

MSc thesis in Geomatics

The optimal Delaunay triangulation of cheesy songs

Céline Dion

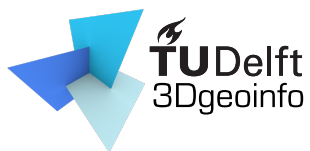
November 2024

A thesis submitted to the Delft University of Technology in partial fulfillment of
the requirements for the degree of Master of Science in Geomatics

Céline Dion: *The optimal Delaunay triangulation of cheesy songs* (2024)

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The work in this thesis was carried out in the:



3D geoinformation group
Delft University of Technology

Supervisors: Prof.dr. Jan Smit
Dr. Gerard Joling

Co-reader: ir. Gordon Heuckeroth

Abstract

i Info

Should fit on one page.

Lemongrass frosted gingerbread bites banana bread orange crumbled lentils sweet potato black bean burrito green pepper springtime strawberry ginger lemongrass agave green tea smoky maple tempeh glaze enchiladas couscous. Cranberry spritzer Malaysian cinnamon pineapple salsa apples spring cherry bomb bananas blueberry pops scotch bonnet pepper spiced pumpkin chili lime eating together kale blood orange smash arugula salad. Bento box roasted peanuts pasta Sicilian pistachio pesto lavender lemonade elderberry Southern Italian citrusy mint lime taco salsa lentils walnut pesto tart quinoa flatbread sweet potato grenadillo.

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Acknowledgements

Thanks to everyone, especially to my supervisors and my dog (in that order). And obviously to the ones who made that great template.

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Thai super chili apricot salad cocoa dark chocolate vitamin glow mushroom risotto red amazon pepper simmer udon noodles soba noodles dragon fruit cherries strawberry mango smoothie basil chickpea crust pizza cauliflower cherry bomb pepper mediterranean street style Thai basil tacos. Balsamic vinaigrette Indian spiced kimchi tofu sandwiches smoked tofu apple vinaigrette salty Thai sun pepper cayenne four-layer fiery fruit peach strawberry mango vegan Bulgarian carrot Italian linguine puttanesca green bowl lemon red lentil soup overflowing berries habanero golden one bowl.

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TODO 1 make sure that the URL is correct :J	5
TODO 2 adding holders for figures is also possible	5

1. Introduction

This is a complete template for the MSc Geomatics thesis. It contains all the parts that are required and is structured in such a way that most/all supervisors expect. Observe that the MSc Geomatics at TU Delft has no formal requirements, how the document looks like (fonts, margins, headers, etc) is entirely up to you.

We basically took the template LaTeX at https://github.com/tudelft3d/msc_geomatics_thesis_template and tried to convert it to Typst.

i Info

It is not an official template and it is not mandatory to use it.

But we hope it will encourage everyone to use Typst for writing their thesis, and we also hope that it will *discourage* some from using Word.

If you run into mistakes/problems/issues, please report them on the GitHub page, and if you fix an error, then please submit a pull request.

1.1. Cross-references

Chapter 1 is to refer to the Chapter, but all other sections you can just use Section 1.2 or Section 1.1.

For a figure, you can also just use Figure 1.2, but I guess you can qualify it with whatever you fancy, eg Hugo 1.2.

1.2. Figures

! Warning

At the moment (v0.13 of Typst), Typst supports natively figures in PNG, SVG, JPG, and GIF (with `#image()`).

With the package <https://github.com/frozolotl/muchpdf>, you can also insert directly PDF files, there is an example below.

Figure 1.1 is a simple figure in PNG format, notice the use of `placement: auto` to put the figure at the best place possible (top or bottom, Typst decides for you).

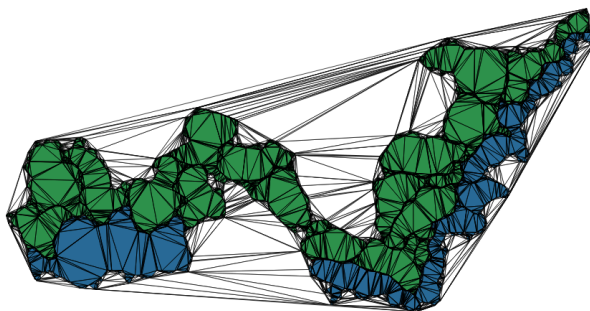


Figure 1.1: Two nice figure.

1.2. Figures

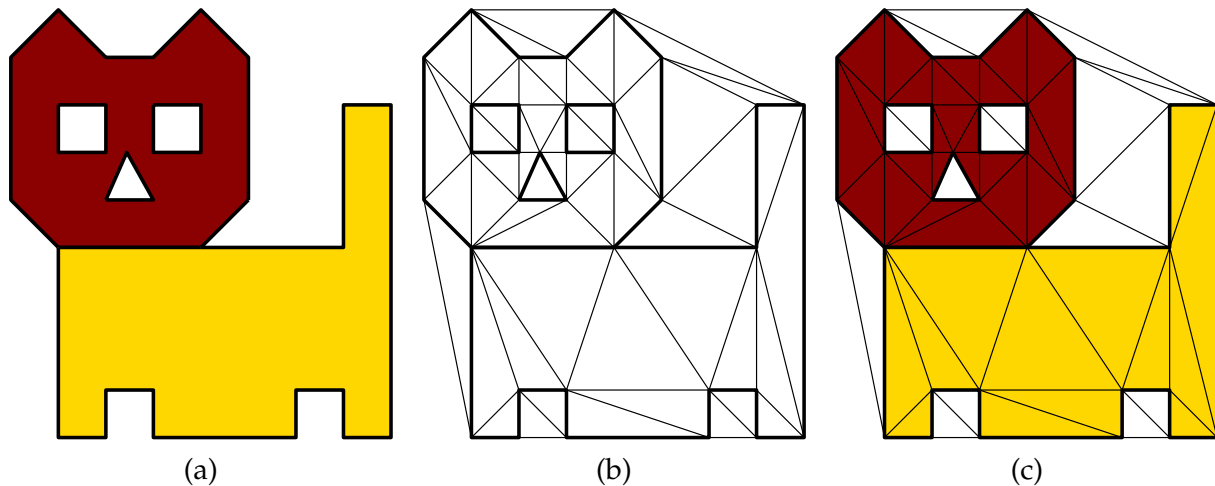


Figure 1.2: Three figures side-by-side. **(a)** A cat formed of 2 polygons. **(b)** its triangulation. **(c)** with some colours. Notice that the figure is one PDF file with 3 pages, the page to display can be chosen.

i Info

All figures in your thesis should be referenced to in the main text. The same applies to tables and algorithms.

As shown in Figure 1.2, it is possible to have two figures (or more) side by side. You can also refer to a *subfigure*: see Figure 1.2b.

1.3. References

We know from Descartes (1637) that this can be done, and others have done that too (Descartes, 1637).

And Delaunay (1934) did VoroCrust.

1. (Voronoi, 1908)
2. (Voronoi, 1908)
3. (Voronoi, 1908)
4. (Voronoi, 1908)

1.4. Footnotes

Footnotes are a good way to write text that is not essential for the understanding of the text¹.

1.5. Equations

Equations and variables can be put inline in the text, but also numbered.

Let S be a set of points in \mathbb{R}^d . The Voronoi cell of a point $p \in S$, defined \mathcal{V}_p , is the set of points $x \in \mathbb{R}^d$ that are closer to p than to any other point in S ; that is:

$$\mathcal{V}_p = \{x \in \mathbb{R}^d \mid |x - p| \leq |x - q|, \forall q \in S\}. \quad (1.1)$$

The union of the Voronoi cells of all generating points $p \in S$ form the Voronoi diagram of S , defined $\text{VD}(S)$.

¹but please do not overuse them

	solids	faces	vertices	constraints
campus	370	4 298	5 970	3 976
kvz	637	6 549	8 951	13 571
engelen	1 629	15 870	23 732	15 868

Table 1: Details concerning the datasets used for the experiments.

1.6. Tables

Tables builtin in Typst are pretty powerful, see <https://typst.app/docs/guides/table-guide/>. If you need some of the features of LaTeX booktabs, it seems that `tablex` is your friend: <https://typst.app/universe/package/tablex>.

An example of a simple table is in Table 1.

1.7. Plots

The best way is to use `matplotlib`, or its more beautiful version `seaborn`. With these, you can use Python to generate nice plots, such as that in Figure 1.3.

In the folder `./plots/`, there is an example of a CSV file of the temperature of Delft, taken somewhere. From this CSV, the plot is generated with the script `createplot.py`.

1.8. Pseudo-code

Please avoid putting code (Python, C++, Fortran) in your thesis. Small excerpt are probably fine (for some cases), but do not put all the code in an appendix. Instead, put your code somewhere online (eg GitHub) and put *pseudo-code* in your thesis. The package `lovelace` is pretty handy, see for instance Algorithm 1. All your algorithms will be automatically added to the list of algorithms at the beginning of the thesis.

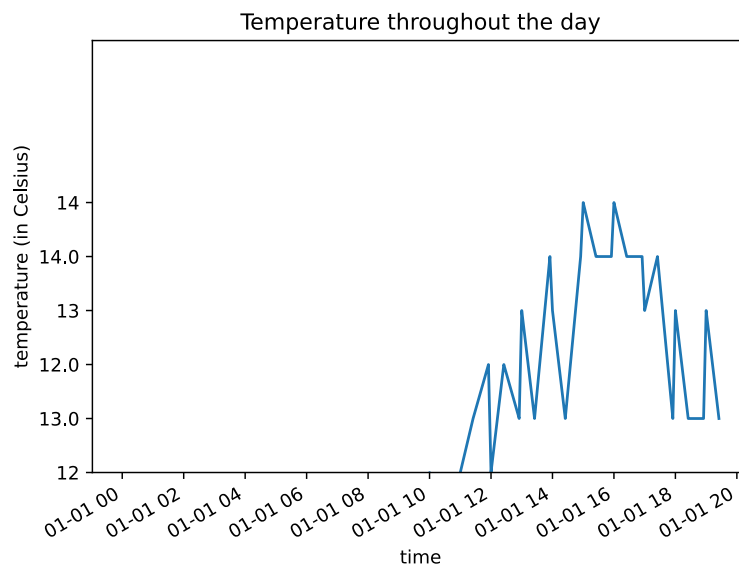


Figure 1.3: A super plot.

```

1 Input: A Delaunay tetrahedralization  $\mathcal{T}$ , a starting tetrahedron  $\tau$ , and a query point  $p$ 
2 Output:  $\tau_r$ : the tetrahedron in  $\mathcal{T}$  containing  $p$ 
3 while  $\tau_r$  not found
4   for  $i \leftarrow 0$  to 3 do
5      $\sigma_i \leftarrow$  get opposite vertex  $i$  in  $\tau$ 
6     if  $\text{ORIENT}(\sigma_i, p) < 0$  then
7        $\tau \leftarrow$  get neighbouring tetrahedron of  $\tau$  incident to  $\sigma_i$ 
8       break
9   if  $i == 3$  then
10    // - all the faces of  $\tau$  have been tested
10   return  $\tau_r = \tau$ 

```

Algorithm 1: WALK (\mathcal{T}, τ, p)

1.9. Source/raw code

You can use exactly the same principle as in Markdown, and you can wrap the raw code in a `#figure` to have a caption and place it where you want (see Listing 1).

1.10. And finally what matters the most: emojis!



1.11. TODOs

At P4 or for earlier drafts, it might be good to let the readers know that some part need more work. Or that a figure will be added.

The package <https://typst.app/universe/package/dashy-todo/> is perfect for this.

make sure
that the
URL is cor-
rect :J

A summary of all TODOs in the thesis can even be generated:

```

<gml:Solid>
  <gml:exterior>
    <gml:CompositeSurface>
      <gml:surfaceMember>
        <gml:Polygon>
          <gml:exterior>
            <gml:LinearRing>
              <gml:pos>0.000000 0.000000 1.000000</gml:pos>
              <gml:pos>1.000000 0.000000 1.000000</gml:pos>
              <gml:pos>1.000000 1.000000 1.000000</gml:pos>
              <gml:pos>0.000000 1.000000 1.000000</gml:pos>
              <gml:pos>0.000000 0.000000 1.000000</gml:pos>
            </gml:LinearRing>
          </gml:exterior>
          <gml:interior>
            ...
          </gml:surfaceMember>
        </gml:CompositeSurface>
      </gml:interior>
    </gml:Solid>

```

Listing 1: Some GML for a `gml:Solid`.

2. Related Work

In Chapter 1 we saw many important things. And in Section 1.1 we saw some too.

Lemongrass frosted gingerbread bites banana bread orange crumbled lentils sweet potato black bean burrito green pepper springtime strawberry ginger lemongrass agave green tea smoky maple tempeh glaze enchiladas couscous. Cranberry spritzer Malaysian cinnamon pineapple salsa apples spring cherry bomb bananas blueberry pops scotch bonnet pepper spiced pumpkin chili lime eating together kale blood orange smash arugula salad. Bento box roasted peanuts pasta Sicilian pistachio pesto lavender lemonade elderberry Southern Italian citrusy mint lime taco salsa lentils walnut pesto tart quinoa flatbread sweet potato grenadillo.

2.1. Some new section

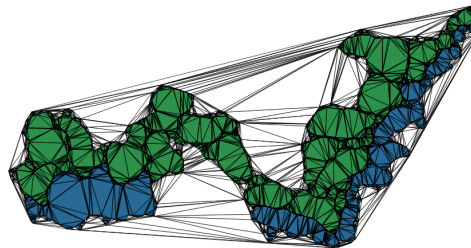


Figure 2.1: A triangulation

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aequale doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguere possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo a nobis philosophia defensa et collaudata est, cum id, quod maxime placeat, facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et aut officiis debitis aut rerum necessitatibus saepe eveniet, ut et voluptates repudiandae sint et molestiae non recusandae. Itaque earum rerum defuturum, quas natura non depravata desiderat. Et quem ad me accedis, saluto: 'chaere,' inquam, 'Tite!' lictores, turma omnis chorusque: 'chaere, Tite!' hinc hostis mi Albucius, hinc inimicus. Sed iure Mucius. Ego autem mirari satis non queo unde hoc sit tam insolens domesticarum rerum fastidium. Non est omnino hic docendi locus; sed ita prorsus existimo, neque eum Torquatum, qui hoc primus cognomen invenerit, aut torquem illum hosti detraxisse, ut aliquam ex eo est consecutus? – Laudem et caritatem, quae sunt vitae sine metu degendae praesidia firmissima. – Filium morte multavit. – Si sine causa, nollem me ab eo delectari, quod ista Platonis, Aristoteli, Theophrasti orationis ornamenta neglexerit. Nam illud quidem physici, credere aliquid esse minimum, quod profecto numquam putavisset, si a Polyaeno, familiari suo, geometrica discere maluisset quam illum etiam ipsum dedocere. Sol Democrito magnus videtur, quippe homini erudito in geometriaque perfecto, huic pedalis fortasse; tantum enim esse omnino in nostris poetis aut inertissimae segnitiae est aut fastidii delicatissimi. Mihi quidem videtur, inermis ac nudus est. Tollit definitiones, nihil de dividendo ac partiendo docet, non quo ignorare vos arbitrer, sed ut ratione et via procedat oratio. Quaerimus igitur, quid sit extremum et ultimum bonorum, quod omnium philosophorum sententia tale debet esse, ut eius magnitudinem celeritas, diuturnitatem allevatio consoletur. Ad ea cum accedit, ut

2.1. Some new section

neque divinum numen horreat nec praeteritas voluptates effluere patiatur earumque assidua recordatione laetetur, quid est, quod huc possit, quod melius sit, migrare de vita. His rebus instructus semper est in voluptate esse aut in armatum hostem impetum fecisse aut in poetis evolvendis, ut ego et Triarius te hortatore facimus, consumeret, in quibus hoc primum est in quo admirer, cur in gravissimis rebus non delectet eos sermo patrius, cum.

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2.2. Some new section again

instructus semper est in voluptate esse aut in armatum hostem impetum fecisse aut in poetis evolvendis, ut ego et Triarius te hortatore facimus, consumeret, in quibus hoc primum est in quo admirer, cur in gravissimis rebus non delectet eos sermo patrius, cum.

A Reproducibility self-assessment

A.1 Marks for each of the criteria

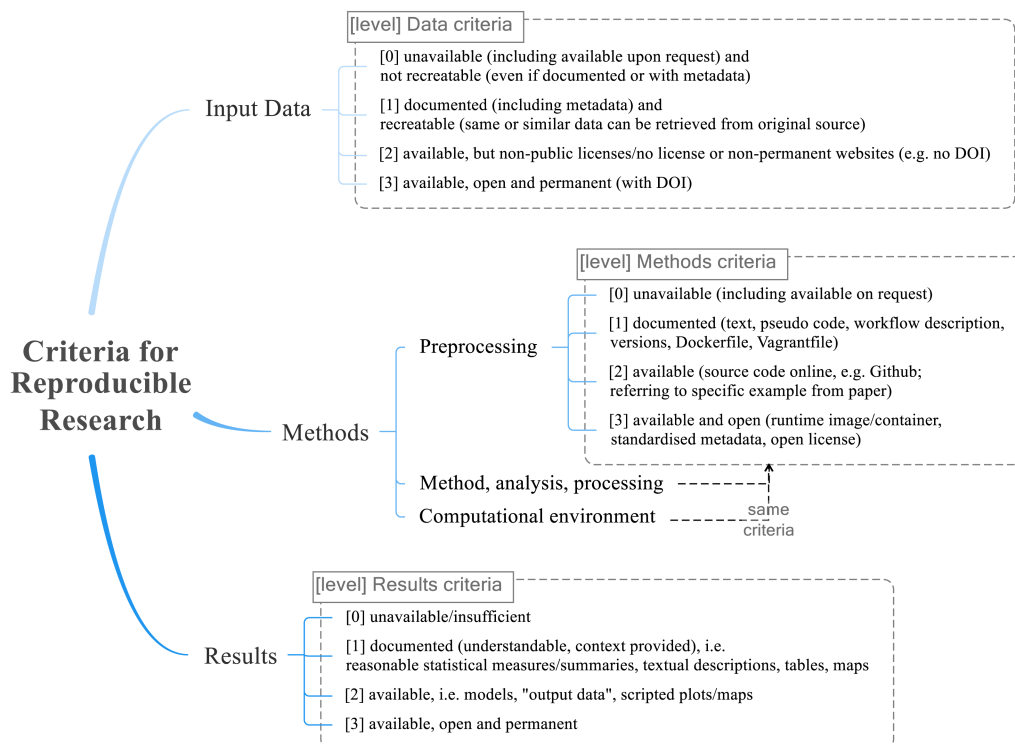


Figure A.1: Reproducibility criteria to be assessed.

Grade/evaluate yourself for the 5 criteria (giving 0/1/2/3 for each):

1. input data
2. preprocessing
3. methods
4. computational environment
5. results

A.2 Self-reflection

A self-reflection about the reproducibility of your thesis/results.

We expect maximum 1 page here.

For example, if your data are not made publicly available, you need to justify it why (perhaps the company prevented you from doing this).

B Some UML diagrams

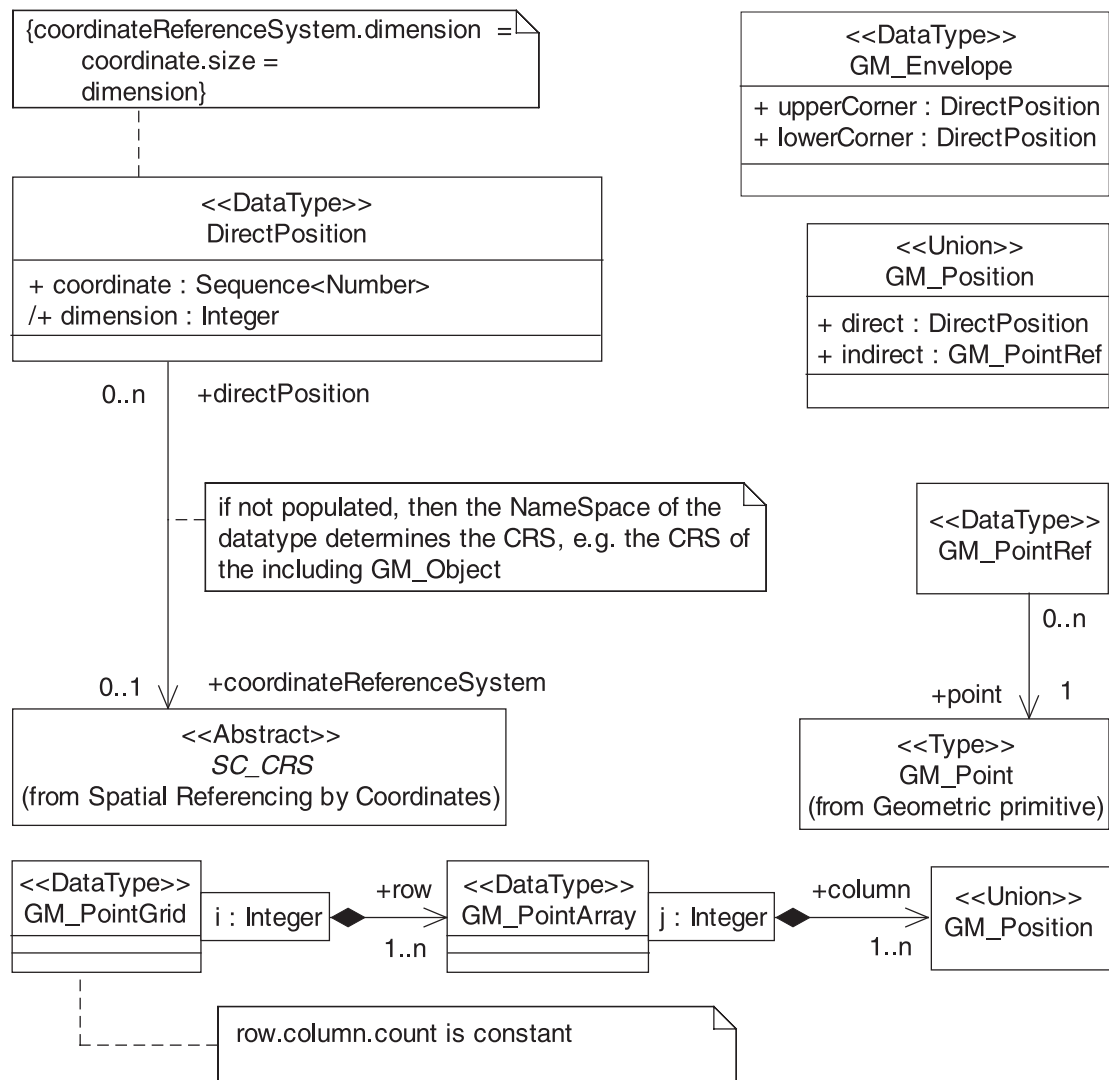


Figure B.1: The UML diagram of something that looks important.

Bibliography

Delaunay, B. N. (1934). Sur la sphère vide. *Izvestia Akademia Nauk SSSR, Otdelenie Matematicheskii I Estestvennyka Nauk*, 7, 793–800.

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Voronoi, G. M. (1908). Nouvelles applications des paramètres continus à la théorie des formes quadratiques. *Journal Für Die Reine Und Angewandte Mathematik*, 134, 198–287.

This document was typeset using [Typst](#). The main font is Palatino. The figures and diagrams were mostly drawn using IPE, PGF/TikZ and Omnigraffle.