Choose those points where λ_- is a local maximum as features



I



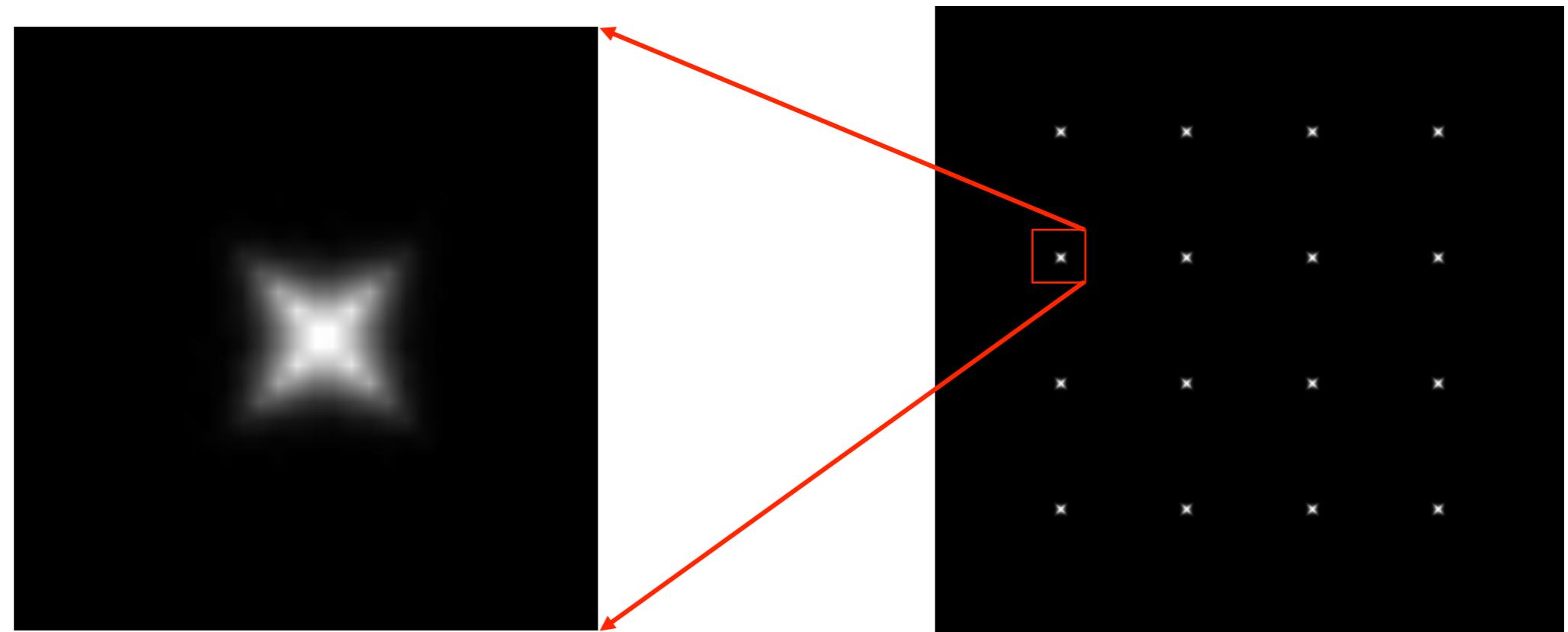
λ_+



λ_-

Feature detection summary

- ◆ Here's what you do
 - ◆ Compute the gradient at each point in the image
 - ◆ Create the H matrix from the entries in the gradient
 - ◆ Compute the eigenvalues.
 - ◆ Find points with large response ($\lambda_>$ threshold)
 - ◆ Choose those points where $\lambda_>$ is a local maximum as features



The Harris operator

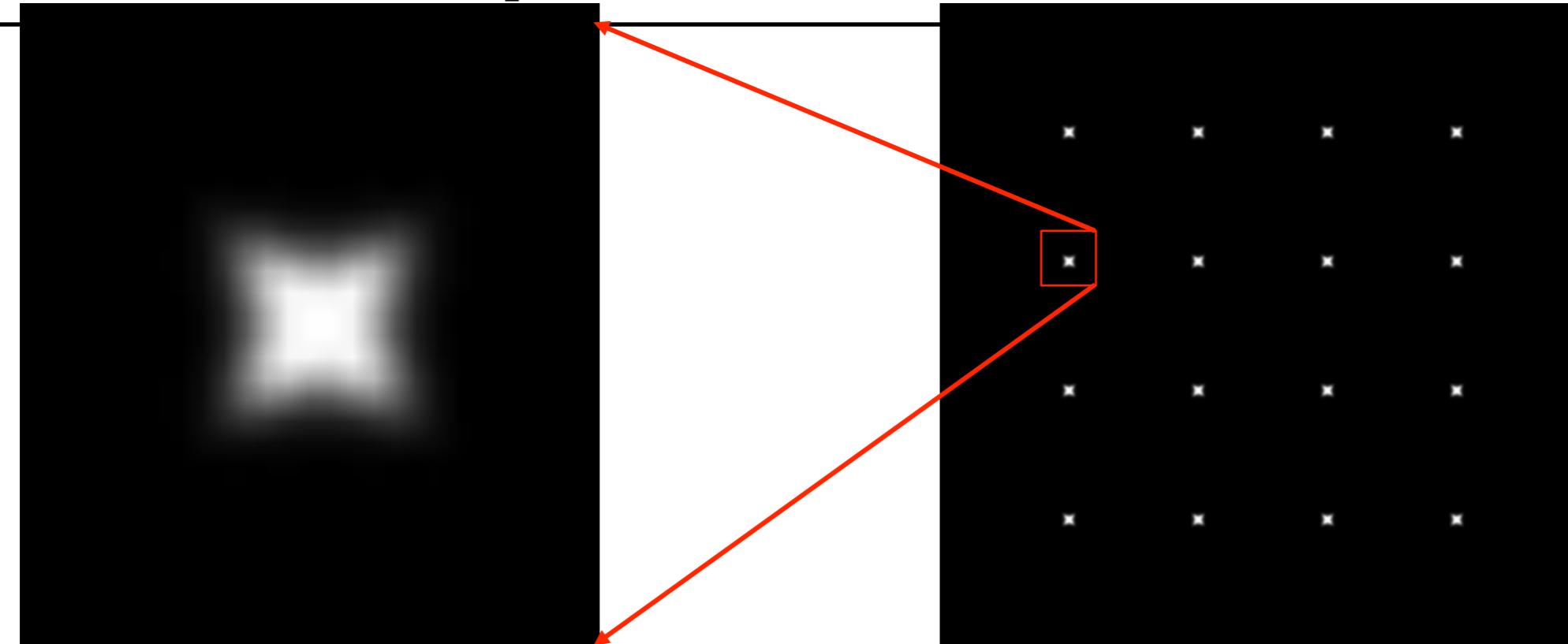
- ◆ λ_- is a variant of the “Harris operator” for feature detection

$$\begin{aligned} f &= \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2 \\ &= \det(H) - \alpha \cdot \text{trace}(A)^2 \end{aligned}$$

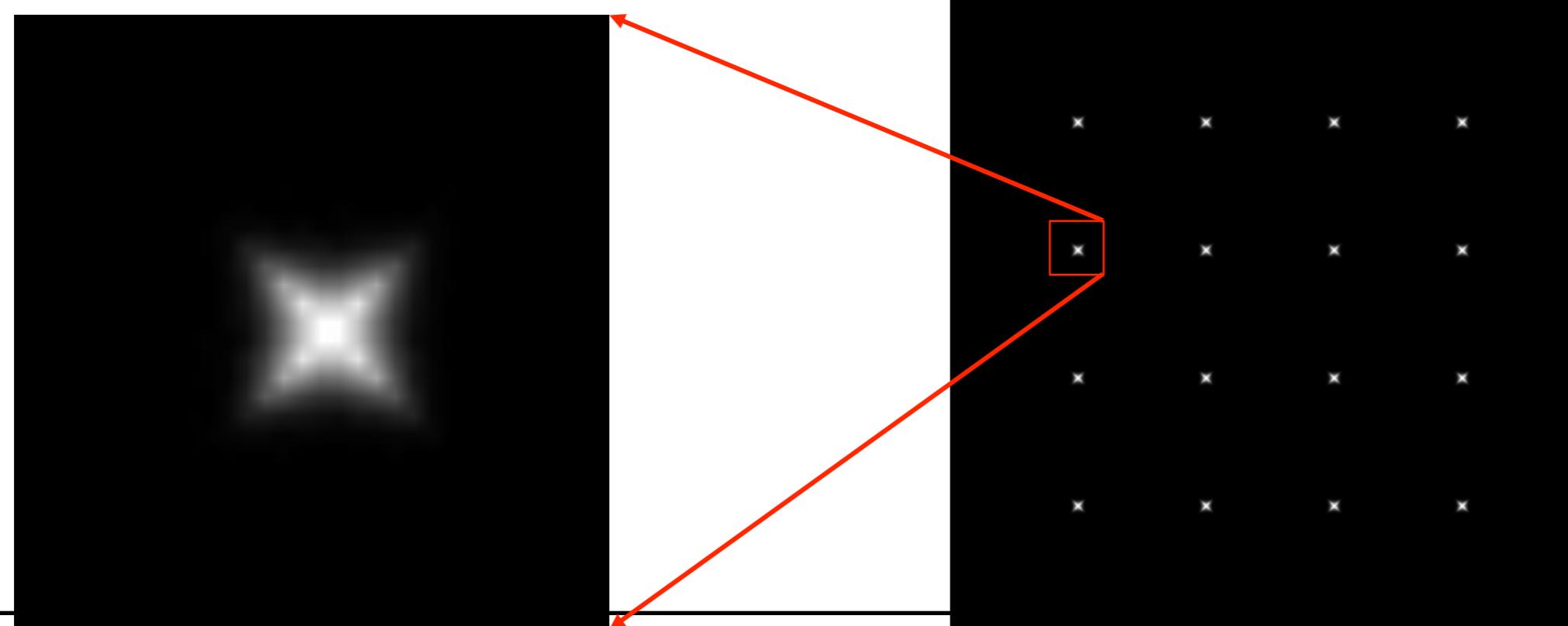
- ◆ The trace is the sum of the diagonals, i.e., $\text{trace}(H) = h_{11} + h_{22}$
- ◆ Very similar to λ_- but less expensive (no square root)
- ◆ Called the “Harris Corner Detector” or “Harris Operator”
- ◆ Lots of other detectors, this is one of the most popular

[Szeliski & Seitz]

The Harris operator



Harris
operator



λ_-

[Szeliski &
Seitz]

Harris detector example

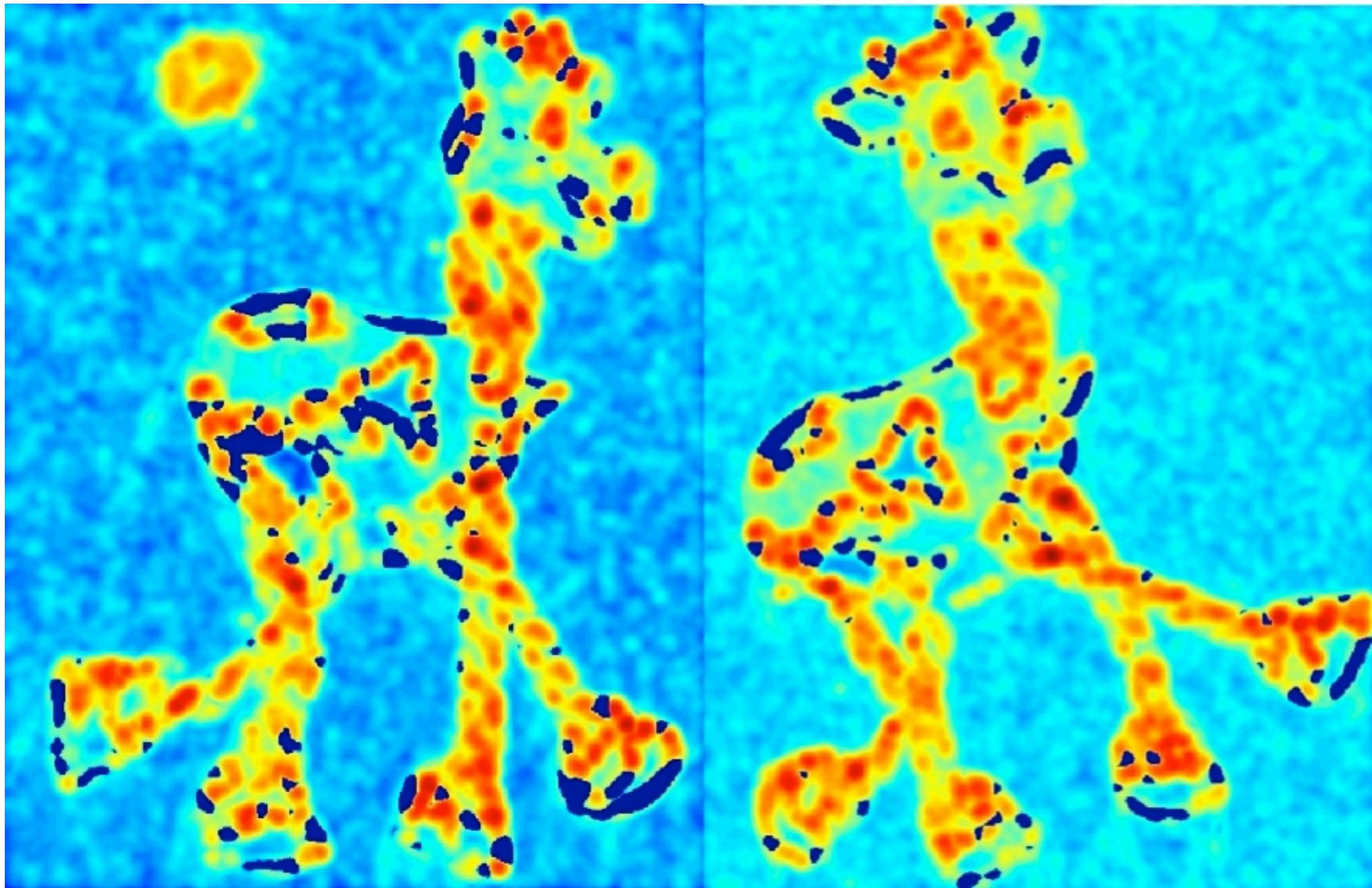


[Szeliski & Seitz]

f value (red high, blue low)

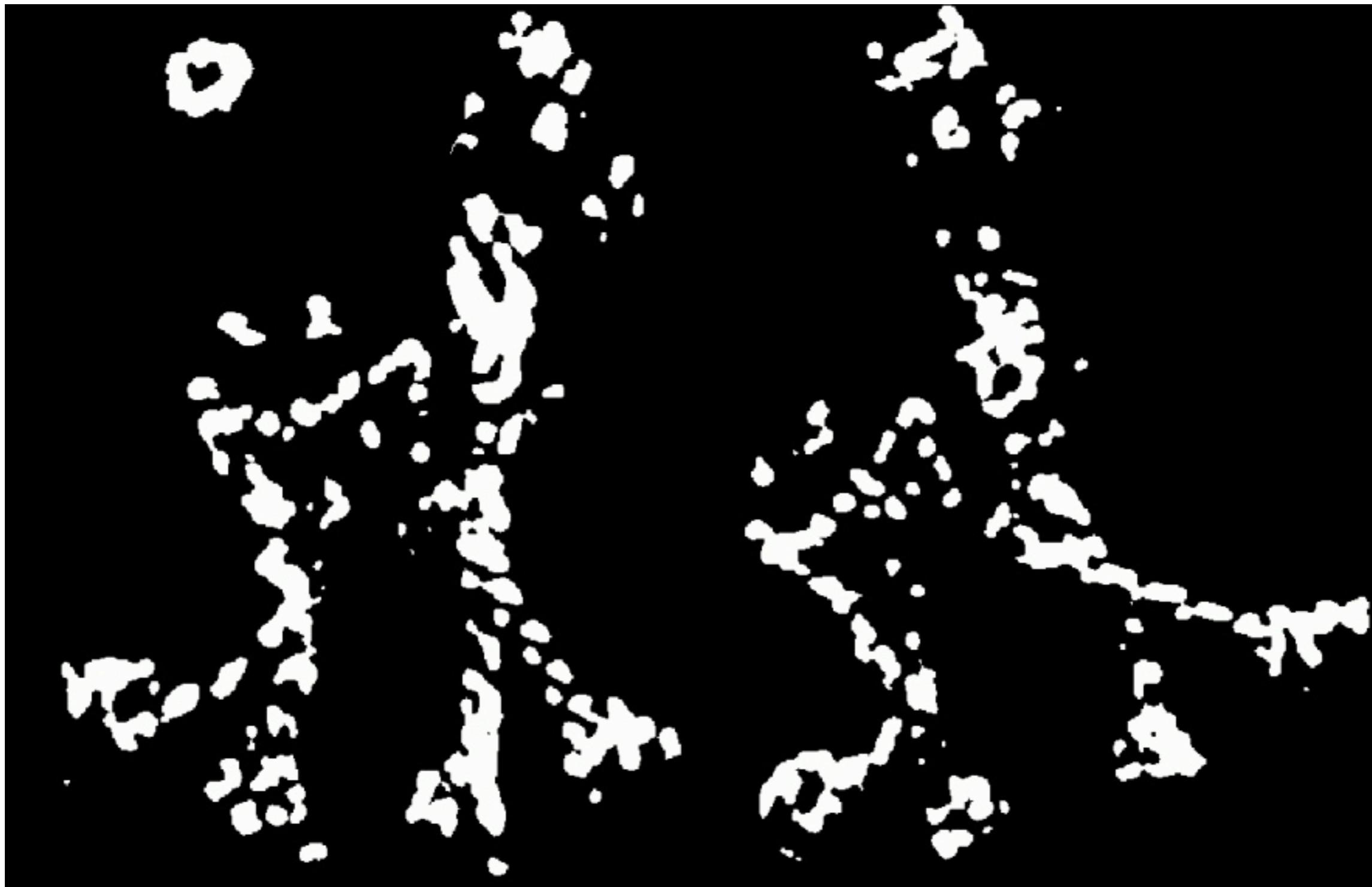


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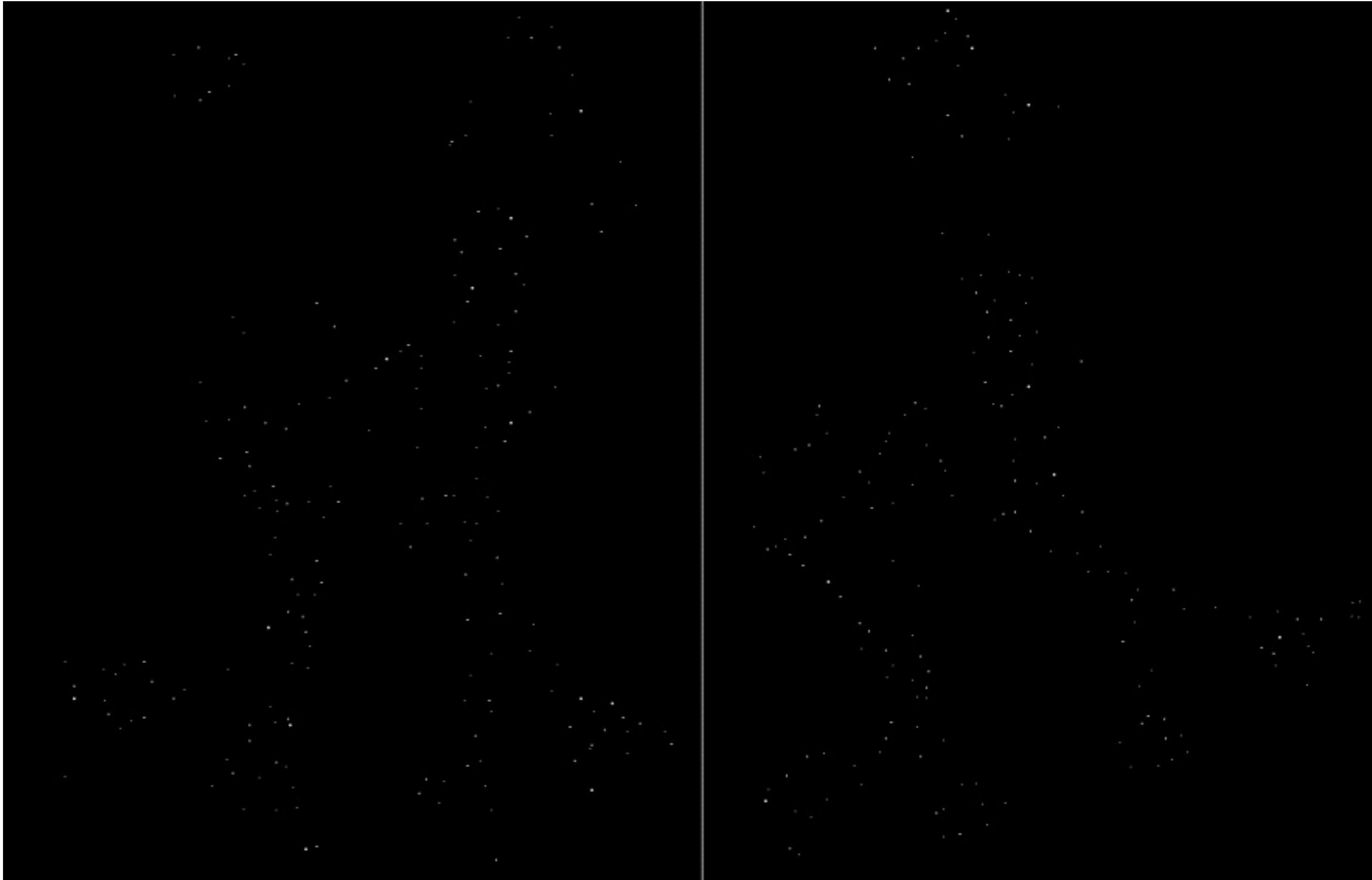
[Szeliski & Seitz]

Threshold ($f > \text{value}$)



[Szeliski & Seitz]

Find local maxima of f



[Szeliski & Seitz]

Harris features (in red)

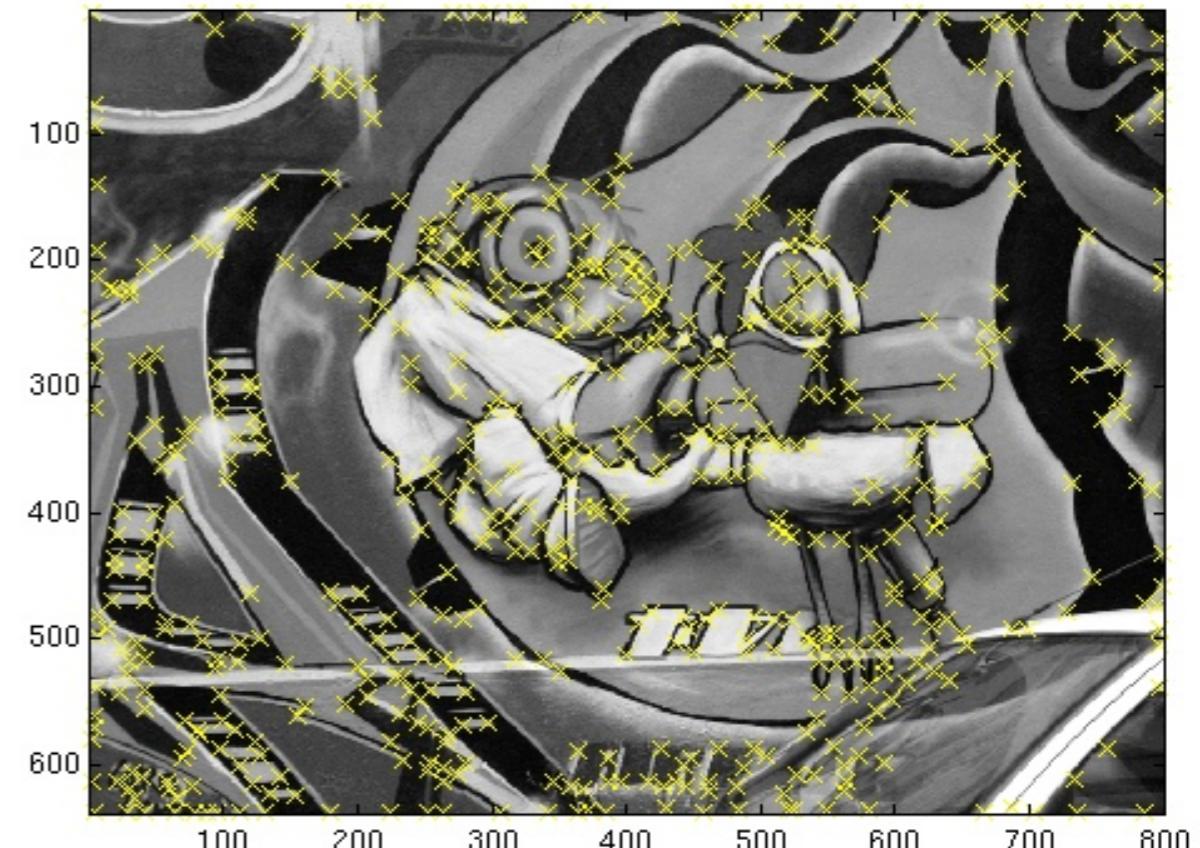
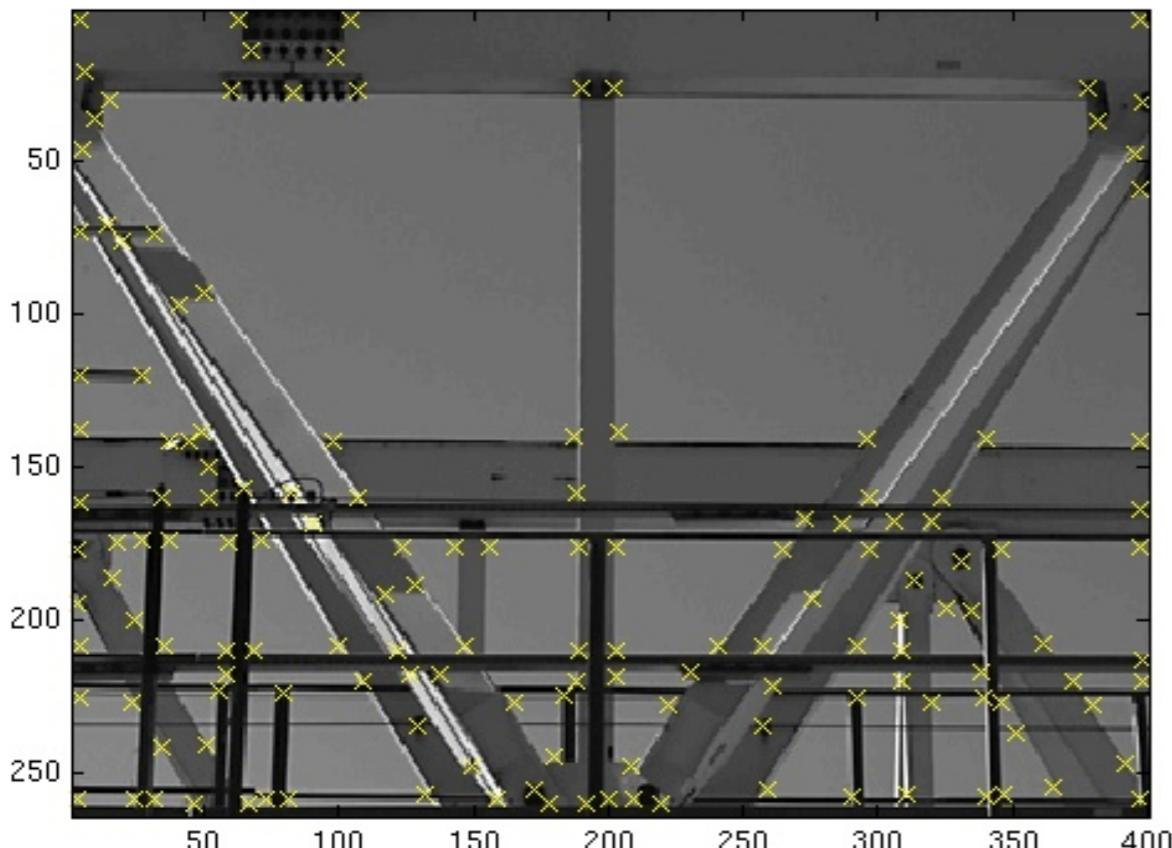


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[Szeliski & Seitz]

Harris interest points



Other interest point detectors

- ◆ Hessian determinant

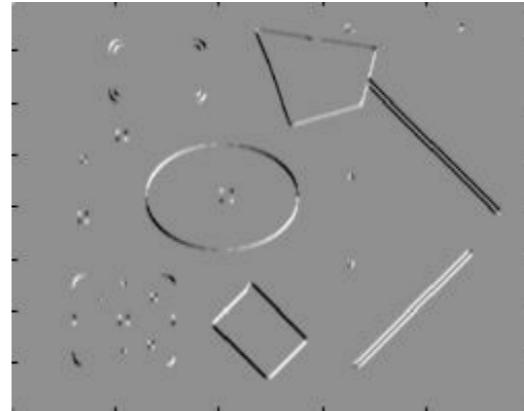
$$H(I) = \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix}$$

$$\det(H(I)) = I_{xx}I_{yy} - I_{xy}^2$$

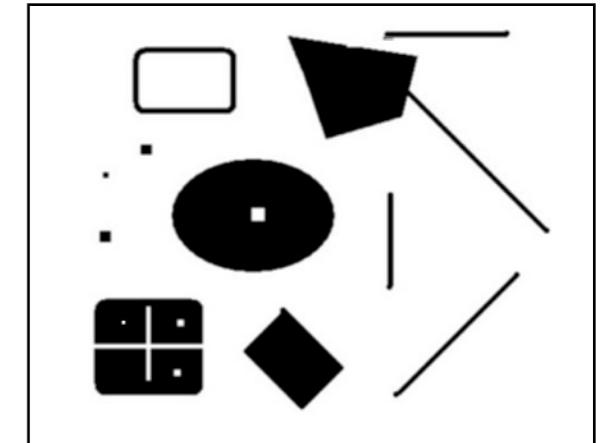
I_{xx}



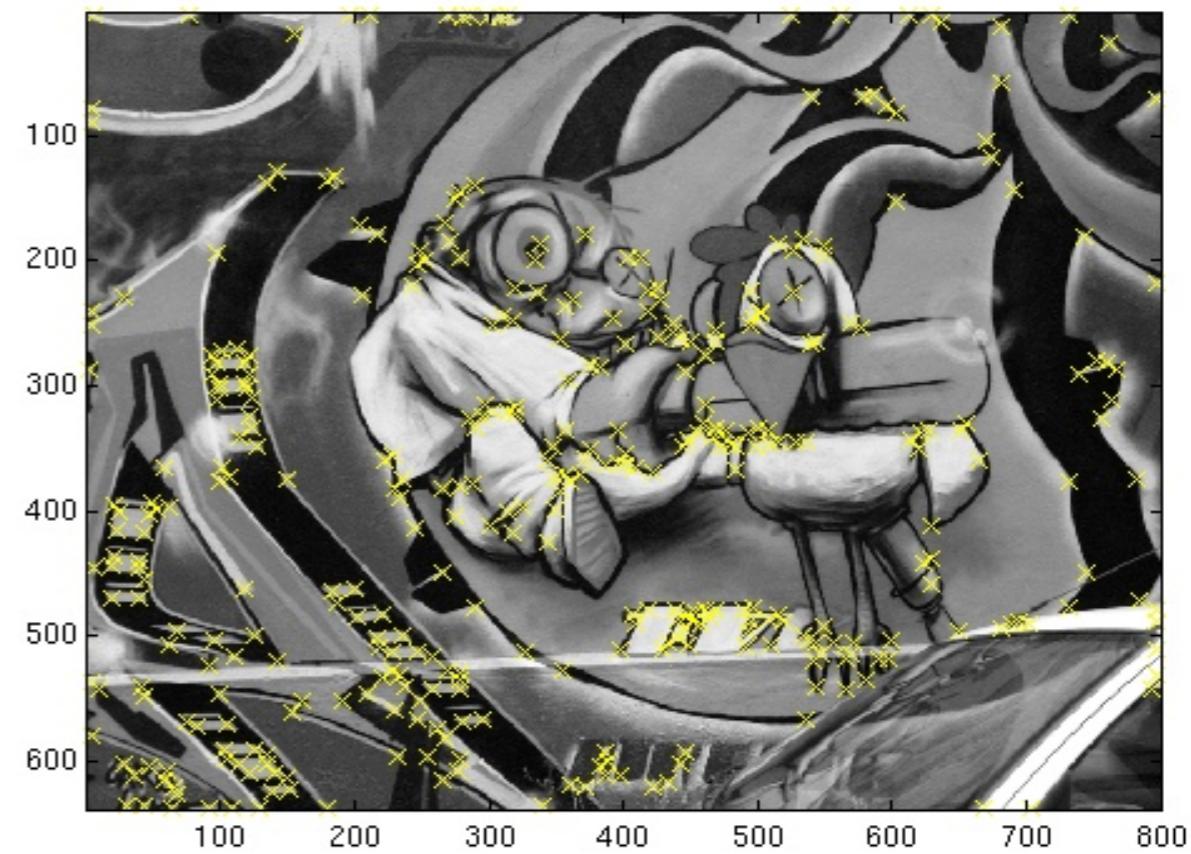
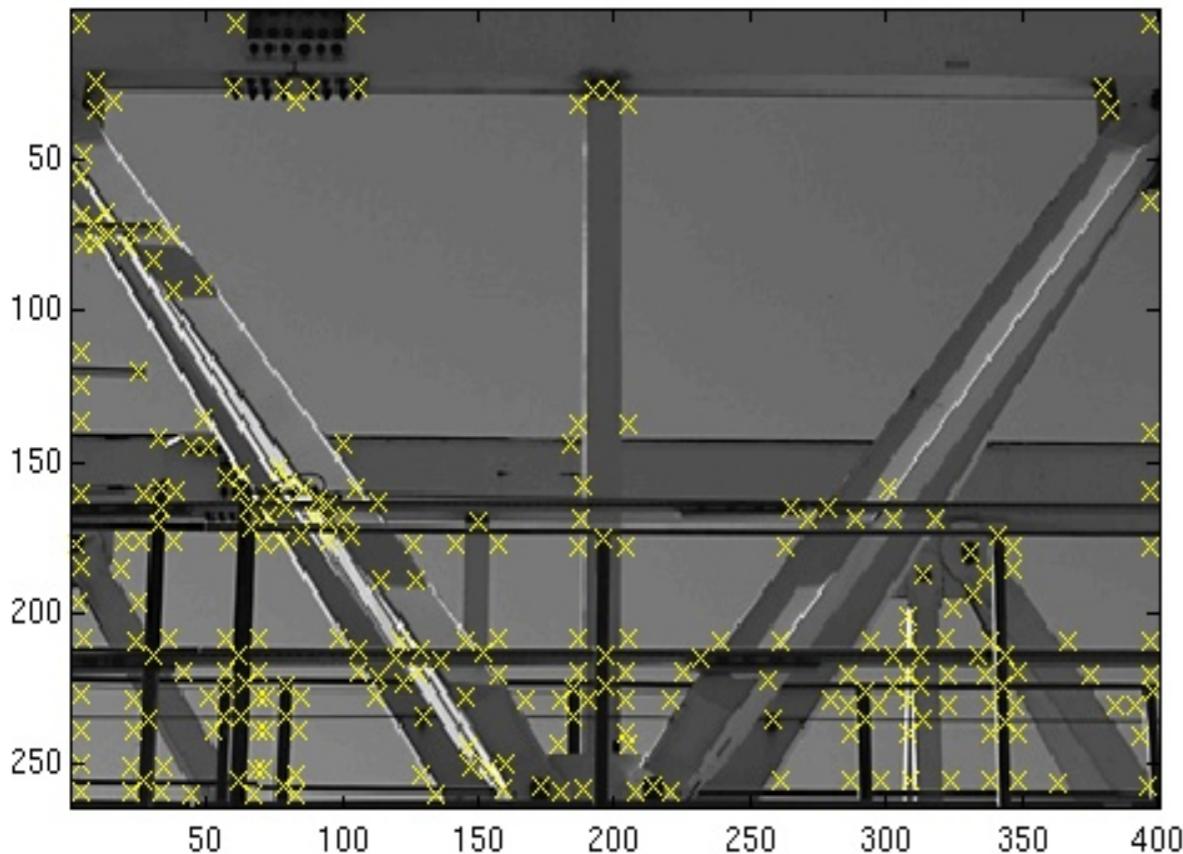
I_{xy}



I_{yy}



Hessian interest points



Discussion

- ◆ Interest-point detection so far:
 - ◆ Based on Harris or Hessian detector
 - ◆ Finds interesting, i.e. discriminative points
(Harris detector was the “de-facto” standard for a long time)
 - ◆ Used for recognition, correspondence for stereo, sparse optical flow/motion, etc.
- ◆ Remember the goals of interest point detection:
 - ◆ Distinctiveness vs. invariance to transformation
 - ◆ Harris & Hessian find distinctive points - but they are **not** invariant to scale, affine and projective transformations