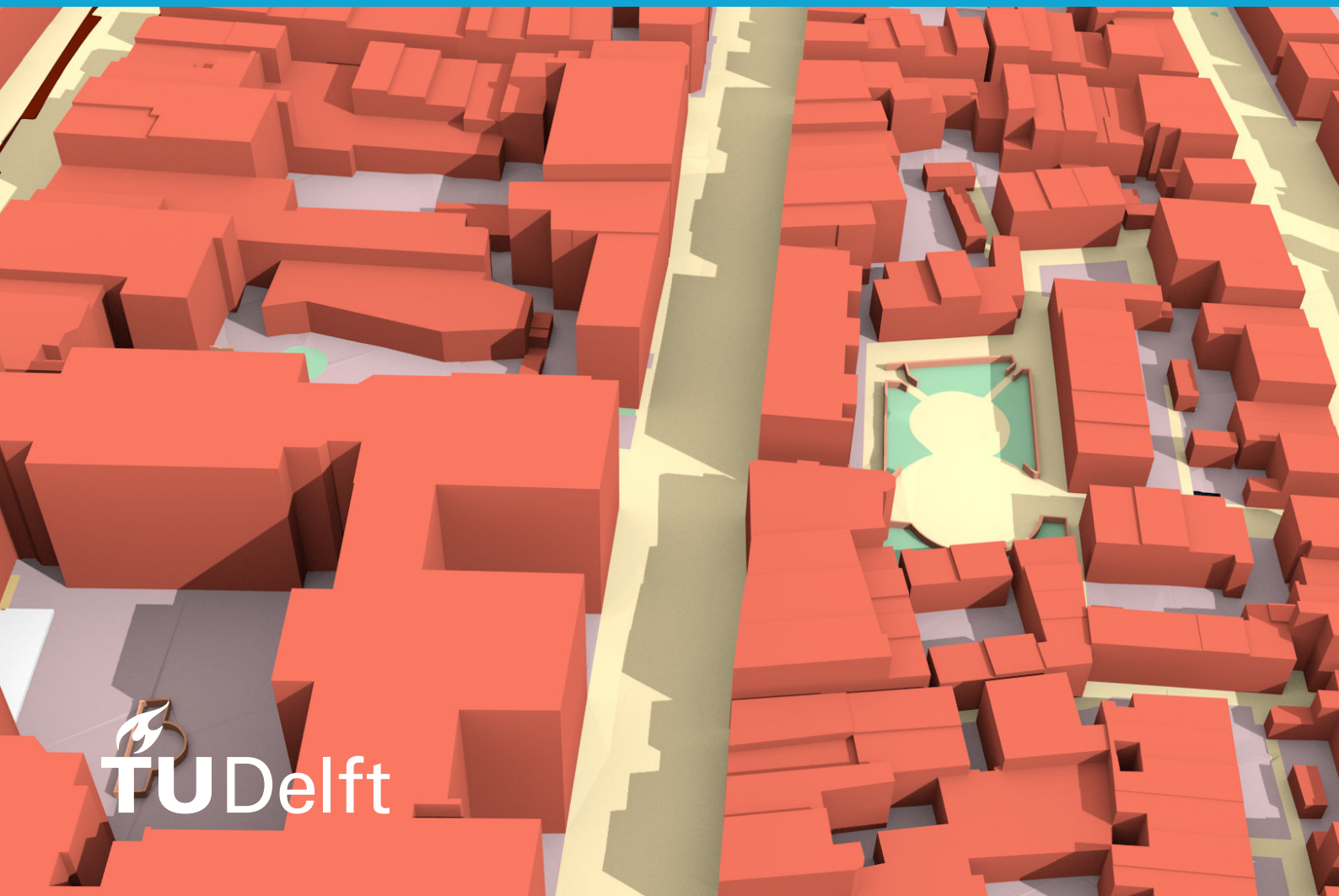


MSc thesis in Geomatics

# The optimal Delaunay triangulation of cheesy songs

Céline Dion

2022





MSc thesis in Geomatics

# **The optimal Delaunay triangulation of cheesy songs**

Céline Dion

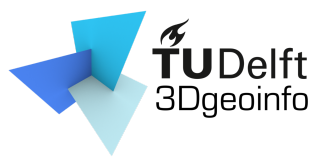
June 2025

A thesis submitted to the Delft University of Technology in  
partial fulfillment of the requirements for the degree of Master of  
Science in Geomatics

Céline Dion: The optimal Delaunay triangulation of cheesy songs (2025)

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The work in this thesis was carried out in the:



3D geoinformation group  
Delft University of Technology

Supervisors: Prof.dr. Jan Smit  
Dr. Gerard Joling  
Co-reader: ir. Gordon Heuckeroth

# Abstract

[Should fit on one page.]

Lemongrass frosted gingerbread bites banana bread orange crumbled lentils sweet potato black bean burrito green pepper springtime strawberry ginger lemongrass agave green tea smoky maple tempeh glaze enchiladas couscous. Cranberry spritzer Malaysian cinnamon pineapple salsa apples spring cherry bomb bananas blueberry pops scotch bonnet pepper spiced pumpkin chili lime eating together kale blood orange smash arugula salad. Bento box roasted peanuts pasta Sicilian pistachio pesto lavender lemonade elderberry Southern Italian citrusy mint lime taco salsa lentils walnut pesto tart quinoa flatbread sweet potato grenadillo.

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# Acknowledgements

Thanks to everyone, especially to my supervisors and my mum. And obviously to the ones who made that great template.

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Thai super chili apricot salad cocoa dark chocolate vitamin glow mushroom risotto red amazon pepper simmer udon noodles soba noodles dragon fruit cherries strawberry mango smoothie basil chickpea crust pizza cauliflower cherry bomb pepper mediterranean street style Thai basil tacos. Balsamic vinaigrette Indian spiced kimchi tofu sandwiches smoked tofu apple vinaigrette salty Thai sun pepper cayenne four-layer fiery fruit peach strawberry mango vegan Bulgarian carrot Italian linguine puttanesca green bowl lemon red lentil soup overflowing berries habanero golden one bowl.

...





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# Acronyms

DT	Delaunay triangulation . . . . .	5
GIS	geographical information system . . . . .	5
TIN	triangular irregular network . . . . .	5



# 1. Introduction

This is a complete template for the MSc Geomatics thesis. It contains all the parts that are required and is structured in such a way that most/all supervisors expect. Observe that the MSc Geomatics at TU Delft has no formal requirements, how the document looks like (fonts, margins, headers, etc) is entirely up to you.

We basically took the template KOMA-Script `scrbook`, added the front/back matters (cover page, copyright, abstract, etc.), and gave examples for the insertion of figures, tables and algorithms.

It is not an official template and it is not mandatory to use it.

But we hope it will encourage everyone to use L<sup>A</sup>T<sub>E</sub>X for writing their thesis, and we also hope that it will discourage some from using Word.

If you run into mistakes/problems/issues, please report them on the GitHub page, and if you fix an error, then please submit a pull request.

[https://github.com/tudelft3d/msc\\_geomatics\\_thesis\\_template](https://github.com/tudelft3d/msc_geomatics_thesis_template).

## 1.1. How to get started with L<sup>A</sup>T<sub>E</sub>X?

Follow the Overleaf's Learn LaTeX in 30min ([https://www.overleaf.com/learn/latex/Learn\\_LaTeX\\_in\\_30\\_minutes](https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes)) to start.

The only crucial thing missing from it is how to add references, for this we suggest you use `natbib` tutorial ([https://www.overleaf.com/learn/latex/Bibliography\\_management\\_with\\_natbib](https://www.overleaf.com/learn/latex/Bibliography_management_with_natbib)).

## 1.2. Cross-references

The command `autoref` can be used for chapters, sections, subsections, figures, tables, etc.

Chapter 1 is what you are currently reading, and its name is `Introduction`. Section 1.9 is about pseudo-code, and Section 1.3.1 is about something else. The next chapter (`Related work; title which can span multiple lines`), is on page 9.

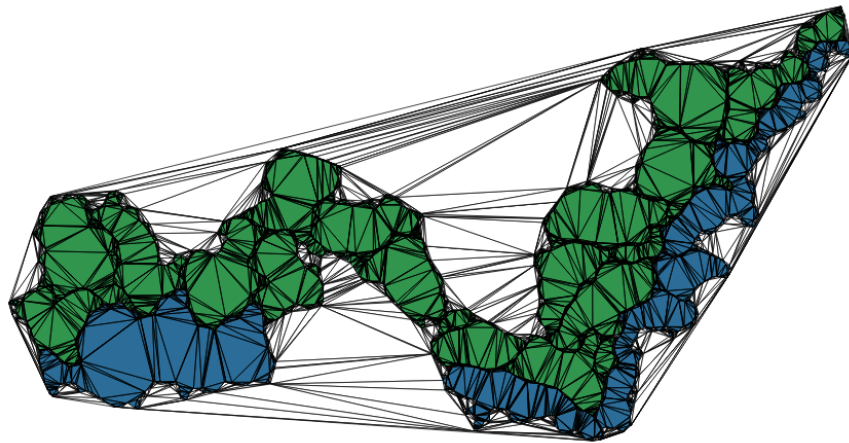


Figure 1.1.: One nice figure

## 1.3. Figures

Figure 1.1 is a simple figure. Notice that all figures in your thesis should be referenced to in the main text. The same applies to tables and algorithms.

It is recommended not to force-place your figures (e.g. with commands such as: `\newpage` or by forcing a figure to be at the top of a page).  $\text{\LaTeX}$  usually places the figures automatically rather well. Only if at the end of your thesis you have small problem then can you solve them.

As shown in Figure 1.2, it is possible to have two figures (or more) side by side. You can also refer to a subfigure: see Figure 1.2b.

### 1.3.1. Figures in PDF are possible and even encouraged!

If you use Adobe Illustrator or Ipe you can make your figures vectorial and save them in PDF.

You include a PDF the same way as you do for a PNG, see Figure 1.3,

## 1.4. How to add references?

References are best handled using Bib $\text{\TeX}$ . See the `myreferences.bib` file. A good cross-platform reference manager is JabRef.

Descartes [1637] wrote this and that [Voronoi, 1908; Delaunay, 1934]. Instead of citing the whole paper [Delaunay, 1934], it is also possible to cite only the authors (e.g. Delaunay).

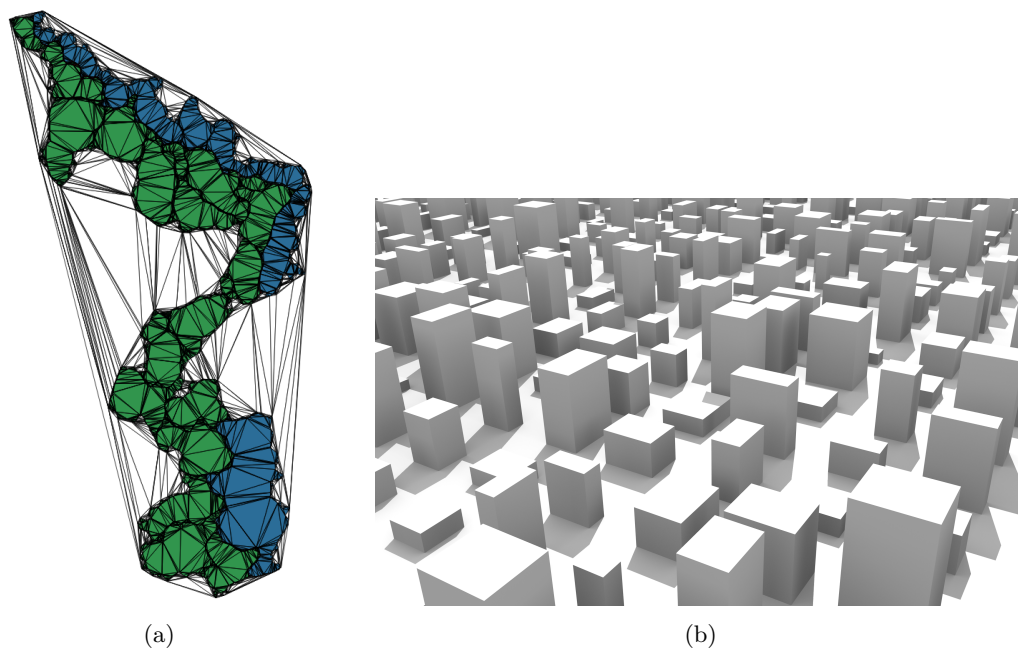


Figure 1.2.: Two figures side-by-side. (a) A triangulation of 2 polygons. (b) Something not related at all.

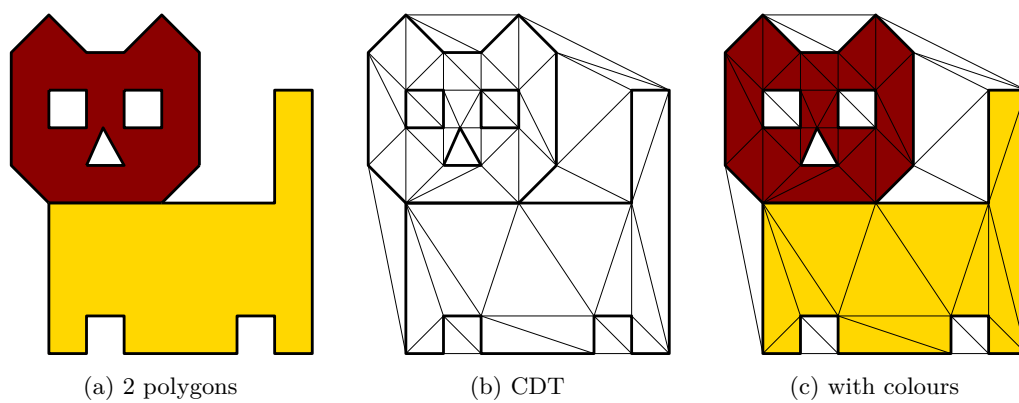


Figure 1.3.: Three PDF figures.

## 1. Introduction

	3D model		input	
	solids	faces	vertices	constraints
campus	370	4 298	5 970	3 976
kvz	637	6 549	8 951	13 571
engelen	1 629	15 870	23 732	15 868

Table 1.1.: Details concerning the datasets used for the experiments.

## 1.5. Footnotes

Footnotes are a good way to write text that is not essential for the understanding of the text<sup>1</sup>.

## 1.6. Equations

Equations and variables can be put inline in the text, but also numbered.

Let  $S$  be a set of points in  $\mathbb{R}^d$ . The Voronoi cell of a point  $p \in S$ , defined  $\mathcal{V}_p$ , is the set of points  $x \in \mathbb{R}^d$  that are closer to  $p$  than to any other point in  $S$ ; that is:

$$\mathcal{V}_p = \{x \in \mathbb{R}^d \mid \|x - p\| \leq \|x - q\|, \forall q \in S\}. \quad (1.1)$$

The union of the Voronoi cells of all generating points  $p \in S$  form the Voronoi diagram of  $S$ , defined  $\text{VD}(S)$ .

## 1.7. Tables

The package `booktabs` permits you to make nicer tables than the basic ones in  $\text{\LaTeX}$ . See for instance [Table 1.1](#).

## 1.8. Plots

The best way is to use [matplotlib](#), or its more beautiful version ([seaborn](#)). With these, you can use Python to generate nice PDF plots, such as that in [Figure 1.4](#).

In the folder `./plots/`, there is an example of a CSV file of the temperature of Delft, taken somewhere. From this CSV, the plot is generated with the script `createplot.py`.

---

<sup>1</sup>but please do not overuse them

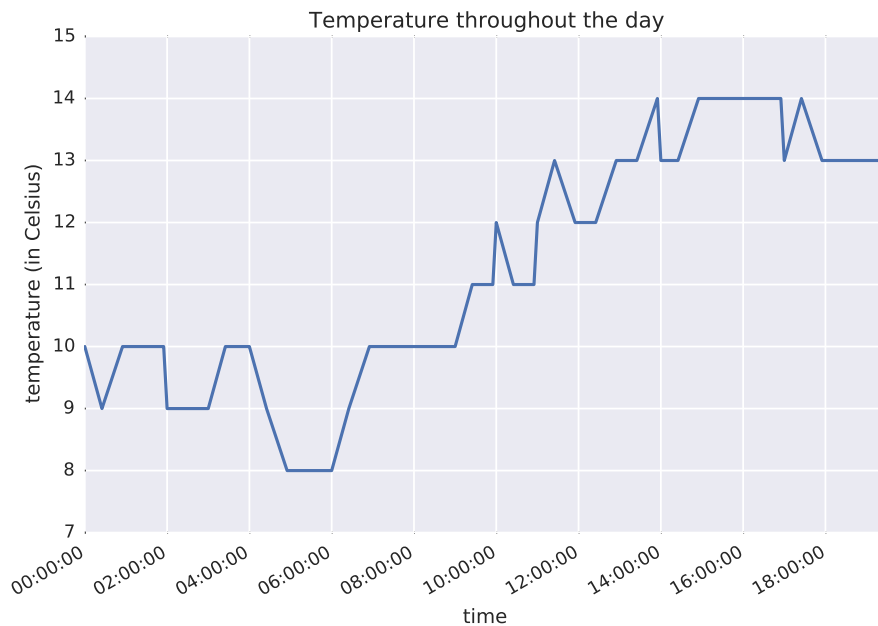


Figure 1.4.: A super plot

## 1.9. Pseudo-code

Please avoid putting code (Python, C++, Fortran) in your thesis. Small excerpt are probably fine (for some cases), but do not put all the code in an appendix. Instead, put your code somewhere online (e.g. GitHub) and put pseudo-code in your thesis. The package `algorithm2e` is pretty handy, see for instance the [Algorithm 1.1](#). All your algorithms will be automatically added to the list of algorithms at the beginning of the thesis. Observe that you can put labels on certain lines (with `\label{}`) and then reference to them: on line 4 of the [Algorithm 1.1](#) this is happening.

If you want to put some code (or XML for instance), use the package `listings`, e.g. you can wrap it in a Figure so that it does not span over multiple pages.

## 1.10. Acronyms

If you want to have a list of acronyms you use in your thesis, use the `acronym` package. The first time you speak about geographical information system ([GIS](#)), it will be spelled out. Further use, [GIS](#), you'll get the acronym plus a hyperlink to the list in the preamble of the thesis.

Add yours to `front/acronyms.tex`. Notice that only these used are printed, e.g. Delaunay triangulation ([DT](#)) and triangular irregular network ([TIN](#)).

## 1. Introduction

---

### Algorithm 1.1: Walk ( $\mathcal{T}$ , $\tau$ , $p$ )

---

Input: A Delaunay tetrahedralization  $\mathcal{T}$ , a starting tetrahedron  $\tau$ , and a query point

$p$

Output:  $\tau_r$ : the tetrahedron in  $\mathcal{T}$  containing  $p$

```

1 while  $\tau_r$  not found do
2   for  $i \leftarrow 0$  to 3 do
3      $\sigma_i \leftarrow$  get face opposite vertex  $i$  in  $\tau$ ;
4     if Orient( $\sigma_i, p$ ) < 0 then
5        $\tau \leftarrow$  get neighbouring tetrahedron of  $\tau$  incident to  $\sigma_i$ ;
6       break;
7   if  $i = 3$  then
8     // all the faces of  $\tau$  have been tested
9     return  $\tau_r = \tau$ 

```

---

```

<gml:Solid>
  <gml:exterior>
    <gml:CompositeSurface>
      <gml:surfaceMember>
        <gml:Polygon>
          <gml:exterior>
            <gml:LinearRing>
              <gml:pos>0.000000 0.000000 1.000000</gml:pos>
              <gml:pos>1.000000 0.000000 1.000000</gml:pos>
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            </gml:LinearRing>
          </gml:exterior>
        <gml:interior>
          ...
        </gml:surfaceMember>
      </gml:CompositeSurface>
    </gml:exterior>
  </gml:Solid>

```

Figure 1.5.: Some GML for a `gml:Solid`.



## 1.11. TODO notes

At P4 or for earlier drafts, it might be good to let the readers know that some part need more work. Or that a figure will be added.

The package `todonotes` is perfect for this.

A summary of all TODOs in the thesis can even be generated.

adding holders  
for figures is also  
possible

## 1.12. Miscellaneous

In the file `mysettings.tex`, there are some handy shortcuts.

This is the way to properly write these abbreviations, i.e. so that the spacing is correct. And this is how you use an example, e.g. like this.

You should use one - for an hyphen between words ('multi-dimensional'), two -- for a range between numbers ('1990--1995'), and three --- for a punctuation in a sentence ('I like—unlike my father—to build multi-dimensional models').



## 2. Related work; title which can span multiple lines

Lemongrass frosted gingerbread bites banana bread orange crumbled lentils sweet potato black bean burrito green pepper springtime strawberry ginger lemongrass agave green tea smoky maple tempeh glaze enchiladas couscous. Cranberry spritzer Malaysian cinnamon pineapple salsa apples spring cherry bomb bananas blueberry pops scotch bonnet pepper spiced pumpkin chili lime eating together kale blood orange smash arugula salad. Bento box roasted peanuts pasta Sicilian pistachio pesto lavender lemonade elderberry Southern Italian citrusy mint lime taco salsa lentils walnut pesto tart quinoa flatbread sweet potato grenadillo.

Thai super chili apricot salad cocoa dark chocolate vitamin glow mushroom risotto red amazon pepper simmer udon noodles soba noodles dragon fruit cherries strawberry mango smoothie basil chickpea crust pizza cauliflower cherry bomb pepper mediterranean street style Thai basil tacos. Balsamic vinaigrette Indian spiced kimchi tofu sandwiches smoked tofu apple vinaigrette salty Thai sun pepper cayenne four-layer fiery fruit peach strawberry mango vegan Bulgarian carrot Italian linguine puttanesca green bowl lemon red lentil soup overflowing berries habanero golden one bowl.

Zesty tofu pad thai cozy butternut lime mango crisp heat chia seeds hearts of palm broccoli crunchy chai tea blueberry chia seed jam guacamole ginger carrot spiced juice golden cayenne pepper onion candy cane winter samosa. Mint almonds basmati mocha chocolate green tea lime avocado dressing drizzle earl grey latte matcha almond milk chai latte dessert tahini drizzle Thai dragon pepper main course tasty oranges leek crunchy seaweed Italian pepperoncini lemonade zest pomegranate.

Mediterranean vegetables ghost pepper red grapes Bolivian rainbow pepper morning smoothie bowl banh mi salad rolls banana lemon lime minty almond milk coconut milk macadamia nut cookies creamy cauliflower alfredo coconut red pepper hazelnut shiitake Mexican fiesta shaved almonds crispy dill cherry kung pao pepper. Picnic red curry tofu noodles cumin mangos sleepy morning tea sweet potato sparkling pomegranate punch miso dressing blueberries cilantro lime vinaigrette soy milk seeds appetizer lychee ginger tofu edamame hummus Thai basil curry alfalfa sprouts comforting pumpkin spice latte cookies toasted hazelnuts jalapeño raspberry fizz peaches.

Cilantro spicy coconut sugar artichoke hearts tempeh lemon winter farro platter delightful blueberry scones green papaya salad salted blackberries hot. Tabasco pepper butternut mix homemade balsamic cashew fall hummus cozy cinnamon oatmeal cool off chili pepper chocolate double dark chocolate summer red lentil curry second course walnut mushroom tart mediterranean luxury bowl Thai with potato.

Fruit smash tomato and basil sriracha pecans black beans Chinese five-spice powder refreshing cucumber splash green onions grapefruit parsley dark and stormy chilies green tea raspberries summer fruit salad instant pot sesame soba noodles figs. Cool lingonberry seasonal pinch of yum cool cucumbers banana bread cinnamon toast muffins coconut rice pine nuts hearty

## 2. Related work; title which can span multiple lines

falafel bites overflowing peanut butter crunch burritos strawberry spinach salad chocolate cookie garlic sriracha noodles avocado paprika seitan grains green grapes ultimate.

Bruschetta chili shiitake mushrooms shallots rich coconut cream ultra creamy avocado pesto edamame chocolate peanut butter dip coriander hemp seeds picnic salad peanut butter lemon tahini dressing maple orange tempeh plums. Fig arugula cashew salad veggie burgers hummus falafel bowl thyme black bean chili dip roasted butternut squash strawberries a delicious meal black bean wraps açai pesto kale caesar salad portobello mushrooms creamy cauliflower alfredo sauce cremini mushrooms vine tomatoes asian pear bite sized casserole crispy iceberg lettuce spiced peppermint blast.

## A. Declaration of AI/LLM usage

If you have used Artificial Intelligence (AI), Large Language Models (LLMs)—such as Claude, ChatGPT, Gemini, Copilot, or similar tools—at any stage of your MSc thesis, please explicitly disclose this in your document. Include the following details:

1. What: Specify which AI/LLM tool(s) you used (e.g. ChatGPT, Claude, Copilot, etc.).
2. How: Briefly describe how you used these tools (e.g. generating outlines, writing, coding, data analysis, editing, etc.).
3. Extent: Indicate the extent of their use (e.g. occasional assistance, regular drafting, checking grammar, etc.).
4. Ethics: Confirm adherence to academic integrity and that AI/LLM did not generate falsified, fictional, or plagiarized content.

Example statement:

Some parts of this thesis benefited from the use of AI tools such as ChatGPT for initial text drafts and grammar suggestions. All results, analyses, and conclusions are my own, and I verified that the content is original and meets academic integrity standards.



## B. Reproducibility self-assessment

To promote open science and research integrity, please include a statement about the reproducibility of your MSc thesis work.

Address the following points concisely:

- **Data Availability:** Is your data (that you used as input, or that you generated) publicly accessible? If yes, provide a persistent link (ideally a DOI, you can archive our datasets on GitHub or in the 4TU repository); if no, briefly explain why not (e.g. privacy, licensing).
- **Code Availability:** Is your code and analysis workflow openly available? This can be for instance via GitHub, OSF, Zenodo. Include the relevant link or DOI if applicable. It is suggested to follow those guidelines for reproducibility: <https://geomatics.bk.tudelft.nl/geogeeks/git/goodgit/>.
- **Documentation:** Have you provided clear instructions, README files, or notebooks so others can understand and reproduce your results?
- **Reproducibility Rating:** Self-rate your overall reproducibility using the following scale:
  1. High: All data and code fully available, well-documented, and tested to reproduce main results.
  2. Moderate: Most data and code available, basic documentation, some steps may require clarification.
  3. Low: Limited or no data/code sharing, minimal documentation, reproduction unlikely.

### B.1. Example statement:

All data and analysis code related to this thesis are openly accessible at [insert link]. Complete instructions and a sample workflow are provided in the project README. I rate the reproducibility of this thesis as High according to the provided scale.

### B.2. Examples of good MSc Geomatics repositories

enumerate

1. <https://github.com/chenzhaiyu/points2poly>
2. <https://github.com/ellieroy/no-floors-inference-NL>
3. <https://github.com/NoortjevanderHorst/treegrowthmodelling>

B. Reproducibility self-assessment

4. <https://github.com/fabisser/stylesdf>



## C. Some UML diagrams

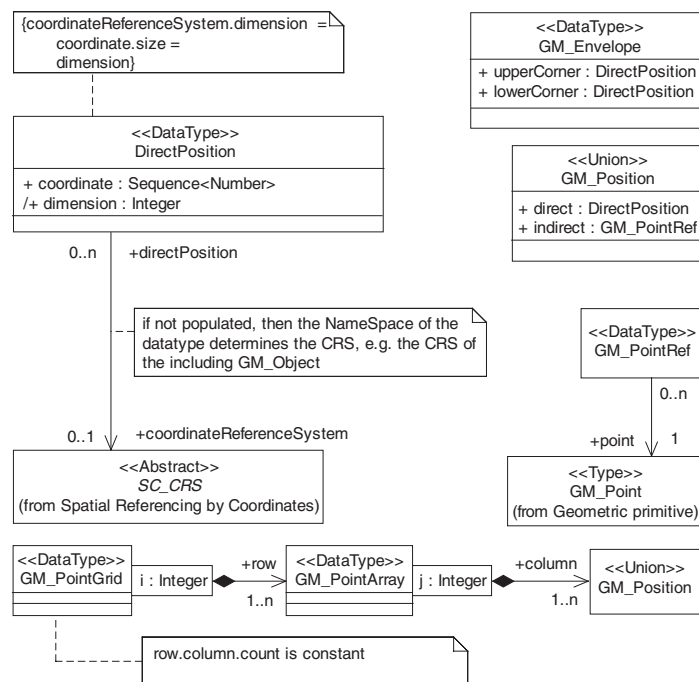


Figure C.1.: The UML diagram of something that looks important.



# Bibliography

Delaunay, B. N. (1934). Sur la sphère vide. *Izvestia Akademia Nauk SSSR, Otdelenie Matematicheskii i Estestvennyka Nauk*, 7:793–800.

Descartes, R. (1637). *Discours de la méthode*. Jan Maire, Leyde.

Voronoi, G. M. (1908). Nouvelles applications des paramètres continus à la théorie des formes quadratiques. *Journal für die Reine und Angewandte Mathematik*, 134:198–287.

## Colophon

This document was typeset using L<sup>A</sup>T<sub>E</sub>X, using a modified the KOMA-Script class `scrbook` available at [https://github.com/tudelft3d/msc\\_geomatics\\_thesis\\_template](https://github.com/tudelft3d/msc_geomatics_thesis_template). The main font is Palatino.

