FACULTY
OF MECHANICAL
ENGINEERING
CTU IN PRAGUE

Semestral project:

Reduction of the ruleset of a Fuzzy Logic system

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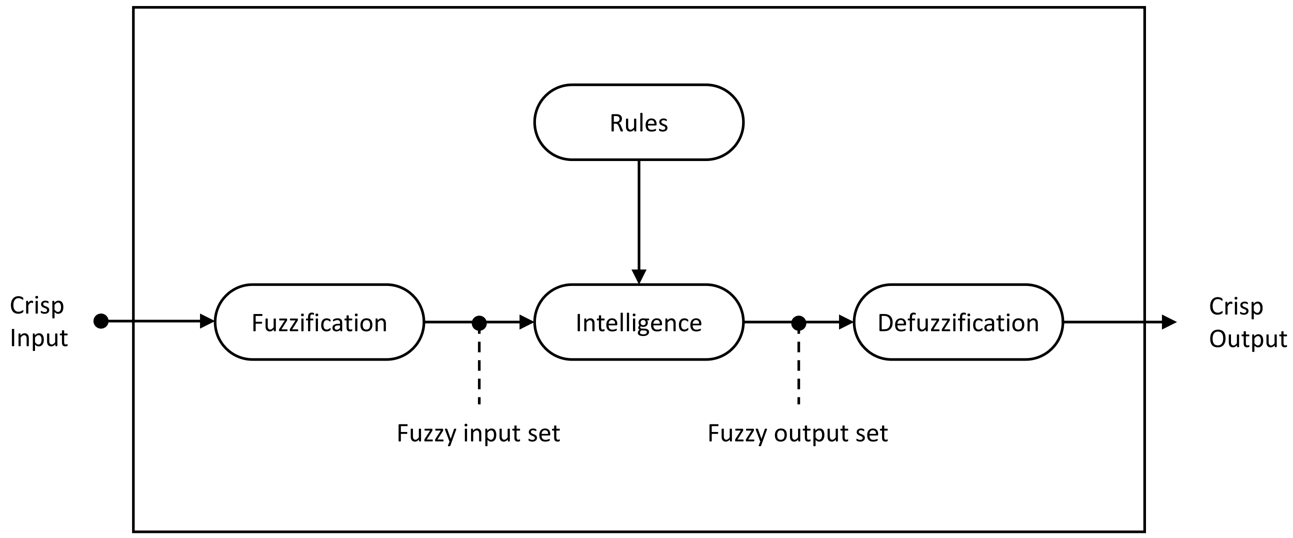
1 Introduction and presentation of Task

Fuzzy Logic has been widely used for applications in control and decision systems. It is a way to distinct between partial true values. In Fuzzy Logic the truth value of a variable might be somewhere between 0 and 1. In contrast to that a variable expressed in Boolean Logic can only have a truth value of 0 or 1. This difference is the reason why Fuzzy Logic is often used to replicate human-based decision making which in many cases is imprecise or vague. In Fuzzy Logic the non-numerical values are expressed as linguistic values.

The general approach of translating a system into a fuzzy system is based on the Mamdani rule-based system. The following image displays the steps that need to be executed:



General approach of translating a system into a fuzzy system

In the first step of Fuzzification the crisp numerical inputs of a system are assigned to fuzzy sets with some degree of membership anywhere between 0 and 1. Any value between 0 and 1 represents the degree of uncertainty that the value belongs in the set. Based on the ruleset the fuzzificated inputs are then computed by the intelligence resulting in the Fuzzy output set. This set is then translated back to crisp output variables by applying the defuzzification.

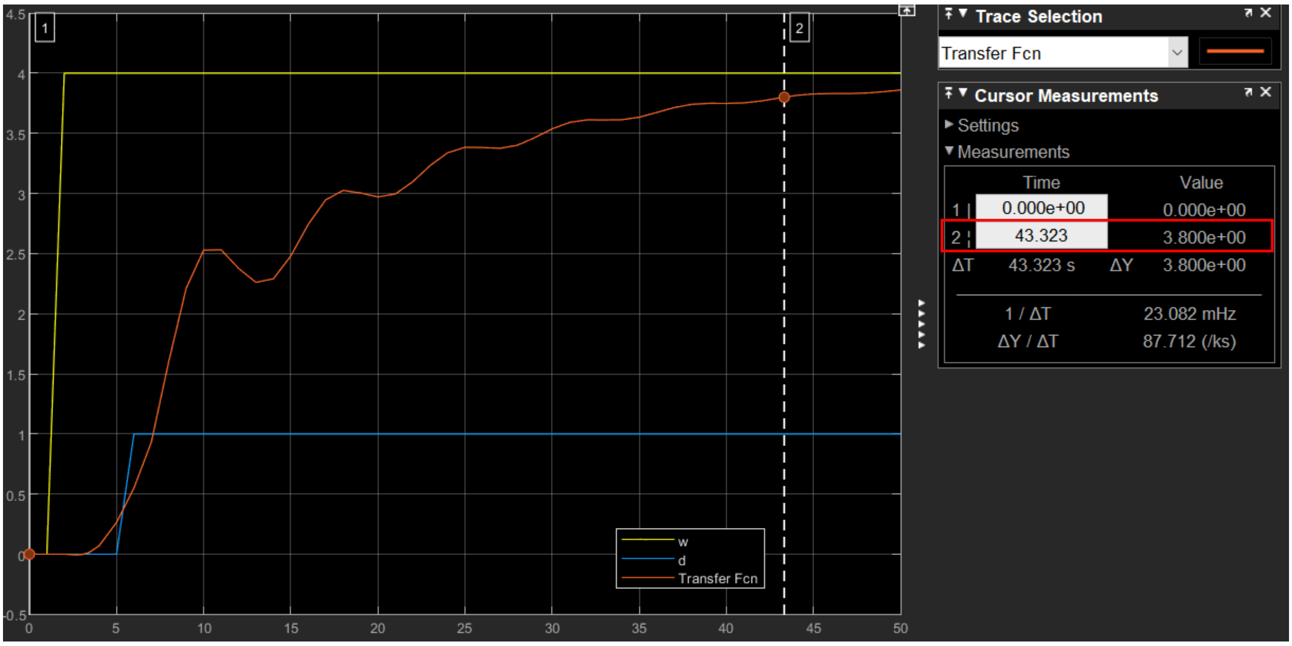
This semestral project in the course “Artificial Intelligence and Neural Networks” at CTU Prague deals with the simplification of a given fuzzy ruleset. In the task a system PT1 is given with the following model:

$$\text{PT1: } \mathrm{G(s)} = \mathrm{Y(s)} / \mathrm{U(s)} = (-0.4\,\mathrm{s} + 1) / (0.2\,\mathrm{s}2 + 1.2\,\mathrm{s} + 1)$$

The model consists of two input variables e and de/dt as well as one output variable du. It contains a ruleset of 25 rules. As a quality criterion the time T_R in which the output variable of the system reaches 95% of the value of the required variable is considered. The task is to minimize the number of rules in the ruleset without the time $T_{R,new}$ exceeding 1,2 times T_R. In general, a reduction of a fuzzy ruleset is used to minimize the need for computational power in environments where it is either not sufficiently available or the task is performed many times and therefore a more efficient computation is needed. In principle a reduction of a fuzzy ruleset is associated with decrease in performance. The Reduction of the given ruleset using two different approaches is described in the next chapter.

2 Application

To calculate the quality criterion T_R the given fuzzy system is executed. The following image shows the resulting plot. At point 2 and after an execution time of 43,323 s the transfer function (orange) reaches 95% of the value of the variable w (yellow).



Calculation of $T_{R,2}$

The new time $T_{R,2,max}$ can be calculated as follows:

$$T_{R,2,max} = 1{,}2 \cdot T_{R,1} = 1{,}2 \cdot 43{,}323 = 51{,}988\ s$$

The time $T_{R,2,max}$ is the quality criterion for the fuzzy system with the reduced ruleset. In this chapter two different approaches to reduce the given ruleset while not exceeding $T_{R,2,max}$ are described.

2.1 First approach: Intuitive reduction

The first approach to reduce the number of rules in the given ruleset is based on an intuitive approach. In the following image the given ruleset is displayed on the left side. A pattern in the output variable can be seen. By combining the negative and positive input and output variables to one each a reduction of the overall rules can be achieved. The reduced ruleset is displayed on the right side of the following image:

	NV	NN	Nu	PN	PV
NV	1	1	5	5	2
NN	1	5	5	2	3
Nu	5	5	2	3	3
PN	5	2	3	3	4
PV	2	3	3	4	4

	N	Nu	P
N	1	1	2
Nu	1	2	3
P	2	3	3

Intuitive approach of reducing the number of rules

This reduction from 25 rules to 9 rules is essentially a coarse mapping of the table that keeps the pattern. However, it does not only reduce the number of rules but also the number of input variables as well as the number of output variables from five to three. It is therefore assumed that the first approach changes the given system itself. While the number of rules could be reduced significantly a general change of the given system is not required by the task. Hence the first approach does not lead to a satisfying result and a second approach to solve the task is approached.

2.2 Second approach: Boolean minimization

A second approach of the problem consists of minimizing the rules based on Karnaugh maps. For each output value there will be calculated the minterms and then there will be applied the Boolean rules for minimization. We used for reference the following Karnaugh map:

	NV	NN	Nu	PN	PV
NV	1	1	5	5	2
NN	1	5	5	2	3
Nu	5	5	2	3	3
PN	5	2	3	3	4
PV	2	3	3	4	4

Karnaugh map of the given ruleset

As a conclusion the values for the output NV will be:

$$(-Va * -Vb) + (-Va * -Nb) + (-Na * -Vb) = NV$$

$$(Na * Vb) + (Va * Nb) + (Va * Vb) = PV$$

$$(-Na * Vb) + (Ua * Nb) + (Ua * Vb) + (Na * Ub) + (Na * Nb) + (Va * -Nb) + (Va * Ub) = PS$$

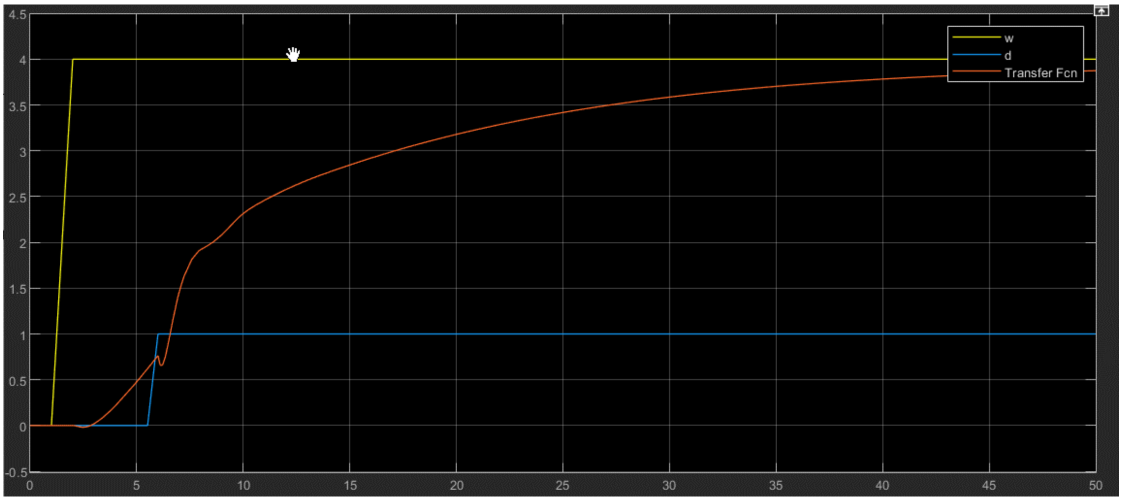
$$(-Va * Ub) + (-Va * Nb) + (-Na * -Nb) + (-Na * Ub) + (Ua * -Vb) + (Ua * -Nb) + (Na * -Vb) = NS$$

$$(-Va * Vb) + (-Na * Nb) + (Ua * Ub) + (Na * -Nb) + (Va * -Vb) = NU$$

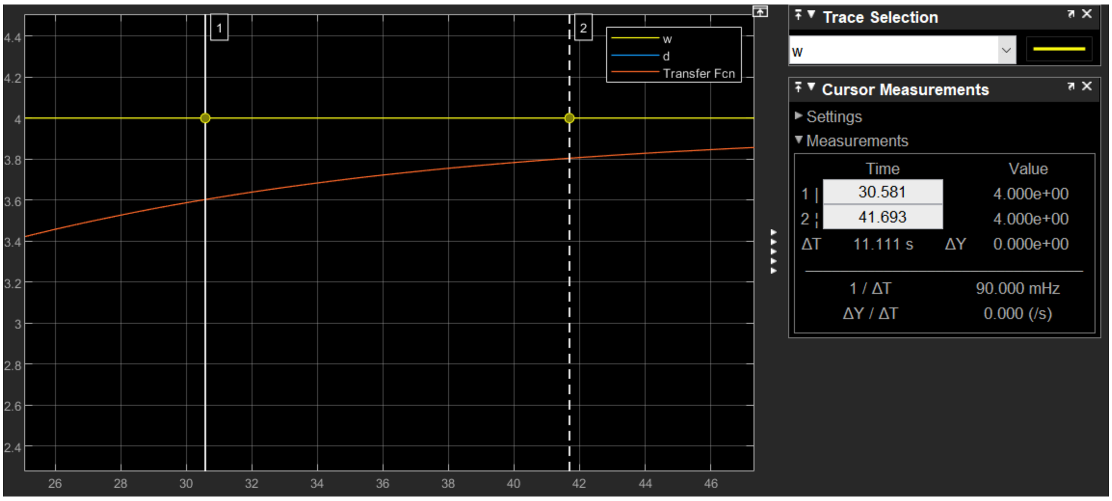
$$(-Va * Vb) + (-Na * Nb) + (Ua * Ub) + (Na * -Nb) + (Va * -Vb) = NU$$

In order to be able to reduce the number of rules, it was used a special notation “intuitive”. The notation presumes that values starting with P are positive. As a consequence they will be noted with either + or nothing at all. On the other hand, values starting with N such as “NN” will be noted with -N as a sign to show it’s a negative value of the initial N value. Furthermore, for readability we introduced notations for the specific input. For example, if we want to interpret a rule such as “If A is NV and B is NN then C is NU” we can simply note it as $-Va * -Nb = Uc$ where U is the notation for NU.

Another consequence of the Karnaugh map is that the only possible value to be minimized is NU on diagonal. After the minimization is done, the rule set of NU will be reduced from 5 rules to just only one being “If A is NU and B is NU then C is NU”. The result of this rule set can be observed in the two following graphs:



Transfer Function of the system with the reduced ruleset



Calculation of $T_{R,2}$

The time of control $T_{R,2}$ is calculated from graph displayed above. When the transfer function of the system with the reduced ruleset reaches 80% of the variable w the time $T_{R,2}$ is reached. It can be seen that:

$$T_{R,2} = 41{,}693\ s < 50{,}036\ s = T_{R,2,max}$$

The second approach of using the Boolean minimization to reduce the number of rules therefore leads to a satisfying result.

3 Conclusion

For this project, the process of designing the rule set had the purpose to approximate the knowledge base for a system. The entire project focused on minimizing an existing rule set and making it more efficient in terms of time control and number of rules. The first intuitive approach led to a significant reduction of the number of rules. However, it also changed the number of variables and therefore the system itself. The second approach employing Boolean minimization successfully led to a reduction of the number of rules. The time $T_{R,2}$ does not exceed the maximum time $T_{R,2,max}$ and therefore the quality criterion is fulfilled. We also conclude that the computation time is reduced with respect to the number of rules.