Problem 2.

bits to display/second = $1920 \times 1080 \times 24$ bits $\times 30$ Hz $= 1{,}492{,}992{,}000$ bits/s

$= 178$ MBytes/s

→ # seconds of videos/single layer HD-DVD $= 15 \times 1024/178 = 86$ seconds

The result does not make sense. With this calculation, even with a single-layer bluray disc of 25 GB, only $86 \times 1.6 = 144$ seconds can fit on the disc. Thus, it should not be a reason in this sense contributing to the failure of HD-DVD to its rival Blure ray.

Blue ray offers more storage capacity than HD-DVD. Video signals are compressed to much smaller bit rate so that long videos can fit on the disc, either HD-DVD or Blue ray. In this sense, obviously large storage capacity has its advantage in providing higher quality videos.

Problem 3.

The pixels returned by two algorithms, Bresenham and diamond exit rule, are the same in general.

Assumption: slope $m \in [0,1]$. Consider a pixel at (x_p, y_p), the diamond within the pixel has the highest corner at $(x_p, y_p + \frac{1}{2})$ and the lowest corner at $(x_p, y_p - \frac{1}{2})$. A line through (x_1, y_1) to (x_2, y_2) intersects the diamond of pixel (x_p, y_p) if and only if the line is below the highest corner and above the lowest corner. Note for two pixels E and NE, the highest corner of E is the lowest corner of NE, which is also the test midpoint to calculate d in Bresenham algorithm.

When the line is above the midpoint, it will intersect the NE diamond and NE pixel is drawn. On the other hand, if the line is below the midpoint, it will intersect the E diamond and E pixel is drawn.

Thus, the results from both algorithm should be the same.

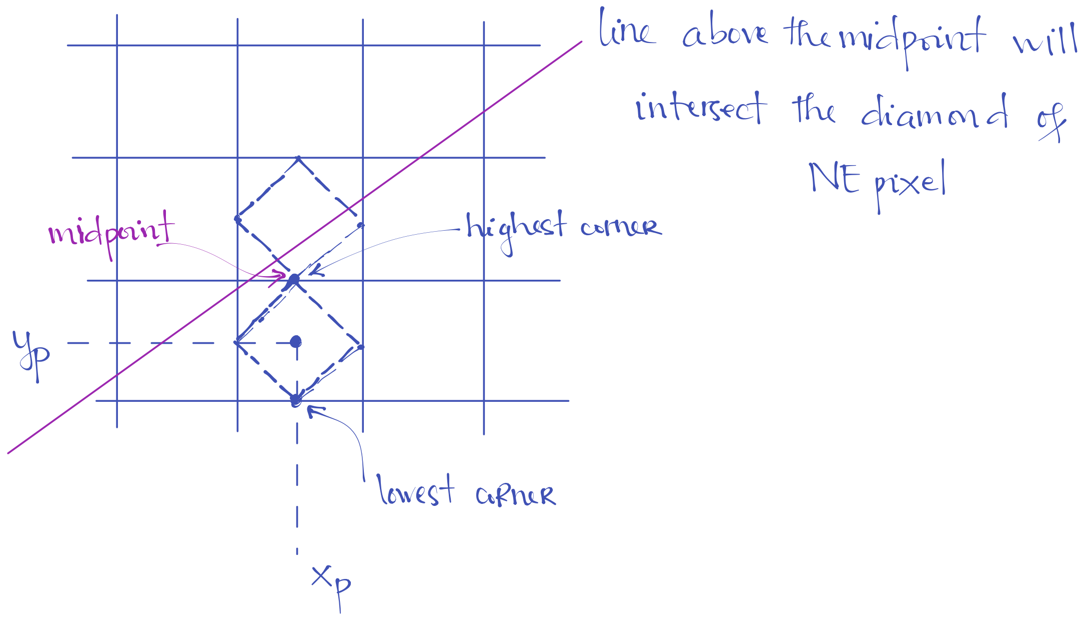


Figure 1. The E pixel will not be drawn but NE

Problem 4.

$$M = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

Let $M_{11} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$; $M_{12} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$; $M_{21} = [0 \ 0 \ 0]$; $M_{22} = 1$

1. $M^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$; $M.M^{-1} = I = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\rightarrow \begin{aligned} M_{11} B_{11} + M_{12} B_{21} &= I_{11} \\ M_{11} B_{12} + M_{12} B_{22} &= I_{12} \\ M_{21} B_{11} + M_{22} B_{21} &= I_{21} \\ M_{21} B_{12} + M_{22} B_{22} &= I_{22} \end{aligned} \quad \rightarrow \quad \begin{aligned} B_{11} &= M_{11}^{-1} = M_{11}^T \\ B_{12} &= -M_{11}^{-1} M_{12} = -M_{11}^T M_{12} \\ B_{21} &= [0 \ 0 \ 0] \\ B_{22} &= M_{22}^{-1} = [1] \end{aligned}$$

$$\rightarrow M^{-1} = \begin{bmatrix} r_{11} & r_{21} & r_{31} & -(r_{11}t_1 + r_{21}t_2 + r_{31}t_3) \\ r_{12} & r_{22} & r_{32} & -(r_{12}t_1 + r_{22}t_2 + r_{32}t_3) \\ r_{13} & r_{23} & r_{33} & -(r_{13}t_1 + r_{23}t_2 + r_{33}t_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Preserve length/angle:

Let $P(x_1, y_1, z_1, 1)^T$ and $Q(x_2, y_2, z_2, 1)$

$\rightarrow$ $u = P - Q$.

After transformation : $P \xrightarrow{M} P'$, $Q \xrightarrow{M} Q'$

$\rightarrow$ $u' = P' - Q' = M_{11} . u$

Presene length:

$$|u'|^2 = u'.u' = M_{11}.u . M_{11}.u = (M_{11}.u)^T. M_{11}u = u^T. \underbrace{M_{11}^T. M_{11}}_{I \ (M_{11}:\ \text{orthornomal})}.u$$

$$= u^T.u = u.u = |u|^2$$

Preserve angle:

Let $S(x_3, y_3, z_3, 1)^T$, and $v = P - S$. We have:

$$u.v = |u|.|v|.\cos t \rightarrow \cos t = u.v / |u||v|$$

After transformation: $\cos t' = (M_{11}.u \ M_{11}v) / (|M_{11}u||M_{11}.v|)$

$$= (M_{11}.v)^T (M_{11}.u) / (|u||v|)$$

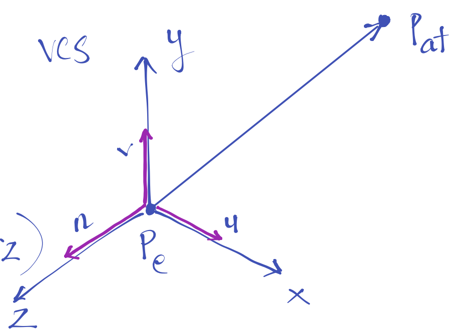
$$= v^T. \underbrace{M_{11}^T M_{11}}_{I} u / |u||v| = v^T.u / |u||v| = u.v / |u||v|$$

$\rightarrow \cos t = \cos t'$

Problem 5.

$$\text{gluLookAt}(\underbrace{ex, ey, ez}_{P_e}, \underbrace{rx, ry, rz}_{P_{at}}, \underbrace{ux, uy, uz}_{v_{up}})$$

$$P_e = \begin{bmatrix} ex \\ ey \\ ez \end{bmatrix} \quad P_{at} = \begin{bmatrix} rx \\ ry \\ rz \end{bmatrix}$$



From the implementation of the function:

$\rightarrow vpn = P_e - P_{at} = (ex, ey, ez) - (rx, ry, rz)$

$$\rightarrow n = \frac{vpn}{|vpn|}$$

$$u = \frac{v_{up} \times n}{|v_{up} \times n|}$$

$$v = \frac{n \times u}{|n \times u|}$$

Rotation matrix:

$$\rightarrow A = \begin{bmatrix} u_x & v_x & n_x & 0 \\ u_y & v_y & n_y & 0 \\ u_z & v_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ref. (Textbook, pp. 231) and math

$\rightarrow$ transform matrix (wcs $\rightarrow$ vcs) : $M = T.A$

$$= \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x & v_x & n_x & 0 \\ u_y & v_y & n_y & 0 \\ u_z & v_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ for vcs $\rightarrow$ wcs : $M^{-1} = (T.A)^{-1} = A^{-1}.T^{-1} = A^{T}.T^{-1}$

$\rightarrow$ transformation matrix : M^{-1}

$$= \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -ex \\ 0 & 1 & 0 & -ey \\ 0 & 0 & 1 & -ez \\ 0 & 0 & 0 & 1 \end{bmatrix}$$