

# Disambiguation and Layout-Sensitive Syntax

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# Disambiguation and Layout-Sensitive Syntax

## Syntax Definition Summary

### Derivations

- Generating sentences and trees from context-free grammars

### Ambiguity

### Declarative Disambiguation Rules

- Associativity and priority

### Grammar Transformations

- Eliminating ambiguity by transformation

### Layout-Sensitive Syntax

- Disambiguation using layout constraints

# Structure

# Syntax = Structure

```
module structure  
  
imports Common  
  
context-free start-symbols Exp
```

context-free syntax

Exp.Var = ID

Exp.Int = INT

Exp.Add = Exp "+" Exp

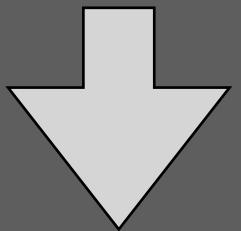
Exp.Fun = "function" "(" {ID ","}\* ")" "{" Exp "}"

Exp.App = Exp "(" {Exp ","}\* ")"

Exp.Let = "let" Bnd\* "in" Exp "end"

Bnd.Bnd = ID "=" Exp

```
let  
  inc = function(x) { x + 1 }  
  in  
  inc(3)  
end
```



```
Let(  
  [ Bnd(  
    "inc"  
    , Fun(["x"], Add(Var("x"), Int("1"))))  
  ]  
, App(Var("inc"), [Int("3")]))
```

# Token = Character

```
module structure  
  
imports Common  
  
context-free start-symbols Exp
```

## context-free syntax

```
Exp.Var = ID  
  
Exp.Int = INT  
  
Exp.Add = Exp "+" Exp  
  
Exp.Fun = "function" "(" {ID ","}* ")" " {" Exp "}"  
  
Exp.App = Exp "(" {Exp ","}* ")"  
  
Exp.Let = "let" Bnd* "in" Exp "end"  
  
Bnd.Bnd = ID "=" Exp
```

```
let  
  inc = function(x) { x + 1 }  
in  
  inc(3)  
end
```

```
module Common
```

## lexical syntax

```
ID   = [a-zA-Z] [a-zA-Z0-9]*  
  
INT = [\-\]? [0-9]+
```

Lexical Syntax = Context-Free Syntax  
(But we don't care about structure of lexical syntax)

# Literal = Non-Terminal

```
module structure  
  
imports Common  
  
context-free start-symbols Exp
```

## context-free syntax

```
Exp.Var = ID  
  
Exp.Int = INT  
  
Exp.Add = Exp "+" Exp  
  
Exp.Fun = "function" "(" {ID ","}* ")" " {" Exp "}"  
  
Exp.App = Exp "(" {Exp ","}* ")"  
  
Exp.Let = "let" Bnd* "in" Exp "end"  
  
Bnd.Bnd = ID "=" Exp
```

```
let  
  inc = function(x) { x + 1 }  
in  
  inc(3)  
end
```

## syntax

"+"	= [\u0043]
"function"	= [\u00102] [\u00117] [\u00110] [\u00199] [\u00116] [\u00105] [\u00111] [\u00110]
"{"	= [\u00123]
= [\u00125]	
"("	= [\u00140]
= [\u00144]	
= [\u00141]	
"let"	= [\u00108] [\u00101] [\u00116]
"in"	= [\u00105] [\u00110]
"end"	= [\u00101] [\u00110] [\u00100]
"="	= [\u00161]

# Layout = Whitespace & Comments

```
module Common

lexical syntax

LAYOUT      = [\t\n\r]
LAYOUT      = /* InsideComment* */
InsideComment = ~[*]
InsideComment = CommentChar
CommentChar  = [*]

LAYOUT      = // ~[\n\r]* NewLineEOF
NewLineEOF   = [\n\r]
NewLineEOF   = EOF
```

```
let
  inc = function(x) { x + 1 }
in
// function application
inc /* function position */ (
  3 // argument list
)
end
```

# Layout = (Almost) Everywhere

```
module Common

lexical syntax

LAYOUT      = [\t\n\r]
LAYOUT      = /* InsideComment* */
InsideComment = ~[*]
InsideComment = CommentChar
CommentChar  = [*]

LAYOUT      = // ~[\n\r]* NewLineEOF
NewLineEOF   = [\n\r]
NewLineEOF   = EOF
```

```
let
  inc = function(x) { x + 1 }
in
// function application
inc /* function position */ (
  3 // argument list
)
end
```

```
Exp.App = Exp "(" {Exp ","}* ")"
```

```
Exp-CF.App = Exp-CF LAYOUT?-CF "(" LAYOUT?-CF {Exp ","}* -CF LAYOUT?-CF ")"
```

# Extension

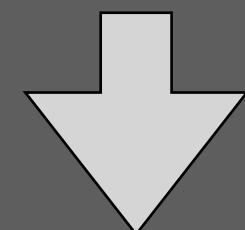
# Language Composition ⇒ Grammar Composition

```
module extension
imports functional query
context-free start-symbols Exp
context-free syntax
Exp = Query
Cond = Exp
```

```
module functional
imports Common
context-free syntax
Exp = <(<Exp>)> {bracket}
...
```

```
module query
imports Common
context-free syntax
Query.Query = <
  select <QID*> from <QID*> where <Cond>
>
Cond.And = <<Cond> and <Cond>> {left}
Cond.Eq = <<Cond> == <Cond>> {non-assoc}
```

```
let
  select = 1
  fs = select f from A where test f = select
in
  print fs
```



```
Let(
  [ Bnd("select", Int("1"))
  , Bnd(
    "fs"
  , Query(
    ["f"]
    , ["A"]
    , Eq(App(Var("test"), Var("f")), Var("select"))
    )
  )
  ]
  , App(Var("print"), Var("fs"))
)
```

**Parsing = Formatting<sup>-1</sup>**

# Parsing = Formatting<sup>-1</sup>

context-free syntax

Exp.Var = <<ID>>

Exp.Int = <<INT>>

Exp.Add = <<Exp> + <Exp>>

Exp.Fun = <  
function(<{ID ", "}\*>){  
 <Exp>  
}  
>

Exp.App = <<Exp>(<{Exp ", "}\*>)>

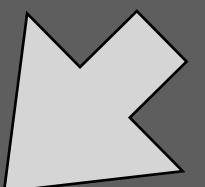
Exp.Let = <  
let  
 <{Bnd "\n"}\*>  
 in  
 <Exp>  
end  
>

Bnd.Bnd = <<ID> = <Exp>>

```
let
  inc = function(x) { x + 1 }
in
inc(3)
end
```



```
Let(
  [ Bnd(
    "inc"
    , Fun(["x"], Add(Var("x"), Int("1")))
  )
  ]
, App(Var("inc"), [Int("3")])
)
```



```
let
  inc = function(x){
    x + 1
  }
in
inc(3)
end
```

# Completion = Rewrite(Incomplete Structure)

```
class A {  
  
    public int m() {  
        int x;  
        x = $Exp;  
        return+$Add  
            +$Sub  
            +$Mul  
            +$Lt  
            +$VarRef
```

```
class A {  
  
    public int m() {  
        int x;  
        x = $Exp + $Exp;  
        retu+$Add  
            +$Sub  
            +$Mul  
            +$Lt  
            +$VarRef
```

```
class A {  
  
    public int m() {  
        int x;  
        x = 21 + $Exp;  
        return x;+$Add  
            +$Sub  
            +$Mul  
            +$Lt  
            +$VarRef
```

```
class A {  
  
    public int m() {  
        int x;  
        x = 21 + 21;  
        return x;  
    }
```

# Context-Free Grammars

## Terminals

- Basic symbols from which strings are formed

## Nonterminals

- Syntactic variables that denote sets of strings

## Start Symbol

- Denotes the nonterminal that generates strings of the languages

## Productions

- $A = X \dots X$
- Head/left side ( $A$ ) is a nonterminal
- Body/right side ( $X \dots X$ ) zero or more terminals and nonterminals

# Example Context-Free Grammar

```
grammar
  start S
  non-terminals E T F
  terminals "+" "*" "(" ")" ID
  productions
    S = E
    E = E "+" T
    E = T
    T = T "*" F
    T = F
    F = "(" E ")"
    F = ID
```

# Abbreviated Grammar

```
grammar
start S
non-terminals E T F
terminals "+" "*" "(" ")" ID
productions
S = E
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```

```
grammar
productions
S = E
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```

Nonterminals, terminals can be derived from productions

First production defines start symbol

# Notation

A, B, C: non-terminals

l: terminals

a, b, c: strings of non-terminals and terminals  
(alpha, beta, gamma in math)

w, v: strings of terminal symbols

# Meta: Syntax of Grammars

context-free syntax // grammars

```
Grammar.Grammar = <  
    grammar  
    <Start?>  
    <Sorts?>  
    <Terminals?>  
    <Productions>  
>
```

context-free syntax

```
Production.Prod = <  
    <Symbol><Constructor?> = <Symbol*>  
>
```

```
Symbol.NT = <<ID>>  
Symbol.T = <<STRING>>  
Symbol.L = <<LCID>>
```

```
Constructor.Con = <.<ID>>
```

context-free syntax

```
Start.Start = <  
    start <ID>  
>
```

```
Sorts.Sorts = <  
    sorts <ID*>  
>
```

```
Sorts.NonTerminals = <  
    non-terminals <ID*>  
>
```

```
Terminals.Terminals = <  
    terminals <Symbol*>  
>
```

```
Productions.Productions = <  
    productions  
    <Production*>  
>
```

# Derivations: Generating Sentences from Symbols

# Derivations

```
grammar  
productions  
E = E "+" E  
E = E "*" E  
E = "-" E  
E = "(" E ")"  
E = ID
```

```
// derivation step: replace symbol by rhs of production  
// E = E "+" E  
// replace E by E "+" E  
//  
// derivation:  
// repeatedly apply derivations
```

```
derivation  
E  
⇒ "-" E  
⇒ "-" "(" E ")"  
⇒ "-" "(" ID ")"
```

```
derivation // derives in zero or more steps  
E ⇒* "-" "(" ID "+" ID ")"
```

# Meta: Syntax of Derivations

context-free syntax // derivations

```
Derivation.Derivation = <  
    derivation  
        <Symbol> <Step*>  
>
```

```
Step.Step      = [⇒ [Symbol*]]  
Step.Steps    = [⇒* [Symbol*]]  
Step.Steps1   = [⇒+ [Symbol*]]
```

# Left-Most Derivation

grammar  
productions

```
E = E "+" E
E = E "*" E
E = "-" E
E = "(" E ")"
E = ID
```

derivation // left-most derivation

```
E
⇒ "-" E
⇒ "-" "(" E ")"
⇒ "-" "(" E "+" E ")"
⇒ "-" "(" ID "+" E ")"
⇒ "-" "(" ID "+" ID ")"
```

Left-most derivation: Expand left-most non-terminal at each step

# Right-Most Derivation

grammar  
productions

```
E = E "+" E
E = E "*" E
E = "-" E
E = "(" E ")"
E = ID
```

derivation // left-most derivation

```
E
⇒ "-" E
⇒ "-" "(" E ")"
⇒ "-" "(" E "+" E ")"
⇒ "-" "(" ID "+" E ")"
⇒ "-" "(" ID "+" ID ")"
```

derivation // right-most derivation

```
E
⇒ "-" E
⇒ "-" "(" E ")"
⇒ "-" "(" E "+" E ")"
⇒ "-" "(" E "+" ID ")"
⇒ "-" "(" ID "+" ID ")"
```

Right-most derivation: Expand right-most non-terminal at each step

# Meta: Tree Derivations

```
context-free syntax // tree derivations
```

```
Derivation.TreeDerivation = <  
    tree derivation  
    <Symbol> <PStep*>  
>
```

```
PStep.Step      = [⇒ [PT*]]  
PStep.Steps    = [⇒* [PT*]]  
PStep.Steps1   = [⇒+ [PT*]]
```

```
PT.App = <<Symbol>[<PT*>]>  
PT.Str = <<STRING>>  
PT.Sym = <<Symbol>>
```

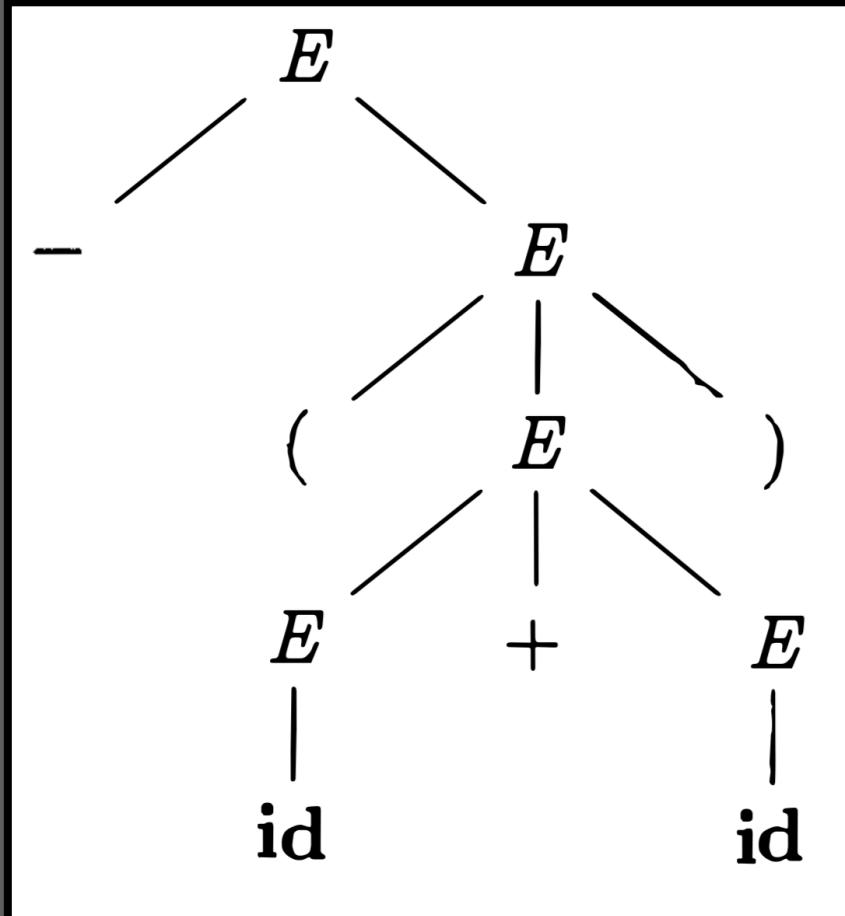
# Left-Most Tree Derivation

grammar  
productions

$E.A = E "+" E$   
 $E.T = E "*" E$   
 $E.N = "-" E$   
 $E.P = "(" E ")"$   
 $E.V = ID$

derivation // left-most derivation

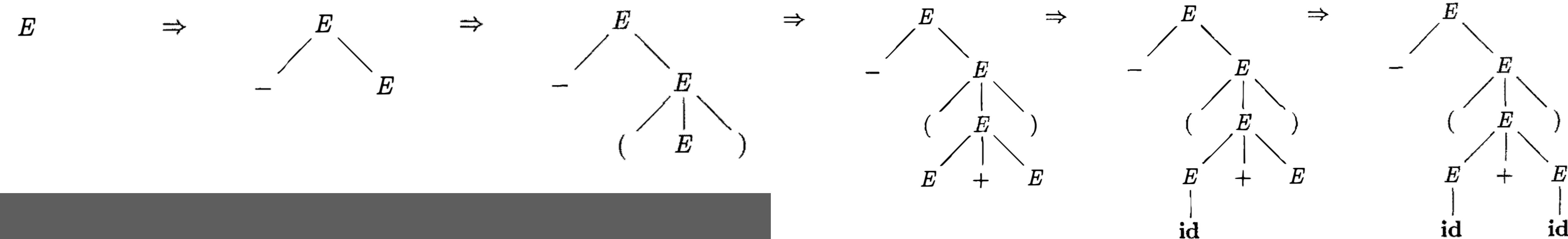
$E$   
 $\Rightarrow "-" E$   
 $\Rightarrow "-" "(" E ")"$   
 $\Rightarrow "-" "(" E "+" E ")"$   
 $\Rightarrow "-" "(" ID "+" E ")"$   
 $\Rightarrow "-" "(" ID "+" ID ")"$



tree derivation // left-most

$E$   
 $\Rightarrow E[- E]$   
 $\Rightarrow E[- E[(" E ")]]$   
 $\Rightarrow E[- E[(" E[E "+" E ] ")]]$   
 $\Rightarrow E[- E[(" E[E[ID] "+" E ] ")]]$   
 $\Rightarrow E[- E[(" E[E[ID] "+" E[ID]] ")]]$

# Left-Most Tree Derivation



tree derivation // left-most

```
E
⇒ E["- E"]
⇒ E["- E[(" E ")"]"]
⇒ E["- E[(" E[E "+" E] ")"]"]
⇒ E["- E[(" E[E[ID] "+" E] ")"]"]
⇒ E["- E[(" E[E[ID]] "+" E[ID]) ")"]"]
```

# Meta: Term Derivations

```
context-free syntax // term derivations
```

```
Derivation.TermDerivation = <  
    term derivation  
    <Symbol> <TStep*>  
>
```

```
TStep.Step      = [⇒ [Term*]]  
TStep.Steps    = [⇒* [Term*]]  
TStep.Steps1   = [⇒+ [Term*]]
```

```
Term.App = <<ID>(<{Term ", }*>)>  
Term.Str = <<STRING>>  
Term.Sym = <<Symbol>>
```

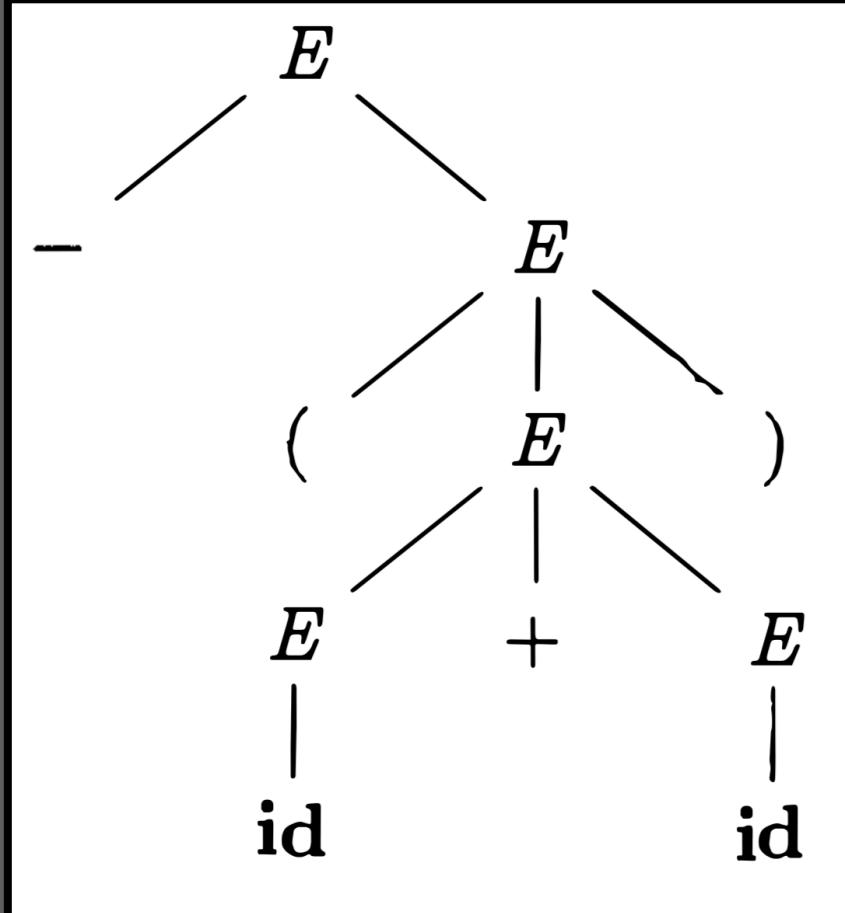
# Left-Most Term Derivation

grammar  
productions

$E.A = E "+" E$   
 $E.T = E "*" E$   
 $E.N = "-" E$   
 $E.P = "(" E ")"$   
 $E.V = ID$

derivation // left-most derivation

$E$   
 $\Rightarrow "-" E$   
 $\Rightarrow "-" "(" E ")"$   
 $\Rightarrow "-" "(" E "+" E ")"$   
 $\Rightarrow "-" "(" ID "+" E ")"$   
 $\Rightarrow "-" "(" ID "+" ID ")"$



term derivation // left-most

$E$   
 $\Rightarrow N(E)$   
 $\Rightarrow N(P(E))$   
 $\Rightarrow N(P(A(E, E)))$   
 $\Rightarrow N(P(A(V(ID), E)))$   
 $\Rightarrow N(P(A(V(ID), V(ID))))$

# Parse Trees Represent Derivations

```
List<String> YIELD(T : Tree) {  
    T match {  
        [A = Ts] => YIELDS(Ts);  
        Str => [Str];  
    };  
}  
  
List<String> YIELDS(Ts : List<Tree>) {  
    Ts match {  
        [] => "";  
        [T | Ts] => YIELD(T) + YIELDS(Ts);  
    };  
}
```

$$S \xrightarrow{*} PT$$

iff

$$S \xrightarrow{*} \text{YIELD}(PT)$$

# Language Defined by a Grammar

$$L(G) = \{ w \mid S \xrightarrow{*} w \}$$

Language: sentences

$$T(G) = \{ T \mid S \xrightarrow{*} T \}$$

Language: trees

$$L(G) = \text{YIELD}(T(G))$$

# Ambiguity

# Ambiguity: Deriving Multiple Parse Trees

grammar

productions

$E \cdot A = E \ " + " \ E$   
 $E \cdot T = E \ " * " \ E$   
 $E \cdot N = " - " \ E$   
 $E \cdot P = "(" \ E \ ")"$   
 $E \cdot V = ID$

derivation

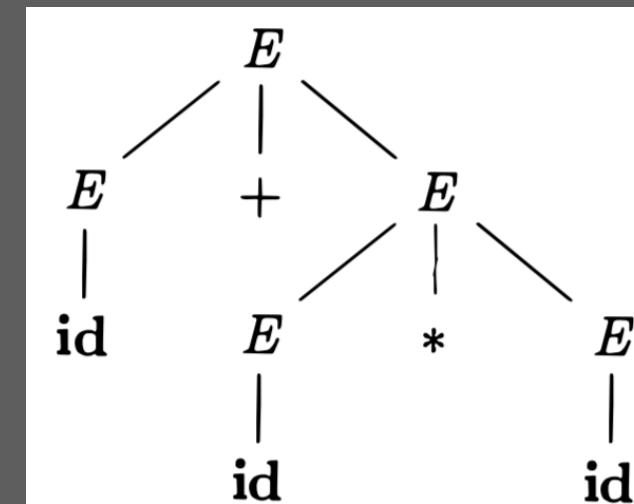
$E \xrightarrow{*} ID \ " + " \ ID \ " * " \ ID$

derivation

$E \xrightarrow{*} E \ " + " \ E$   
 $\Rightarrow ID \ " + " \ E$   
 $\Rightarrow ID \ " + " \ E \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ ID$

tree derivation

$E \xrightarrow{*} E[E[ID]] \ " + " \ E[E[ID]] \ " * " \ E[E[ID]]$

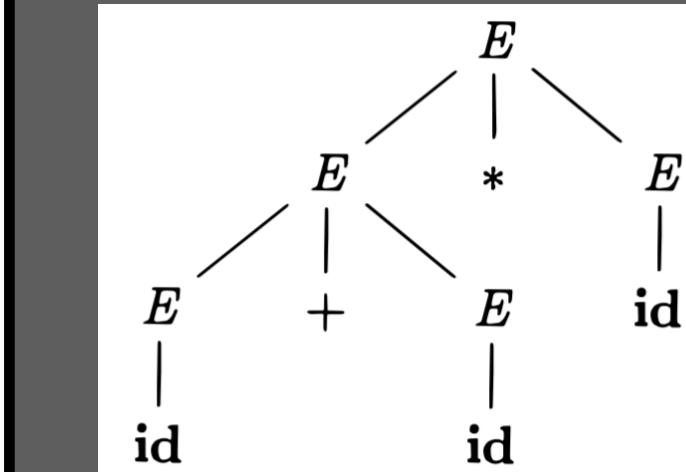


derivation

$E \xrightarrow{*} E \ " * " \ E$   
 $\Rightarrow E \ " + " \ E \ " * " \ E$   
 $\Rightarrow ID \ " + " \ E \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ ID$

tree derivation

$E \xrightarrow{*} E[E[E[ID]] \ " + " \ E[ID]] \ " * " \ E[ID]]$



Ambiguous grammar: produces >1 parse tree for a sentence

# Ambiguity: Deriving Abstract Syntax Terms

grammar

productions

$E \cdot A = E \ " + " \ E$   
 $E \cdot T = E \ " * " \ E$   
 $E \cdot N = " - " \ E$   
 $E \cdot P = "(" \ E \ ")"$   
 $E \cdot V = ID$

derivation

$E \xrightarrow{*} ID \ " + " \ ID \ " * " \ ID$

derivation

$E$   
 $\Rightarrow E \ " + " \ E$   
 $\Rightarrow ID \ " + " \ E$   
 $\Rightarrow ID \ " + " \ E \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ ID$

term derivation

$E$   
 $\Rightarrow A(E, E)$   
 $\Rightarrow A(V(ID), E)$   
 $\Rightarrow A(V(ID), T(E, E))$   
 $\Rightarrow A(V(ID), T(V(ID), E))$   
 $\Rightarrow A(V(ID), T(V(ID), V(ID)))$

derivation

$E$   
 $\Rightarrow E \ " * " \ E$   
 $\Rightarrow E \ " + " \ E \ " * " \ E$   
 $\Rightarrow ID \ " + " \ E \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ E$   
 $\Rightarrow ID \ " + " \ ID \ " * " \ ID$

term derivation

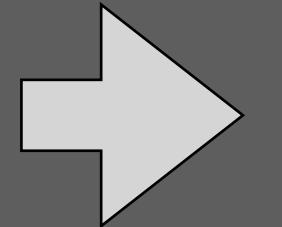
$E$   
 $\Rightarrow T(E, E)$   
 $\Rightarrow T(A(E, E), E)$   
 $\Rightarrow T(A(V(ID), E), E)$   
 $\Rightarrow T(A(V(ID), V(ID)), E)$   
 $\Rightarrow T(A(V(ID), V(ID)), V(ID))$

# Disambiguation

# Traditional: Ambiguity = Parse Table Conflict

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>>  
  
Exp.Fun     = <function(<{ID "," }*>) <Exp>>  
Exp.App     = <<Exp> <Exp>>  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>>
```



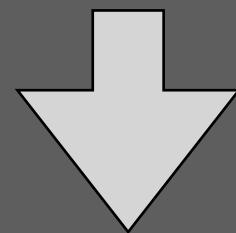
No can parse

# Ambiguity = Multiple Possible Parses

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>>  
  
Exp.Fun     = <function(<{ID "," }*>) <Exp>>  
Exp.App     = <<Exp> <Exp>>  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
Bnd.Bnd     = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>>
```

a + b + c



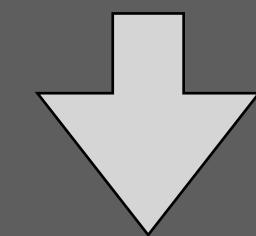
```
amb(  
  [ Add(Var("a"), Add(Var("b"), Var("c")))  
  , Add(Add(Var("a"), Var("b")), Var("c"))  
  ]  
)
```

# Disambiguation = Select(Structure)

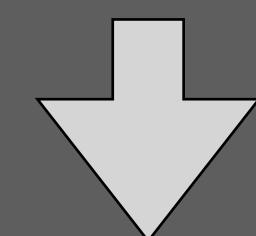
## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>>  
  
Exp.Fun     = <function(<{ID "," }*>) <Exp>>  
Exp.App     = <<Exp> <Exp>>  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
Bnd.Bnd     = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>>
```

a + b + c



```
amb(  
  [ Add(Var("a"), Add(Var("b"), Var("c")))  
  , Add(Add(Var("a"), Var("b")), Var("c"))  
  ]  
)
```



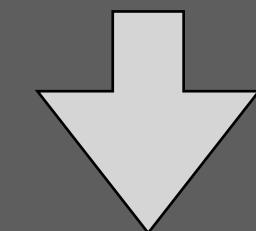
Add(Add(Var("a"), Var("b")), Var("c"))

# Brackets = Explicit Disambiguation

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>>  
  
Exp.Fun     = <function(<{ID ","}>*)> <Exp>>  
Exp.App     = <<Exp> <Exp>>  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
Bnd.Bnd     = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>>
```

a + (b + c)

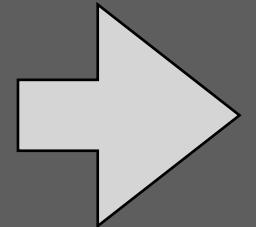


Add(Var("a"), Add(Var("b"), Var("c")))

# Disambiguation by Manual Transformation = Bad

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>>  
  
Exp.Fun     = <function(<{ID "," }*>) <Exp>>  
Exp.App     = <<Exp> <Exp>>  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>>
```



Big ugly grammar

# Declarative Disambiguation = Separate Concern

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}*>) <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}
```

## context-free priorities

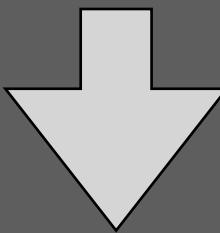
```
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

# Associativity = Solve Intra Operator Ambiguity

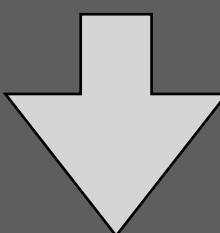
## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}*>) <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

a + b + c



```
amb(  
  [ Add(Var("a"), Add(Var("b"), Var("c")))  
  , Add(Add(Var("a"), Var("b")), Var("c"))  
  ]  
)
```



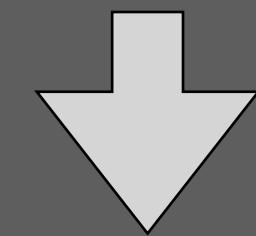
Add(Add(Var("a"), Var("b")), Var("c"))

# Priority = Solve Inter Operator Ambiguity

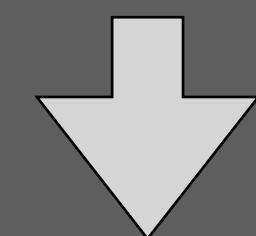
## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}*>) <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

f a + b



```
amb(  
  [ Add(App(Var("f")), Var("a")), Var("b"))  
  , App(Var("f")), Add(Var("a"), Var("b")))  
)
```



Add(App(Var("f")), Var("a")), Var("b"))

# Dangling Else = Operators with Overlapping Prefix

## context-free syntax

```
Exp      = <(<Exp>)> {bracket}  
  
Exp.Int  = INT  
Exp.Var  = ID  
Exp.Add  = <<Exp> + <Exp>> {left}  
  
Exp.Fun  = <function(<{ID ","}>*)> <Exp>>  
Exp.App  = <<Exp> <Exp>> {left}  
  
Exp.Let  = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd = <<ID> = <Exp>>  
  
Exp.If   = <if(<Exp>) <Exp>>  
Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match = <match <Exp> with <{Case " | "}>+>  
           {longest-match}  
Case.Case = [[Pat] → [Exp]]  
  
Pat.PVar = ID  
Pat.PApp = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

if(1) if(2) 3 else 4

amb( [ IfElse(  
 Int("1")  
 , If(Int("2"), Int("3"))  
 , Int("4")  
 )  
 , If(  
 Int("1")  
 , IfElse(Int("2"), Int("3"), Int("4"))  
 )  
]

If(  
 Int("1")  
 , IfElse(Int("2"), Int("3"), Int("4"))  
)

# Safe Disambiguation = Do Not Reject Unambiguous Sentences

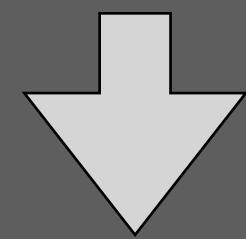
## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}*>) <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}
```

## context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

4 + if(y) x



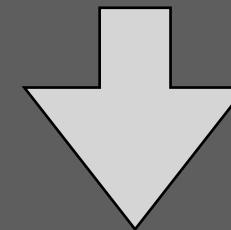
Add(Int("4"), If(Var("y"), Var("x")))

# Deep Priority Conflict

## context-free syntax

```
Exp      = <(<Exp>)> {bracket}  
  
Exp.Int   = INT  
Exp.Var   = ID  
Exp.Add   = <<Exp> + <Exp>> {left}  
  
Exp.Fun   = <function(<{ID ","}*>) <Exp>>  
Exp.App   = <<Exp> <Exp>> {left}  
  
Exp.Let   = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd   = <<ID> = <Exp>>  
  
Exp.If    = <if(<Exp>) <Exp>>  
Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match = <match <Exp> with <{Case " | "}>+>  
           {longest-match}  
Case.Case = [[Pat] → [Exp]]  
  
Pat.PVar  = ID  
Pat.PApp  = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

4 + if(y) x + 3



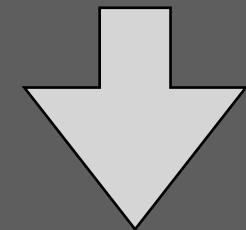
```
amb(  
  [ Add(  
    Int("4")  
  , amb(  
    [ Add(If(Var("y"), Var("x")), Int("3"))  
    , If(Var("y"), Add(Var("x"), Int("3")))  
    ]  
  )  
  , Add(  
    Add(Int("4"), If(Var("y"), Var("x")))  
    , Int("3")  
  )  
]  
)
```

# Deep Priority Conflict (Solved)

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}*>) <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

4 + if(y) x + 3



```
Add(  
  Int("4")  
, If(Var("y"), Add(Var("x"), Int("3"))))  
)
```

# Longest Match = Solve Repetition Ambiguity

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}>*)> <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

```
match x with  
  a → match 5 with  
    b → 3  
  | c → 4
```

```
Match(  
  Var("x"))  
, amb(  
  [ [ Case(  
        PVar("a"))  
    , Match(  
        Int("5"))  
    , [ Case(PVar("b"), Int("3"))]  
    ]  
  )  
  , Case(PVar("c"), Int("4"))  
  , [ Case(  
        PVar("a"))  
    , Match(  
        Int("5"))  
    , [ Case(PVar("b"), Int("3"))  
        , Case(PVar("c"), Int("4"))  
    ]  
  ]  
)
```

# Longest Match = Solve Repetition Ambiguity

## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int     = INT  
Exp.Var     = ID  
Exp.Add     = <<Exp> + <Exp>> {left}  
  
Exp.Fun     = <function(<{ID ","}>*)> <Exp>>  
Exp.App     = <<Exp> <Exp>> {left}  
  
Exp.Let     = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd    = <<ID> = <Exp>>  
  
Exp.If      = <if(<Exp>) <Exp>>  
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match   = <match <Exp> with <{Case " | "}>+>  
              {longest-match}  
Case.Case   = [[Pat] → [Exp]]  
  
Pat.PVar    = ID  
Pat.PApp    = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

```
match x with  
  a → match 5 with  
    b → 3  
  | c → 4
```

```
Match(  
  Var("x")  
, [ Case(  
    PVar("a")  
, Match(  
      Int("5")  
, [ Case(PVar("b"), Int("3"))  
      , Case(PVar("c"), Int("4"))  
    ]  
)  
)  
)  
)
```

# Parenthesize

# Parenthesize = Disambiguate<sup>-1</sup> (Insert Necessary Parentheses)

## context-free syntax

Exp = <(<Exp>)> {bracket}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {left}

Exp.Fun = <function(<{ID ","}\*>) <Exp>>

Exp.App = <<Exp> <Exp>> {left}

Exp.Let = <let <Bnd\*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case " | "}>+>  
{longest-match}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

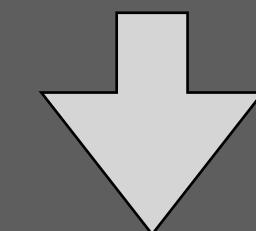
Pat.PApp = <<Pat> <Pat>> {left}

## context-free priorities

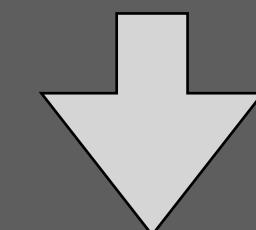
Exp.App > Exp.Add > Exp.IfElse > Exp.If

> Exp.Match > Exp.Let > Exp.Fun

(a + b) + c



Add(Add(Var("a"), Var("b")), Var("c"))



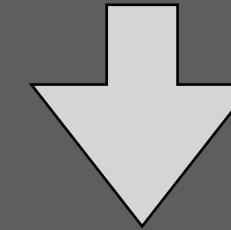
a + b + c

# Parenthesize = Disambiguate<sup>-1</sup> (Insert Necessary Parentheses)

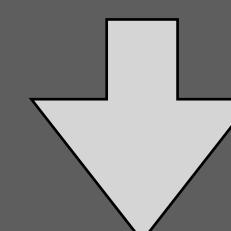
## context-free syntax

```
Exp          = <(<Exp>)> {bracket}  
  
Exp.Int      = INT  
Exp.Var      = ID  
Exp.Add      = <<Exp> + <Exp>> {left}  
  
Exp.Fun      = <function(<{ID ","}>*)> <Exp>>  
Exp.App      = <<Exp> <Exp>> {left}  
  
Exp.Let      = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd      = <<ID> = <Exp>>  
  
Exp.If        = <if(<Exp>) <Exp>>  
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match    = <match <Exp> with <{Case " | "}>+>  
               {longest-match}  
Case.Case    = [[Pat] → [Exp]]  
  
Pat.PVar     = ID  
Pat.PApp     = <<Pat> <Pat>> {left}  
  
context-free priorities  
Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

a + (let x = b in (c + d))



```
Add(  
  Var("a")  
, Let(  
  [Bnd("x", Var("b"))]  
, Add(Var("c"), Var("d"))  
)  
)
```



a + let  
 x = b  
in  
 c + d

# Parenthesize = Disambiguate<sup>-1</sup> (Insert Necessary Parentheses)

## context-free syntax

Exp = <(<Exp>)> {bracket}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {left}

Exp.Fun = <function(<{ID ","}>\*)> <Exp>>

Exp.App = <<Exp> <Exp>> {left}

Exp.Let = <let <Bnd\*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case " | "}>+>  
{longest-match}

Case.Case = [[Pat] → [Exp]]

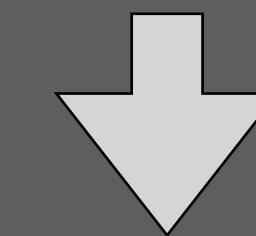
Pat.PVar = ID

Pat.PApp = <<Pat> <Pat>> {left}

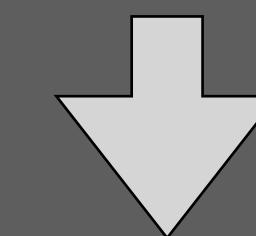
## context-free priorities

Exp.App > Exp.Add > Exp.IfElse > Exp.If  
> Exp.Match > Exp.Let > Exp.Fun

(a + (let x = b in c)) + d



Add(  
  Add(  
    Var("a")  
    , Let([Bnd("x", Var("b"))], Var("c"))  
  )  
  , Var("d")  
)



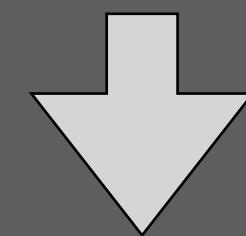
a + (let  
  x = b  
  in  
  c) + d

# Parenthesize = Disambiguate<sup>-1</sup> (Insert Necessary Parentheses)

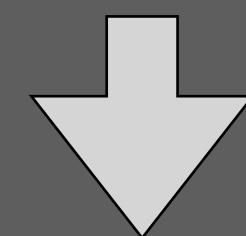
## context-free syntax

```
Exp      = <(<Exp>)> {bracket}  
  
Exp.Int  = INT  
Exp.Var  = ID  
Exp.Add   = <<Exp> + <Exp>> {left}  
Exp.Sub   = <<Exp> - <Exp>> {left}  
  
Exp.Fun   = <function(<{ID ","}>*)> <Exp>>  
Exp.App   = <<Exp> <Exp>> {left}  
  
Exp.Let   = <let <Bnd*> in <Exp>>  
  
Bnd.Bnd  = <<ID> = <Exp>>  
  
Exp.If    = <if(<Exp>) <Exp>>  
Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>  
  
Exp.Match = <match <Exp> with <{Case "|"}+>>  
            {longest-match}  
Case.Case = [[Pat] → [Exp]]  
  
Pat.PVar  = ID  
Pat.PApp  = <<Pat> <Pat>> {left}  
  
context-free priorities  
{left: Exp.Sub Exp.App} > Exp.Add > Exp.IfElse >  
Exp.If  
> Exp.Match > Exp.Let > Exp.Fun
```

(a + (let x = b in c)) + d



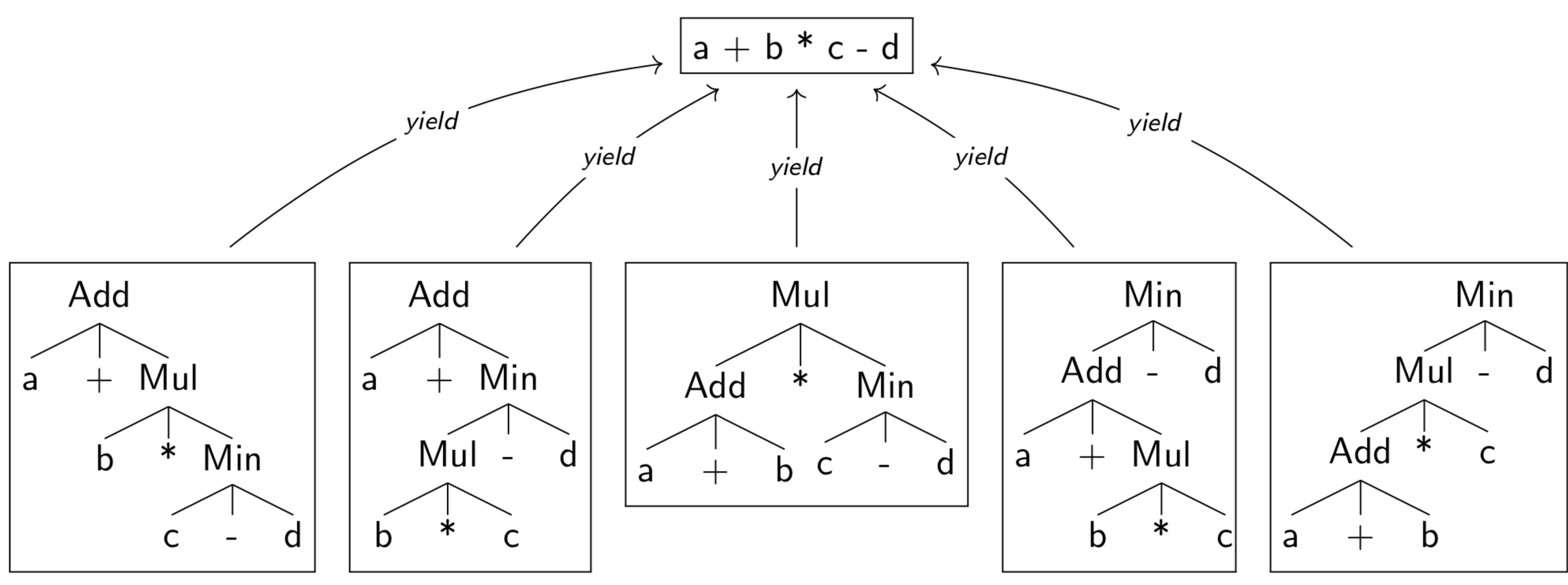
```
Add(  
  Add(  
    Var("a")  
  , Let([Bnd("x", Var("b"))], Var("c"))  
  )  
, Var("d"))  
)
```



a + (let  
 x = b  
 in  
 c) + d

# Semantics of Associativity and Priority

# Ambiguous Sentence has Multiple Parse Trees



# Associativity and Priority as Subtree Exclusion Rules [SDF2 (1997)]

Rules

$$\frac{A.C_1 > A.C_2}{C_1}$$

```
graph TD; C1 --- alpha; C1 --- C2; C2 --- beta; C2 --- gamma;
```

$$\frac{A.C_1 \text{ left } A.C_2}{C_1}$$

```
graph TD; C1 --- alpha; C1 --- C2; C2 --- beta;
```

$$\frac{A.C_1 \text{ right } A.C_2}{C_1}$$

```
graph TD; C1 --- C2; C1 --- gamma; C2 --- beta;
```

Disambiguation rules generate subtree exclusion patterns (aka conflict patterns)

Instances

$$\frac{E.Mul > E.Add}{Mul}$$

```
graph TD; Mul --- Add; Mul --- star; Add --- E1; Add --- E2; E1 --- E3; E1 --- E4;
```

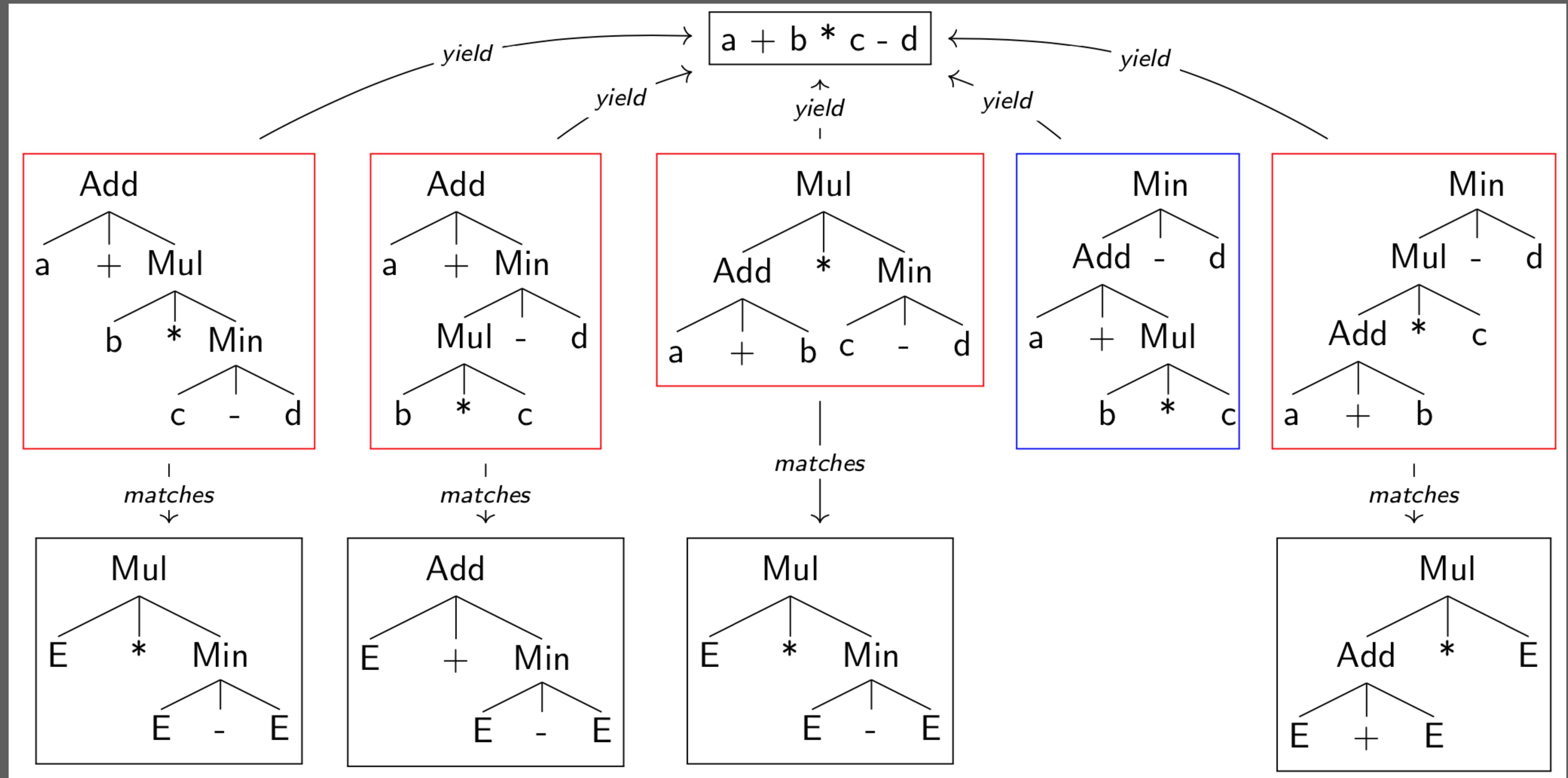
$$\frac{E.Mul > E.Add}{Mul}$$

```
graph TD; Mul --- E1; Mul --- star; Mul --- Add; E1 --- E3; E1 --- E4; star --- E5; star --- E6;
```

$$\frac{E.Add \text{ left } E.Add}{Add}$$

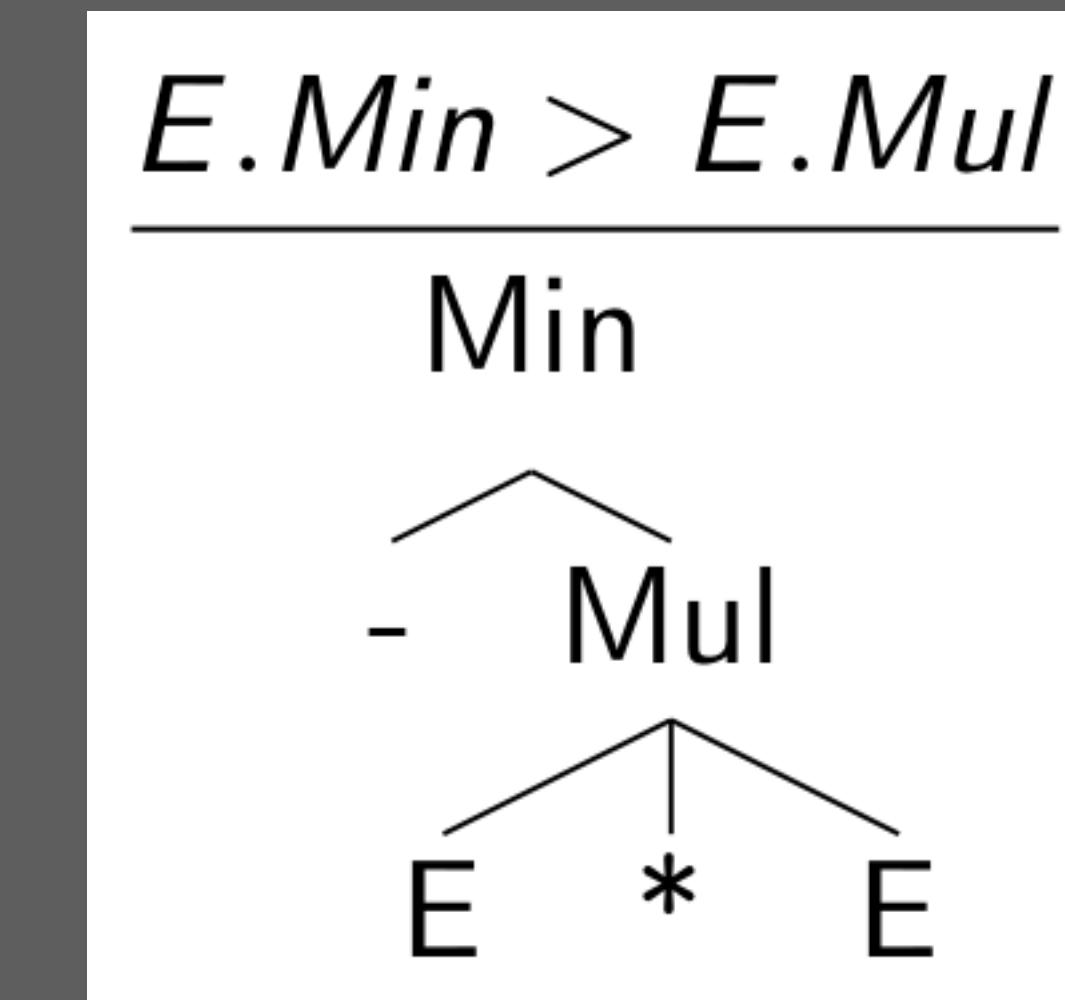
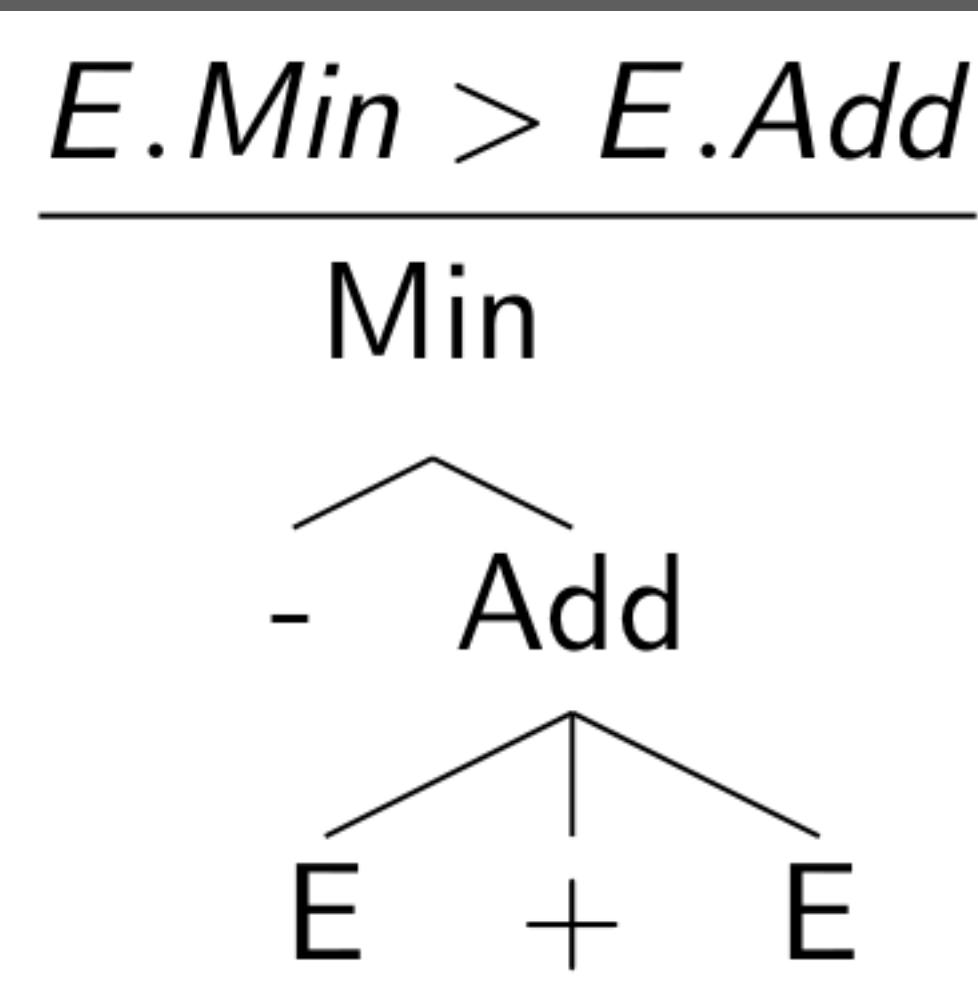
```
graph TD; Add --- E1; Add --- Add2; E1 --- E3; E1 --- E4; Add2 --- E5; Add2 --- E6;
```

# Disambiguation by Subtree Exclusion

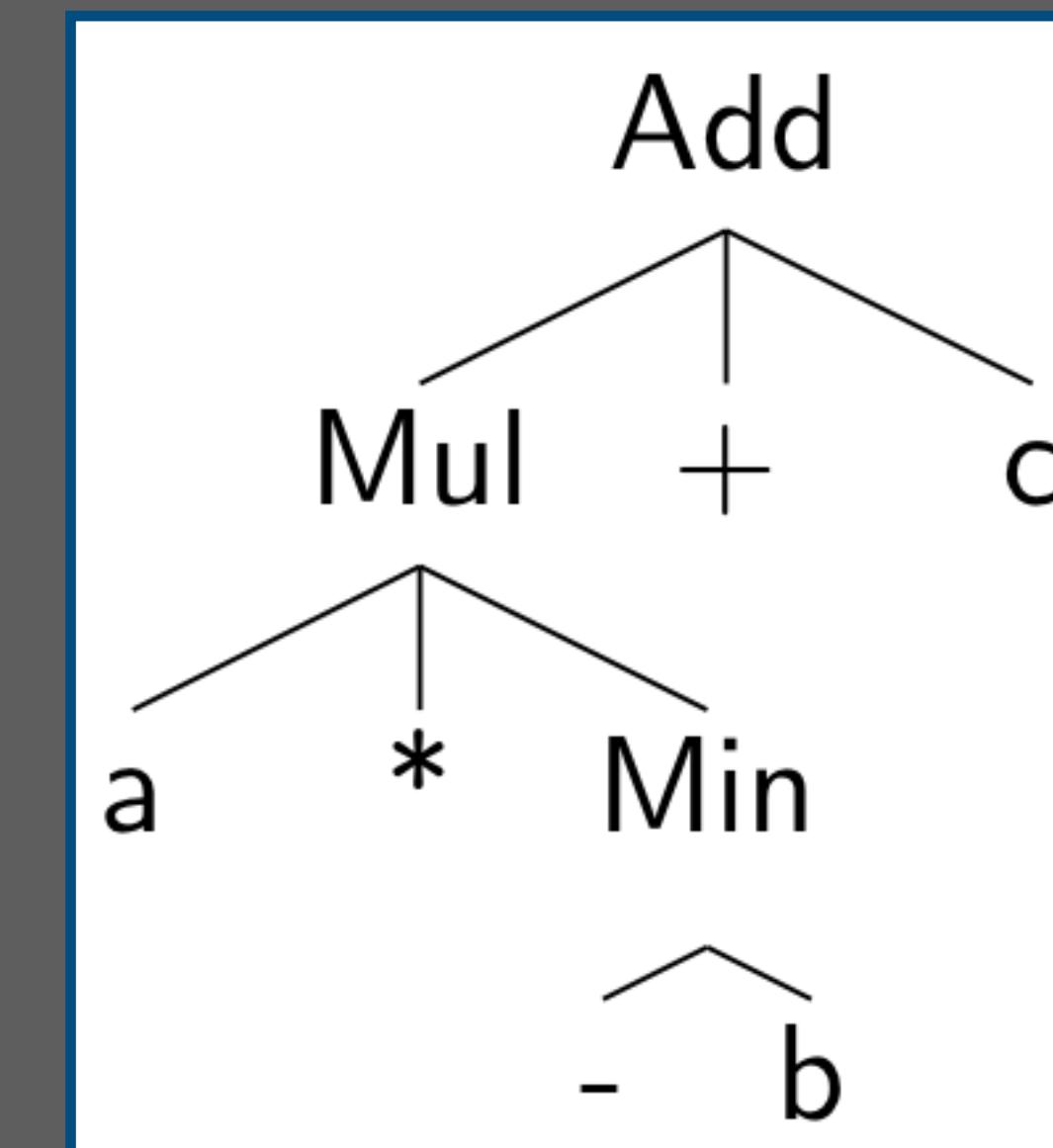
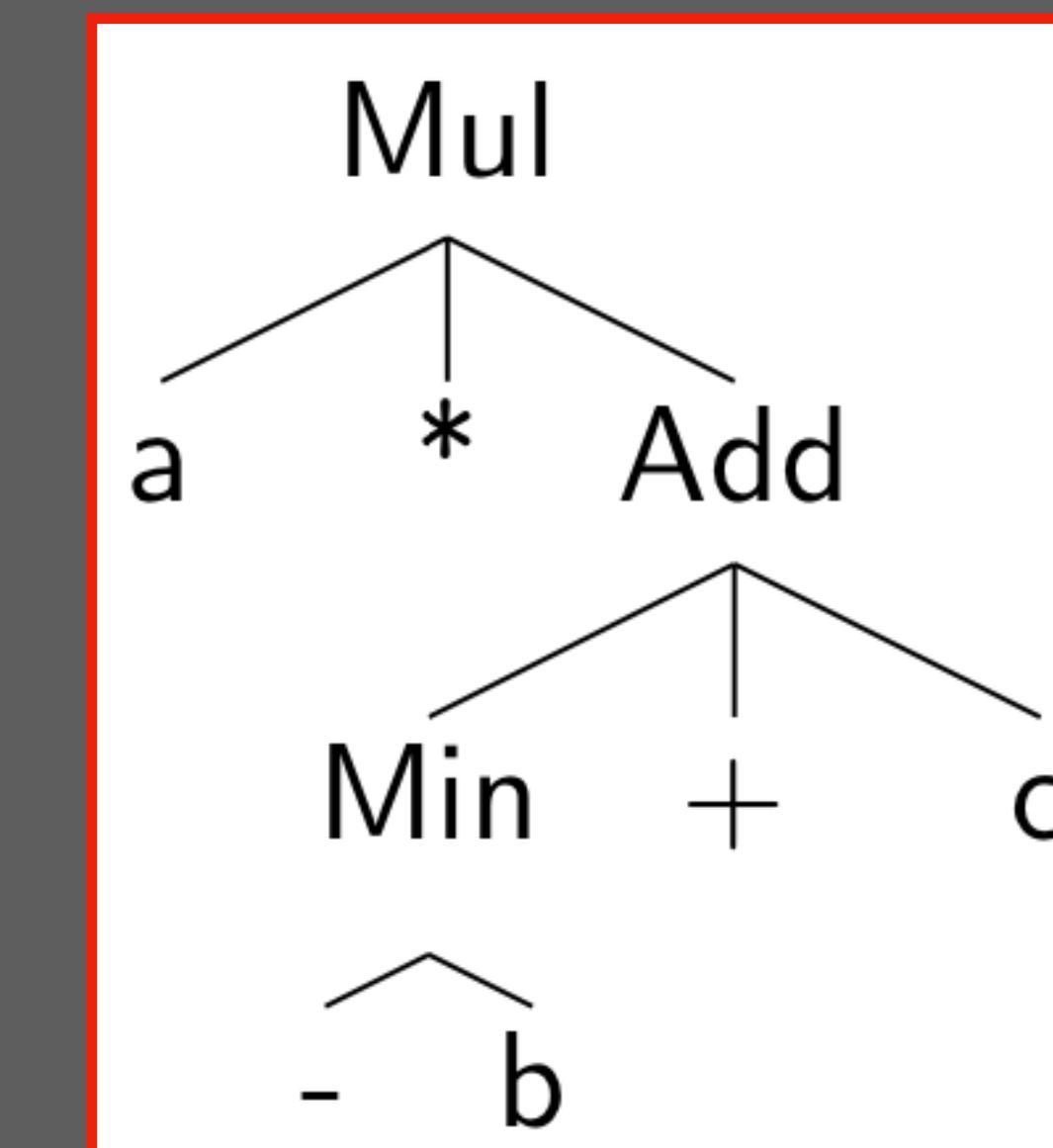
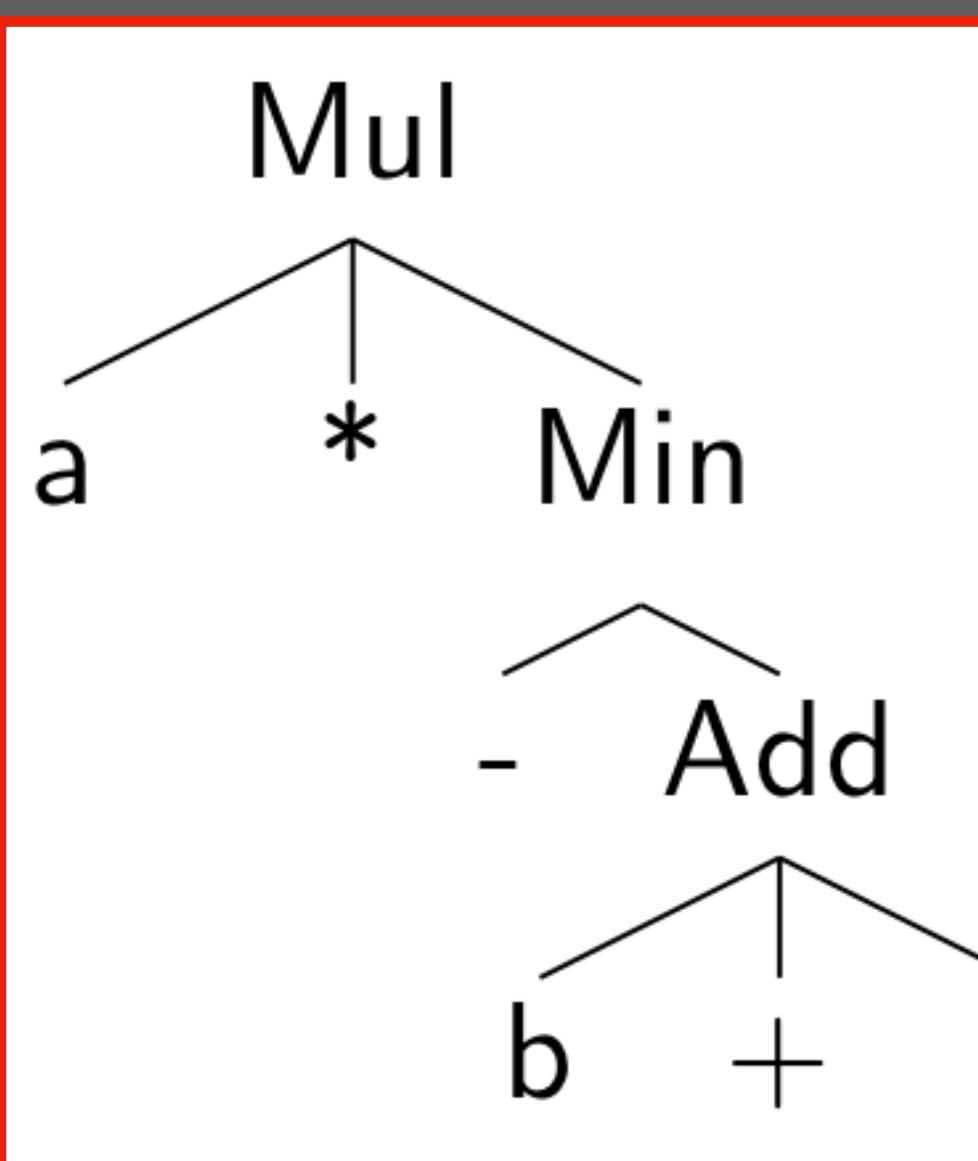


# Safe for High Priority Prefix Operators

Conflict Patterns

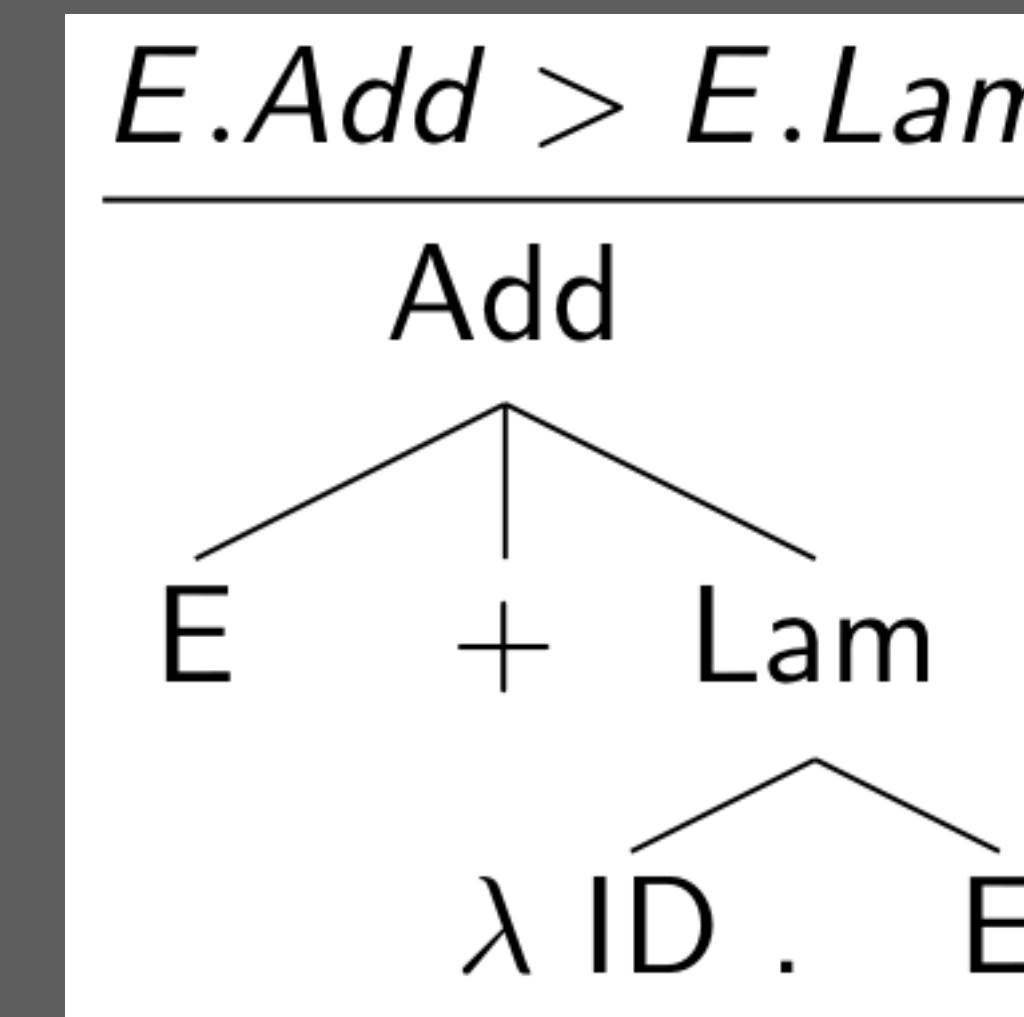
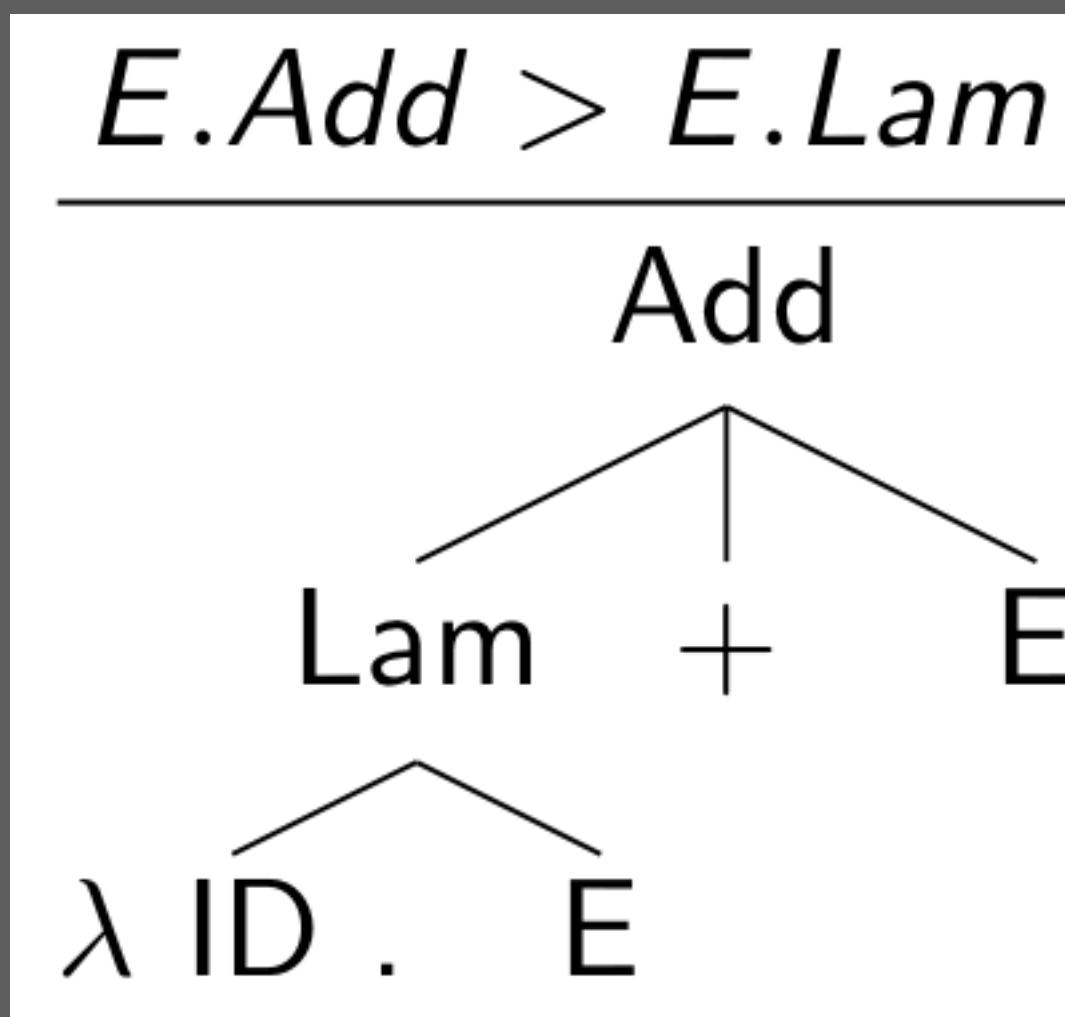


Trees

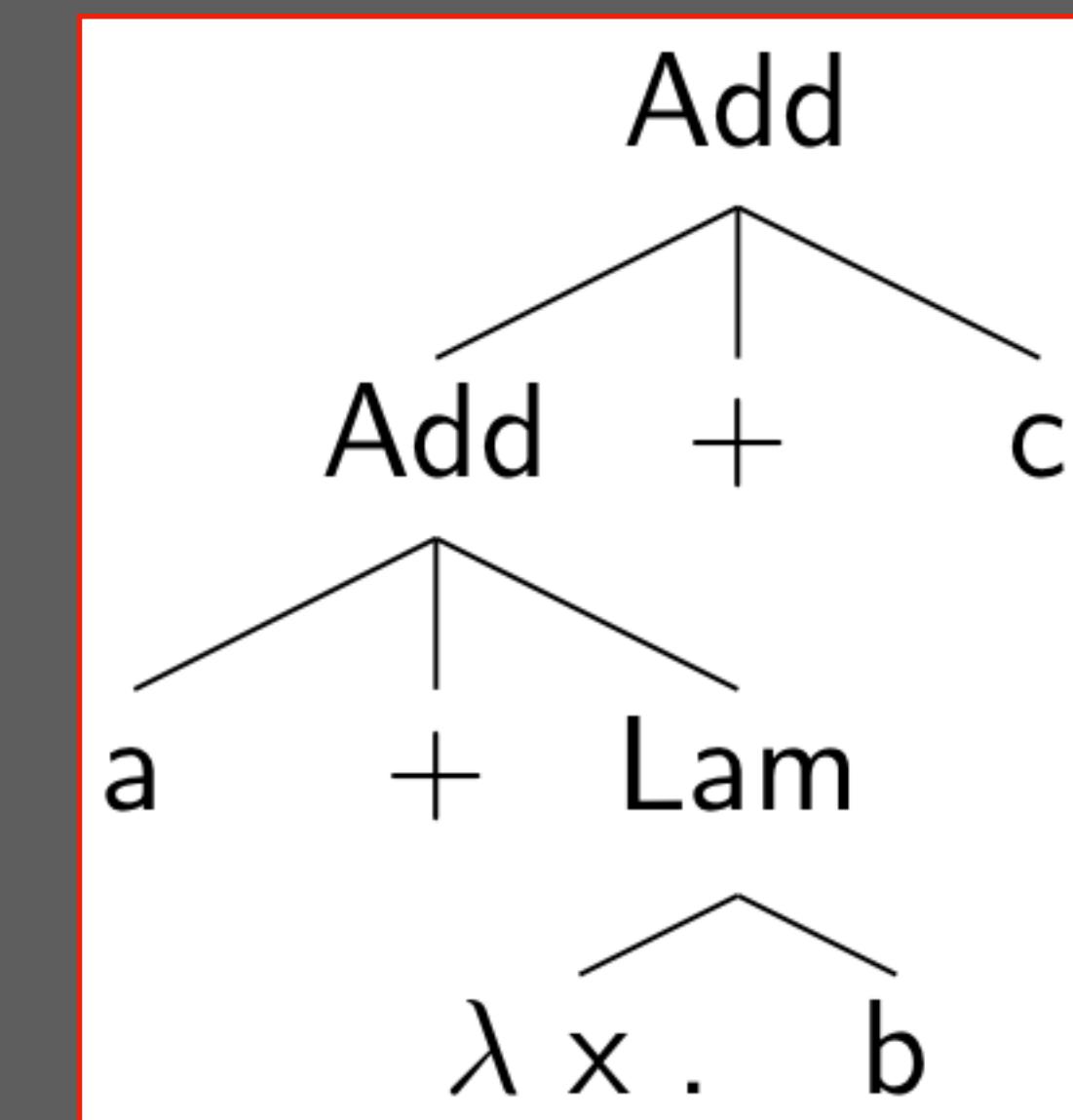
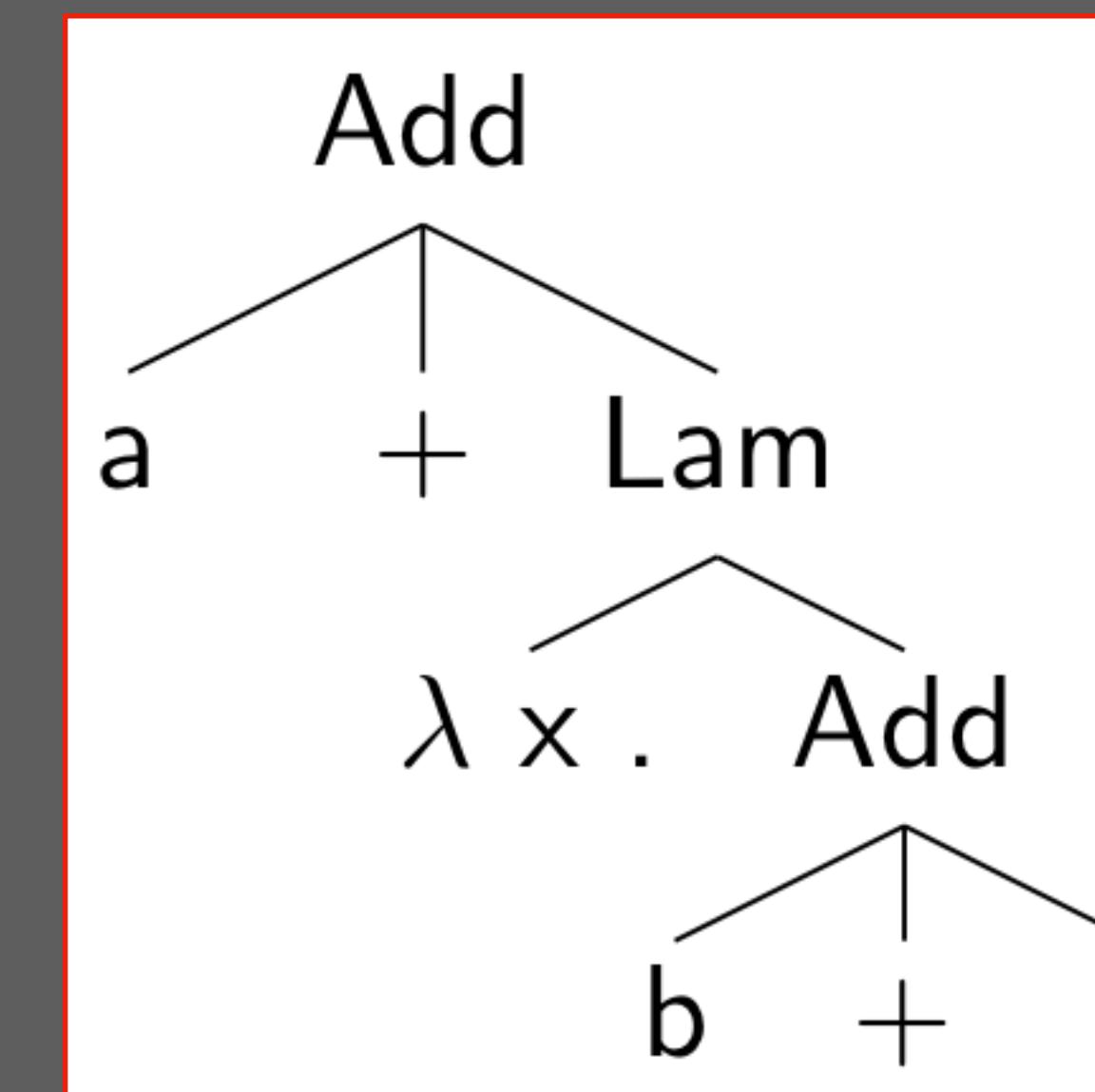
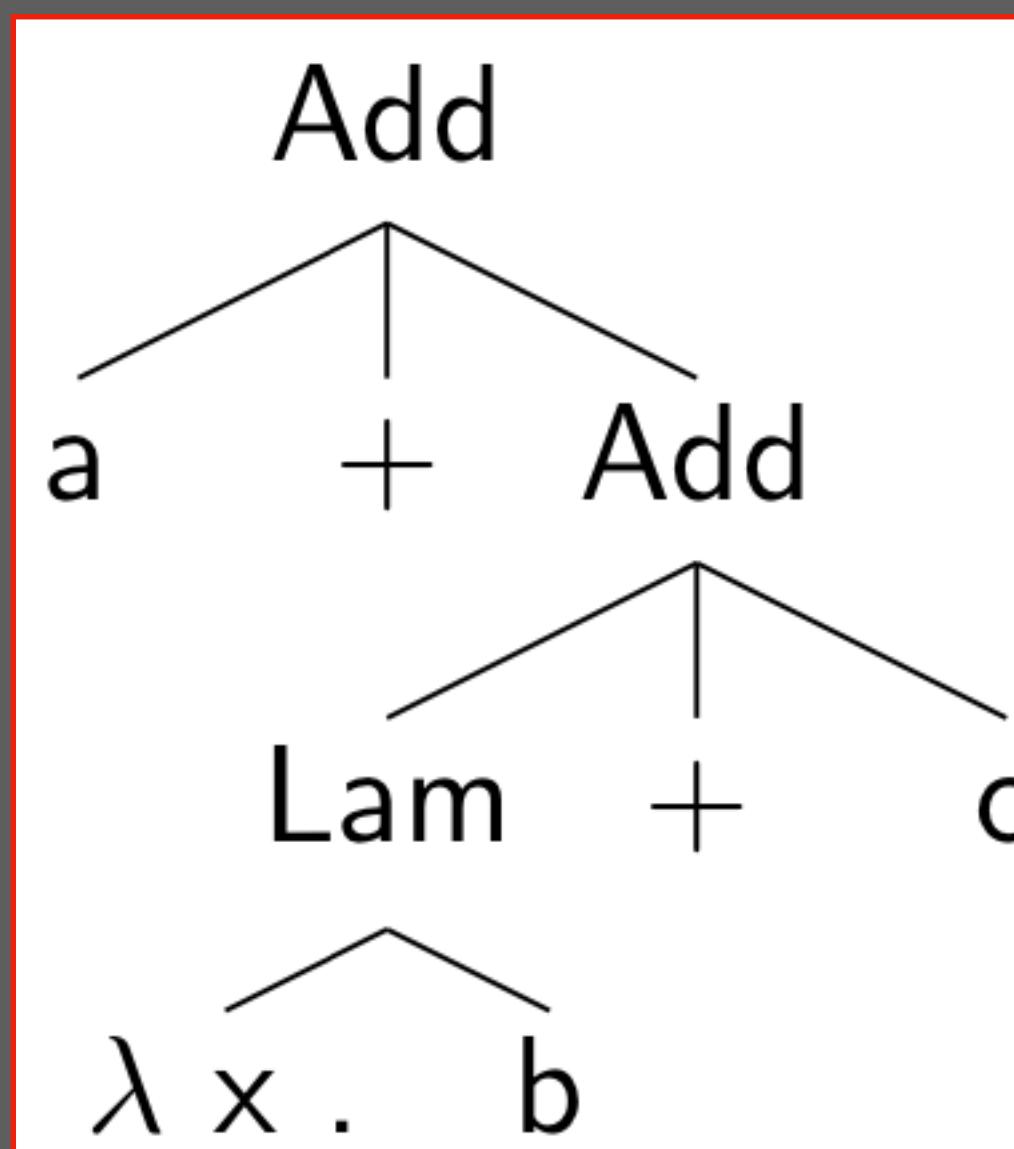


# Unsafe for Low Priority Prefix Operators [SDF2]

Conflict Patterns



Trees



# Safe Subtree Exclusion Rules [SDF3 (2019)]

Rules

$$\frac{A.C_1 > A.C_2}{C_1}$$

```

graph TD
    C1[C1] --- C2[C2]
    C1 --- beta[β]
    C2 --- alpha[α]
    C2 --- A[A]
  
```

Right Recursive in  
Left Recursive Position

$$\frac{A.C_1 > A.C_2}{C_1}$$

```

graph TD
    C1[C1] --- alpha[α]
    C1 --- C2[C2]
    C2 --- A[A]
    C2 --- beta[β]
  
```

Left Recursive in  
Right Recursive Position

$$\frac{A.C_1 \text{ left } A.C_2}{C_1}$$

```

graph TD
    C1[C1] --- A[A]
    C1 --- alpha[α]
    A --- C2[C2]
    A --- beta[β]
    C2 --- A1[A]
    C2 --- A2[A]
  
```

Associativity

Conflict  
Patterns

$$\frac{E.Add > E.Lam}{Add}$$

```

graph TD
    Add[Add] --- Lam[Lam]
    Add --- plus[+]
    Lam --- lambda[λ]
    Lam --- ID[ID]
    plus --- E[E]
    E --- E2[E]
  
```

conflict pattern:  
\ right recursive

$$\frac{E.Add > E.Lam}{Add}$$

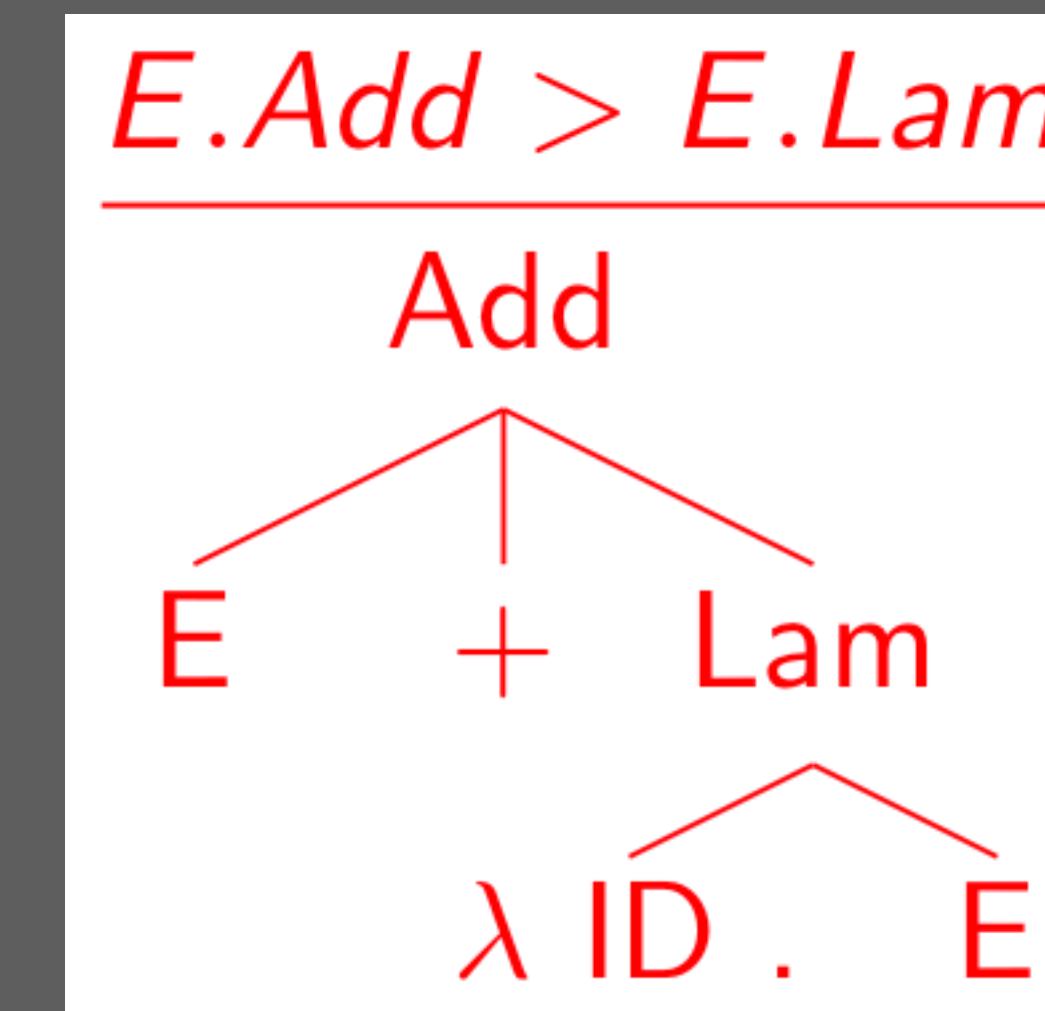
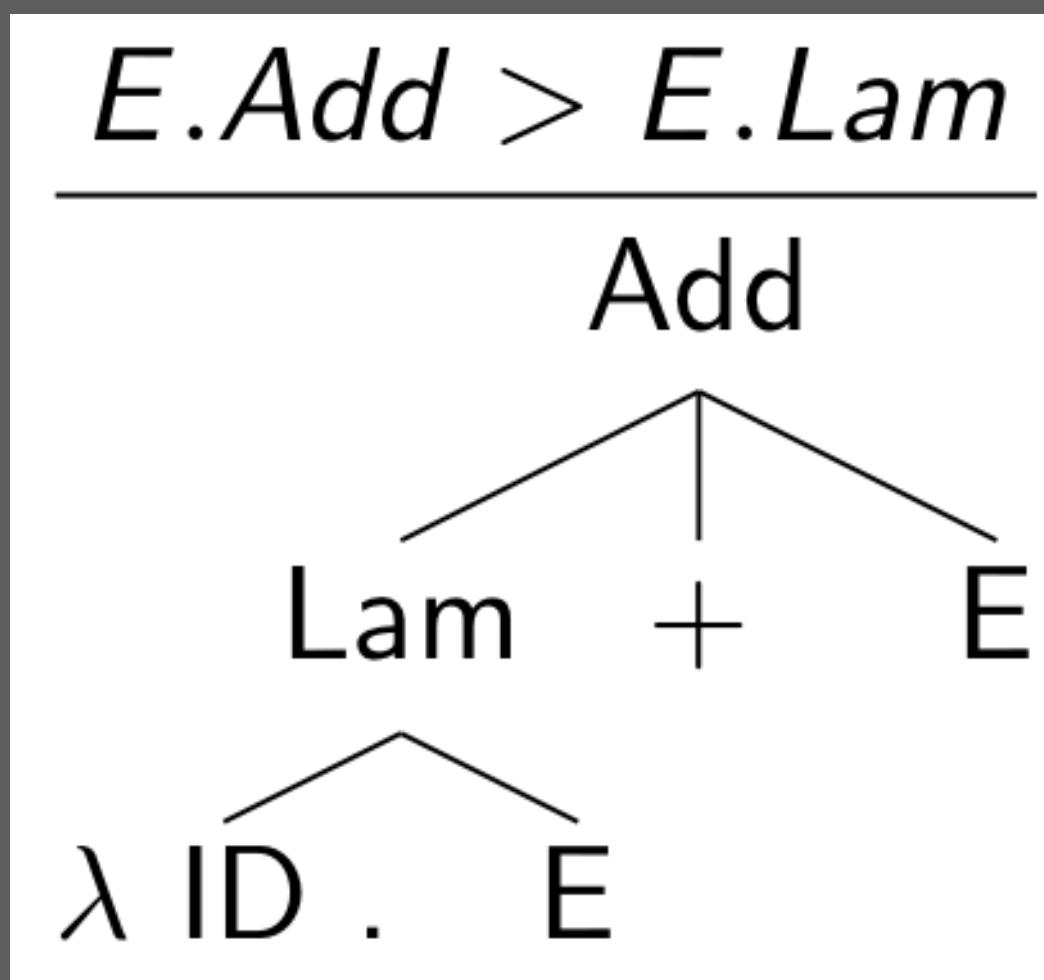
```

graph TD
    Add[Add] --- E[E]
    Add --- plus[+]
    Add --- Lam[Lam]
    E --- E2[E]
    plus --- E3[E]
    Lam --- lambda[λ]
    Lam --- ID[ID]
    E --- E4[E]
  
```

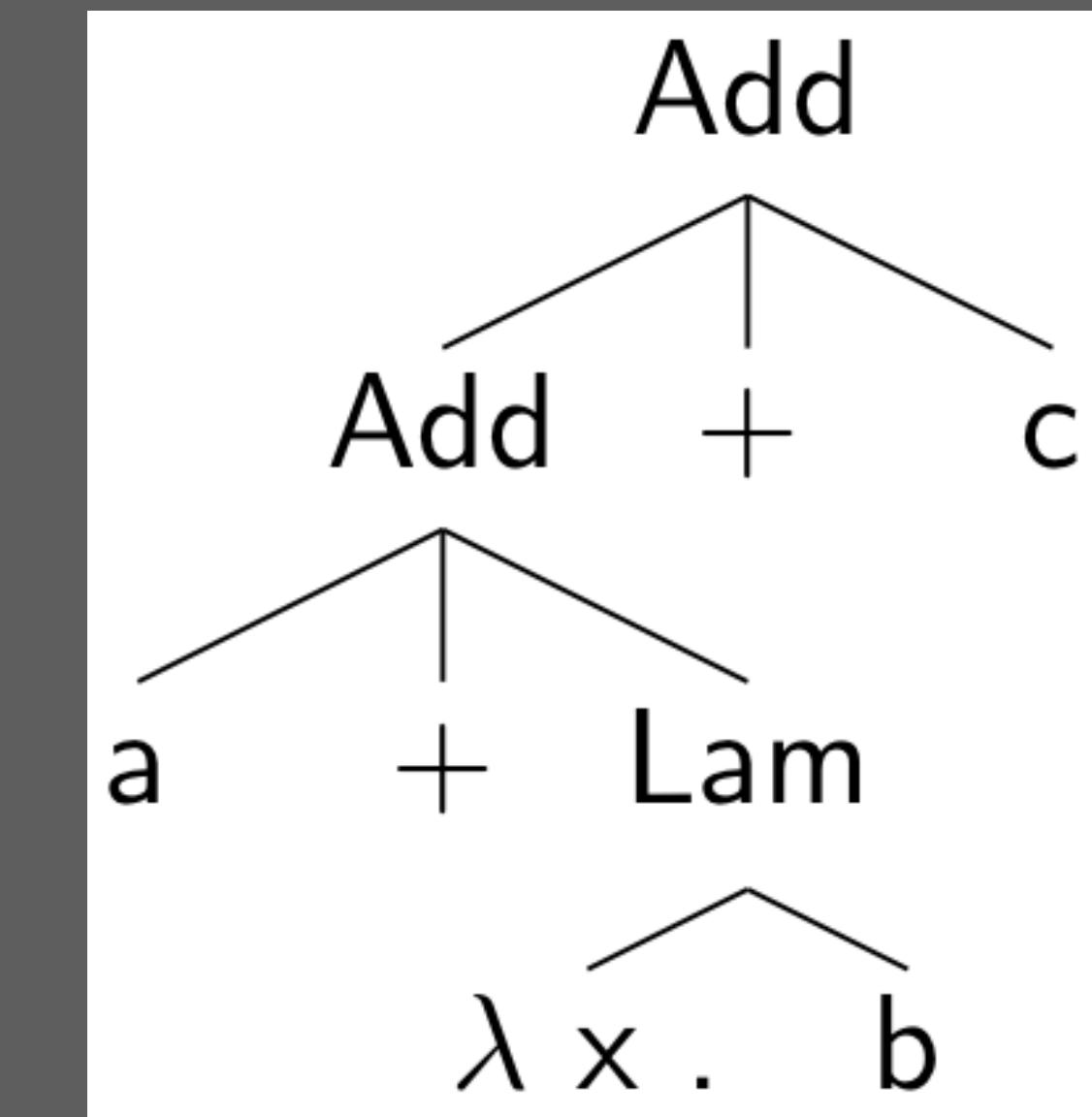
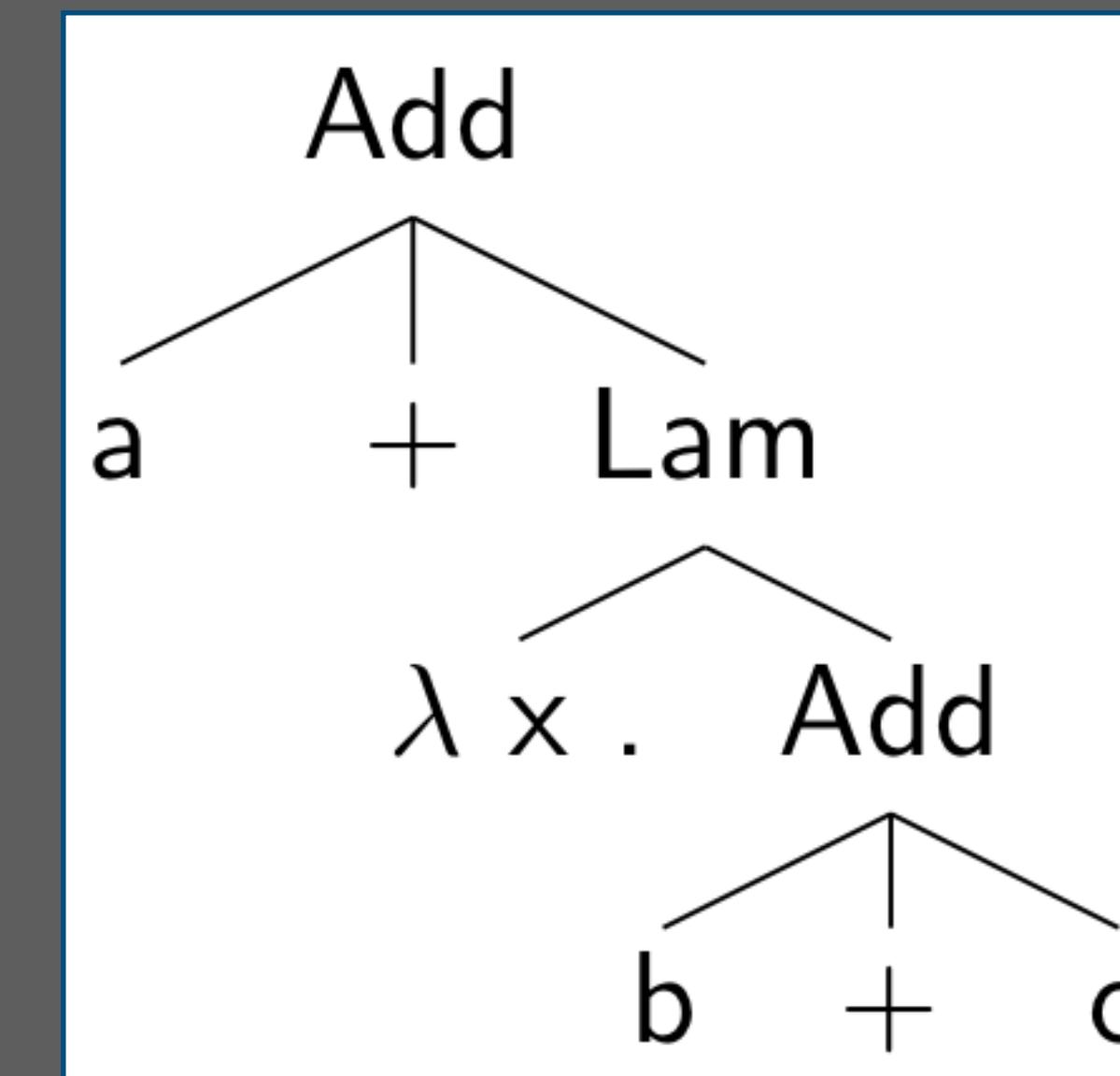
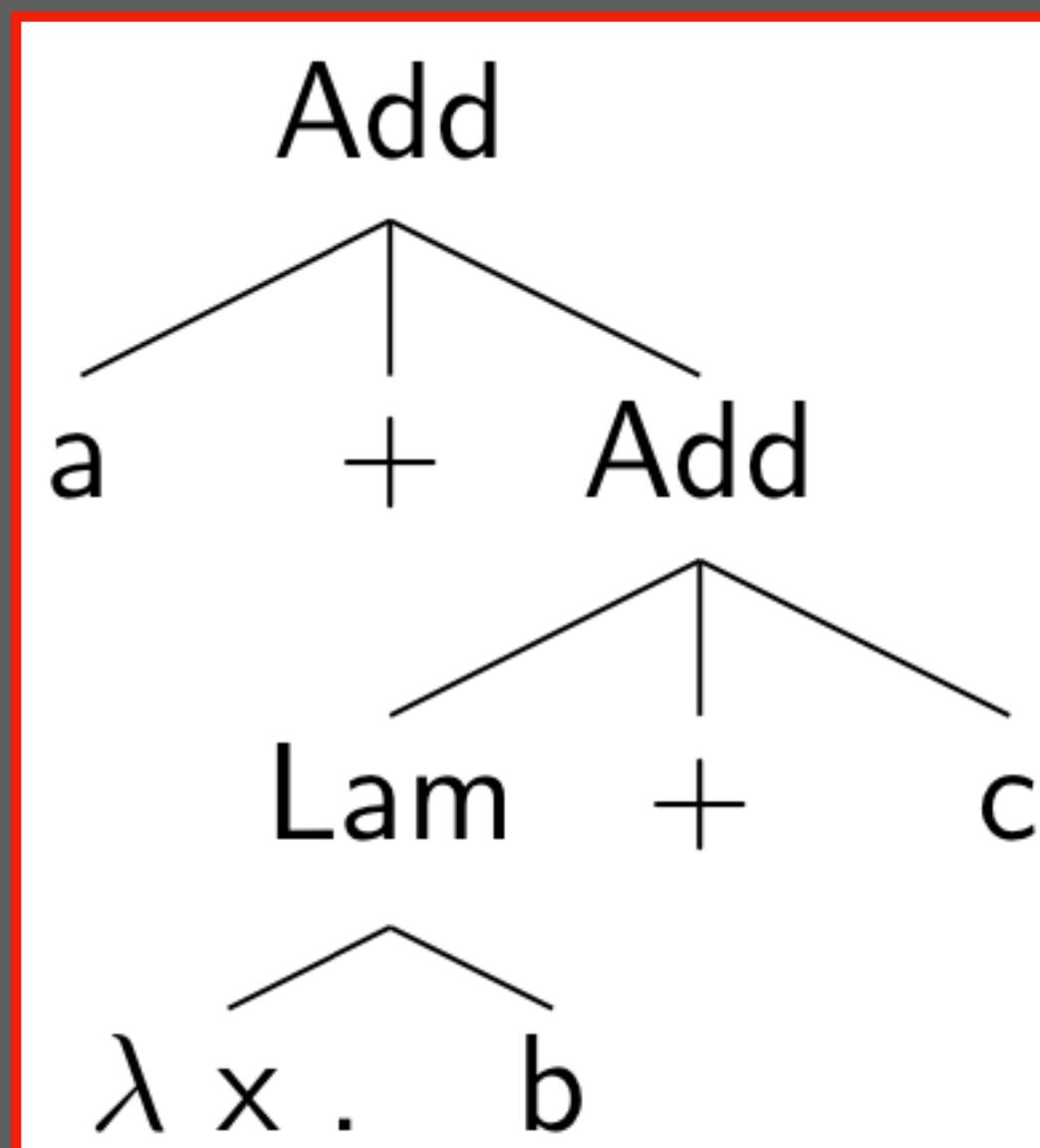
not a conflict pattern:  
\ not left recursive

# Shallow Interpretation: Safe for Low Priority Prefix Operators

Conflict Patterns

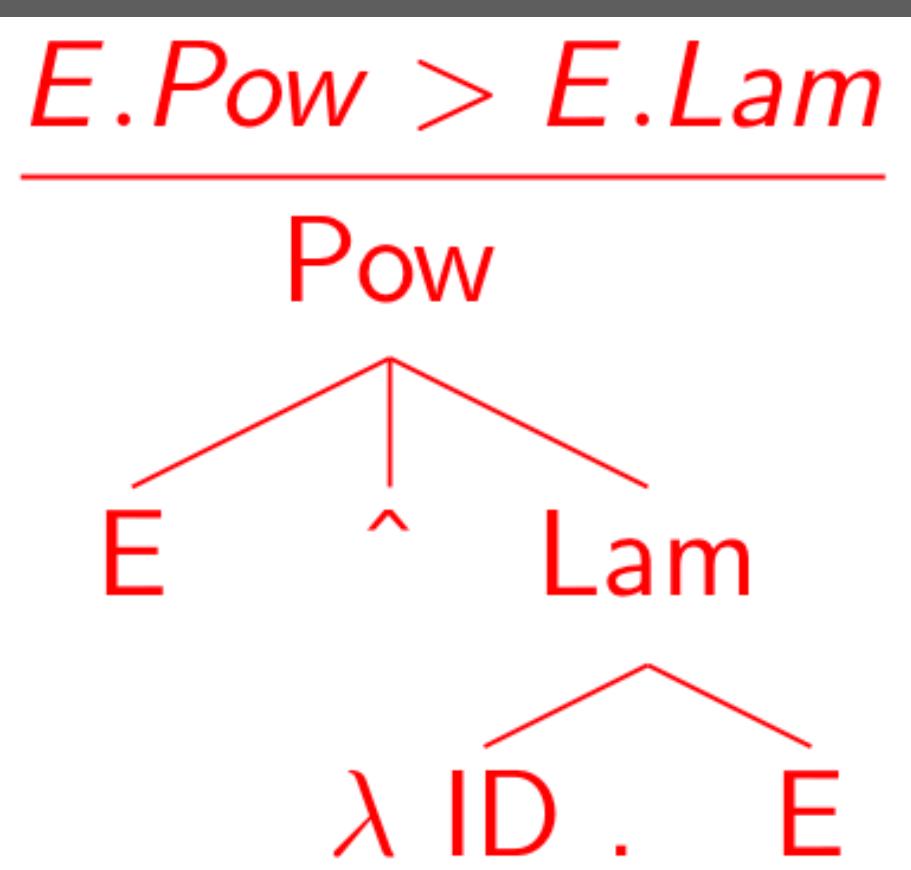
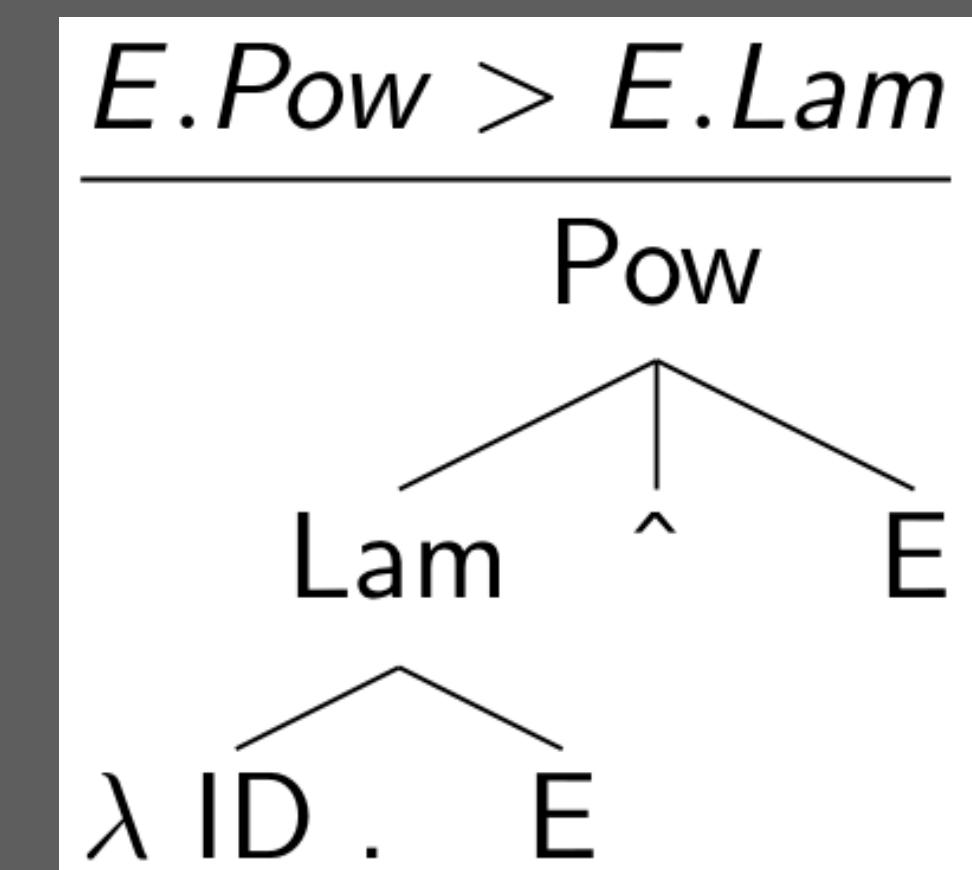
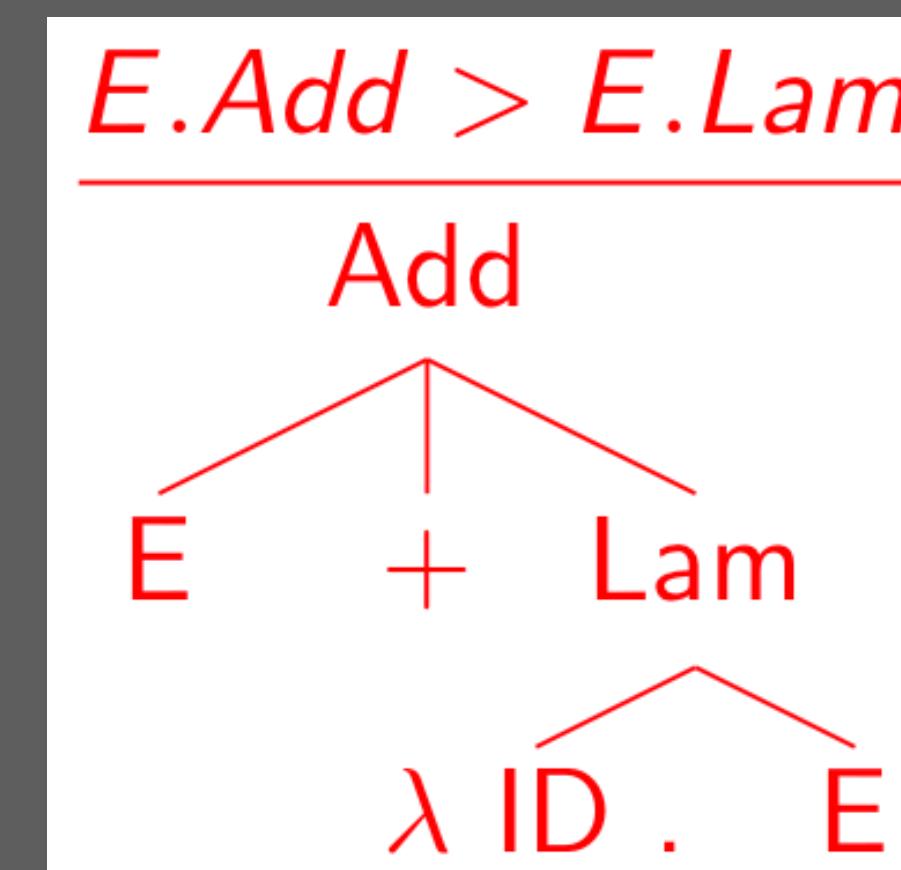
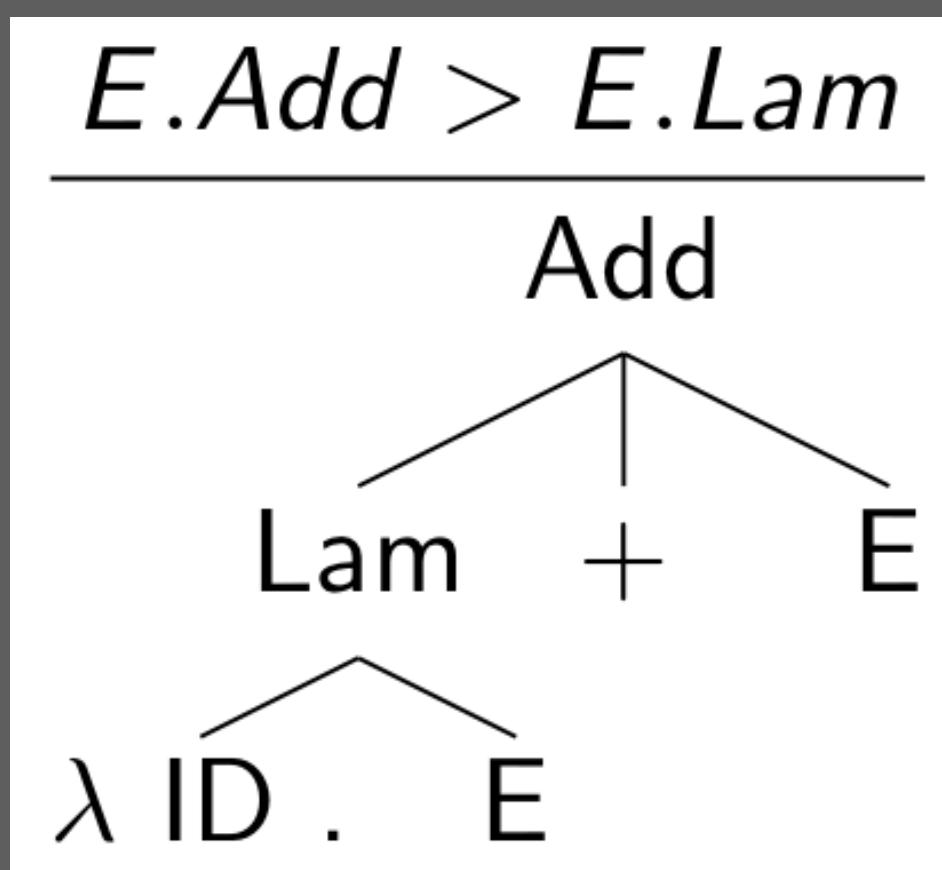


Trees



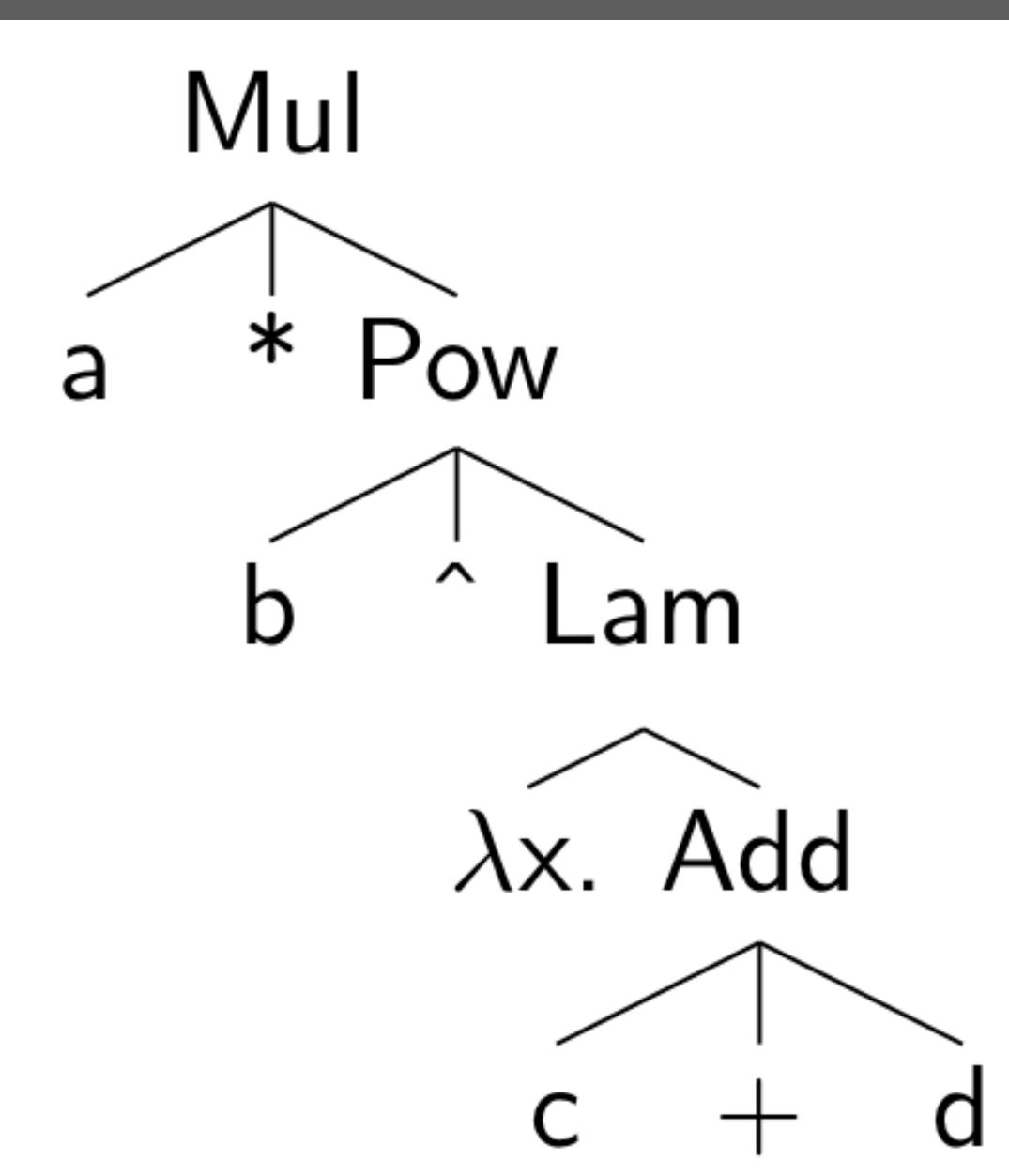
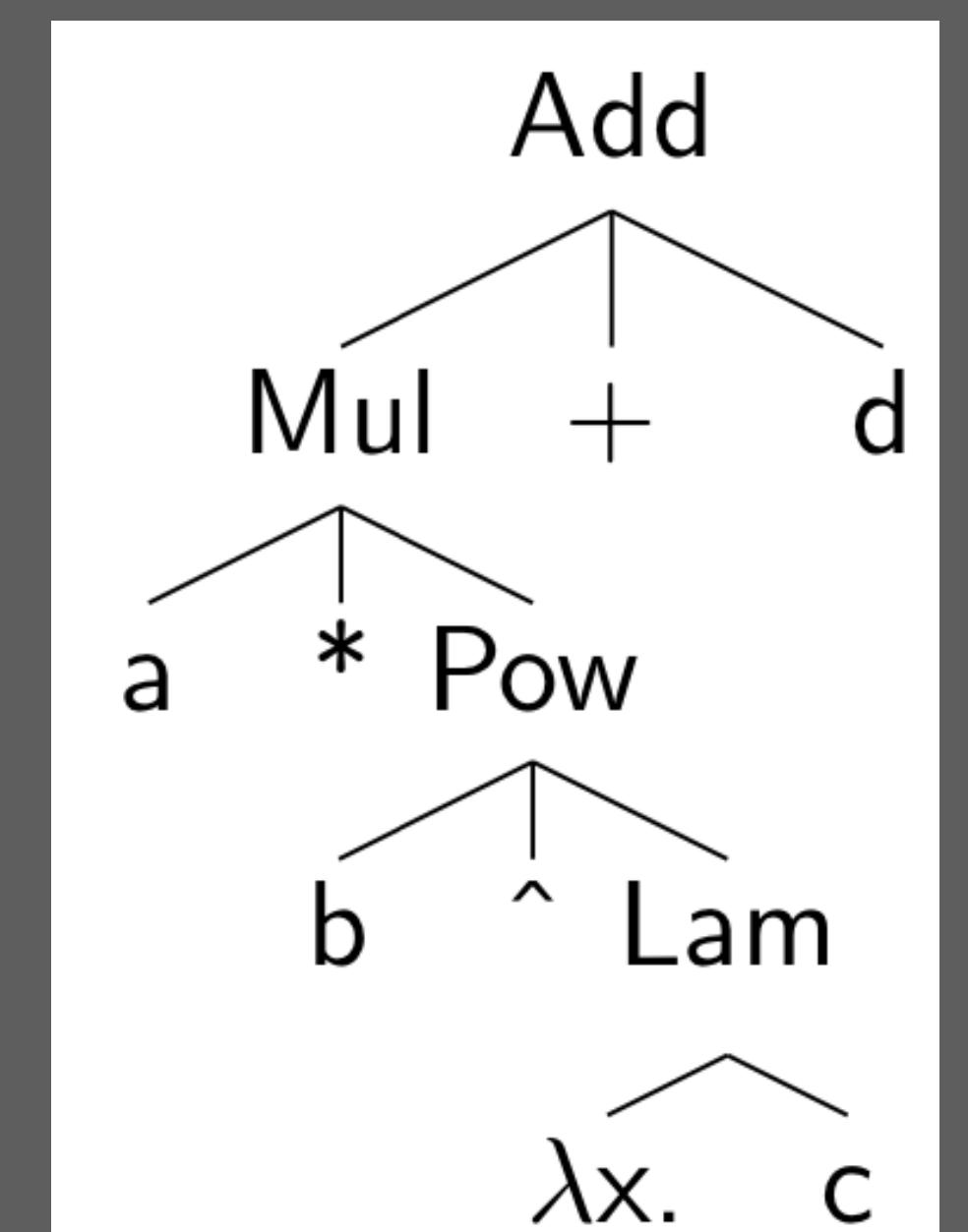
# Shallow Interpretation: Incomplete for Low Priority Prefix Operators

Conflict Patterns



Trees

$a * b ^ \lambda x. c + d$



# Deep Priority Conflicts: Match Subpattern in Right-Most Subtree

$$\frac{E.\text{Add} > E.\text{Lam}}{\text{Add}}$$

Add

```
graph TD; Add --> Lam1[Lam]; Lam1 --> Lambda1["λ ID ."]; Lam1 --> E1[E]
```

$$\frac{E.\text{Add} > E.\text{Lam}}{\text{Add}}$$

Add

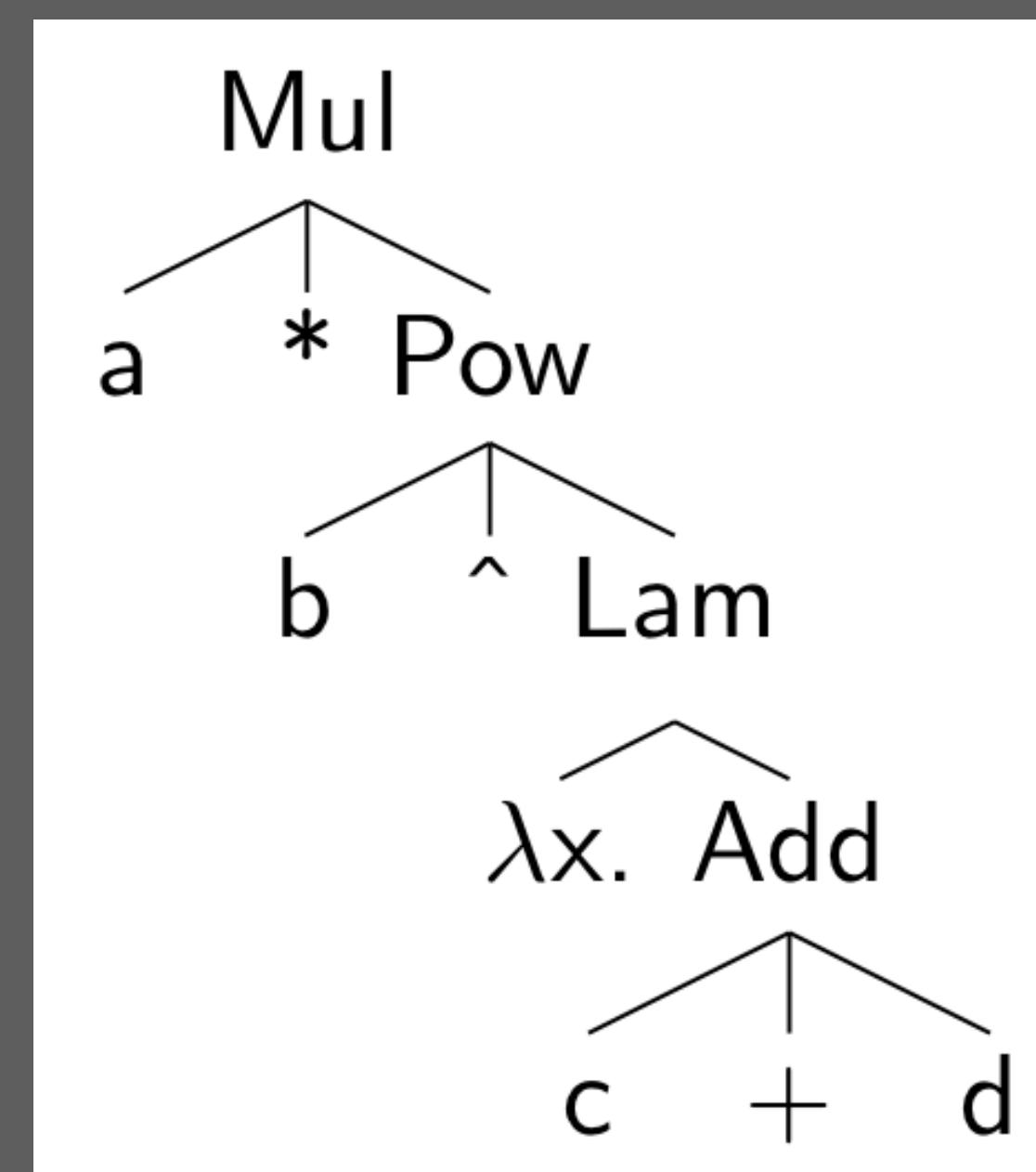
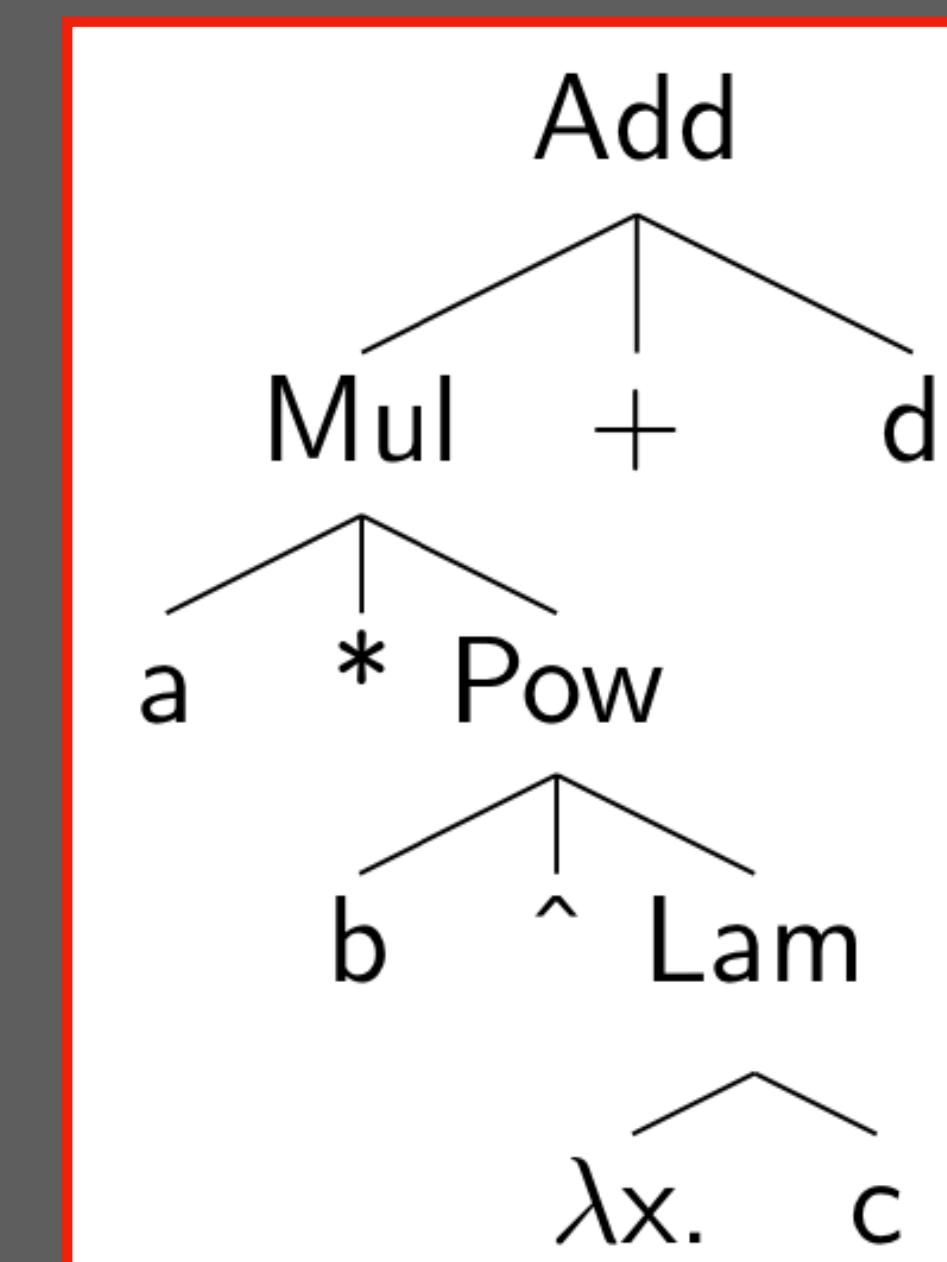
```
graph TD; Add --> C1[C1]; C1 --> Alpha1["α"]; C1 --> Lam1[Lam]; Lam1 --> Lambda1["λ ID ."]; Lam1 --> E1[E]
```

$$\frac{E.\text{Add} > E.\text{Lam}}{\text{Add}}$$

Add

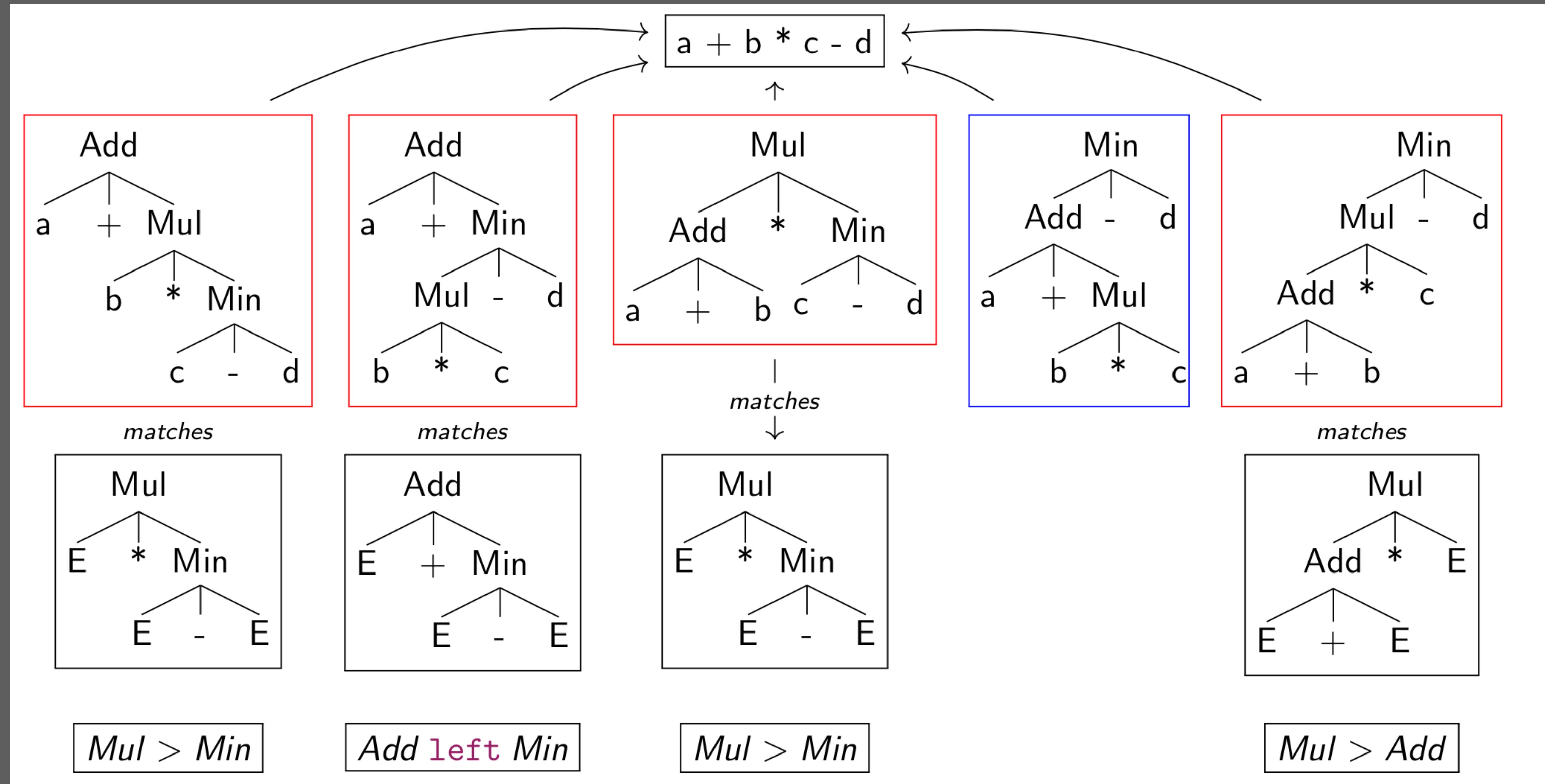
```
graph TD; Add --> C1[C1]; C1 --> Alpha2["α"]; C1 --> C2[C2]; C2 --> Beta2["β"]; C2 --> Lam2[Lam]; Lam2 --> Lambda2["λ ID ."]; Lam2 --> E2[E]
```

...

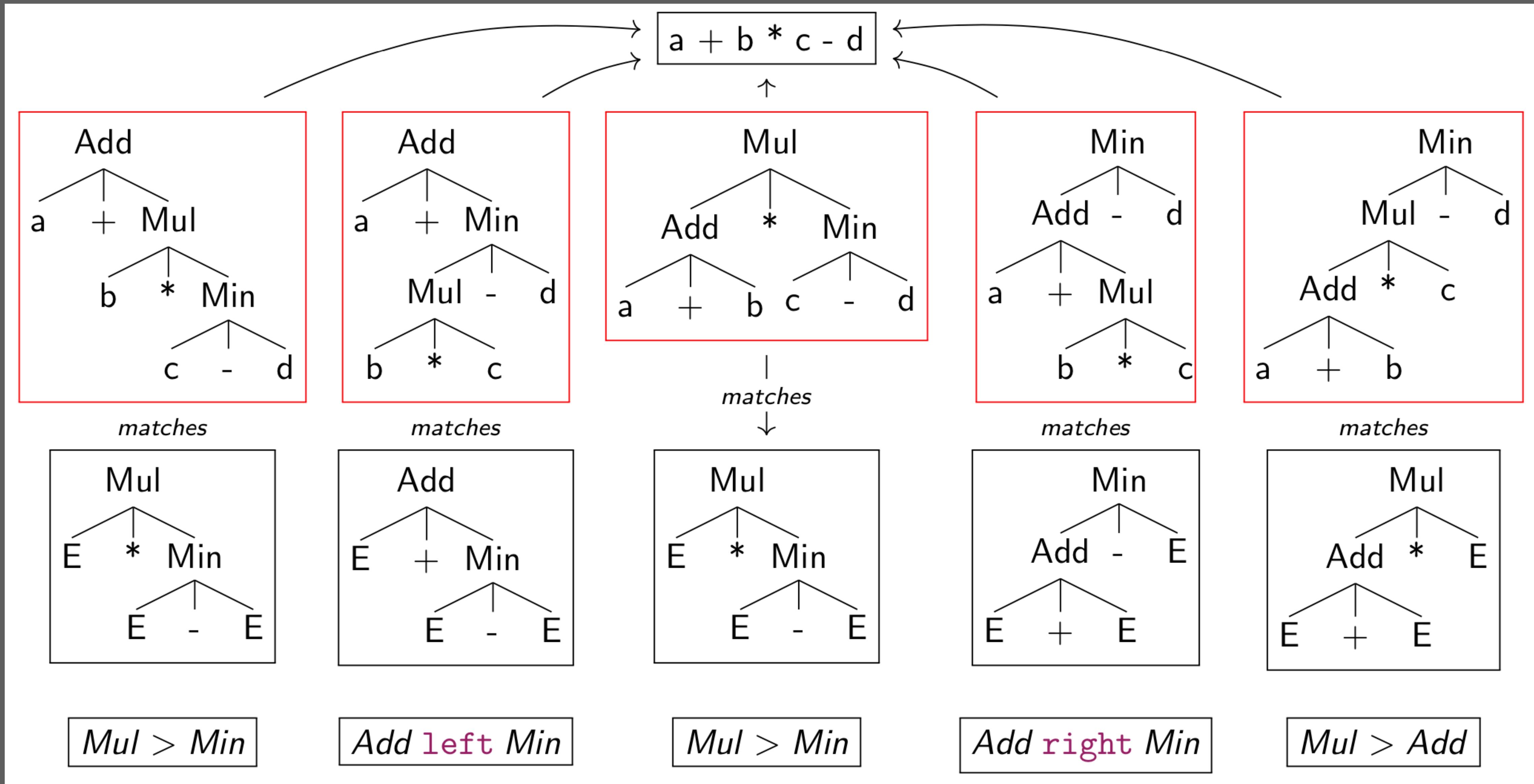


Infinite set of conflict patterns

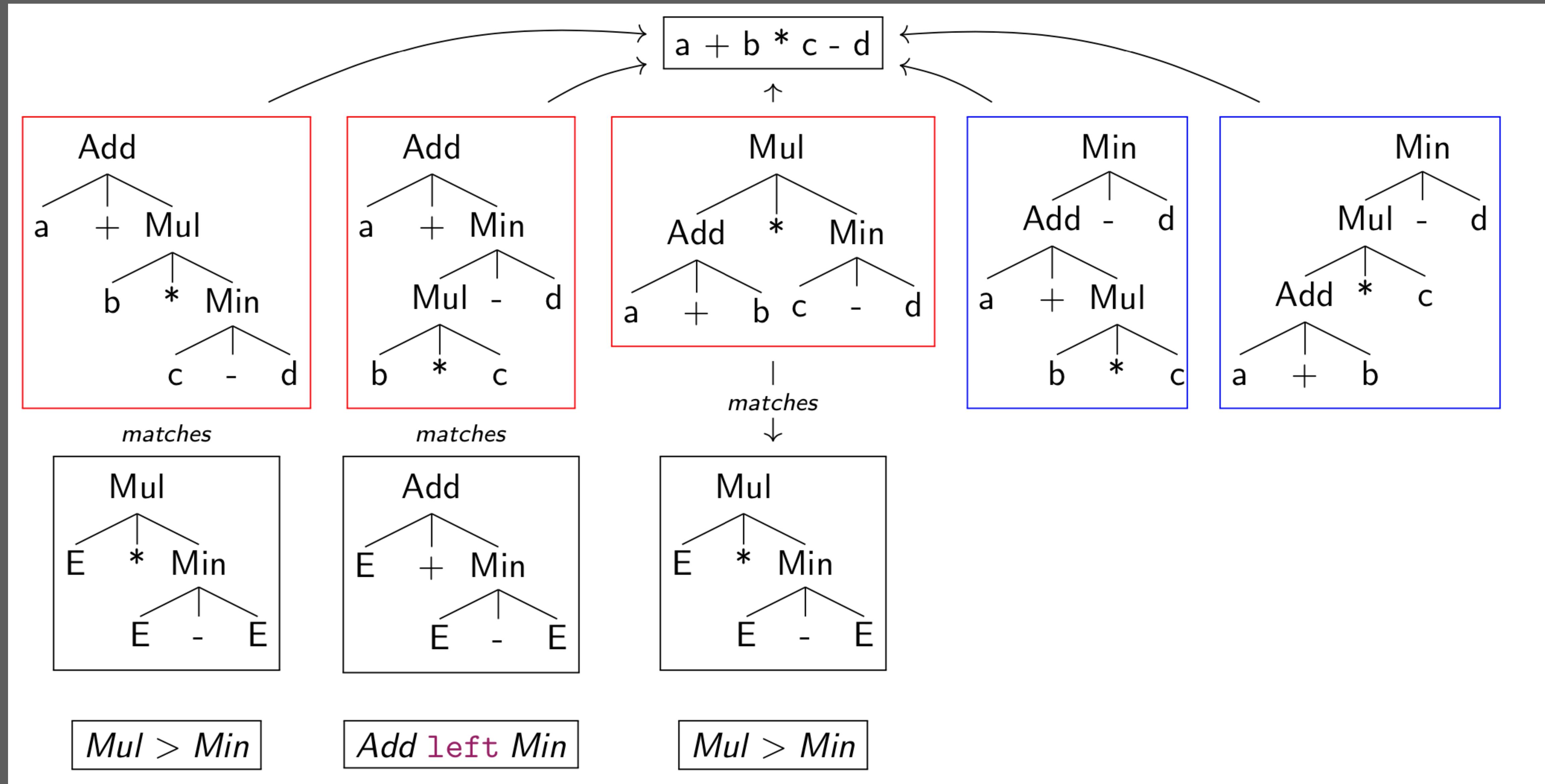
# Safe and Complete Disambiguation Rules



# Unsafe: Too Many Disambiguation Rules



# Incomplete: Too Few Disambiguation Rules



# Semantics of Associativity and Priority

## What is the semantics of associativity and priority rules?

- subtree exclusion: tree patterns that are forbidden

## Is a set of disambiguation rules safe?

- At most one rule for each pair of productions
- + some well-formedness criteria

## Is a set of disambiguation rules complete?

- At least one rule for each pair of productions
- + some well-formedness criteria

## How to implement?

# Grammar Transformations

# Grammar Transformations

## Why?

- Disambiguation
- For use by a particular parsing algorithm

## Transformations

- Eliminating ambiguities
- Eliminating left recursion
- Left factoring

## Properties

- Does transformation preserve the language (set of strings, trees)?
- Does transformation preserve the structure of trees?

# Ambiguous Expression Grammar

grammar

productions

$E.A = E "+" E$   
 $E.T = E "*" E$   
 $E.M = "-" E$   
 $E.B = "(" E ")"$   
 $E.V = ID$

derivation

$E \Rightarrow^* ID "*" ID "+" ID$

term derivation

$E$   
 $\Rightarrow A(E, E)$   
 $\Rightarrow A(T(E, E), E)$   
 $\Rightarrow A(T(E, E), E)$   
 $\Rightarrow A(T(V(ID), E), E)$   
 $\Rightarrow A(T(V(ID), V(ID)), E)$   
 $\Rightarrow A(T(V(ID), V(ID)), V(ID))$

term derivation

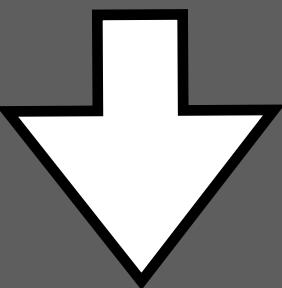
$E$   
 $\Rightarrow T(E, E)$   
 $\Rightarrow T(E, E)$   
 $\Rightarrow T(V(ID), E)$   
 $\Rightarrow T(V(ID), A(E, E))$   
 $\Rightarrow T(V(ID), A(V(ID), E))$   
 $\Rightarrow T(V(ID), A(V(ID), V(ID)))$

# Associativity and Priority Filter Ambiguities

grammar

productions

```
E.A = E "+" E  
E.T = E "*" E  
E.M = "-" E  
E.B = "(" E ")"  
E.V = ID
```



derivation

$E \Rightarrow^* ID \ast ID + ID$

term derivation

```
E  
⇒ A(E, E)  
⇒ A(T(E, E), E)  
⇒ A(T(E, E), E)  
⇒ A(T(V(ID), E), E)  
⇒ A(T(V(ID), V(ID)), E)  
⇒ A(T(V(ID), V(ID)), V(ID))
```

grammar

productions

```
E.A = E "+" E {left}  
E.T = E "*" E {left}  
E.M = "-" E  
E.B = "(" E ")"  
E.V = ID
```

priorities

$E.M > E.T > E.A$

term derivation

```
E  
⇒ T(E, E)  
⇒ T(E, E)  
⇒ T(V(ID), E)  
⇒ T(V(ID), A(E, E))  
⇒ T(V(ID), A(V(ID), E))  
⇒ T(V(ID), A(V(ID), V(ID)))
```

# Define Associativity and Priority by Transformation

grammar

productions

$E.A = E "+" E \{left\}$

$E.T = E "*" E \{left\}$

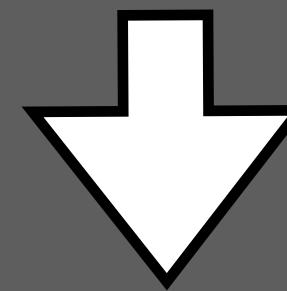
$E.M = "-" E$

$E.B = "(" E ")"$

$E.V = ID$

priorities

$E.M > E.T > E.A$



derivation

$E \Rightarrow^* ID "*" ID "+" ID$

term derivation

$E$

$\Rightarrow A(E, E)$

$\Rightarrow A(T(E, E), E)$

$\Rightarrow A(T(E, E), E)$

$\Rightarrow A(T(V(ID), E), E)$

$\Rightarrow A(T(V(ID), V(ID)), E)$

$\Rightarrow A(T(V(ID), V(ID)), V(ID))$

grammar

productions

$E.A = E "+" T$

$E = T$

$T.T = T "*" F$

$T = F$

$F.V = ID$

$F.B = "(" E ")"$

term derivation

$E$

$\Rightarrow T(E, E)$

$\Rightarrow T(E, E)$

$\Rightarrow T(V(ID), E)$

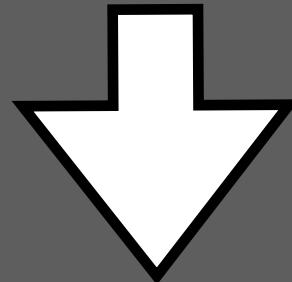
$\Rightarrow T(V(ID), A(E, E))$

$\Rightarrow T(V(ID), A(V(ID), E))$

$\Rightarrow T(V(ID), A(V(ID), V(ID)))$

# Define Associativity and Priority by Transformation

```
grammar  
productions  
E.A = E "+" E {left}  
E.T = E "*" E {left}  
E.M = "-" E  
E.B = "(" E ")"  
E.V = ID  
priorities  
E.M > E.T > E.A
```



```
grammar  
productions  
E.A = E "+" T  
E = T  
T.T = T "*" F  
T = F  
F.V = ID  
F.M = "-" F  
F.B = "(" E ")"
```

Define new non-terminal for each priority level:

E, T, F

Add 'injection' productions to include priority level n+1 in n:

E = T  
T = F

Change head of production to reflect priority level

T = T "\*" F

Transform productions

Left: E = E "+" T  
Right: E = T "+" E

# Dangling Else Grammar

grammar

sorts S E

productions

S.If = if E then S

S.IfE = if E then S else S

S = other

derivation

$S \Rightarrow^* \text{if } E_1 \text{ then } S_1 \text{ else if } E_2 \text{ then } S_2 \text{ else } S_3$

term derivation

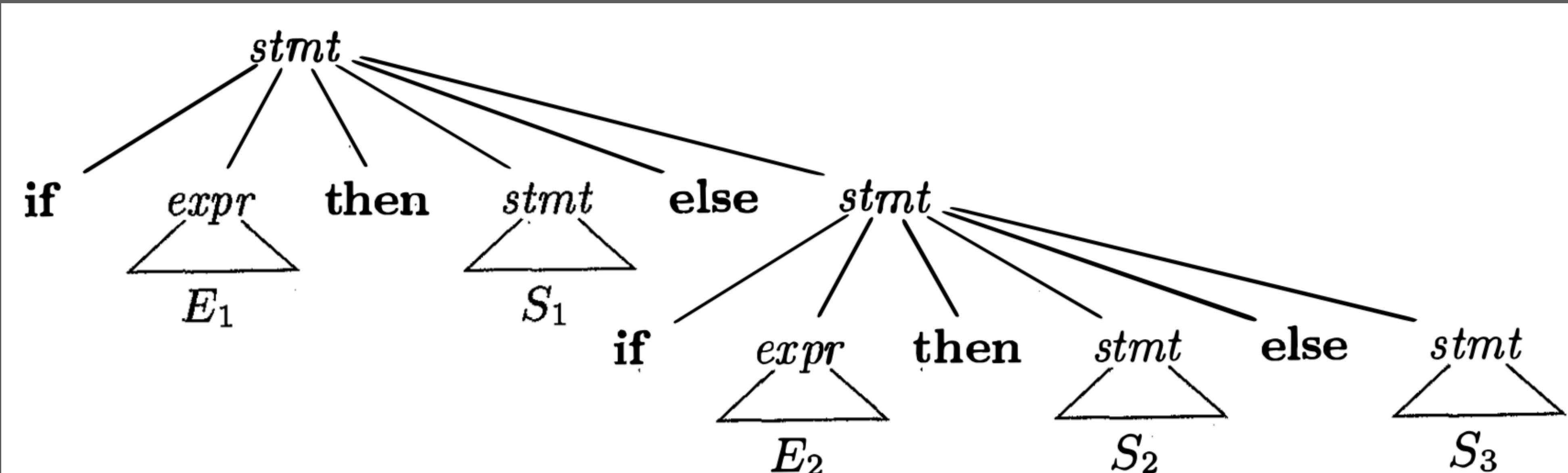
$S \Rightarrow^* \text{IfE}(E_1, S_1, \text{IfE}(E_2, S_2, S_3))$

term derivation

S

$\Rightarrow \text{IfE}(E_1, S_1, S)$

$\Rightarrow \text{IfE}(E_1, S_1, \text{IfE}(E_2, S_2, S_3))$



# Dangling Else Grammar is Ambiguous

grammar

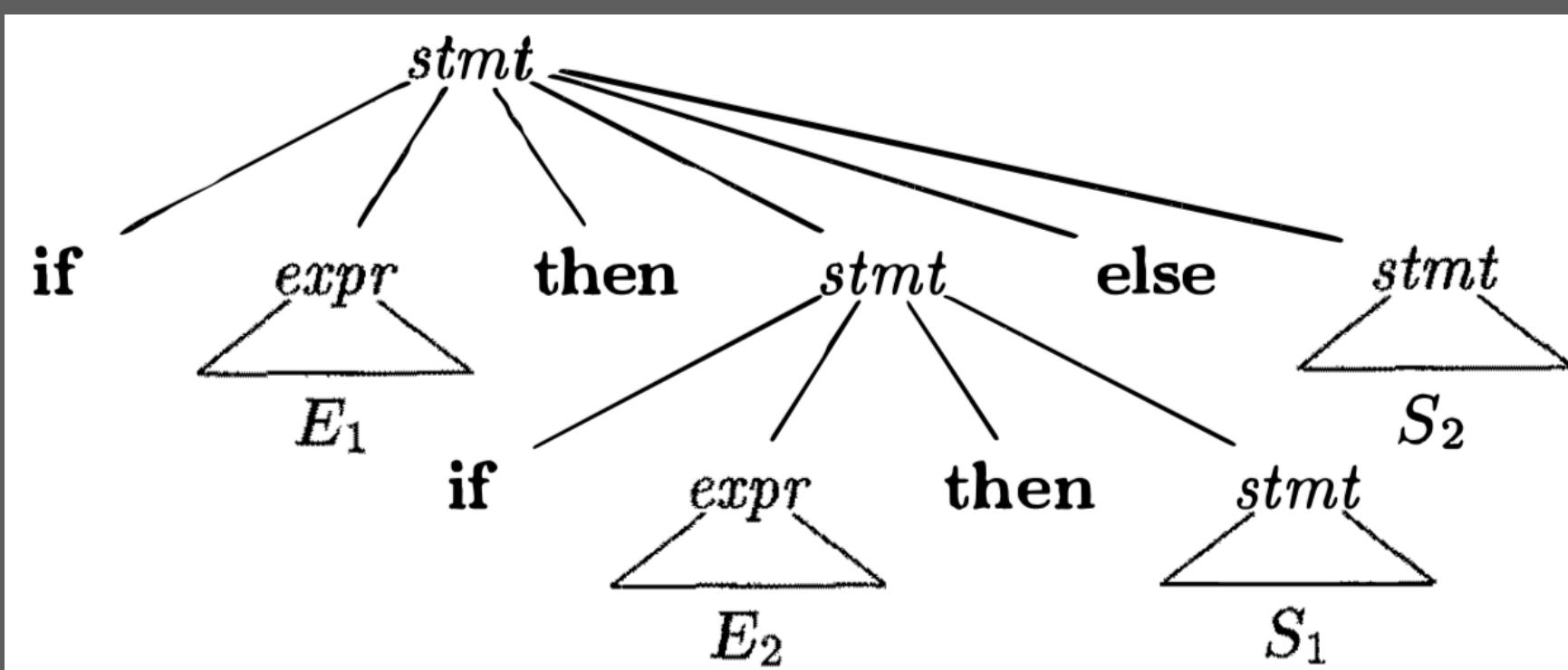
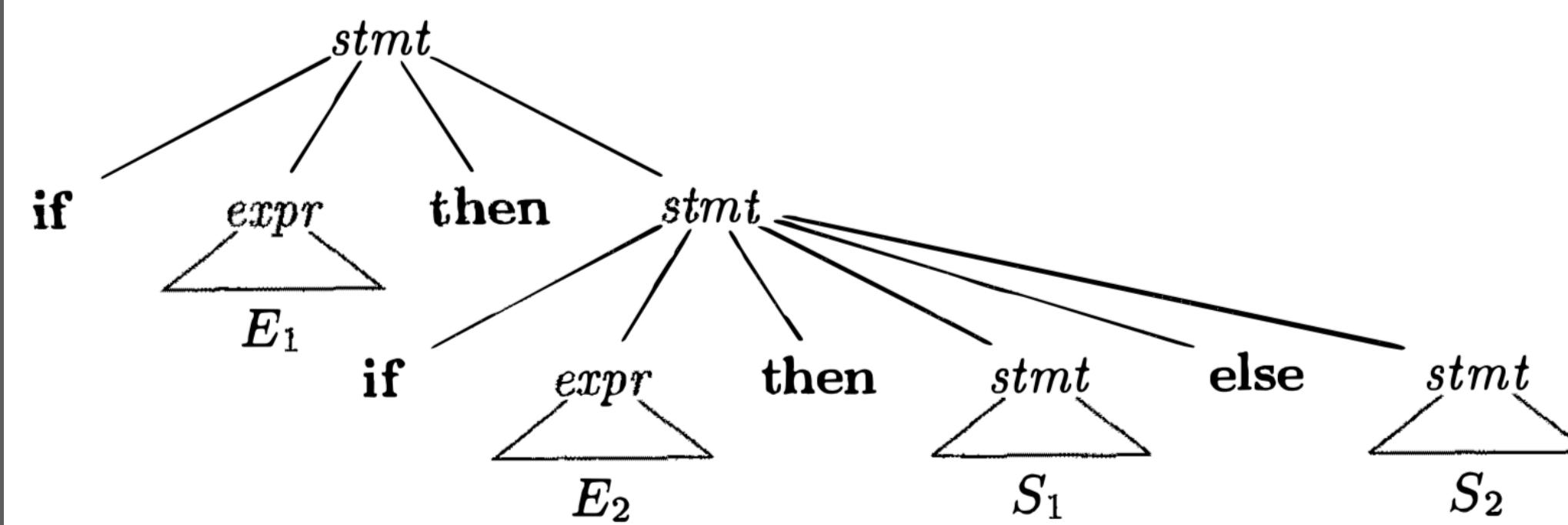
sorts S E

productions

S.If = if E then S

S.IfE = if E then S else S

S = other



derivation

$S \Rightarrow^* \text{if } E_1 \text{ then if } E_2 \text{ then } S_1 \text{ else } S_2$

term derivation

$S \Rightarrow \text{If}(E_1, S)$   
 $\Rightarrow \text{If}(E_1, \text{IfE}(E_2, S_1, S_2))$

derivation

$S \Rightarrow \text{if } E_1 \text{ then } S$   
 $\Rightarrow \text{if } E_1 \text{ then if } E_2 \text{ then } S_1 \text{ else } S_2$

term derivation

$S \Rightarrow \text{IfE}(E_1, S, S_2)$   
 $\Rightarrow \text{IfE}(E_1, \text{If}(E_2, S_1), S_2)$

derivation

$S \Rightarrow \text{if } E_1 \text{ then } S \text{ else } S_2$   
 $\Rightarrow \text{if } E_1 \text{ then if } E_2 \text{ then } S_1 \text{ else } S_2$

# Eliminating Dangling Else Ambiguity

grammar

sorts S E

productions

S.If = if E then S

S.IfE = if E then S else S

S = other

grammar

productions

S.If = if E then S

S.IfE = if E then SE else S

S = other

SE.IfE = if E then SE else SE

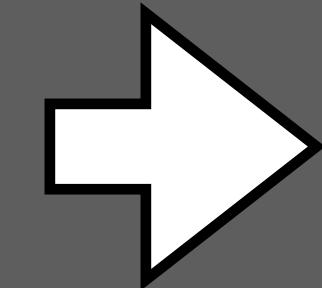
SE = other

Generalization of this transformation: contextual grammars

# Eliminating Left Recursion

grammar  
productions

```
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```

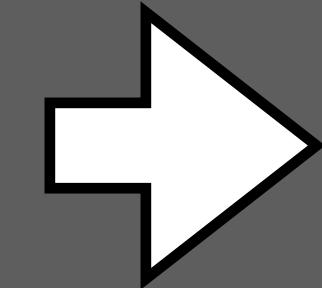


grammar  
productions

```
E = T E'
E' = "+" T E'
E' =
T = F T'
T' = "*" F T'
T' =
F = "(" E ")"
F = ID
```

grammar  
productions

```
A = A a
A = b
```



grammar  
productions

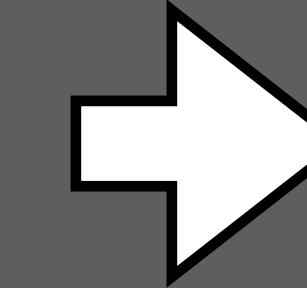
```
A = b A'
A' = a A'
A' = // empty
```

// b followed by a list of as

# Eliminating Left Recursion using Regular Expressions

grammar  
productions

```
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```

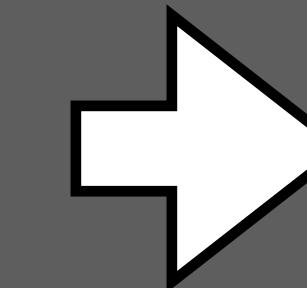


grammar  
productions

```
E = T ("+" T)*
T = F ("*" F)*
F = "(" E ")"
F = ID
```

grammar  
productions

```
A = A a
A = b
```



grammar  
productions

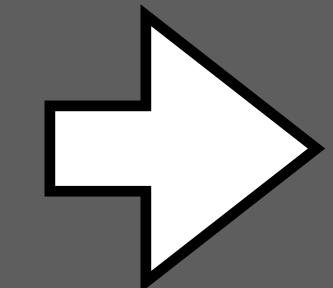
```
A = b a*
```

// b followed by a list of as

# Left Factoring

grammar  
productions

S.If = if E then S  
S.IfE = if E then S else S  
S = other

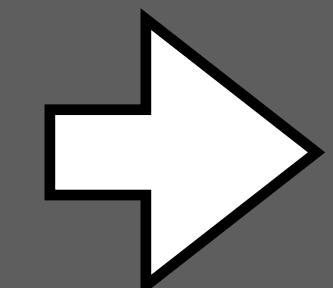


grammar  
sorts S E  
productions

S.If = if E then S S'  
S'.Else = else S  
S'.NoElse = // empty  
S = other

grammar  
productions

A = a b1  
A = a b2  
A = c



grammar  
productions

A = a A'  
A' = b1  
A' = b2  
A = c

# Properties of Grammar Transformations

## Preservation

- Preserves set of sentences
- Preserves set of trees
- Preserves tree structure

## Systematic

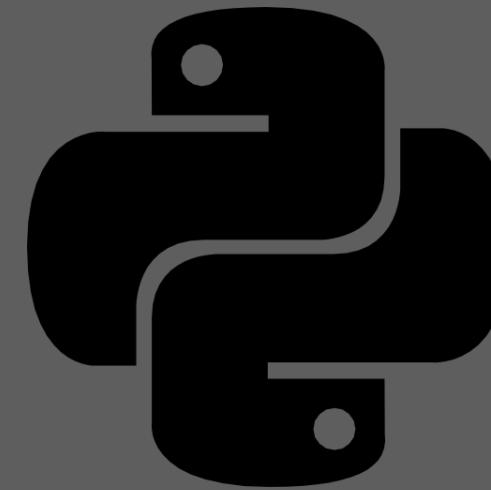
- Algorithmic
- Heuristic

# Layout-Sensitive Syntax

# Layout-Sensitive Syntax

```
if x ≠ y:  
    if x > 0:  
        y = x  
else:  
    y = -x
```

```
guessValue x = do  
    putStrLn "Enter your guess:"  
    guess ← getLine  
    case compare (read guess) x of  
        EQ → putStrLn "You won!"  
        _ → do putStrLn "Keep guessing."  
              guessValue x
```



# Disambiguation with Indentation Sensitive Context-free Grammars

```
case → 'case'> exp= 'of'> altBlock=
  -- Reset indentation for delimited blocks

altBlock → '{'> alts* close*
close   → '}'>
  -- Increase indentation for non-delimited blocks

altBlock → altLayout>
altLayout → |alts≡ altLayout=
altLayout → |alts≡
  -- Clause sequences

alts → alt=
alts → alt= ';'> alts=
```

Michael Adams. Principled Parsing for Indentation-Sensitive Languages: Revisiting Landin's Offside Rule. In POPL'13.

# SDF3: Disambiguation with Layout Constraints

## context-free syntax

```
Impl          = Stm {layout(1.first.col < 1.left.col)}  
  
Impls.StmSeq = Impl Impl {layout(1.first.col = 2.first.col)}  
  
Impls        = Impl  
Expls        = Stm  
Expls.StmtSeq = Stm ";" Expls  
Stms.Stms    = Impl  
  
Stms.Stms    = "{" Expls "}" {ignore-layout}
```

# Layout Constraints: Token Selectors

```
x = do 9 + 4
        * 3
main = do putStrLn $
        show (x *
              2) right
        last
```

*first* → `putStrLn $`  
*left* → `show (x *`  
*right* → `2)`  
*last* → `last`

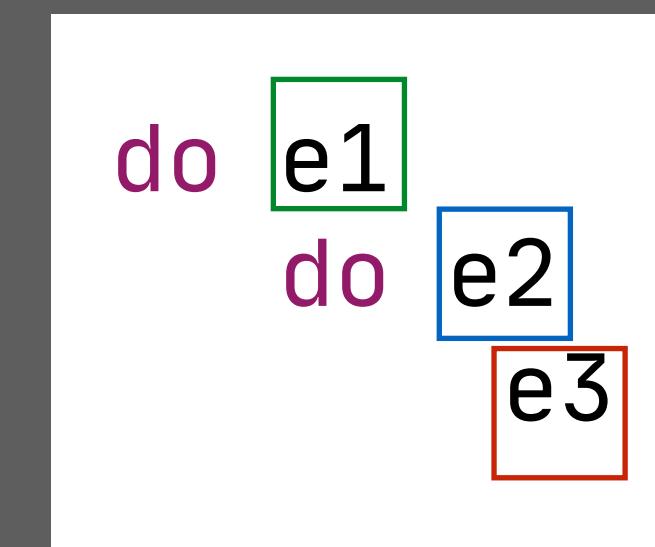
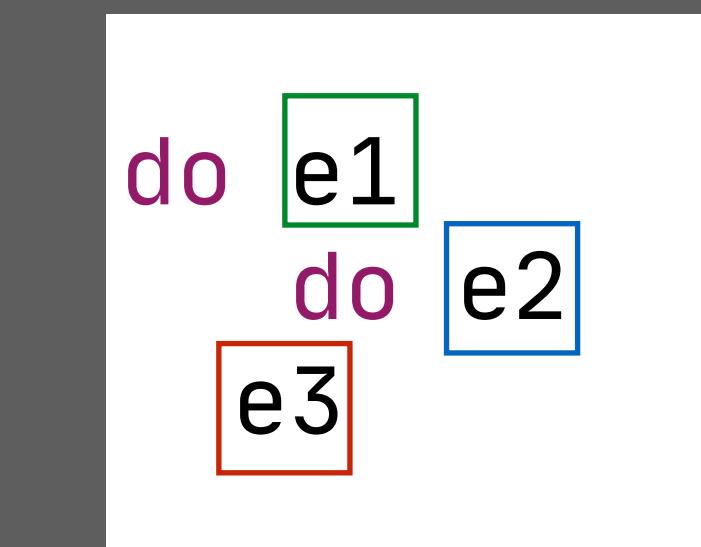
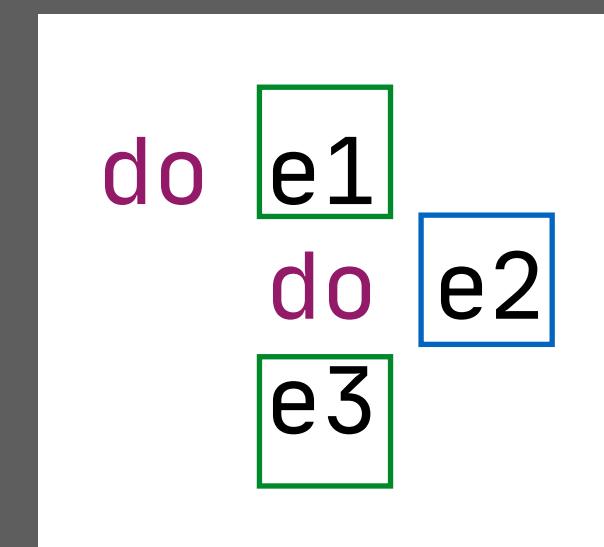
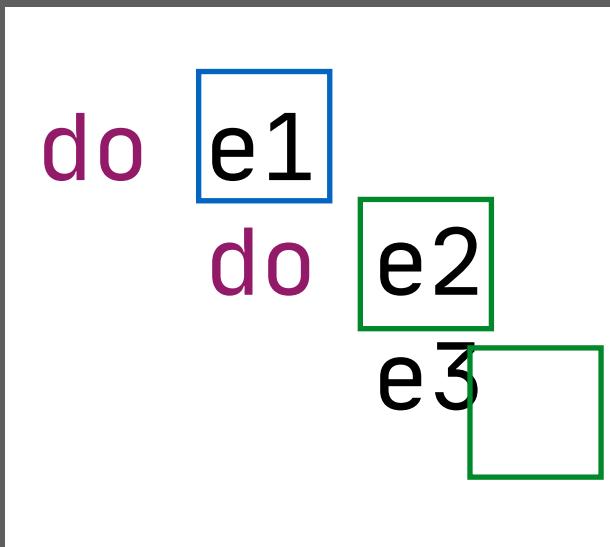
```
lComp = do
    x <- xRange
    return $ do
        y <- yRange
        return (x, y)
```

Interpret indentation using shapes around  
the tokens that belong to a subtree.

# Layout Constraints: Encoding Alignment

context-free syntax

```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}
Exp.Id      = ID
```



# Layout Constraints: Encoding Indentation

```
Exp.IfElse = "if" Exp "then" Exp "else" Exp
{layout(4.first.line > 1.last.line && // then clause in next line
      4.first.col > 1.first.col && // and indented
      1.first.col == 5.first.col && // "if" and "else" aligned
      4.first.col == 6.first.col)} // then and else clauses aligned
```

```
if e1 then
  if e2 then
    e3
  else
    e4
else
  e5
```

Low-level declarations

# Layout Declarations

# Tree Selectors

context-free syntax

```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}
Exp.Id      = ID
```

context-free syntax

```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = exps:ExpList exp:Exp {layout(... exps exp)}
Exp.Id      = ID
```

# Tree Selectors

context-free syntax

```
Exp.Do      = "do" ExpList {layout(2.first.col > 1.first.col)}
ExpList.Cns = Exp
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}
Exp.Id     = ID
```

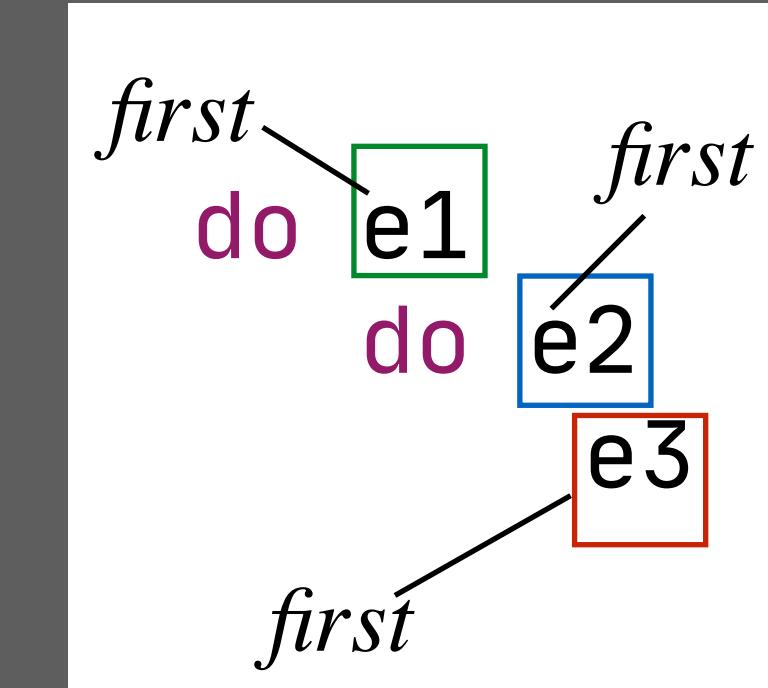
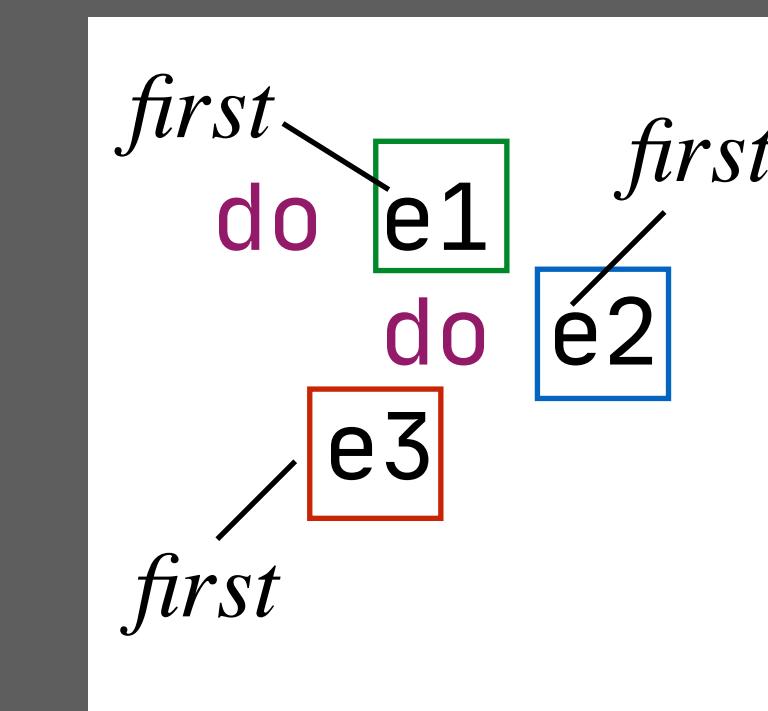
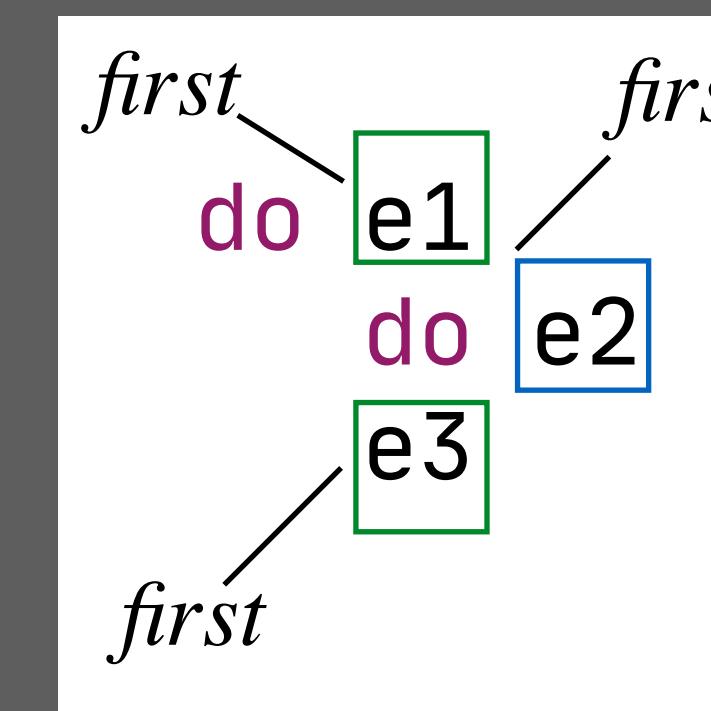
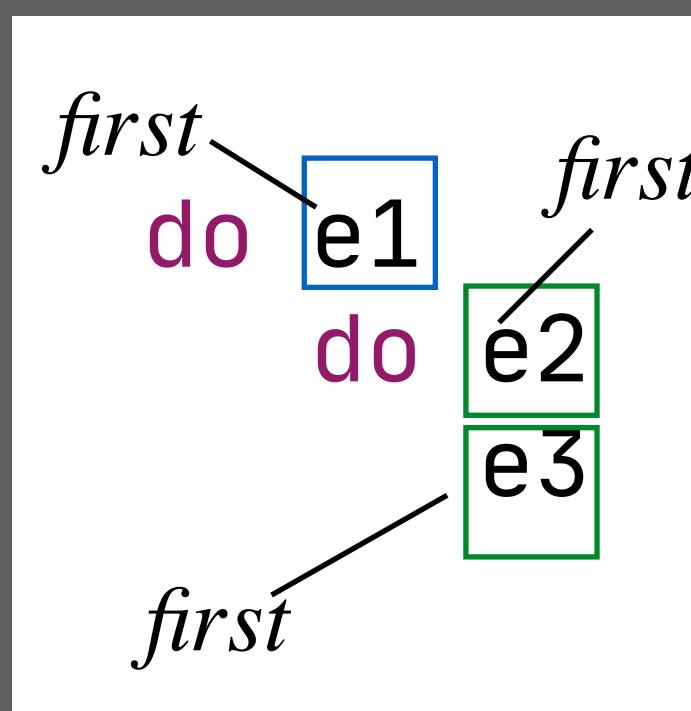
context-free syntax

```
Exp.Do      = "do" ExpList {layout(... "do" 1)}
ExpList.Cns = Exp
ExpList.Lst = exps:ExpList exp:Exp {layout(... exps exp)}
Exp.Id     = ID
```

# Alignment with Layout Constraints

context-free syntax

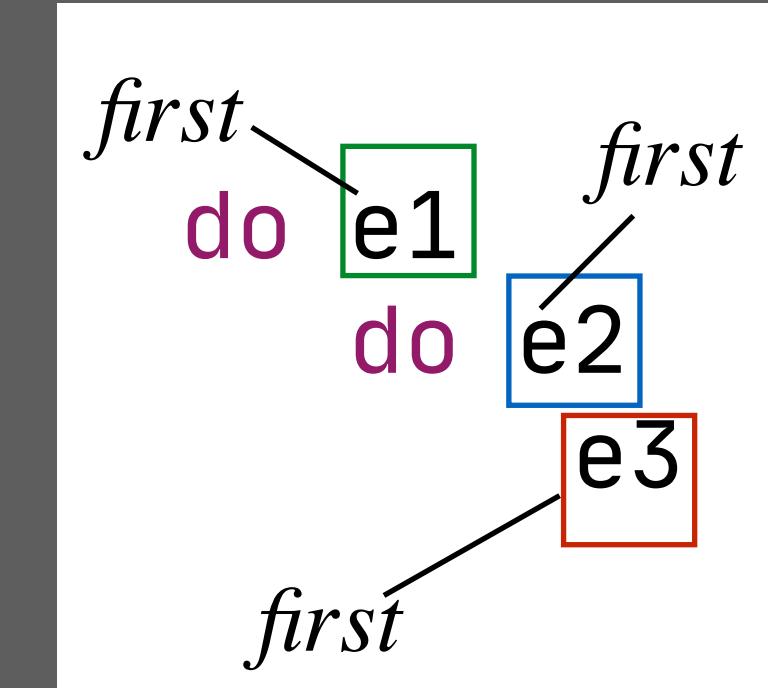
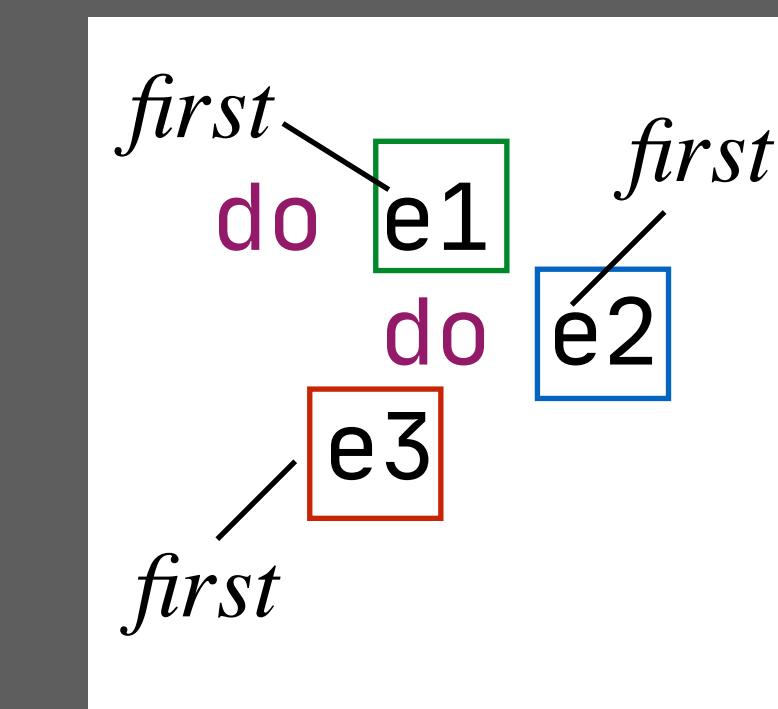
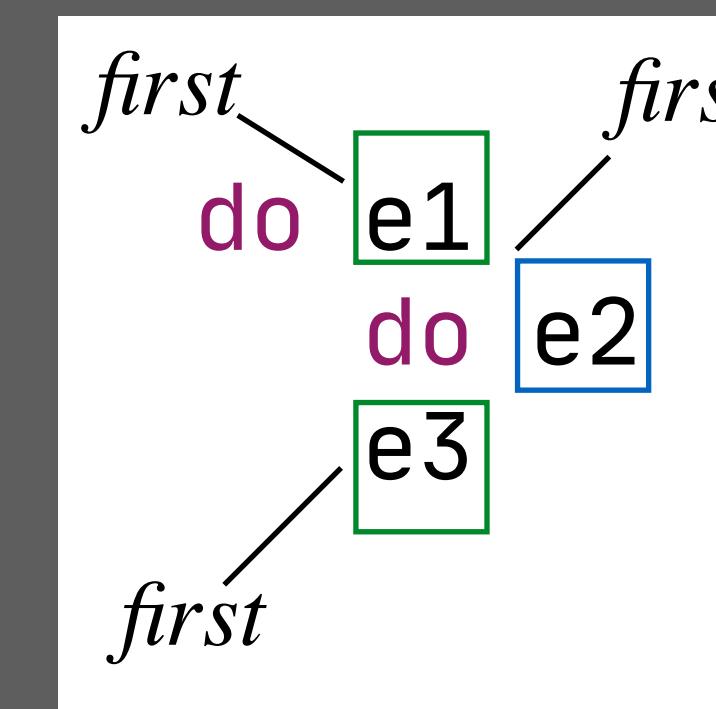
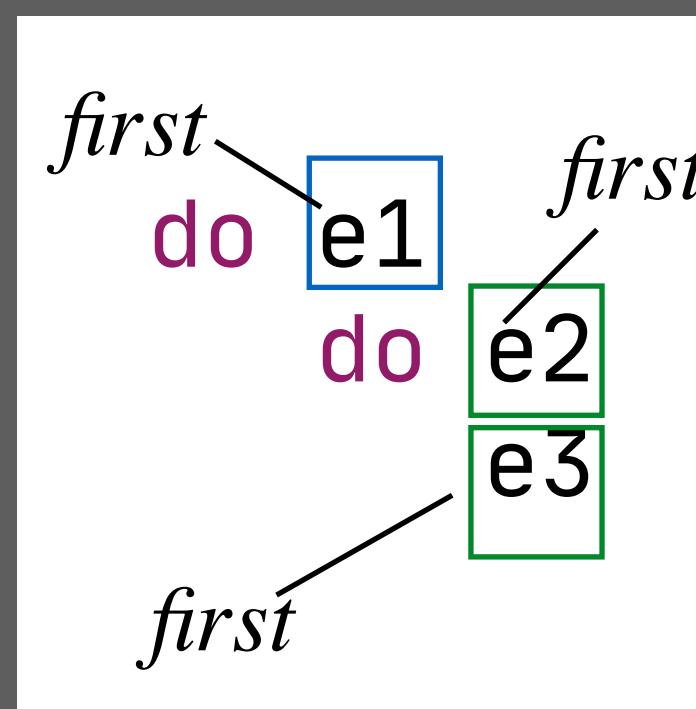
```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}
Exp.Id     = ID
```



# Alignment Declaration

context-free syntax

```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = exps:ExpList exp:Exp {layout(align exps exp)}
Exp.Id     = ID
```



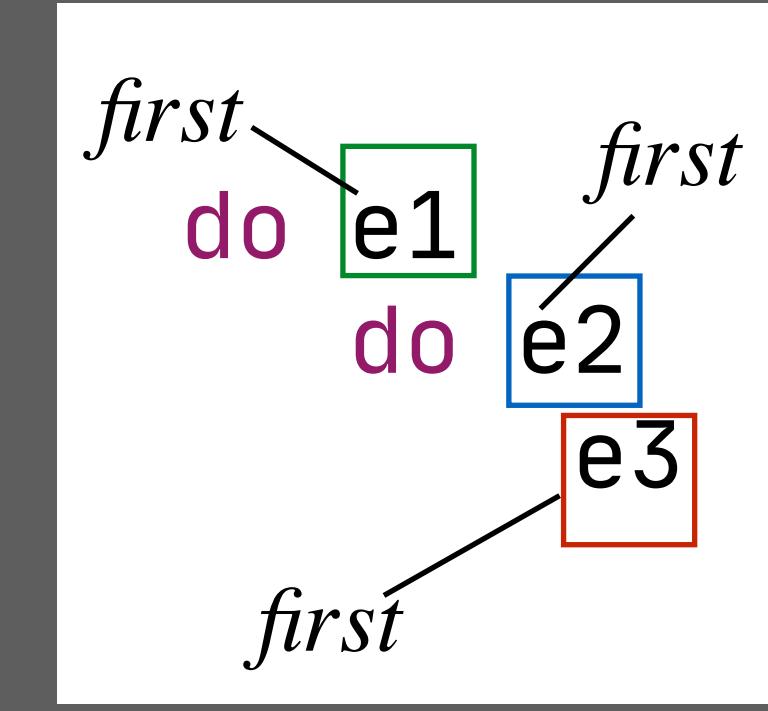
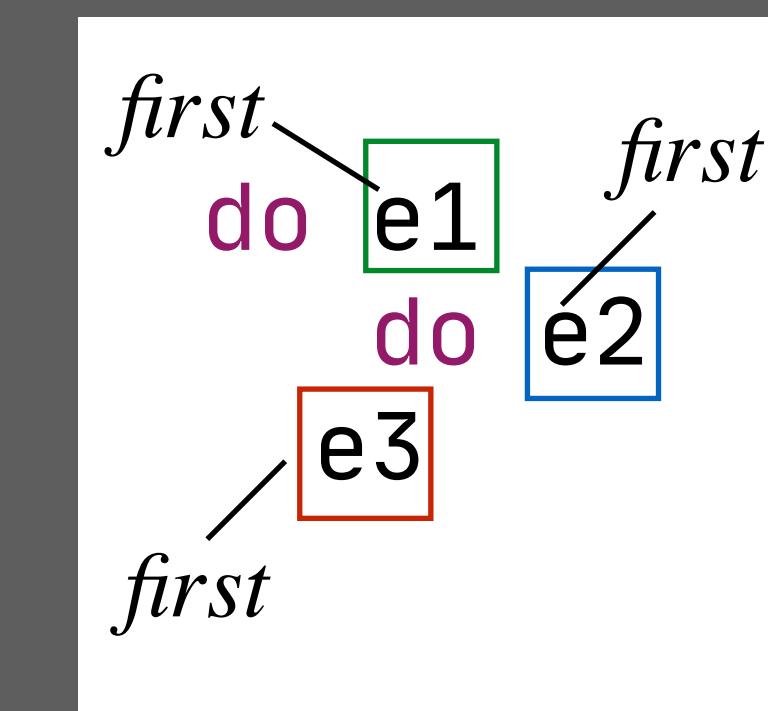
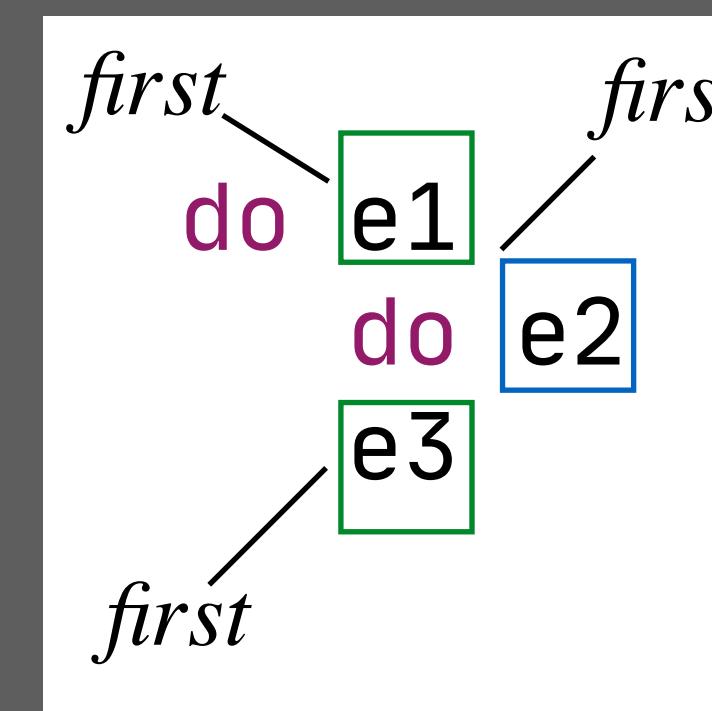
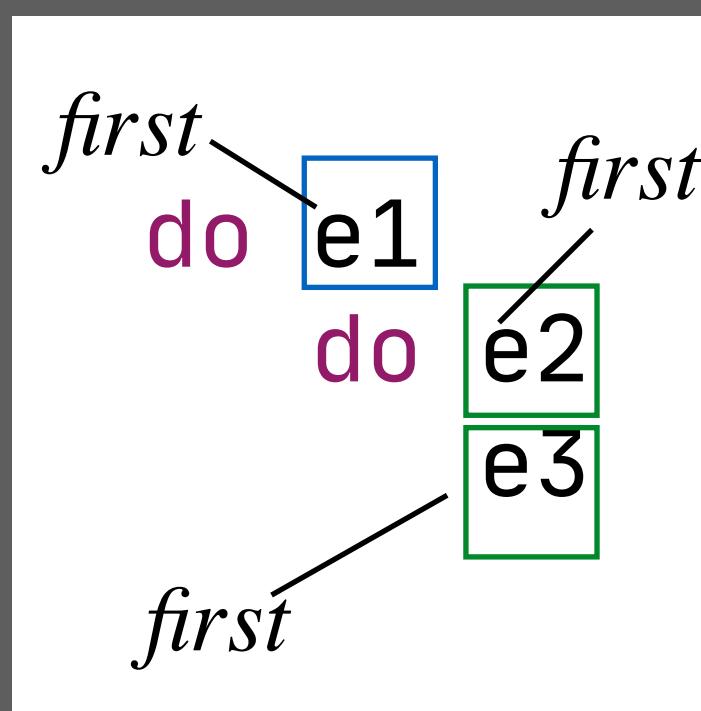
Semantics

$$\frac{x.\text{first.col} = y.\text{first.col}}{\text{align } x \text{ } y}$$

# List Alignment Declaration

context-free syntax

```
Exp.Do  = "do" exps:Exp+ {layout(align-list exps)}  
Exp.Id  = ID  
Exp+    = Exp+ Exp // normalized  
Exp+    = Exp          // productions
```



Semantics

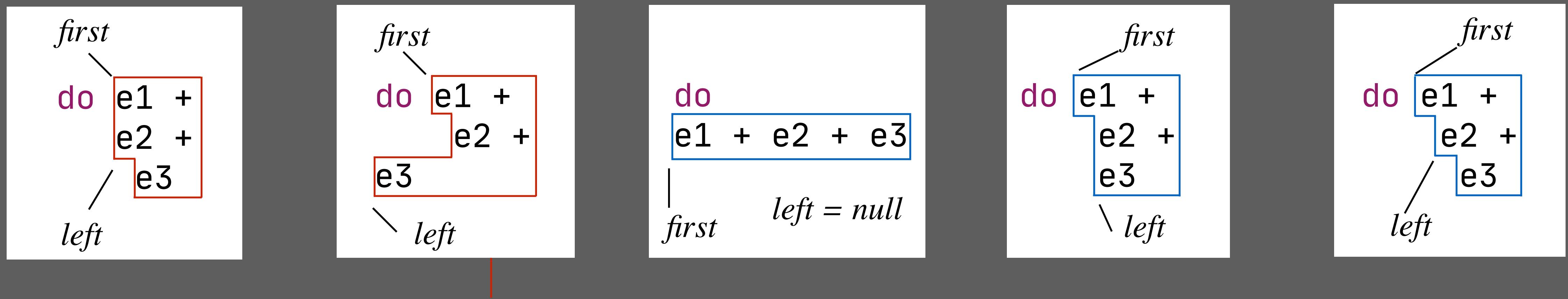
---

$A^+ = A^+ \ A \ layout(1.\text{first}.col = 2.\text{first}.col)$

---

align-list x

# Offside Rule



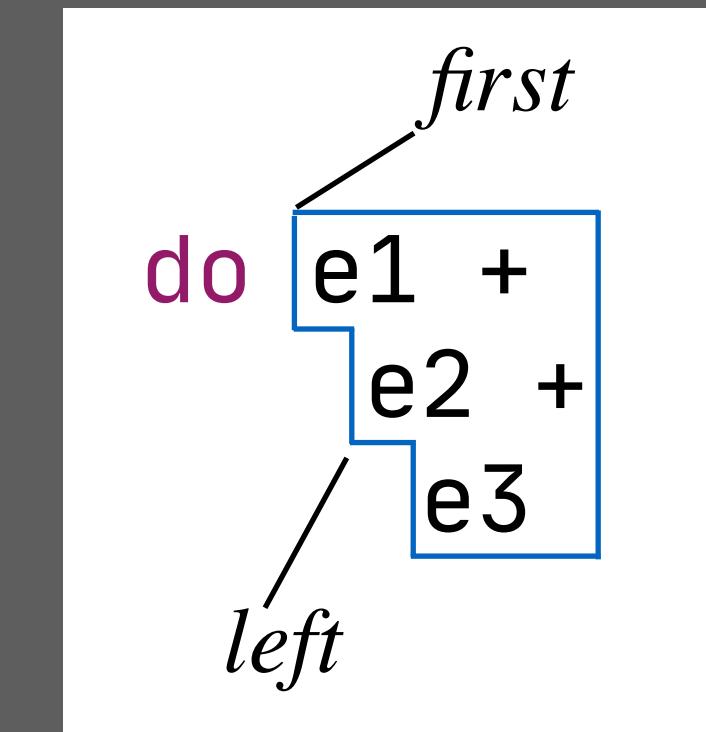
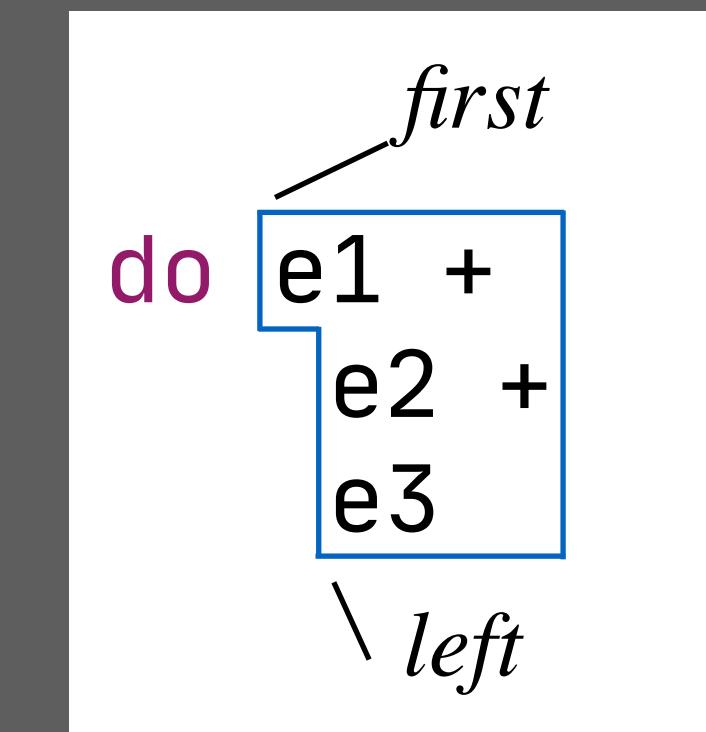
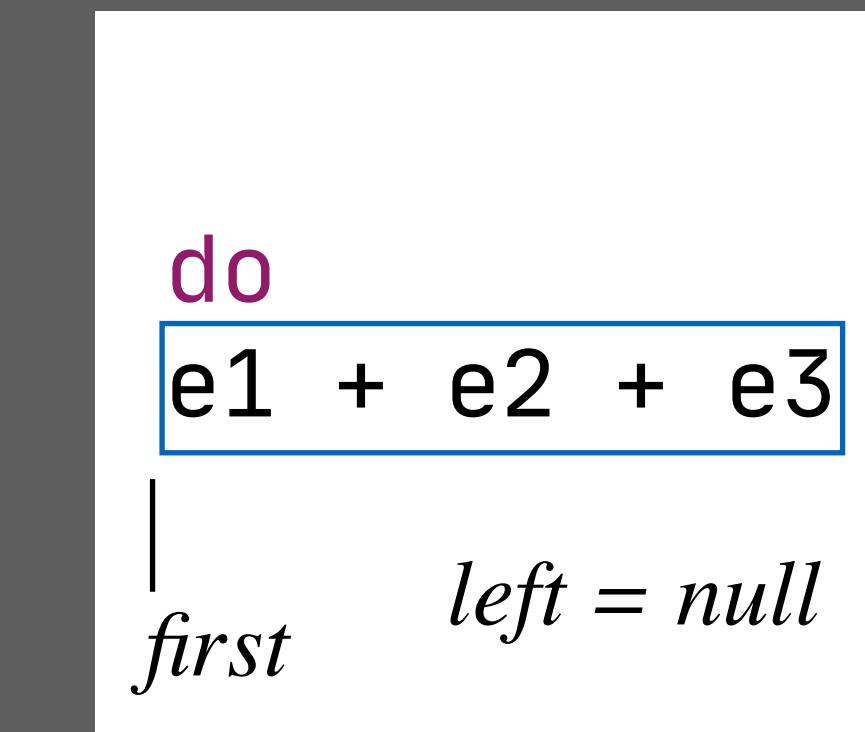
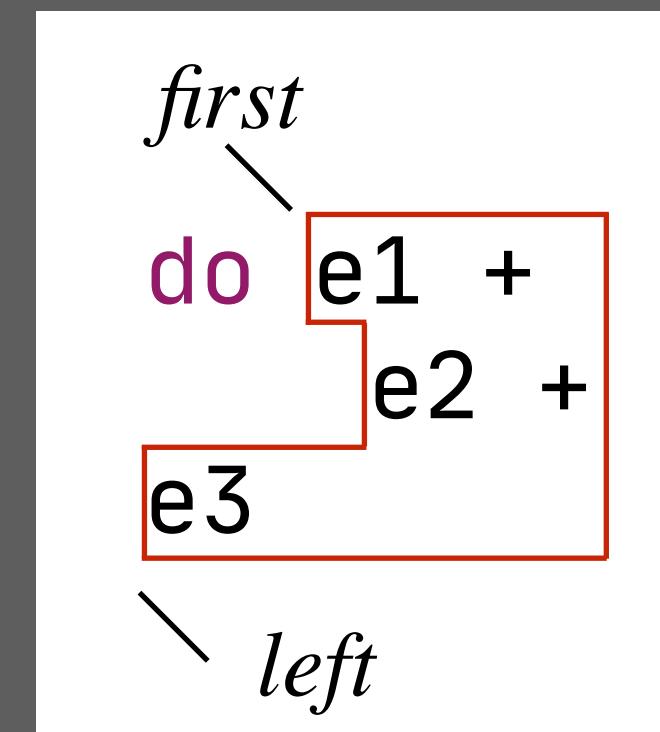
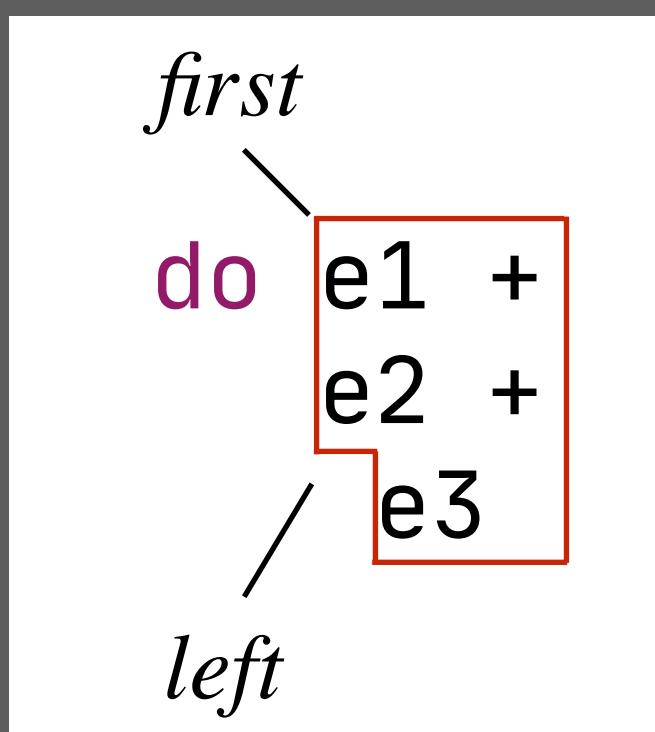
“The offside rule prescribes that all non-whitespace tokens of a structure must be further to the right than the token that starts the structure.”

Erdweg et. al.. Layout-Sensitive Generalized Parsing. In SLE’12.

# Offside with Layout Constraints

context-free syntax

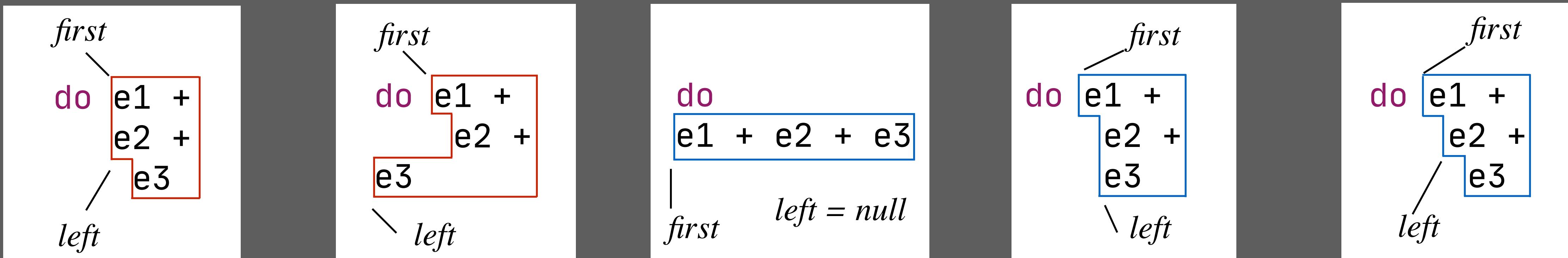
```
Exp.Do  = "do" Exp {layout(2.left.col > 2.first.col)}  
Exp.Add = Exp "+" Exp {left}  
Exp.Id  = ID
```



# Offside

context-free syntax

```
Exp.Do  = "do" exp:Exp {layout(offside exp)}  
Exp.Add = Exp "+" Exp {left}  
Exp.Id  = ID
```



Semantics

$x.left.col > x.first.col$

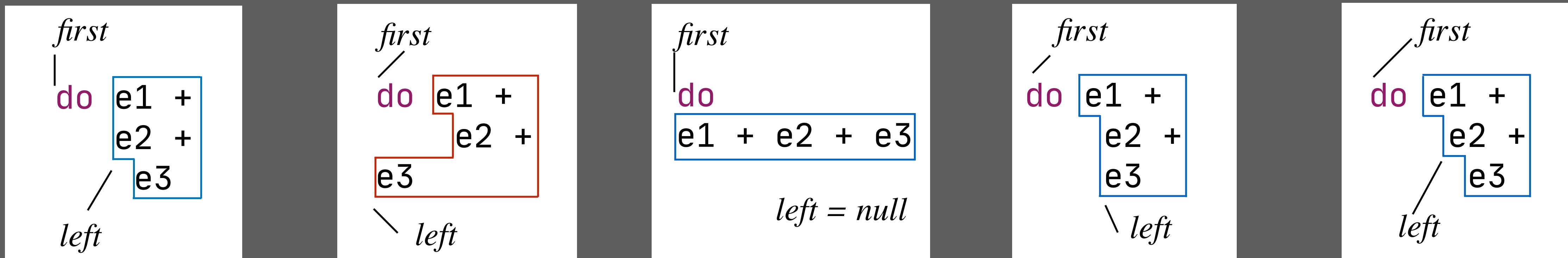
---

offside  $x$

# Relative Offside

context-free syntax

```
Exp.Do  = "do" exp:Exp {layout(offside "do" exp)}  
Exp.Add = Exp "+" Exp {left}  
Exp.Id  = ID
```



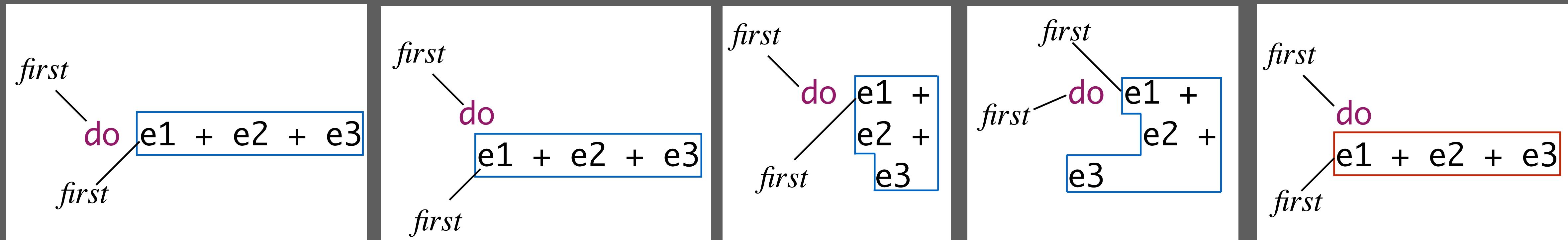
Semantics

$$\frac{y.\text{left.col} > x.\text{first.col}}{\text{offside } x \ y}$$

# Indentation

## context-free syntax

```
Exp.Do  = "do" exp:Exp {layout(indent "do" exp)}  
Exp.Add = Exp "+" Exp {left}  
Exp.Id  = ID
```



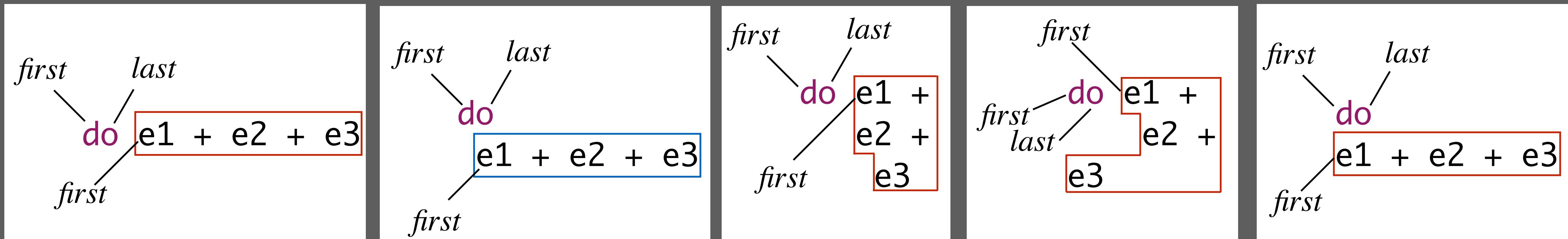
Semantics

$$\frac{y.\text{first.col} > x.\text{first.col}}{\text{indent } x \ y}$$

# Newline + Indentation

context-free syntax

```
Exp.Do  = "do" exp:Exp {layout(newline-indent "do" exp)}  
Exp.Add = Exp "+" Exp {left}  
Exp.Id  = ID
```



Semantics

$y.\text{first.col} > x.\text{first.col} \& \& y.\text{first.line} > x.\text{last.line}$   
newline-indent  $x \cdot y$

# Parsing and Pretty-Printing

{S} spoofax

The diagram illustrates the process of transforming source code into an abstract syntax tree (AST) and then into pretty-printed output. It consists of four windows arranged vertically:

- LayoutSens.sdf3**: Shows the SDF grammar definition for the LayoutSens module, defining rules for context-free syntax, priorities, and lexical syntax.
- example1.lsn**: Shows the input source code:

```
1 if e1 then if e2 then e3
2           else e4
3 else      e5
4
5 do x = 1
6   y = 2
7   x
8   + y
```
- example1.aterm**: Shows the resulting Abstract Syntax Tree (AST) in ATERM format, generated by the parser:

```
1 Exps(
2   [ IfElse("e1", IfElse("e2", "e3", "e4"))
3     , Do([Assign("x", "1"), Assign("y", "2")
4       , Add("x", "y")]
5   )
6 )
```
- example1.pp.lsn**: Shows the pretty-printed version of the source code:

```
1 if e1 then
2   if e2 then
3     e3
4   else
5     e4
6 else
7   e5
8
9 do x = 1
10  y = 2
11
12 x + y
```

Two large blue arrows indicate the flow: a downward-pointing arrow labeled "Parse" between the second and third windows, and another downward-pointing arrow labeled "Pretty-Print" between the third and fourth windows.

Bottom status bar: Writable Insert 1 : 1

# Reading Material

# Introduces layout constraints

[https://doi.org/10.1007/978-3-642-36089-3\\_14](https://doi.org/10.1007/978-3-642-36089-3_14)

# Layout-Sensitive Generalized Parsing

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**Abstract.** The theory of context-free languages is well-understood and context-free parsers can be used as off-the-shelf tools in practice. In particular, to use a context-free parser framework, a user does not need to understand its internals but can specify a language *declaratively* as a grammar. However, many languages in practice are not context-free. One particularly important class of such languages is layout-sensitive languages, in which the structure of code depends on indentation and whitespace. For example, Python, Haskell, F#, and Markdown use indentation instead of curly braces to determine the block structure of code. Their parsers (and lexers) are not declaratively specified but hand-tuned to account for layout-sensitivity.

To support *declarative* specifications of layout-sensitive languages, we propose a parsing framework in which a user can annotate layout in a grammar. Annotations take the form of constraints on the relative positioning of tokens in the parsed subtrees. For example, a user can declare that a block consists of statements that all start on the same column. We have integrated layout constraints into SDF and implemented a layout-sensitive generalized parser as an extension of generalized LR parsing. We evaluate the correctness and performance of our parser by parsing 33 290 open-source Haskell files. Layout-sensitive generalized parsing is easy to use, and its performance overhead compared to layout-insensitive parsing is small enough for practical application.

## 1 Introduction

Most computer languages prescribe a textual syntax. A parser translates from such textual representation into a structured one and constitutes the first step in processing a document. Due to the development of parser frameworks such as lex/yacc [15], ANTLR [18,17], PEGs [6,7], parsec [13], or SDF [8], parsers can be considered off-the-shelf tools nowadays: Non-experts can use parsers, because language specifications are declarative. Although many parser frameworks support some form of context-sensitive parsing (such as via semantic predicates in ANTLR [18]), one particularly relevant class of languages is not supported declaratively by any existing parser framework: layout-sensitive languages.

Layout-sensitive languages were proposed by Landin in 1966 [12]. In layout-sensitive languages, the translation from a textual representation to a structural one depends on the code's layout and its indentation. Most prominently, the *off-side rule* prescribes that all non-whitespace tokens of a structure must be further to the right than the token that starts the structure. In other words, a token

Introduces layout declarations to abstract from low-level layout constraints and pretty-printing based on layout declarations/constraints.

Includes summary of layout constraints

Won distinguished paper award at SLE'18.

<https://doi.org/10.1145/3276604.3276607>



## Declarative Specification of Indentation Rules

A Tooling Perspective on Parsing and Pretty-Printing Layout-Sensitive Languages

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### Abstract

In layout-sensitive languages, the indentation of an expression or statement can influence how a program is parsed. While some of these languages (e.g., Haskell and Python) have been widely adopted, there is little support for software language engineers in building tools for layout-sensitive languages. As a result, parsers, pretty-printers, program analyses, and refactoring tools often need to be handwritten, which decreases the maintainability and extensibility of these tools. Even state-of-the-art language workbenches have little support for layout-sensitive languages, restricting the development and prototyping of such languages.

In this paper, we introduce a novel approach to declarative specification of layout-sensitive languages using *layout declarations*. Layout declarations are high-level specifications of indentation rules that abstract from low-level technicalities. We show how to derive an efficient layout-sensitive generalized parser and a corresponding pretty-printer automatically from a language specification with layout declarations. We validate our approach in a case-study using a syntax definition for the Haskell programming language, investigating the performance of the generated parser and the correctness of the generated pretty-printer against 22191 Haskell files.

**CCS Concepts** • Software and its engineering → Syntax; Parsers;

**Keywords** parsing, pretty-printing, layout-sensitivity

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### 1 Introduction

Layout-sensitive (also known as indentation-sensitive) languages were introduced by Landin [17]. The term characterizes languages that must obey certain *indentation rules*, i.e., languages in which the indentation of the code influences how the program should be parsed. In layout-sensitive languages, alignment and indentation are essential to correctly identify the structures of a program. Many modern programming languages including Haskell [11], Python [23], Markdown [14] and YAML [4] are layout-sensitive. To illustrate how layout can influence parsing programs in such languages, consider the Haskell program in Figure 1, which contains multiple *do*-expressions:

```
1 guessValue x = do
2   putStrLn "Enter your guess:"
3   guess <- getLine
4   case compare (read guess) x of
5     EQ -> putStrLn "You won!"
6     _ -> do putStrLn "Keep guessing."
7   guessValue x
```

Figure 1. *Do*-expressions in Haskell.

In Haskell, all statements inside a *do*-block should be aligned (i.e., should start at the same column). In Figure 1, we know that the statement on line 7 (*guessValue x*) belongs to the inner *do*-block solely because of its indentation. If we modify the indentation of this statement, aligning it with the statements in the outer *do*-block, the program would have a different interpretation, looping indefinitely.

# Next: Static Analysis and Type Checking